An Equivalence Theorem

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The proof of an equivalence theorem was published in Epistemological Letters of June 1978, and delivered at a colloquium on Bell's theorem held March 3-4, 1978, attended by J.S. Bell, A. Shimony, B. D'Espagnat, and myself. It pertains to the need for faster-than-light effects in a physical world in which some simple predictions of quantum theory are valid. The following text makes electronically available a photo-copy of the parts of that article pertaining to this proof. Section one introduces a notional framework for a typical Bell's theorem experiment. A sequence of n pairs of particles is considered. All pairs are prepared in the same quantum state ψ. The two particles of each pair are directed to two different space-time regions that are space-like separated from each other. In each region an experimenter freely chooses, and then performs, one or the other of two alternative possible measurement procedures on the sequence of n particle entering his region. Each individual measurement has two alternative possible outcomes. The theoretical notion that the information about which measurement is *freely chosen* in a region cannot get to the other region is expressed by the following "local causes" condition: for both regions, and for each of the alternative possible experiments there, the outcome that appears must be independent of which experiment is freely chosen in the other region. It had previously been proved that, for certain quantum states ψ, it is mathematically impossible to satisfy both this locality condition and the predictions of quantum theory pertaining to this state to an accuracy of, say, 3%, for n larger than some large number N. The equivalence theorem links this "local causes theorem", which is expressed directly in terms of possible *observable* variables, to certain "local hidden-variable theorems". These latter theorems, known as Bell's Theorems, presume the existence of an underlying structure involving "hidden variables", and the theorems specify detailed mathematical structures related to these variables. The associated theorems can be either local *deterministic* or local *probabilistic* hidden-variable theorems. The latter appear to be more general, but it is shown in section two, by explicit construction, that both are mathematically equivalent to the local causes theorem, which is not restricted by any conditions associated with unobservable (microscopic) "hidden variables".

Specifically, the local probabilistic hidden-variable theories, which encompass the local deterministic hidden-variable theories as a special case, postulate the existence of two probability functions $P_1(r_1, m_1, \lambda)$ and $P_2(r_2, m_2, \lambda)$, and a weight function $\rho(\lambda)$, where r and m specify "result" and "measurement type", respectively, λ specifies the hidden variable, and the subscript 1 or 2 specifies the space-time region in which the measurement is freely chosen and performed, and the result/outcome then appears. The joint probability function is required (by locality) to be the product of the two separate probability functions. The Equivalence Theorem asserts that all expectation values of quantities depending on (r_1, m_1, r_2, m_2) that arise from such a hidden-variable probability structure can be reproduces to arbitrarily fine accuracy e%, by taking average values over a set of possible outcomes satisfying the local causes condition, for n greater than some sufficiently large N(e). Conversely, any expectation values entailed by a structure satisfying the "local causes" condition can be reproduced by (an exemplar of) some local probabilistic hidden-variable theory, and, indeed, by a local deterministic hidden-variable theory. Thus, to prove the theorem that no local probabilistic hidden-variable theory can reproduce the predictions of quantum theory, it is enough to prove that no theory that enforces the notion of no-faster-than-light transfer of information (about the freely made choices) by invoking the "local causes" condition can agree with the predictions of quantum theory. Consequently, the presumption of the existence of the specific mathematical machinery associated with the idea of "local hidden variables" is an unnecessary condition: it can be replaced with the "local causes", condition, which is formulated directly in terms of *observable* variables, rather than *hidden* variables.

The background trial presumptions are that: 1), several pertinent predictions of quantum theory are valid to within 3%; 2) what appears in a region under either of the conditions specified there cannot depend on which experiment is freely chosen and performed *later* in some frame; and 3), in view of the fact that there are myriads of ways in which the experimenters can arrange to have the experimental devices put in place, with no affect on the predictions of quantum theory, we can, within the present context, treat the two choices of the bivalent experimental arrangements as two independent free variables.

The principal conclusion is that these three conditions cannot all be satisfied!

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H.P.Stapp - Non Local Character of Quantum Theory

Abstract

The collapse of the wave function in quantum theory is a manifestly nonlocal change. However, wave functions represent probabilities, and probabilities can change everywhere the instant new information is obtained, even in a theory that is completely local. Thus it is conceivable that the collapse of the wave function in quantum theory is not associated with any genuine nonlocal aspect of the theory.

