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A Bell-type Theorem Without Hidden Variables *

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Abstract

It is shown that no theory that satisfies certain premises can exclude faster-than-light influences. The premises include neither the existence of hidden variables, nor counterfactual definiteness, nor any premise that effectively entails the general existence of outcomes of unperformed local measurements. All the premises are compatible with Copenhagen philosophy, and the principles and predictions of relativistic quantum field theory. The present proof is contrasted with an earlier one with the same objective. Also described is a partial many-worlds way of evading this faster-than-light effect by accepting von Neumann's Process I, but rejecting collapse.

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1. Introduction.

The premises of Bell's original hidden-variable theorem[1] postulate the existence of a substructure that determines in a local manner the outcomes of a set of alternative possible measurements at most one of which can actually be performed. The implementation of the locality condition in this way thus involves a technical "hidden-variable" assumption that goes beyond the locality condition itself. Consequently, Bell's proof of the inconsistency of this local hidden-variable assumption with certain predictions of quantum theory casts no serious doubt on the locality condition: its implementation via the assumed hidden-variable substructure would appear to be the more likely cause of the derived inconsistency.

Bell[2] introduced later a seemingly weaker local hidden-variable assumption. However, this latter form can be shown[3,4] to entail the original one, apart from errors that tend to zero as the number of experiments tends to infinity. Thus both forms of the hidden-variable assumption place strong conditions on the class of theories that are covered by the theorems. These conditions are essentially equivalent, logically, to the assumption that values can be pre-assigned conjunctively and locally to all of the outcomes of all of the alternative possible measurements. That assumption conflicts with what I believe to be the orthodox quantum philosophical attitude that one should not make any assumption that effectively postulates the existence of a well defined outcome of a localized measurement process that is not performed. Thus these hidden-variable theorems place in no serious jeopardy the locality idea that a free choice made by an experimenter in one space-time region has no influence in a second region that is space-like separated from the first.

The present paper shows that this locality idea fails, however, not only under the hidden-variable assumption (or some closely related presumption that effectively ensures the existence, within the theory, of outcomes of unperformed local measurements) but also in a much larger class of theories, namely those that are compatible with the properties of Free Choice and No Backward-in-Time Influence on Observed Outcomes, and that yield certain predictions of quantum theory in experiments of the Hardy type[5]. The first two of these three properties are now described.

Free Choices.
For the purposes of understanding and applying quantum theory, the choice of which experiment is to be performed in a certain space-time region can be treated as an independent free variable localized in that region. Bohr repeatedly stressed the freedom of the experimenter to choose between alternative possible options. This availability of options is closely connected to his "complementarity" idea that the quantum state contains complementary kinds of information pertaining to the various alternative mutually exclusive experiments that might be chosen. Of course, no two mutually incompatible measurements can both be performed, and an outcome of an experiment can be specified only under the condition that that particular experiment be performed.

No Backward-in-Time Influence. (NBITI)
An outcome that has already been observed and recorded in some spacetime region at an earlier time can be considered fixed and settled, independently of which experiment a far-away experimenter will freely choose to perform at some later time. This assumption assigns no value to a local measurement except under the condition that this local measurement be performed. But any such locally observed value is asserted to be independent of which measurement will at some later time be freely chosen and performed in a spacelike separated region.

This NBITI assumption is required to hold in at least one Lorentz frame of reference, hence forth called LF.

This NBITI assumption is compatible with relativistic quantum field theory. In the Tomonaga-Schwinger[6, 7] formulation the evolving state is defined on a forward moving space-like surface. Their work shows that this surface can be defined in a continuum of ways without altering the predictions of the theory, so that no Lorentz frame is singled out as preferred. On the other hand, their formalism *allows* the quantum state to be defined by the constant time surfaces in any one single Lorentz frame that one wishes to choose, and shows that in this one frame the evolution, including all reductions associated with specific outcomes of measurements, proceeds forward in time, with a well defined past that is not influenced either by later free choices made by experimenters or by the outcomes of the later measurements. Thus this NBITI assumption is compatible with the principles and predictions of relativistic quantum field theory. (Included in this assumption is the tacit assumption that if an outcome appears to an observer then the mutually exclusive alternative does not occur: the many-worlds idea that both outcomes occur is excluded. This option will be discussed separately.)

