Quantum Implications

Essays in honour of David Bohm

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Routledge & Kegan Paul
London and New York

This collection first published in 1987 by
Routledge & Kegan Paul Ltd
11 New Fetter Lane, London EC4P 4EE
Published in the USA by
Routledge & Kegan Paul Inc.
in association with Methuen Inc.
29 West 35th Street, New York, NY 10001

Set in Times
by Richard Clay Ltd
and printed in Great Britain
by Richard Clay Ltd,
Bungay, Suffolk

Library of Congress Cataloging in Publication Data

Quantum implications.
Includes index.
1. Quantum theory. 2. Bohm, David. I. Bohm, David.
II. Hiley, B. J. (Basil J.) III. Peat, F. David, 1938–
QC174.125.Q34 1987 530.1′2 86–22032

British Library CIP Data also available
ISBN 0–7102–0806–5

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objective reality. A paper now being written with H. P. Stapp describes space-building quanta which we call 'vacuons'.

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Light as foundation of being

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According to Niels Bohr quantum theory must be interpreted, not as a description of nature itself, but merely as a tool for making predictions about observations appearing under conditions described by classical physics:

> Strictly speaking, the mathematical formalism of quantum theory . . . merely offers rules of calculation for the deduction of expectations about observations obtained under well-defined conditions specified by classical physical concepts.[1]
>
> There can be no question of any unambiguous interpretation of the symbols of quantum mechanics other than that embodied by the well-known rules which allow to predict the results to be obtained by a given experimental arrangement described in a totally classical way.[2]
>
> This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are treated as measuring instruments and those which constitute the object under investigation may indeed be said to form *a principal distinction between classical and quantum description of physical phenomena.*[2]
>
> Indispensable use of classical concepts . . . even though classical physical theories do not suffice.[2]

This indispensable use of the invalidated classical concepts is a troublesome point. So is the intrusion into the theory of the scientist himself; the scientist must make a somewhat arbitrary division of a single unified physical system into two separate parts, and describe them according to mutually incompatible physical theories.

The aim of the present article is to show how recent technical developments in the quantum theory of light may allow quantum theory to be formulated as a unified theory of the physical world itself. The classical aspects of nature would then emerge automatically from the evolution of the fully quantum mechanical system, with no intrusion of observers or scientists. In this theory the electromagnetic field (i.e. light) plays a central role: it is the carrier of both classical properties and actual being itself.

The model presented here has elements of arbitrariness that render it unsatisfactory as a true model of the universe: it is a rudimentary form of such a model, not a finished product.

The technical development mentioned above arose in connection with the famous 'infrared catastrophe': the contributions of infinite numbers of very low-energy photons had led to apparent infinities in the calculation of many physical quantities. The essential feature of the resolution of this problem was discovered in 1937 by Bloch and Nordsieck: one must separate out the classical aspects of the problem. The original work[3] dealt only with simple cases, and involved approximations, but it was developed and extended in an immense collection of works by many authors. But there remained until recently the basic problem of understanding how the observed classical results emerged in all of the appropriate macroscopic limits.

This problem was resolved recently by recognizing that there was an exact separation between the classical and quantum parts of the electromagnetic current.[4] The coordinate-space Feynman path of each charged particle has one set of vertices for the 'quantum' interactions with light, and a different set of vertices for the 'classical' interaction. The radiation from each classical vertex depends only on its own location, and those of the two neighboring *classical* vertices; it is independent of what happens between these classical vertices. Furthermore, an arbitrary number of classical photons, all identical, can be emitted from each classical vertex. These classical photons, whose character depends only on the locations of the *classical* vertices can be summed. The sum is a unitary operator that creates precisely the unique coherent quantum state that corresponds to the light radiated by a classical charge moving on a space-time path defined by the sequence of classical vertices.

Before showing how this identification of the classical part of the electromagnetic field provides the basis for a unified self-governing quantum universe, with automatic emergence of classical reality, some peripheral questions will be addressed.