However, the following nonlocality property can be proved: There are in principle experimental situations involving two space-like-separated regions such that the results obtained in one region must depend on the value of a (dichotomic) variable randomly chosen in the other region: The set of all conceivable results in the four alternative experimental situations corresponding to the two alternative possible settings of the two dichotomic variables contains not even a single set of four conceivable sets of results, one set for each of the four alternative experimental situations, such that the predictions of quantum theory are satisfied to within 3% in all four cases, and the results in each region are independent of the variables randomly chosen in the other region. This nonlocality property of quantum theory entails the result of Bell that no local hidden variable theory can agree with quantum theory, but it is more general because it does not depend on the notion of hidden variables: it is expressed, rather, completely in terms of the observables of quantum theory.

Introduction

The question of the effect of a measurement made in one space-time region upon experimental results occurring in a space-like-separated region was raised already in the famous paper of Einstein, Podolsky and Rosen (1). These authors assumed that no such effect occurs, and Bohr apparently concurred (2): "of course there is in a case like that just considered no question of a mechanical disturbance

of the system under investigation during the last final stages of the measuring procedure". Heisenberg stated, in connection with the closely related question of the nature of the collapse of the wave function (3): "when the old adage 'Natura non facit saltus' is used as a basis for criticism of quantum theory, we can reply that certainly our knowledge can change suddenly and that this fact justifies the use of the term 'quantum jump'". Thus Heisenberg also appears to reject the idea that the collapse of the wave function is associated with any sudden effect of the measurement made in one region upon the results occurring in a space-like-separated region, apart from the familiar dependence of probabilities upon our knowledge.

This presumption that no direct nonlocal physical influence occurs has recently been brought into question by Bell's (4) discovery that a certain nonlocality property is entailed by the assumed validity of the statistical predictions of quantum theory. Bell stated his result in the context of local hidden variable theories. However, it is possible to formulate the result in a broader context that refers only to macroscopic observables, not to hidden variables. This more general formulation of the nonlocality property is described in section one, where it is also compared to Bell's result that no deterministic local hidden variable theory can agree with quantum theory.

Clauser and Horne (5) and Bell (6) have extended Bell's original result to probabilistic local hidden variable theories. In section two it is shown that the general nonlocality property stated in section one entails also the results of Clauser and Horne, and of Bell, that no probabilistic local hidden variable theory can agree with quantum theory.

1. The Nonlocality Property

Consider a situation in which a large number of two-particle collisions take place in a space-time region R_o and in which it is determined by coincidence techniques that a sequence of N pairs of particles has emerged from R_o, that one particle of each pair is going to pass through an apparatus in a space-time region R_1, and that the other particle is going to pass through an apparatus in a space-time region R_2, where R_1 and R_2 are situated space-like relative to each other. Each particle will pass through a Stern-Gerlach device in the appropriate region and be detected in one of two counting devices, assumed 100% efficient. The result in R_1 associated with the i-th pair will be represented by $r_{1i} = +1$ or $r_{1i} = -1$ depending on which of the two counting devices registers, and the result in R_2 will similarly be represented by $r_{2i} = +1$ or $r_{2i} = -1$. The direction of the axis of deflection in R_1 is controlled by a computer in R_1, and it is set at one of two positions a' or a" just before the arrival of the sequence of N particles. Similarly the direction of the axis in R_2 is set at one of two positions b' or b", just before the arrival of the sequence of N particles.

The choice between a' and a" is determined by a number picked from a table of random numbers. The position of the number in this table is determined by a set of irrationally chosen and apparently irrelevant numbers associated with far-away physical systems. The choice between b' and b" is determined in a similar fashion.

We now form tables of conceivable results in the four alternative cases. Each table has a set of conceivable results for each of the four alternative experimental situations. Thus each table has eight columns of N rows each, arranged as follows:

	a'	b'	a'	b''	a''	b'	a''	b''
	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
i = 1	+1	+1	+1	+1	-1	+1	-1	+1
2	+1	-1	-	-	-	-	-	-
3	-1	-1	-	-	-	-	-	-
	-1	-	-	-	-	-	-	-
	-	-	-		-	-	-	-
	-				-			
	-							
i = N	+1							

Fig. 1 Table of Conceivable Results for the Four Alternative Experimental Situations.

