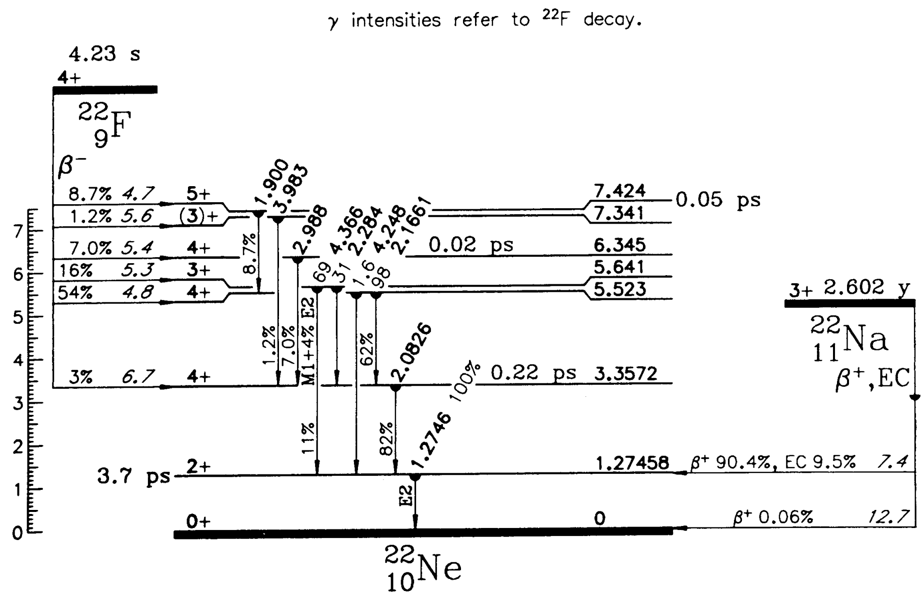
Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 1: due on Tuesday, 26-Jan-99 at begin of lecture.

Solutions:

1. ^{22}Na is commonly used as a calibration source. The level scheme from the *Table of Isotopes* is shown below.



a) Sketch the emission spectrum, showing the relative intensities.

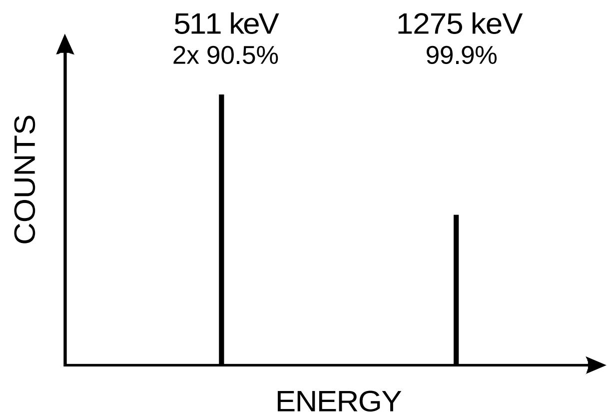
^{22}Na decays by both positron emission and electron capture to ^{22}Ne. Of all decays, 99.9% go to the first excited state of ^{22}Ne, which decays with a lifetime of 3.7 ps to the ground state under emission of a 1275 keV photon.

Higher energy states are populated in the decay of ^{22}F, but since we have a ^{22}Na source, none of these will appear.

A positron is emitted in 90.46% of all ^{22}Na decays. When it interacts with material a pair of 511 keV annihilation photons will be emitted. If the source is large enough (order several mm), nearly all positrons will interact in the source, so each decay yields two 511 keV photons.

The ^{22}Ne daughter atoms will also emit x-rays, but these are low energy, < 1 keV, so they tend to be absorbed in the source.

Emission spectrum in 4π



b) Assuming a source strength of 0.1 μCi, what are the counting rates of the individual spectral lines in a detector with 5 cm diameter placed 50 and 100 cm from the source? Assume an ideal detector with 100% efficiency.

1 Curie corresponds to 3.7×10^{10} decays per second, so a source of 0.1 μCi has a decay rate $n_0 = 3.7 \times 10^3\ s^{-1}$.

Assume a detector of diameter d placed at radius r large enough that the distance from the source doesn't change significantly over the diameter of the detector. The detector will intercept the rate of decays

$$n(r) = n_0 \frac{A_{\text{det}}}{4\pi r^2} = n_0 \frac{\pi (d/2)^2}{4\pi r^2} = n_0 \left(\frac{d}{4r} \right)^2$$

For r = 50 cm, n = 2.31 s^{-1}. At r = 100 cm the rate drops by ¼ to 0.58 s^{-1}. Since 99.9% of decays yield a 1275 keV line, this is the rate of this line. Since the detector can intercept only one of the two annihilation gammas, a 511 keV line of nearly equal intensity to the 1275 keV line will be seen.

The rates in the 511 and 1275 keV lines will be 90.5% and 99.9% of the total rate, i.e. 2.1 and 2.3 s^{-1} at 50 cm and 0.52 and 0.58 s^{-1} at 100 cm.

2. A detector system with a 10-bit digitizer is calibrated with ^{60}Co and ^{137}Cs sources. The ^{60}Co lines at 1173 and 1332 keV are centered at channels 797 and 906 (the numbering starts with 0). The 662 keV line from the Cs source is at channel 447.

   a) Determine the calibration function that relates channels to energy. What are the minimum and maximum energies that can be measured without readjusting the system?

   The desired calibration function is of the form

$$E = E_0 + kN$$

   The measured energies E and channel numbers N are

662 keV	447
1173 keV	797
1332 keV	906

   The most precise technique is to apply a straight line fit to the data, but a simpler "back of the envelope" technique can be applied to get quick results in the lab.

   The slope k in keV/ch for the three possible combinations is

$$(1173 - 662) / (797 - 447) = 1.46$$
$$(1332 - 1173) / (906 - 797) = 1.46$$
$$(1332 - 662) / (906 - 447) = 1.46$$

   Typically, the three results will vary slightly, so one would take the average. Insertion of any of the peak positions into the above equation yields E_0= 8.76 keV, so

$$E = 9.52 + 1.46N \text{ [keV]}$$

   The channel boundaries of the system are N= 0 and N= 1023 (10 bits correspond to 2^{10}= 1024), so the measurement range is 9.52 to 1503 keV.

   b) