**Appendix 1: Proof That Information Must Be Transferred Faster Than Light**

Strengthening Bell’s Theorem: Removing the Hidden-Variable Assumption

**Spooky action at a distance.**

In the context of correlation experiments involving pairs of experiments performed at very nearly the same time in very far-apart experimental regions Einstein famously said [1]:

“But on one supposition we should in my opinion hold absolutely fast: ‘The real factual situation of the system S2 is independent of what is done with system S1 which is spatially separated from the former.’ “

This demand is incompatible with the basic ideas of standard (Copenhagen/Orthodox) quantum mechanics, which makes two relevant claims:

(1) Experimenters in the two labs make “local free choices” that determine which experiments will be performed in their respective labs. These choices are “free” in the sense of not being pre-determined by the prior history of the physically described aspects of the universe, and they are “localized” in the sense that the physical effects of these free choices are inserted into the physically described aspects of the universe only within the laboratory in which the associated experiment is being performed.

(2) These choices of “what is done with the system” being measured in one lab can (due to a measurement-induced global collapse of the quantum state) influence the outcome of the experiment performed at very close to the same time in the very faraway lab.

This influence of ‘*what is done’ with* the system being measured in one region upon the outcome appearing at very nearly the same time in a very faraway lab was called “spooky action at a distance” by Einstein, and was rejected by him as a possible feature of “reality”.

**John Bell’s quasi-classical statistical theory**

Responding to the seeming existence in the quantum world of “spooky actions”, John Bell [2] proposed a possible alternative to the standard approach that might conceivably be able to reconcile quantum spookiness with “reality”. This alternative approach rests on the fact that quantum mechanics is a statistical theory. We already have in physics a statistical theory called “classical statistical mechanics”. In that theory the statistical state of a system is expressed as a *sum* of terms, each of which is a possible *real physical state* λ of the system multiplied by a probability factor.

Bell conjectured that quantum mechanics, being a statistical theory, might have the same kind of structure. Such a structure would satisfy the desired properties of “locality” and “reality” (local realism) if, for each real physical state λ in this sum, the relationships between the chosen measurements in the two regions and the appearing outcomes are expressed as product of two factors, with each factor depending upon the measurement and outcome in just one of the two regions. The question is then whether the statistical properties of such a statistical ensemble can be consistent with the statistical predictions of quantum mechanics.

Bell and his associates proved that the answer is No! They considered, for example, the empirical situation that physicists describe by saying that two spin-1/2 particles are created in the so-called spin-singlet state, and then travel to the two far-apart but nearly simultaneous experimental regions. The experimenter in each region freely chooses and performs one of the two alternative possible experiments available to him. Bell et. al. then prove that the predictions of quantum mechanics cannot be satisfied if the base states λ satisfy the “factorization property” demand of “local realism”. A theory satisfying this demand is called a “local hidden-variable theory” because the asserted underlying “reality” is described by variables that cannot be directly apprehended.

**Two Problems With Bell’s Theorems.**

Bell-type theorems, if *considered as proofs of the logical need for spooky actions in a theory that entails the predictions of quantum mechanics*, have two problems*.* The first is that the theorems postulate a “reality” structure basically identical to that of classical statistical mechanics. Bell’s theorems then show that imposing “locality” (factorizability for each fixed λ) *within this classical-type reality structure* is incompatible with some predictions of quantum mechanics. But that result can be regarded as merely added confirmation of the fact that quantum mechanics is logically incompatible with the conceptual structure of classical mechanics. Simply shifting to a classically conceived “statistical” level does not eliminate the essential conceptual dependence on the known-to-be-false concepts of classical physics.

The second problem is that the condition of “local realism” is implemented by a “factorization” property, described above, that goes far beyond Einstein’s demand for no spookiness. In addition to the non-dependence of outcomes in a region upon *“what is done”* in the faraway region “local realism” entails also what Shimony calls “outcome independence”. That condition goes significantly beyond what Einstein demanded, which is merely a non-dependence of the factual reality (occurring outcome) in one region on *the choice of experiment performed in the faraway region*. “Outcome independence” demands that the outcome in each region be independent also of the *outcome* in the other region.