Each entry is either plus one or minus one. Thus there are 2^{8N} possible tables. These will be labelled by the index I. Thus I represents a set of conceivable individual results for each of the four alternative experimental situations:

$$I \equiv \left\{ (r_{1i}(a,b,I),\ r_{2i}(a,b,I));\quad i \in (1,\ldots,N),\quad a \in (a',a'') \quad b \in (b',b'') \right\}$$

The number N is supposed to be large enough so that almost all sets of results that occur in nature are predicted by quantum theory to conform to the averages predicted by quantum theory to within 3%.

The set Q is the set of I such that the averages

$$\langle r_1 \rangle_{(a,b,I)} \equiv \frac{1}{N} \sum r_{1i}(a,b,I)$$

$$\langle r_2 \rangle_{(a,b,I)} \equiv \frac{1}{N} \sum r_{2i}(a,b,I)$$

$$\langle r_1 \, r_2 \rangle_{(a,b,I)} \equiv \frac{1}{N} \sum r_{1i}(a,b,I) \; r_{2i}(a,b,I)$$

conform to the predictions of quantum theory to within 3% for all four values of (a,b).

In the actual world one particular value of the pair (a,b) is realized, and the other three pairs are not realized. However, quantum theory gives predictions for all four possible values of the pair (a,b): it deals conceptually with the various alternative possibilities on an equal footing. Our intuitive idea of locality says that it should be possible, in some theoretical realm of possible worlds, to change the value of the parameter a in R_1 without disturbing the results in R_2, and to change the value of the parameter b in R_2 without disturbing the results in R_1. Thus we define L to be the subset of the set of all conceivable sets of results I in which the results in R_1 are independent of b and the results in R_2 are independent of a.

That is, L is defined to be the set of I such that for all i

$$r_{1i}\,(a',b',I) \;=\; r_{1i}\,(a',b'',I)$$

$$r_{1i}\,(a'',b',I) \;=\; r_{1i}\,(a'',b'',I)$$

$$r_{2i}\,(a',b',I) \;=\; r_{2i}\,(a'',b',I)$$

and

$$r_{2i}\,(a',b'',I) \;=\; r_{2i}\,(a'',b'',I)\;.$$

We may now state the basic mathematical result:

<u>Theorem I</u> (Nonlocality Property)

There is no I such that I belongs to both L and Q:

$$L \cap Q = \emptyset\,.$$

This result asserts that there is not even a single set of conceivable results for the four al ternative experimental situations such that the predictions of quantum theory are satisfied in al cases and the results in each region are independent of the parameter randomly chosen in the spac like-separated region. This result has been prove in references (7) and (8).

This result states that there is no conceivabl way to independently change the parameters a and in such a way that the results in R_1 are independent of b and the results in R_2 are independent o a, without violating the predictions of quantum theory.

We now compare this result to the result obtained by Bell for deterministic local hidden variable theories. These theories are those in which i is possible to find some set of (hidden) variable λ such that a, b, and λ can be regarded as independent variables and the following locality property is satisfied:

for all i and λ

$$r_{1i}(a',b',\lambda) = r_{1i}(a',b'',\lambda)$$

$$r_{1i}(a'',b',\lambda) = r_{1i}(a'',b'',\lambda)$$

$$r_{2i}(a',b',\lambda) = r_{2i}(a'',b',\lambda)$$

and

$$r_{2i}(a',b'',\lambda) = r_{2i}(a'',b'',\lambda) .$$

We first observe that if one identifies λ with I then these conditions are the same as those that define L. Thus if one takes the hidden variables) to be the sets of variables $I \in L$ then the formal requirement on the hidden variables are satisfied. Our theorem says immediately, in this special case that there is no choice of the hidden variable

$\lambda \equiv I \in L$ that satisfies $\lambda \equiv I \in Q$:

the quantum prediction cannot be satisfied.

More generally, local deterministic hidden-variable theories can be treated as special cases of local probabilistic hidden-variable theories.

2. Connection to Probabilistic Local Hidden Variable Theories

Clauser and Horne (5) and Bell (6) have considered local hidden variable theories in which the local hidden variables (a,b,λ) determine not the individual results themselves but only the probabilities of those results. In principle this sort of theory is no more general than a deterministic local hidden variable theory, since by the addition of stochastic variables one can convert the probabilistic theory to a deterministic one. However, Clauser and Horne and Bell have extracted their locality condition for the probabilistic theories not directly from the individual event locality condition but rather from certain models that are considered to characterize probabilistic local hidden variable theories. The model of Clauser and Horne differs from that of Bell, which is much more general, but the final probabilistic locality condition is the same. Thus the question arises: what is the connection between the probabilistic locality condition of Clauser and Horne and Bell and the individual-event locality condition of section one ? It is shown in this section that these two conditions are essentially equivalent, and that consequently theorem I of section one implies that no deterministic or probabilistic local hidden variable theory can agree with quantum theory.