The first question is whether there is any need for unified formulation of quantum theory. Bohr gave convincing arguments that, in the realm of atomic physics, no theory could give predictions going beyond those attainable from his observer-based formulation of quantum theory. However, the experimental situation encountered in atomic physics is far from universal. It involves large preparing and detecting devices, which are considered to be parts of the full classically-described macroscopic environment, plus a tiny quantum system. This quantum system must be small enough so that during the interval between its preparation and detection its influence upon the macroscopic environment is negligible. For if the quantum system influences the macroscopic environment then phase information is transferred to this environment, and the Schrödinger equation fails. The macroscopic environment must then be described quantum mechanically, which contradicts the requirement that it be described classically. Consequently, as the quantum system is increased in size it must eventually reach the stage where neither the classical nor quantum description is adequate. To deal with such intermediate situations it would appear necessary to treat in a unified way the full physical system of macroscopic environment plus quantum object.

It has been claimed that most physicists accept Bohr's interpretation of quantum theory. Of course, any physicist who uses quantum theory in a practical way in atomic physics is probably interpreting quantum theory as a useful tool, in the way Bohr suggested. But at the level of basic principle the dissenters include most of the founders of quantum theory: Einstein, Schrödinger, de Broglie, Pauli, Heisenberg, Wigner, and von Neumann, to name a few. Gell-Mann said 'Niels Bohr brainwashed a whole generation of physicists into believing that the problem had been solved fifty years ago.'[5] Pauli[6] said: 'I think the important and extremely difficult task of our time is to try to build up a fresh idea of reality.'

I doubt if any physicist can be completely comfortable injecting human observers and invalid classical concepts into atomic theory, and giving up the ideal that basic physical theory should describe the world itself. The two overriding considerations are rather that Bohr's interpretation works fine in atomic physics, and that even Einstein himself, in spite of intensive effort, was unable to propose any alternative.

The present proposal is based on two results that did not exist in Einstein's day. The first is the above-mentioned development of our understanding of the classical part of the electromagnetic field, and the second is the refinement in our ideas of locality and causality that have grown out of Bell's theorem.[7]

The present proposal is in line with some ideas of David Bohm[8] and Werner Heisenberg[9] that will be described next.

Bohm-Heisenberg idea of events

The wave function of quantum theory has many similarities to the probability function of classical statistical mechanics. This latter function represents the probability for the various particles of the system to *be* in certain states at specified times. But if we were to place detectors in certain locations then it could also represent the probability for something to 'happen,' i.e. for the detectors to detect something.

The square of the absolute value of the wave function of quantum theory has an intuitive significance similar to this 'happening' or 'event' interpretation of the classical probability function, and Heisenberg was willing to say that the detection event actually occurs at the level of physical device.[9] Then the probability for 'finding' the particle can be interpreted as the probability for this physical detection event to occur, quite apart from any human participant or observer.

The difficulty with this idea is to know how to describe in a precise way what has 'happened.' If we consider the 'event' to be the observation by a human observer, then we know by education and training how to judge whether this observation conforms to certain specifications. And these specifications have, quite naturally, a certain impreciseness, which allows for the necessary quantum fluctuations. But if we are going to consider the event to be something in the external physical world itself, then we need some sort of mathematical description of what is happening. But what is the precise form of the description of device plus quantum object before and after the 'event'?

If one tries to use only the wave function and the Schrödinger equation of quantum mechanics then one finds that the 'event' never occurs. Rather, every possible event occurs: there is no singling out of the one event that actually occurs from the myriad of possible events that might occur.

The origin of this problem is precisely that the wave function has mathematical properties appropriate to a representation of probabilities, rather than actualities. For a system of n particles the wave, at fixed time, is a function in a space of $3n$ dimensions. But we live in a space of only three dimensions. Thus, the wave function, like a classical probability function, represents all things that possibly can happen; it does not single out the one thing that actually does happen.

To represent the actual thing one appears to have three options:

1 introduce object-like (or field-like) entities to represent the actual things;

2 introduce idea-like entities to fill up all the 'mindful' possibilities corresponding to the multi-branched wave function;

3 introduce action-type entities to 'collapse' (i.e. eliminate) the unrealized branches of the wave function.