This NBITI assumption is a small part of the larger locality condition in question here, which is the demand that what an experimenter freely chooses to do in one region has no effect in a second region that is spacelike separated from the first. The no backward-in-time part of the no-faster-than-light condition can be imposed without generating any difficulties or conflict with relativistic quantum field theory. But a faster-than-light effect then appears elsewhere. In particular the following theorem holds:

Theorem. Suppose a theory or model is compatible with the premises:

1. Free Choices: This premise asserts that the choice made in each region as to which experiment will be performed in that region can be treated as a localized free variable,

2. No Backward in Time Influence: This premise asserts that experimental outcomes that have already occurred in an earlier region (in frame LF) can be considered to be fixed and settled independently of which experiment will be chosen and performed later in a region spacelike separated from the first, and

3. Validity of Predicions of QT: Certain predictions of quantum theory in a Hardy-type experiment are valid.

Then this theory or model violates the following Locality Condition:
The free choice made in one region as to which measurement will be performed there has, within the theory, no influence in a second region that is spacelike separated from the first.

2. Proof of the Theorem.

The theorem refers to the following Hardy-type [5] experimental set-up.

There are two experimental spacetime regions L and R, which are spacelike separated, with L lying earlier than R in LF. The experimenter is L freely chooses either $L1$ or $L2$, and an outcome, either $+$ or $-$, then appears in region L. Then the experimenter in region R freely chooses either $R1$ or $R2$, and one or the other of the two alternative possible outcomes, $+$ or $-$, then appears in R.

The detectors are assumed to be 100% efficient, so that for whichever measurement is chosen in L one of the two alternative possible outcomes of that measurement, either $+$ or $-$, will appear in L, with each of these possibilities occurring about half the time, and for whichever measurement is then chosen in R, some outcome of that measurement, either $+$ or $-$, will appear in R.

For each of the two choices $L1$ or $L2$ available to the experimenter in L, and for each of the two alternative possible outcomes $+$ or $-$ of that experiment, quantum theory makes predictions for *both* of the two alternative choices $R1$ and $R2$ available to the experimenter in R.

In the statements that follow the symbol $L1$ will be an abbreviation of the statement "Experiment $L1$ is performed in L". The symbols $L2$, $R1$, and $R2$ will have analogous meanings.

The symbol $L1+$ will stand for the assertion "Experiment $L1$ is performed in L and outcome $+$ of that experiment appears in L." The symbols $L1-$, $L2+$, $L2-$, $R1+$, $R1-$, $R2+$, and $R2-$ have analogous meanings. Using these abbreviations the first two pertinent predictions of QT for this Hardy setup are these(*):

Under the condition that $L2$ is performed in L,
If $R2+$ then $L2+$ (2.1)
and
If ($L2+$ and $R1$) then $R1-$. (2.2)

If, in accordance with our assumption, the choice made in R does not affect the outcome that has already occurred in L, then these two conditions entail:

Property 1: Quantum theory predicts that if an experiment of the Hardy-type is performed then,

$L2$ implies SR,

where,

$SR =$ If $R2$ is performed and gives outcome $+$ then if, instead, $R1$ had been performed the outcome would have been $-$.

Proof of Property 1: The concept "instead" is given an unambiguous meaning by the combination of the premises of "free choice", and "no backward in time influence": the choice between $R1$ and $R2$ is to be treated, within the theory, as a free variable, and switching between $R1$ and $R2$ is required to leave any outcome in the earlier region L undisturbed. But then (2.1) and (2.2) can be joined in tandem to give the result SR.

