5. THE UNSEEN.

Quantum theory represents our knowledge about the unseen system being probed by means of a mathematical structure called the quantum state. This state normally evolves continuously in accordance with a deterministic law that is closely connected to the "laws of nature" used in classical physics. However, at certain instants this orderly progression is suddenly interrupted by an abrupt "quantum jump". Such a jump occurs each time one of the observers gains new knowledge: the jump brings the quantum state into concordance with the new state of our knowledge. Thus the quantum state of the system being examined represents always the evolving knowledge of the community of communicating observers.

But how can a mathematically state represent human knowledge?. Our knowledge seems to be an ephemeral and ineffable vagary, whereas the mathematically described states of quantum theory are precise structures that allow empirically observed numbers to be computed to an accuracy of one part in a hundred million.

To explain this connection I need to introduce two mathematical ideas: "Hilbert space", and "projection operator". These names may sound intimidating, but the ideas are basically simple, and understanding them will allow you to grasp the essence of quantum theory.

A Hilbert space is a collection of *vectors*, and a vector is a displacement by a specified amount in a specified direction. Two vectors, A and B, can be added together to give a vector C, which is formed by adding together the displacements A and B.

Consider, for example, the displacement from the corner of a room where two walls and the floor meet to a point on one of these two walls. That displacement is the sum of a vertical displacement up from the corner plus a horizontal displacement along the wall that contains the point.

If the two vectors A and B that add to give C are perpendicular to each other, as in this example, then the theorem of Pythagoras asserts that the square of the length of A plus the square of the length of B equals the square of the length of C.

This celebrated theorem is tied to the probability rules of quantum theory: If C is a vector of length one (i.e., unity) and A and B are two perpendicular vectors that sum to C, then the square of the length of A plus the square of the length of B is unity (i.e., one). The two perpendicular vectors will correspond to two *alternative possible outcomes of the probing action*, and the square of the length of A will be the probability for the event associated with A to occur, and the square of the length of B will be the probability for the event associated with B to occur. The sum of these two probabilities is unity by virtue of the theorem. This accords with the fact that the probabilities associated with alternative possibilities must sum to unity.

In the example of the point on the wall, the space of vectors is two-dimensional: any point on the wall can be reached from the corner by a sum of just two displacements, one in each of the two pre-specified perpendicular directions, vertical and horizontal. We can also easily visualize displacements in a three-dimensional space. But it is possible to consider mathematically an N-dimensional vector space in which there are exactly N mutually perpendicular directions, and each vector in the space is a sum of N vectors, one directed along each of these N directions. We allow null displacements and also negative displacements, which are the same as positive displacements in the reverse direction.

A set of N vectors, each perpendicular to every other one, is not easy to visualize, geometrically, for large N. But if one uses an algebraic approach in terms of sets of numbers, then the examples of vector spaces in one, two, and three dimensions are easily generalized to spaces of arbitrarily large but finite dimension N. With a little more effort one can even go to the case where N is infinite. Hilbert spaces include the infinite-N cases, but that is a technical matter that need not concern us here. It will be enough to think of simple cases where N in finite.

If a vector V is composed of a sum of N perpendicular vectors then a generalization of the theorem of Pythagoras shows that the square of the length of V is the sum of the squares of the lengths of these N mutually perpendicular vectors that add up to form V.

The second important concept is the idea of a projection operator. A *projection operator* P acts on a vector V to give a new vector PV. The action of P eliminates a specified subset of a set of perpendicular vectors that add up to give the vector V upon which it acts, but leaves unaffected the remaining vectors in the sum. Thus, for example, the vector V from the corner of a room to any point in the interior of the room would be converted by a certain projection operator P to the vector PV that is the displacement from the corner to the point on the floor that lies directly under that point in the room: the vertical vector is eliminated by the action of this particular projection operator P.