The first alternative leads to the de Broglie-Bohm[8] pilot-wave idea, in which the part of the wave function representing all of the unrealized possibilities awkwardly continues to exist in an objective sense. The second leads to myriads of parallel worlds[10] that are all interpreted as objectively real, but which seem to be simply the consequence of insisting that the wave function represent objective reality itself rather than merely the probabilities for events. The third possibility is the one to be pursued here. It is in general accord with ideas of Bohm,[11] Heisenberg,[9] and von Neumann.[12] The problem with this idea has been the unavailability of any objective way to single out the various 'classically allowed possibilities.' Lacking any objective mechanism for making this selection, physicists have assigned this task to themselves.

Time and quantum process

The model to be proposed here is the embodiment of a *process* conception of nature. By *process* I mean nature conceived as a progressively growing set of things that are fixed and settled. This growing body of accumulating facts is considered to increase in discrete steps; at each step some unsettled things become fixed and settled.

Before Newton the history of the world could have been regarded as a growing body of facts, with the factual content of the region $t < T$ representing what is fixed and settled at time T, and the region $t > T$ representing the realm of the unfixed possibilities. However, Newton's laws altered this picture: they fixed the complete space-time story, once the initial conditions were fixed. Hence the idea of process was effectively banished.

This banishment of process made way for Einstein's theory of relativity. For if the entire space-time story is fixed, then choices of coordinates become purely matters of scientific convenience: there is no need for a scientist to worry about the questions of 'what exists now' or 'what has already taken place.' In a deterministic world the whole notion of 'becoming' becomes so nebulous and shadowy that it drops completely out of the physicist's stock of operative ideas.

The non-deterministic character of quantum theory reopens the whole question of the connection of space and time to the ontological categories of existence, being, becoming, etc. For if initially unsettled things can eventually become fixed and settled, then the order in which different things become fixed might have conceptual and dynamical significance. To expand the structural possibilities we go beyond the pragmatic confines of Bohr's interpretation: we distinguish human knowledge from general existence, and base physical theory on the latter.

As regards the relationship between order as defined by process (i.e. the order in which things become fixed and settled) and temporal order (as defined by space-time coordination) we are, *ab initio*, completely free. The theory of relativity says nothing at all about the order in which things become fixed and settled, because it was set in a framework in which no such order occurred.

As regards causal influences it now recognized[7] that quantum theory requires that what appears in one space-time region cannot in general be required to be independent of what is done (i.e. is chosen) in spatially separated regions. However, this necessity for 'non-local influences' need not provide any possibility for sending signals faster than light.

Coherent states and classical concepts

The model to be proposed here makes essential use of the coherent states of the quantized electromagnetic field. These states play a role in the model similar to those played by the 'observer' in Bohr's formulation of quantum theory.

These coherent states are, as is well known, the quantum-mechanical counterparts of the fields that occur in classical electromagnetic theory. I shall briefly review here this connection between coherent states and classical physics, and then describe some properties of coherent states that will be used in the model. Further details about coherent states, and their uses in quantum electrodynamics and quantum optics, can be found in articles by Kibble[13,14] and Glauber.[15]

The classical electromagnetic field in a cavity can be decomposed into eigenmodes i. Each such mode has an eigen frequency ω_i and a complex amplitude $a_i(t)$, which is related to the real canonical variables $q_i(t)$ and $p_i(t)$ by the equation:

$$a_i(t) = (m_i\omega_i q_i(t) + ip_i(t))/(2\hbar m_i\omega_i)^{\frac{1}{2}}$$

where m_i is a characteristic mass. Thus the real and imaginary parts of the complex amplitude $a_i(t)$ are associated with the canonical variables $q_i(t)$ and $p_i(t)$ respectively.

The possible free motions of the classical electromagnetic field in the cavity are represented by taking each variable $a_i(t)$ to be of the form:

$$a_i(t) = a_i \exp(-i\omega_i t)$$

Thus, the complex variable $a_i(t)$ moves with velocity $-\omega_i$ in a circle about the origin in the complex plane. The real and imaginary parts of $a_i(t)$ correspond to the magnetic and electric parts of the electromagnetic field, and the circular motion corresponds to the familiar oscillation of the energy of the radiation field between the electric and magnetic fields, both in the standing-wave modes, and in the circularly-polarized traveling-wave modes.