The second two pertinent predictions of QT for this Hardy setup are:

Under the condition that $L1$ is performed in L,
If ($L1-$ and $R2$) then $R2+$ (2,3)
and
If ($L1-$ and $R1$) then sometimes $R1+$ (2.4).

If our premises are valid then these two predictions entail:

Property 2. Quantum theory predicts that if an experiment of the Hardy-type is performed then:

$L1$ implies that SR is false.

Proof of Property 2: Quantum theory predicts that if $L1$ is performed then outcome $-$ appears about half the time. Thus if $L1$ is chosen then there are cases where $L1-$ is true. But in a case where $L1-$ is true prediction (2.2) asserts that the premise of SR is true. But (2.3), in conjunction with our two premises that give meaning to "instead", implies that the conclusion of SR is not true: if $R1$ is performed instead of $R2$ the outcome is not necessarily $R1-$, as it was in case $L2$.

Any theory or model that makes SR true or false according to whether $L2$ or $L1$ is freely chosen in region L entails the existence of faster-than-light effects. This conclusion is discussed in the next section.

3. Connection to Earlier Works.

The aim of this work is similiar to that of an earlier work of this author[8]. That work was criticized by Unruh[9], by Mermin[10], and by Shimony and Stein[11] on various grounds. I have answered these objections [12, 13, 14]. However, the very existence of those challenges shows that the approach used in [9] has serious problems, which originate in the fact that it is based on classical modal logic.

That approach provides the possibility of a concise logically rigorous proof based on an established logic. However, that virtue is overshadowed by the following drawbacks:

1. Although the symbolic proof is concise and austere, that brevity is based on a background that most physicists lack. This means that most physicists cannot fully understand it without a significant investment of time.
2. The question arises as to whether the use of classical modal logic begs the question by perhaps being based in implicit ways on the deterministic notions of classical physics.
3. Classical modal logic itself is somewhat of an open question, and it is not immediately clear to what extent these issues undermine the proof.

For these reasons I have in the present formulation relied only on quantum thinking and language thoughout: there is no appeal to concepts unfamiliar to physicists.

But the present proof differs from the 1997 version by more than just the use of the language of physicists. The earlier proof introduced an assumption

LOC2. That assumption was introduce in order to set up a *reducio ad absurdum* argument: it was meant to be proved false. But a lot of the discussion my earlier augument was whether I had adequately justified this "assumption," which, however, I was trying to prove false. The present version gives is a straightforward proof of the key properties 1 and 2, without introducing the false assumption LOC2.

The key objection to the earlier argument was stated most clearly by Shimony and Stein: Although the *explicit* condition for the truth of SR is specified entirely by the truth or falsity of statements about possible events localized in region R—and hence the proven dependence of the truth of SR upon which experiment is freely chosen in L seems *manifestly* to require an influence in R of that choice made in L—the word "instead" that occurs in SR harbors an *implicit* dependence of SR upon the choice made in L, and that implicit dependence upon L that upsets the "manifest" conclusion.

I do not believe that objection is valid. The word "instead" does bring in the explicitly stated assumption that the free choice between $R1$ and $R2$ made in R has *no influence* on the outcome in L. But the combination of this *explicit denial* of the existence of an influence from R to L coupled with a denial of any influence from L to R would leave one at a loss as to how to understand the proven dependence of the truth of SR upon the free choice made in L.

4. Comparison to Orthodox Ideas.

The conclusion obtained here about faster-than-light influences parallels Bohr's reply to the paper of Einstein, Podolsky, and Rosen[15]. The assumption of those authors was that there was no faster-than-light influence of any kind. Bohr's response[16] was a partial rejection of that assumption: he granted that there was no faster-than light "mechanical disturbance" , but argued that "there is an influence on the very conditions that define the possible types of predictions regarding the future behavior of the system."

A "mechanical disturbance" would be one capable of transmitting a signal, whereas the others pertain to "predictions", and hence to the *theoretical structure*, which from Bohr's point of view, was primarily a tool for making predictions about what outcomes will be observed under various alternative conditions.