That property, “*outcome independence”,* is not something that one wants to postulate if a resulting incompatibility with predictions of quantum mechanics is supposed to entail the existence of spooky actions at a distance!

That unwanted independence assumption is not a just a minor fine point. Consider the simple example of two billiard balls, one black, one white, shot out in opposite directions to two far-apart labs. This physical example allows – given the initial symmetrical physical state -- the outcome in one region to be correlated with the *“outcome”* appearing in the other region, without any hint of any spooky action at a distance”: a “black” ball in one region entails a “white” ball in the other, and vice versa, without any spooky action. Hence Bell’s theorems do not address – or claim to address -- the key question of the compatibility of Einstein’s demand for no spookiness with the predictions of (relativistic) quantum mechanics. Bell’s theorems are based on the stronger assumption of local hidden variables.

Bell’s theorems (regarded as proofs of the need for spooky actions) are thus deficient in two ways: they bring in from classical (statistical) mechanics an alien-to-quantum-mechanics idea of “reality”; and they assume, in the process of proving a contradiction, a certain property of “outcome independence” that can lead to a violation of quantum predictions without entailing the lack of spookiness that Einstein demanded.

The question thus arises whether the need for spooky interactions can be proved simply from the validity of some empirically well validated predictions of standard quantum mechanics, without introducing Bell’s essentially classical “hidden variables”? The answer is “Yes”!

**The Proof.**

The following proof of the need for “spooky actions” places no conditions at all on any underlying process or reality, beyond the macroscopic predictions of quantum mechanics: it deals exclusively with connections between macroscopic measurable properties. This change is achieved by taking Bell’s parameter λ to label, now, the different experiments in a very large set of simultaneously performed similar experiments, rather than the different possible basic microscopic states λ of the statistical ensembles. The ontology thereby becomes essentially different, though the mathematics is similar. The macroscopic experimental arrangements are the ones already described above.

In the design of this experiment the physicists are imagining that a certain initial macroscopic preparation procedure will produce a pair of tiny invisible (spin 1/2) particles in what is called the singlet state. These two particles are imagined to fly out in opposite directions to two faraway experimental regions. Each of these experimental regions contains a Stern-Gerlach device that has a directed preferred axis that is perpendicular to the incoming beam. Two detection devices are placed to detect particles deflected either along this preferred axis, or in the opposite direction. Each of these two devices will produce a visible signal (or an auditory click) if the imagined invisible particle reaches it.

The location of the individual detector is specified by the angle Փ of the directed preferred axis such that a displacement along that particular direction locates the detector. Clearly, the two detectors in the same experimental region will then be specified by two angles Փ that differ by 180 degrees. For example, if one detector is displaced “up” (Փ = 90 degrees) then the other is displaced “down” (Փ = minus 90 degrees). The angle Փ = 0 labels in both regions a common deflection to the right: e.g., along the positive x axis in the usual x-y plane.

Under these macroscopic experimental conditions, quantum theory predicts that, if the detectors are 100% efficient, and if, moreover, the geometry is perfectly arranged, then for each created pair of particles -- which are moving in opposite directions to the two different regions -- exactly one of the two detectors in each region will produce a signal (i.e.,“fire”). The key prediction of quantum theory for this experimental setup is that the fraction *F* of the particle pairs for which the detectors that fire in the first and second regions are located at angles Փ1 and Փ2, respectively, is given by the formula

*F = (1-Cosine(Փ1-Փ2))/4*.

In the experiment under consideration there are two alternative possible experiments in the left-hand lab, and two alternative possible experiments in the right-hand lab, making 2X2 = 4 alternative possible pairs of experiments. For each single experiment (on one side) there are two detectors, and hence two angles Փ. Thus there are altogether 4X2X2=16 F’s.