Upon quantization the complex amplitude $a_i(t)$ becomes an operator $\hat{a}_i(t)$. The operators $\hat{a}_i(t)$ obey the familiar commutation relations:

$$[\hat{a}_i(t), \hat{a}_j^{\dagger}(t)] = \delta_{ij}$$

Each mode i has a discrete set of eigenstates $|n_i>$, $n_i \in (0, 1\ldots)$, where n_i is the number of photons in mode i. The eigenmodes of the full electromagnetic field itself are represented as products over the states $|n_i>$ of the individual oscillators.

One may, however, consider also the state $|a_i(t)>$ obtained by shifting the ground-state wave-function of oscillator i from its original position centered at the origin of phase-space to a new position centered at point $a_i(t)$. The equations of motion of the quantum system dictate that this state develop in time according to the classical equations of motion:

$$|a_i(t)> = |a_i \exp(-i\omega_i t)>$$

A coherent state $|A(t)>$ of the electromagnetic field is constructed as a product of these displaced ground states:

$$|A(t)> = |a_1(t)>|a_2(t)> \ldots$$

This state is defined by the set of amplitudes $\{a_i(t)\}$ for the various modes i, and hence by a positive-frequency solution $A^{(+)}(x)$ of the classical electromagnetic field equations. The coherent state $|A(t)>$ in the interaction representation can be labelled by:

$$A(x) = A^{(+)}(x) + (A^{(+)}(x))^* = 2 \operatorname{Re} A^{(+)}(x)$$

The expectation value in the state $|A(x)>$ of the quantum operator $\hat{A}(x')$ corresponding to the vector potential of electromagnetism is:

$$<A(x)|\hat{A}(x')|A(x)> = A(x')$$

More generally, if $\hat{A}^{(-)}(x')$ and $\hat{A}^{(+)}(x') = (\hat{A}^{(-)}(x'))^{\dagger}$ represent the creation and annihilation operator parts of the quantum operator $\hat{A}(x')$ then:

$$<A(x)|\hat{A}^{(-)}(x'_1) \ldots \hat{A}^{(-)}(x'_m)\hat{A}^{(+)}(x''_1) \ldots \hat{A}^{(+)}(x''_n)|A(x)> = A^{(-)}(x'_1) \ldots A^{(-)}(x'_m)A^{(+)}(x''_1) \ldots A^{(+)}(x''_n)$$

Consequently, by virtue of the Dyson-Wick expansion, the S matrix in a coherent state is equal to the S matrix in the corresponding classical electromagnetic field:

$$<A(x)|B^{(+)}SC^{(-)}|A(x)> = <B^{(+)}S(A(x))C^{(-)}>0$$

The right-hand side is a vacuum expectation value, the $C^{(-)}$ and $B^{(+)}$ are particle (i.e. non-photon) operators, and $S(A(x))$ is the S matrix in the presence of the classical electromagnetic field $A(x)$. This result consolidates the close connection between coherent states and classical fields.

A key formula for us will be the matrix element between two coherent states. For a single mode the formula is:

$$\begin{aligned}<a|b> &= \exp[a^*b - (\tfrac{1}{2})a^*a - (\tfrac{1}{2})b^*b] \\ &= \exp[-(\tfrac{1}{2})|a-b|^2 + i \operatorname{Im} a^*b]\end{aligned}$$

where:

$$\begin{aligned}|a-b|^2 &= (a^* - b^*)(a - b) \\ &= (\operatorname{Re}(a-b))^2 + (\operatorname{Im}(a-\mathrm{b}))^2\end{aligned}$$

is the square of the distance between the two complex numbers, considered as points in a two-dimensional space.

This formula generalizes immediately to the coherent states:

$$<A|B> = \exp\left[-(\tfrac{1}{2})|A-B|^2 + i\Phi\right]$$

where:

$$|A-B|^2 = \sum_i (\mathrm{Re}(a_i - b_i))^2 + (\mathrm{Im}(a_i - b_i))^2$$

and

$$\Phi = \mathrm{Im} A^*B = \sum_i \mathrm{I_m}\, a_i^* b_i$$

Thus two coherent states $|A>$ and $|B>$ can be said to be separated from each other by a distance $|A - B|$, and a phase Φ, both of which vanish if $A = B$.