Bohr's conclusions about the existence of faster-than-light influences, like the one obtained here, was made completely within the framework of the Copenhagen interpretation of quantum theory. But the present argument is more direct: it complements the philosophically subtle argument of Bohr.

All the premises of the present nonlocality theorem are compatible with orthodox quantum philosophy, and both the premises and conclusions are compatible with relativistic quantum field theory. The theorem therefore covers orthodox quantum theory as a special case.

The locality condition whose violation is demonstrated here is similar to the one occurring in Bell's theorems in that: (1), the dependence in question is on an experimenter's free choice of which measurement to perform in a certain region, and (2), the property that depends on this free choice are predictions about *both* of the alternative possibilities between which the experimenter in the other region is free to choose. But the present argument is carried out without using any combination of properties that are logically equivalent to Bell's hidden-variable assumptions: the premises used here are significantly weaker than those of Bell's theorems, and hence the class of models covered is significantly larger.

By dispensing in this way with the hidden-variable substructure, the present theorem evades challenges to Bell's theorems of the kind that recently appeared in the Proceedings of the National Academy of Science[17].

5. A Partial Many-Worlds Way Out.

The significance of this faster-than-light theorem is that it places a strong and anti-intuitive condition on any model or theory that seeks to reproduce the predictions of quantum theory. However, there is a interesting way out. The proof given above depends tacitly upon the orthodox idea of the collapse of the wave function. It was first pointed out in reference 18 that if one rejects the collapse idea then the physical conditions that define the correlated states and associated amplitudes do not come into existence until the (essentially classical) signals carrying the information about the outcomes in the two regions R and L come together in a region that lies in the intersection of the forward light cones from these two regions. This is the region where the experimenters compare and correlate the data coming from the two regions. But there is no violation of any faster-than-light condition if the region in which the pertinent correlation information comes into being is confined to the intersection of those two forward light cones. Also, the basic locality condition, which asserts that the choice of outcome in one region does not depend upon which experiment is chosen in the faraway region, cannot be imposed if no choice is ever made between the two possible outcomes.

This consideration casts some weight in favor of adopting a many-worlds (or many-minds) type of ontology. On the other hand, a serious difficulty with the usual many-worlds/minds approach was recently described in reference 19. The problem, basically, is that the traditional *full* many-worlds/minds approach generates continua of overlapping states, rather than the sharp divisions entailed by von Neumann's Process I, and this lack of sharply defined alternatives creates problems in constructing well defined probabilities.

This situation suggests the approach of accepting the objective reality of von Neumann's Process I, but never mentioning the subsequent collapse to one or the other of the two alternatives. Von Neumann[20] himself postulated Process I, and also the Schroedinger Process II, but made no commitment to Process III, the collapse to one of the several nonoverlapping branches specified by Process I. I view this omission as significant. By postulating the existence of Processes I and II, but rejecting Process III, one can avoid the difficulties described in reference 19, and avoid also the faster-than-light effects entailed by the analysis done here. An important point is that the "state" of a subsystem is defined by tracing over "other" variables, and therefore Process I, unlike Process III, has no effect on faraway systems. Thus von Neumann's approach seems to offer a way to get both the definiteness that the many-minds/worlds theories lack, and the absence of long-range faster-than-light influences that many-minds/worlds theories provide.

*[NB: To obtain these four predictions from Hardy's paper, one transforms my notation into Hardy's using

$$(L, R) \rightarrow (1, 2)$$

$$(1, 2) \rightarrow (D, U)$$

$$(+, -)_L \rightarrow (0.1)$$

$$(+.-)_R \rightarrow (1, 0)$$

and uses the three zero's connecting my pairs of states $(R1+, L1-)$, $(L2-, R1-)$ and $(L1+, R2+)$ that arise from his Eqs. $(13.a, b, c)$ to obtain my (2.1),(2.2), and (2.3), respectively, and uses his $(13.d)$, which says that my matrix element $(L2-, R2+)$ is positive, to obtain my (2.4).]

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