I take the large set of similar experiment to have 1000 experiments. Then the fractions F of 1000 are entered into the 16 associated boxes of the following diagram.



In Diagram 1, the first and second *rows* correspond to the two detectors in the *first* possible set-up in the left-hand region. The third and fourth rows correspond to the two detectors in the *second* possible set-up in the left-hand region. The four *columns* correspond in the analogous way to the detectors in the right-hand region. The arrows on the periphery show the directions of the displacements of the detectors associated with the corresponding row or column.

For example, in the top-left 2-by-2 box if the locations of the two detectors (one in each region) that fire together are both specified by the same angle, Փ1 = Փ2, then, because *Cosine 0 =1*, each specified pair of detectors will *never* both fire together: if one of these two specified detectors fires, then the other will not fire. If Փ1 is some fixed angle and Փ2 differs from it by 180 degrees then, because *Cosine 180 degrees = minus 1*, these two specified detectors will, under the ideal measurement conditions, fire together for *half* of the created pairs. If Փ1 is some fixed angle and Փ2 differs from it by 90 degrees then these two specified detectors will fire together for ¼ of the pairs. If Փ1 is some fixed angle and Փ2 differs from it by 45 degrees then these two specified detectors will fire together, in a long run, for *close to 7.3%* of the pairs. If Փ1 is some fixed angle and Փ2 differs from it by 135 degrees then these two specified detectors will fire together, in a long run, for *close to 42.7%* of the created pairs.

I have listed these particular predictions because they are assumed to be valid in the following proof of the need for near-instantaneous transfer of information between the two far-apart, but nearly simultaneous, experimental space-time regions. These particular predictions have been massively confirmed empirically.

The second assumption is “localized free choices”. The point here is that physical theories make predictions about experiments performed by experimenters with devices that detect or measure properties of the systems whose properties are being probed by these devices. The theory entails that the various settings of the devices will correspond to probe-associated properties of the system being probed.

Of course, in an actual situation these specified parts of the experimental setup are all parts of a universe that includes also the experimenter and whatever the experimenter uses to actually fix the experimental settings. Such a “choosing” part of the universe *could*, however, *conceivably* causally affect not only to the setting of the associated measuring device but, say, via the distant past, also other aspects of the experiment. Those unsuspected linkages via the past could then be responsible for systematic correlations between the empirical conditions in the two regions -- correlations that are empirically dependent on which experiments are chosen and performed but are empirically independent of how the experimental setups are chosen.

In view of the limitless number of ways one could arrange to have the experimental setup specified, and the empirically verified fact that the predictions are found to be valid independently of *how* the setup is chosen, it is reasonable to assume that the choices of the experimental setups can be arranged so that they are not systematically connected to the specified empirical aspects of the experiment except via these choices of the experimental setup. This is the assumption of “localized free choices.” It is needed to rule out the (remote) possibility that the choice of the setup is significantly and systematically entering the dynamics in some way other than as just the localized fixing of the experimental setup.

Suppose, then, that we have the two far-apart experimental regions, and in each region an experimenter who can freely choose one or the other of two alternative possible experimental set-ups. Suppose we have, in a certain region called the source region, a certain mechanical procedure to which we give the name “creation of N individual experimental instances, where N is a large number, say a thousand. At an appropriate later time the experimenters in the two regions make and implement their “localized free choices” pertaining to which of the two alternative possible experiments will be set up in their respective experimental regions. At a slightly later time each of the two experimenters looks at and sees, in each of the N individual instances, which one of his two detection devices has fired, and then records the angle Փ that labels that detector, thereby recording the outcome that occurs in that individual instance.