Observers and classical concepts

In the Bohr (Copenhagen) interpretation of quantum theory the observer plays a central role; he distinguishes between the classically distinct results of measurements. If before the measurement the state of atomic object plus measuring device is a pure quantum state ψ, then after the measurement the state must (if the environment is ignored) still be a pure state. If the experiment has, for example, two possible results then this pure state ψ' will have two 'classically distinguishable' components, which correspond to the two distinct possible results of the experiment. Thus the process of measurement produces a change.

$$\psi \longrightarrow \psi' = |\varphi_1> + |\varphi_2>$$

where $|\varphi_1>$ and $|\varphi_2>$ represent two (non-normalized) 'classically distinguishable' results.

The state ψ' can, however, be written in an infinite number of ways as a sum of two non-normalized vectors. So the problem is this: What distinguishes this particular separation from all of the other possibilities?

In the Copenhagen interpretation this separation is defined by means of 'conditions specified by classical concepts.' These conditions are reasonably well-defined in terms of what human observers can see and do, but they are not precisely defined in terms that are either completely compatible with quantum theory itself, or are objective in the sense that they do not refer in any way to human observers.

Once the decomposition of ψ' into its classically distinguishable components $|\varphi_1>$ and $|\varphi_2>$ has been specified, then the quantum rules say that observed state will be either $|\varphi_1>$ or $|\varphi_2>$ and that the probability that it will be $|\varphi_1>$ is $<\varphi_1|\varphi_1>$, whereas the probability that it will be $|\varphi_2>$ is $<\varphi_2|\varphi_2>$.

The idea of the present model is to replace the 'observer' by an objective mechanism based on coherent states. This mechanism produces 'events' with probabilities specified by the quantum formalism.

Mechanism of event generation

The mechanism of event generation is constructed as follows. Each event i corresponds to the specification of the Schrödinger state vector ψ_i on a spacelike surface σ_i. The region lying between σ_{i-1} and σ_i is a cell i that is bounded in both time and space. The spatial extent of cell i defines a cavity that specifies the modes that participate in the temporal development of the state from σ_{i-1} to σ_i.

The operator $U(\sigma_i, \sigma_{i-1})$ that takes the quantum system from σ_{i-1} to σ_i can be written[4]:

$$U(\sigma_i,\sigma_{i-1}) = \sum_P U(P)F(P)$$

Here P represents a multi-particle Feynman classical path, and $U(P)$ is a unitary operator that creates from the vacuum the coherent state corresponding to the classical electromagnetic field radiated by the charged particles moving along the multi-particle set of classical paths P.

The summation over all Feynman paths P tends to wash out these coherent states, but if there is a large-scale collective motion of matter then some of the coherent states having characteristic distances similar to those of the collective modes should remain prominent.

Suppose that in the unitary development generated in cell j the coherent states in the modes $i\in(1, 2 \ldots n)$ remain prominent. Then the full Hilbert space can be separated into a product of two spaces, one, S_{j1} corresponding to the modes $i\in(1, 2 \ldots n)$, and the other, S_{j2} corresponding to both the rest of the electromagnetic field plus the matter fields.

The coherent states are an overcomplete set of states: any state in the subspace S_{j1} can be expressed as a linear combination of coherent states in S_{j1}. Thus the state $\psi_i' = U(\sigma_i, \sigma_{i-1})\, \psi_{i-1}$ can be expressed as $\sum_k |A_\kappa > |\varphi_\kappa>$ with all $|A_\kappa>$ in S_{i1}. The corresponding density matrix is $\rho_i' = |\psi_i' > \cdot < \psi_i'|$ where the dot is placed between a ket and a bra to indicate no summation.