The work of Clauser and Horne (5) concerns "objective local theories". These are theories in which the two-particle system at a given time can be characterized by a "state" λ, and there is a relationship

$$p_{12}(\lambda,a,b) = p_1(\lambda,a)\ p_2(\lambda,b), \tag{2.1}$$

where $p_1(\lambda,a)$ and $p_2(\lambda,b)$ are the probabilities of

certains counts in R_1 and R_2, respectively, if the state at a certain time is λ, and where $p_{12}(\lambda,a,b)$ is the probability of coincident counts in both R_1 and R_2. These probabilities depend on the parameters a and b.

One considers ensembles of systems all characterized by the same value of λ in order to relate probabilities to averages over individual results. For each state λ one considers alternative possible values of the parameters a and b. That is, the theory treats a and b as free variables.

The motivation for the factorized form (2.1) of the probability is that this form emerges if one considers a classical picture in which the two-particle system consists of two localized objects.

The requirement that the two-particle system should be characterized by such a time-dependent state λ is very restrictive. Quantum theory suggests that particles are intimately associated with the devices that produce and detect them. Thus it may not be possible in a fundamental theory, to represent particles as separate and distinct time-dependent entities. Also the results in R_λ might depend on things located throughout the backward light-cone of R_λ, rather than on a state that represents things at a particular time.

Bell (6) considers a much more general type of local theory in which the probability of any result is <u>completely</u> determined by all real things that lie in the backward light-cone of the region in which the result appears. From this condition he derives a locality condition

$$(2.2)\quad \left\{A,B \,\middle|\, \Lambda,M,N\right\} = \left\{A \,\middle|\, \Lambda,N\right\} \left\{B \,\middle|\, M,N\right\}$$

This locality condition, like that of Clauser and Horne, says that for fixed hidden variable the probability of result A <u>and</u> B (together) is the <u>product of the probabilities of each result separately</u>. The quantities Λ, M, N represent the real things (called local beables) in the regions indi-

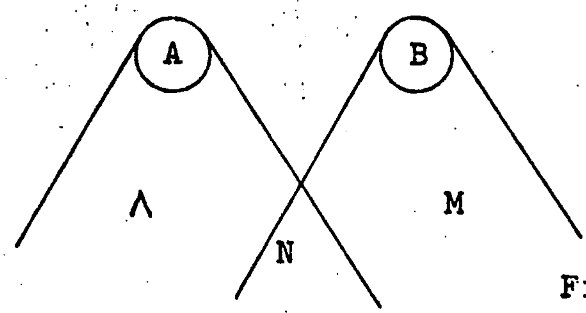

Fig. 2

The probabilities referred to by the above equation are the probabilities associated with a single instance i, or, in any case, with a set of instances i for which the probabilities are all identical.

Bell then considers situations in which the parameters a and b are changed, for the same instances i Thus he also considers alternative situations. Later, as emphasized by Shimony, Horne and Clauser (9), Bell (10) and Shimony (11), Bell makes the essential assumption that the parameters a and b can be regarded as effectively free, i.e., as independent variables.

Both models discussed above deal only with the probabilities, not with the individual results themselves. Consequently the locality property is expressed as a condition on the probabilities, namely that the probabilities can be expressed as a certain sum of products of two factors, one depending on r_1 and a, and the other depending on r_2 and b, rather than as the condition that the individual results in each region be independent of the parameter chosen in the other region. However, these two forms of the locality condition are equivalent, apart from terms that tend to zero as N tends to infinity. In particular, if the individual results in each region satisfy the locality properties

$$r_{1i}\,(a,b) = r_{1i}\,(a) \quad \text{and} \quad r_{2i}\,(a,b) = r_{2i}\,(b)$$

then the probabilities

$$(2.3)\quad \left\{r_1, r_2 \middle| a, b\right\}_{av} \equiv \frac{1}{N} \sum_i \theta_{1i}(a, r_1) \quad \theta_{2i}(b, r_2)$$

$$(2.4)\quad \left\{r_1 \middle| a\right\}_{av} \equiv \frac{1}{N} \sum_i \theta_{1i}\ (a, r_1)$$