There are altogether two times two, or four, alternative possible experimental setups. Diagram 1 gives, for each of these four alternative possible setups, the number of individual instances, from the full set of 1000, that produce firings in the pair of detectors located at the pair of angles Փ specified along the left-hand and top boundaries of the full diagram. For example, the four little boxes in the first two rows and the first two columns correspond to the case in which the experimenter in the left-hand region sets his two detectors at “up” (Փ1=90 degrees) and “down” (Փ1= minus 90 degrees)., while the experimenter in the right-hand region sets his two detectors also at “up” (Փ2= 90 degrees) and “down” (Փ2= minus 90 degrees). In this case the expected distribution (modulo fluctuations) of the thousand instances is 500 in the box in which Փ1= 90 degrees and Փ2= minus 90 degrees and the other 500 in the box in which Փ1= minus 90 degrees and Փ2= 90 degrees.

The fluctuations become relatively smaller and smaller as N get larger and larger. So I will, for simplicity, ignore them in this discussion and treat the predictions to be exact already for N=1000.

The two experimental regions are arranged to be essentially simultaneous, *very* far apart, and very tiny relative to their separation. These two regions will be called the “left” and “right” regions.

The “no-essentially-instantaneous-transfer of information about localized free choices” assumption made here is that, no matter which experiment is performed in a region, the outcome appearing there is independent of which experiment is freely chosen and performed in the faraway region. This means, for example, that if the experiment on the right is changed from the case represented by the left-hand two columns to the case represented by the right-hand two columns, then the *particular set* of 500 instances – from the full set of 1000 -- that are represented by the 500 in the top row second column get shifted into the two boxes of the top row in the second two columns.

More generally, a change in the experiment performed on the right shifts the individual instances – in the set of 1000 individual instanced – horizontally, in the same row; whereas a change in the experiment performed on the left shifts the individual instance vertically. The diagram 1 then shows how, by a double application of the “no FTL condition”, a subset of the set of 500 instances occupying box A gets shifted via box B to box C, which must then contain at least 427 -73 = 354 of the original 500 instances in A. However, the applying of the two changes in the other order, via D, demands that the subset of instances in A that can be in C can be no greater than 250. That is a contradiction. Thus one cannot maintain simultaneously both the general rule of no FTL transfers of information and four very basic and empirically confirmed predictions of quantum mechanics.

In more detail the argument then goes as follows. Let the pairs (individual instances) in the ordered sequence of the 1000 created pairs be numbered from 1 to 1000. Suppose that the actually chosen pair of measurements corresponds to the first two rows and the first two columns in the diagram. This is the experiment in which, in each region, the displacements of the two detectors are “up” and “down”. Under this condition, quantum theory predicts that for some particular 500-member subset of the full set of 1000 individual instances (created pairs) the outcomes conform to the specifications associated with the little box labeled A. The corresponding 500 member subset of the full set of 1000 positive integers is called Set A. This Set A is a particular subset of 500 integers from set {1, 2, …,1000}. The first 4 elements in Set A might be, for example, {1, 3, 4, 7}.

If the local free choice in the right-hand region had gone the other way, then the prediction of quantum mechanics is that the thousand integers would be distributed in the indicated way among the four little boxes that lie in one of the first two rows and also in one of the *second* two columns, with the integer in each of these four little boxes specifying the number of instances in the subset of the original set of 1000 individual instances that lead to that specified outcome. Each such outcome consists, of course, of a pair of outcomes, one in the left-hand experimental region, and specified by the row, the other in the right-hand experimental region and specified by the column.

If we now add the Locality Condition, then the demand that the macroscopic situation in the left-hand region be undisturbed by the reversal of the localized free choice made by the experimenter in the (faraway) right-hand region means that the set of 500 integers in Set A must be distributed between the two little boxes standing directly to the right of the little box A. Thus the Set B, consisting of the 427 integers in box B, would be a 427 member subset of the 500 integers in Set A.