The mechanism of event generation is represented as follows. Let $|A>$ represent a coherent state in S_{i1} (a Schrödinger state on σ_i). Then define:

$$\rho_i(A) = |A> \cdot <A|\rho_i'|A> \cdot <A|$$

The ith event is then represented by the following transformation:

$$\rho_i' \rightarrow \rho_i = \rho_i(A)/\mathrm{Tr}\rho_i(A)$$

The probability density for this event is:

$$P(A) = \mathrm{Tr}\rho_i(A)$$

This probability density is defined relative to the measure that appears in coherent state theory:

$$\prod_{i=1}^{n} \mathrm{d}(\mathrm{Re}\ a_i)\mathrm{d}(\mathrm{Im}\ a_i)/\pi$$

Relative to this measure the coherent states satisfy a completeness property:

$$I = |a><a| = \sum_a |a>\cdot<a|$$

where the sum over a means an integral over d(Re a) d(Im a)/π. This completeness property entails that:

$$\begin{aligned}\sum_A P(A) &= \sum_A \mathrm{Tr}\ |A>\cdot<A|\rho_i'|A>\cdot<A| \\ &= \sum_A \mathrm{tr}\ <A|\rho_i'|A> \\ &= \sum_A \mathrm{Tr}\ \rho_i'|A>\cdot<A| \\ &= \mathrm{Tr}\ \rho_i' = 1\end{aligned}$$

where Tr represents trace in the full space and tr represents trace in S_{i2}.

To see how this mechanism works in a traditional measurement situation consider the simple example:

$$\psi_i' = |A_1>|\varphi_1> + |A_2>|\varphi_2>$$

Here $|\varphi_1>$ and $|\varphi_2>$ correspond to the two possible results of the measurement discussed earlier, and $|A_1>$ and $|A_2>$ represent the coherent states generated by the interaction of these two states with the electromagnetic field.

The event-generation mechanism takes the normalized state ψ_i' to some normalized state $|A>|\varphi_A>$. The probability density $P(A)$ associated with $|A>|\varphi_A>$ is:

$$\begin{aligned}P(A) &= \mathrm{Tr}[|A>\cdot<A|(|A_1>|\varphi_1> + |A_2>|\varphi_2>)\cdot(<\varphi_1|<A_1| + \\ &\qquad\qquad <\varphi_2|<A_2|)|A>\cdot<A|] \\ &= \mathrm{tr}<A|(|A_1>|\varphi_1> + |A_2>|\varphi_2>)\cdot(<\varphi_1|<A_1| + <\varphi_2|<A_2|)|A> \\ &= <\varphi_1|\varphi_1> \exp - |A - A_1|^2 \\ &+ <\varphi_2|\varphi_2> \exp - |A - A_2|^2 \\ &+ <\varphi_1|\varphi_2> \exp[-\tfrac{1}{2}|A - A_1|^2 - \tfrac{1}{2}|A - A_2|^2 \\ &\qquad\qquad + i\,\mathrm{Im}(A_1^*A + A^*A_2)] \\ &+ <\varphi_2|\varphi_1> \exp[-\tfrac{1}{2}|A - A_1|^2 - \tfrac{1}{2}|A - A_2|^2 \\ &\qquad\qquad + i\,\mathrm{Im}(A_2^*A + A^*A_1)]\end{aligned}$$

If the two coherent states $|A_1>$ and $|A_2>$ are very different, so that $|A_1 - A_2|$ is very large, then the exponential factors in $P(A)$, and the triangle inequality, ensure that A will, with very high probability, lie very close to either A_1 or A_2. Furthermore, the total probability that $|A>|\varphi_A>$ will be approximately $|A_1>|\varphi_1>$ is $<\varphi_1|\varphi_1>$, and the total probability that $|A>|\varphi_A>$ will be approximately $|A_2>|\varphi_2>$ is $<\varphi_2|\varphi_2>$. Thus in this case the event-generation mechanism gives results that conform to the Copenhagen interpretation rules.

Note, however, that the mechanism produces a classical state $|A>|\varphi_A>$ also in the cases where $|A_1>$ and $|A_2>$ and $|\varphi_1>$ and $|\varphi_2>$ are not very different. And it gives the probability density $P(A)$ also in these more complex situations where the Copenhagen rules would not apply.