and

$$(2.5)\quad \left\{r_2 \middle| b\right\}_{av} \equiv \frac{1}{N} \sum_i \theta_{2i}\ (b, r_2)$$

where

$$(2.6)\quad \theta_{1i}(a, r_1) = \begin{cases} 1 & \text{if } r_{1i}(a) = r_1 \\ 0 & \text{if } r_{1i}(a) \neq r_1 \end{cases}$$

and

$$(2.7)\quad \theta_{2i}(b, r_2) = \begin{cases} 1 & \text{if } r_{2i}(b) = r_2 \\ 0 & \text{if } r_{2i}(b) \neq r_2 \end{cases}$$

can be written in the forms

$$(2.8)\quad \left\{r_1, r_2 \middle| a, b\right\} = \sum_\lambda \rho(\lambda)\ p_1(\lambda, a, r_1)\ p_2(\lambda, b, r_2)$$

$$(2.9)\quad \left\{r_1 \middle| a\right\} = \sum_\lambda \rho(\lambda)\ p_1\ (\lambda, a, r_1)$$

$$(2.10)\quad \left\{r_2 \middle| b\right\} = \sum_\lambda \rho(\lambda)\ p_2\ (\lambda, b, r_2)$$

~~where the range of λ is finite and~~

$$(2.11)\quad \sum_\lambda \rho(\lambda) = 1$$

$$(2.12)\quad \sum_{r_1 = \pm 1} p_1\ (\lambda, a, r_1) = 1 \qquad (\text{all } \lambda,\ a)$$

$$(2.13)\quad \sum_{r_2 = \pm 1} p_2\ (\lambda, b, r_2) = 1 \qquad (\text{all } \lambda,\ b)$$

and

(2.14) $\rho = |\rho|$, $p_1 = |p_1|$, $p_2 = |p_2|$

To see this one identifies λ with i, sets $p_1(\lambda,a,r_1) = \theta_{1i}(a,r_1)$ and $p_2(\lambda,b,r_2) = \theta_{2i}(b,r_2)$ and notes that each $r_{1i}(a)$ and $r_{2i}(b)$ is either plus one or minus one.

Conversely, any probabilities that can be expressed in the form (2.8) - (2.14) can be expressed also in the form (2.3) - (2.7), up to terms that tend to zero as N tends to infinity.

To see this introduce a set of four signs

$$\sigma \equiv (\sigma_{a'}, \sigma_{a''}, \sigma_{b'}, \sigma_{b''})$$
$$\equiv \{\sigma_\nu\} \qquad \sigma_\nu = \pm 1$$

where ν runs over the four parameter values a', a", b', and b". The four probability functions $p_1(\lambda,a,r)$ and $p_2(\lambda,b,r)$ can be expressed in a symmetrical form as $p(\lambda,\nu,r)$. Then properties (2.12) and (2.13) give

(2.15) $$\sum_{r=\pm 1} p(\lambda,\nu,r) = 1 \qquad (\text{all } \lambda,\nu).$$

The set of four signs σ has sixteen possible values. Sixteen weight functions $w(\lambda,\sigma)$ are defined by

(2.16) $$w(\lambda,\sigma) = \rho(\lambda) \prod_\nu p(\lambda,\nu,\sigma_\nu).$$

Note that

(2.17) $$\sum_{\lambda,\sigma} w(\lambda,\sigma) = 1$$

by virtue of (2.11) and (2.15).

Take a large number N of indices i and apportion them among cells $C_{\lambda\sigma}$ in accordance with the weights $w(\lambda,\sigma)$. For finite N there may be discrepancies but these will tend to zero as N tends to infinity.

Let $\sigma(i) \equiv (\sigma_{a'}(i), \sigma_{a''}(i), \sigma_{b'}(i), \sigma_{b''}(i))$ and $\lambda(i)$ label the cell in which index i is placed. Then

$$(2.18) \qquad w(\lambda, \sigma) \simeq \frac{N(\lambda, \sigma)}{N}$$

where

$$(2.19) \qquad N(\lambda, \sigma) \equiv (\#\text{ of } i\text{'s in which } \lambda(i) = \lambda \text{ and } \sigma(i) = \sigma)$$

$$= (\#\text{ of } i\text{'s in which } \lambda(i) = \lambda \text{ and } \sigma_\nu(i) = \sigma_\nu \text{ for all } \nu).$$