The above conclusions were based on the supposition that the actual choice of experiment on the left was the option, represented by the top two rows and the leftmost two columns in diagram 1. However, having changed the choice in the right-hand region to the one that is represented by the *rightmost* two columns – the possibility of which is which is entailed by Einstein’s reference to a dependence on “what is done with” the faraway system -- we next apply the locality hypothesis to conclude that changing the choice on the left must leave the outcomes on the right undisturbed. That means changing the top two rows to the bottom two rows, leaving the integers that label the particular experiment in the set of 1000 experiments in the same column. This means that the 427 elements in the box B must get distributed among the two boxes that lie directly beneath it. Thus box C must include at least 427-73=354 of the 500 integers in Set A.

Repeating the argument, but reversing the order in which the two reversals are made, we conclude, from exactly the same line of reasoning, that box C can contain no more than 250 of the 500 integers box A, Thus the conditions on Set C that arise from the two different possible orderings of the two reversals are contradictory!

A contradiction is thus established between the consequences of the two alternative possible ways of ordering these two reversals of localized free choices. Because, due to the locality hypothesis being examined, no information about the choice made in either region is present in the other region, no information pertaining to the order in which the two experiments are performed is available in either region. Hence nothing pertaining to outcomes can depend upon the relative ordering of these two space-like separated reversals of the two choices.

This argument uses only macroscopic predictions of quantum mechanics -- without any conditions on, or mention of, any micro-structure from whence these macroscopic properties come -- to demonstrate the logical inconsistency of combining a certain 16 (empirically validated) predictions of quantum mechanics with the locality hypothesis that for each of the two experimental regions there is no faster-than-light transfer to the second region of information about macroscopically localized free choices made in the first .

The Bell’s theorem proofs are rightly identified as proofs of the incompatibility of “local realism” with the predictions of quantum mechanics. But “local realism” brings in both alien-to-quantum-theory classical concepts and also an “outcome independence” condition whose inclusion nullifies those theorems as possible proofs of the need for spooky actions at a distance. Both of these features are avoided in the present proof.

As regards Einstein’s reality condition, namely that the no-spooky-action condition pertains to the “real factual situation” one must, of course, use the quantum conception of the “real factual situation”, not an invalid classical concept. In ontologically construed orthodox quantum mechanics (in the contemporary relativistic quantum field theory version that I use) the “real factual situation” evolves in a way that depends upon the experimenter’s free choices and nature’s responses to those choices. The no-spooky-action condition is a condition on these choice-dependent real factual situations – namely outcomes observed under the chosen conditions-- that is inconsistent with certain basic predictions of the theory. That is what has just been proved. In classical mechanics there are no analogous free choices: the physical past alone uniquely determines the physical present and future.

The Einstein idea of no spooky actions involves comparing two or more situations only one of which can actually occur. This is the kind of condition that occurs in modal logic considerations involving “counterfactuals”. But here this modal aspect does not bring in any of the subtleties or uncertainties that plague general modal logic. For in our case the specified condition is a completely well defined and unambiguous (trial) mathematical assumption of the non-dependence of a nearby outcome upon a faraway free choice between two alternative possible probing actions. The proof does not get entangled with the subtle issues that arise in general modal logic. Everything is just as well defined as in ordinary logic.

In this proof there is no assumption of a “hidden variable” of an essentially classical kind lying “behind” the ontologically construed orthodox quantum theory. The phenomena are rationally understandable in terms of an evolving quantum state of the universe that represents “potentialities for experiences” that evolve via a Schroedinger-like equation punctuated by an ordered sequence of psycho-physical events each of which is an observer’s personal experience accompanied by a “collapse of the quantum state of the universe” that brings that evolving state into conformity with the observer-initiated experience of that observer.

The bottom line is that, given the validity of some basic macroscopic predictions of quantum mechanics, there is no way that the macroscopic phenomena can conform to the predictions of quantum mechanics without allowing violations of the general notion that the information about the local free choices cannot get essentially instantaneously to faraway regions and affect outcomes appearing in those regions.

**References**

1. Albert Einstein, in *Albert Einstein:Philosopher-Scientist,* Ed. P.A. Schilpp, Tudor, New York, 1949. p. 85.

2.John Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Gambridge UK, 1987, p. 1, 15.