More generally, suppose that:

$$\psi_i' = \sum_k |A_k>|\varphi_k>$$

and that the set of $|A_k>$ can be separated into N subsets such that all of the $|A_k>$ in each subset are far away from all of the $|A_k>$ in each of the other subsets. This separation of the $|A_k>$ induces a separation:

$$\psi_i' = \sum_{j=1}^{N} \psi_j''$$

The event mechanism will cause the state ψ_i' to jump into some state ψ_i that is 'close' to one of the state ψ_j''. And the total probability that the state ψ_i' will jump to a state close to ψ_j'' is $<\psi_j''|\psi_j''>$. So the result is again compatible with the Copenhagen rules, but more general.

The essential point behind this mechanism is that, generally, linear combinations of coherent states are not coherent states. Consequently, for example, the second of the two following decompositions does not give states into which the quantum state ψ_i' can jump:

$$\begin{aligned}\psi_i' &= |A_1>|\varphi_1> + |A_2>|\varphi_2> \\ &= \tfrac{1}{2}[(|A_1> + |A_2>)(|\varphi_1> + |\varphi_2>) + (|A_1> - |A_2>)(|\varphi_1> - \\ &\qquad\qquad |\varphi_2>)]\end{aligned}$$

Thus the special role played by coherent states in the event-generation mechanism has the effect of specifying very special modes of decomposition of ψ_i' into its 'classically distinct' components.

To convert the properties described above into a complete theory one needs to specify the rules for determining (statistically at least) the placement of the surfaces σ_i. And one must specify the precise rule for identifying the subspace S_{i1} associated with σ_i. However, by introducing even arbitrary rules one generates at least a conceptual framework for replacing the human observers of the Copenhagen interpretation by an objective mechanism (based on light) that could give precision to the Bohm-Heisenberg idea of objective events as the foundation of classical reality.

In the specifications of the sequence of surfaces σ_i, and the subspaces

S_{j1}, it is important to recognize that the principles of the theory of relativity pertain to the general laws, and hence to descriptions of the *probabilities*, rather than the actualities: the actual situations do not possess the general symmetries. Thus one should not specify the sequence of surfaces σ_i independently of the developing actual situation; that would give preferences to certain space-time structures, independently of the actual. Rather, each σ_i should be specified by the prior actualities. Then there is no conflict with the general relativistic principle that different frames and coordinate systems are *intrinsically* equivalent.

This work was begun as a contribution to this volume honoring David Bohm. The deadline has now arrived and the task is unfinished. I hope, however, that even in its present rudimentary form the model described herein will serve to clarify and stimulate the thinking of readers of this volume about a subject that has filled a great part of the scientific life of David Bohm, and to which he has contributed immensely.

Acknowledgment

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the US Department of Energy under contract DE-ACO3-76SF00098.

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The automorphism group of C_4

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1 Introduction

One of the most pressing mathematical problems thrown up by the highly original work of David Bohm[1] is to construct numerous detailed examples that will help in the understanding of the important concepts of implicate and explicate order. Recently Bohm and Hiley[2] have themselves stressed the important role played by the Clifford algebras C_n here, since the automorphisms of the (even) Clifford algebras C_{2r} are all inner and 'any theory based on an algebra can always be put in an implicate order by an inner automorphism of the algebra'[3]. To make the notation precise, I am using C_n for the algebra generated by n anticommuting elements, which I usually denote by $E_i (i = 1 \ldots n)$, but in the case of quaternions, C_2, I use e_1, e_2, and set $e_3 = e_1 e_2$. Thus C_n has a basis of 2^n elements, including the unit, which I call 1. Unless explicitly stated otherwise, the algebra is over the field of reals, R.

The simplest example of this is C_2 which, since Hamilton's original paper[4], is known to fit three-dimensional space perfectly (as was intended) and in which the automorphism group $G_2 = \{T_q\}$:

$$T_q : v \longrightarrow v' = qvq^{-1}$$

is exactly $O^+(3)$, the proper orthogonal group. This fact is easily proved since, on the one hand, for any vector (i.e. 3-vector) v, v^2 is minus the square of the magnitude of v, and is evidently invariant under G_2 and, on the other, G_2 is evidently a 3-parameter group and so is the whole of $O^+(3)$. But it is to some extent a surprise, and another way of looking at quaternions makes it more natural. This