We now specify the results corresponding to i in all four alternative cases (a,b) by the equations

$$(2.20) \qquad r_{1i}(a,b) = \sigma_a(i)$$

$$(2.21) \qquad r_{2i}(a,b) = \sigma_b(i)$$

Then

$$(2.22) \qquad \left\{r_1 \middle| a'\right\}_{av} \equiv \frac{1}{N} (\#\text{ of } i\text{'s such that } r_{1i}(a') = r_1)$$

$$= \frac{1}{N} (\#\text{ of } i\text{'s such that } \sigma_{a'}(i) = r_1)$$

$$= \frac{1}{N} \sum_{\{\lambda, \sigma;\, \sigma_{a'} = r_1\}} N(\lambda, \sigma)$$

$$\approx \sum_{\{\lambda, \sigma;\, \sigma_{a'} = r_1\}} w(\lambda, \sigma)$$

$$= \sum_{\lambda} \rho(\lambda) \sum_{\{\sigma;\, \sigma_{a'} = r_1\}} \prod_{\nu} p(\lambda, \nu, \sigma_\nu)$$

(# means "number")

$$= \sum_{\lambda} \rho(\lambda)\, p(\lambda, a', r_1) = \{r_1 \mid a'\}$$

as required. The similar equations with a' replaced by the other values of ν follow in the same way.

Similarly

$$(2.23)\quad \{r_1, r_2 \mid a', b'\}_{av}$$

$$\equiv \frac{1}{N} \left\{ \begin{array}{l} \#\text{ of } i\text{'s such that} \\ r_{1i}(a',b') = r_1 \text{ and} \\ r_{2i}(a',b') = r_2 \end{array} \right\}$$

$$= \frac{1}{N} \sum_{\{\lambda, \sigma;\ \sigma_{a'} = r_1,\ \sigma_{b'} = r_2\}} N(\lambda, \sigma)$$

$$\simeq \sum_{\{\lambda, \sigma;\ \sigma_{a'} = r_1,\ \sigma_{b'} = r_2\}} w(\lambda, \sigma)$$

$$= \sum_{\lambda} \rho(\lambda) \sum_{\{\sigma;\ \sigma_{a'} = r_1 \text{ and } \sigma_{b'} = r_2\}} \prod_{\nu} p(\lambda, \nu, \sigma_\nu)$$

$$= \sum_{\lambda} \rho(\lambda)\ p_1(\lambda, a', r_1)\, p_2(\lambda, b', r_2)$$

$$= \{r_1, r_2 \mid a', b'\}$$

as required.

The result just proved can be summarized as follows:

Let $P \equiv (\{r_1|a\}, \{r_2|b\}, \{r_1, r_2 \mid a,b\})$ represent any set of probabilities, for the four va-

lues of (a,b). Let L_P be the set of P that satisfy the probabilistic locality conditions (2.8) - (2.14). Let P(I) be the set of values P obtained by averaging over the sets of individual results in I.

Let $\cong$ mean equal to within any preassigned difference $\varepsilon > 0$. Then we have the following
<u>Theorem II</u> (Equivalence Theorem)

a) If $I \in L$ then $P(I) \in L_P$

b) If $P \in L_P$ then there is an
$I \in L$ such that $P(I) \cong P$.

Let Q_P be the set of P that conform to the predictions of quantum theory to within 3%. Then Theorem I can be expressed in the form

<u>Theorem I'</u> (Nonlocality Property)

There is no I such that $I \in L$ and $P(I) \in Q_P$.

These two theorems entail Theorem III (Nonlocality Property - P form)

$$L_P \cap Q_P = \emptyset .$$

That is, there is no set of probabilities P that satisfy the probabilistic locality condition and the predictions of quantum theory to within 3% (i.e., to < 3%).

<u>Proof</u>. Suppose there were a P such that $P \in L_P$ and $P \in Q_P$. Since $P \in L_P$ we conclude from theorem II(b) that there is an $I \in L$ such that $P(I) \cong P$. Since P is in Q_P, the P(I) will be within Q_P, if we chose ε appropriately. Thus there must be an I such that $I \in L$ and $P(I) \in Q_P$. However, this possibility is excluded by theorem I'. Thus there is no

P such that $P \in L_P$ and $P \in Q_P$:

$$L_P \cap Q_P = \emptyset \qquad \text{Q.E.D.}$$

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