That example is a very special case. For one thing the three perpendicular vectors were very special, involving one vertical vector and two particular horizontal ones. But one can imagine replacing the room by a cubic box, and consider the infinity of ways that this box could be oriented relative to the room. For each of these orientations the three edges that meet at a corner define three perpendicular directions. Then one can go from N = 3 to arbitrary N, and select any subset of the set of N perpendicular directions to be the set that is not set to zero. This obviously gives a huge set of logically possible projection operators P.

For each projection operator P there is a unique complementary projection operator P' that does *not* set to zero exactly the subset of the N perpendicular vectors that *is* set to zero by P. Thus for any vector V, it is true that PV + P'V = V. The vectors PV and P'V are two perpendicular vectors that sum to V.

Given this simple idea of a vector, and how a vector in an N dimensional space can be considered to be a sum of a set of N vectors, each of which is perpendicular to every other one, we can now state the basic idea of quantum theory: Our knowledge about the unseen system, gleaned from earlier experience about things we *can* see, is represented, under certain ideal conditions, by a vector V of unit length. This vector evolves under the action of a rule called "the Schroedinger equation", which alters the direction that V points, but leaves its length unchanged.

When the outcome of a probing action appears the vector V suddenly jumps to the vector PV or to P'V, where P is the projection operator associated with the probing action, and

P' is the complementary projection operator. The probability of V jumping to PV is the square of the length of PV and the probability that the jump will be to P'V is the square of the length of P'V. These two probabilities add to unity, by virtue of the theorem of Pythagoras. This property matches the property of probabilities that their sum over any set of alternative possibilities must be unity.

The essential point here is that our knowledge of the unseen system being probed can, according to quantum theory, be associated with a vector V in a Hilbert space, and this association gives simple rules for the probabilities for the alternative possible outcomes of our probing action to appear, once the form of the projection operator P is known.

With this general picture in mind we can now return to the question of how our knowledge is represented mathematically.

According to quantum theory the polarization of a photon is represented by a vector V in the two dimensional space that is perpendicular to the photon's line of flight. Suppose a photon is allowed to fall on a crystal that splits the beam so that the part polarized along direction A1 is deflected to a photon detector D1 and the part polarized in the direction A2, perpendicular to A1, is deflected to a photon detector D2. If the detectors are 100% efficient then one or the other of the two detectors will fire, but not both.

In this example the probing action is associated with the projection operator P such that the vector PV is directed along A1 and P'V is directed along A2. The vectors PV and P'V represent the *alternative* possible outcomes of the probing action. If the observer sees detector D1 fire that he knows that the system being probed is in state PV; if he sees detector D2 fire then he knows that the system being probed is in the state P'V. Thus quite accurate information about the new state of the unseen system can be gleaned from the empirically discernible fact of whether D1 fires or D2. Discarding the part of the state V=PV +P'V that is incompatible with the empirical fact that D1 fires and D2 does not, or vice versa, is non problematic.

I specified at the beginning that V represented of *our knowledge* of the system being probed. Thus there is no problem with the fact that V suddenly changes when an observer acquires new knowledge by seeing one of the two the detectors, D1 or D2, fire and the other one not fire. However, this facile way of speaking glosses over some deep problems. This vector V seems to be connected more closely to the state of the photon itself than to human consciousness. The very fact that the photon could be represented by a vector, and that this vector should evolve normally in accordance with the Schroedinger equation was a consequence of incorporating Planck's constant into the equations of classical physics. That quantization procedure converted the old classical-type of reality into the new cloud-like (or vector-type) replacement. This transformation seems to be an objective change, not related specifically to human consciousness. Moreover, the stability of matter itself, and the formation of the elements are all understood in terms of these quantum equations of motion. All of this structure and process predates human existence. The original Copenhagen pragmatic way of understanding the quantum mathematics, while tremendously useful as a stepping stone, closes the door to any real understanding of the reality that replaces the one that empirical phenomena has ruled out.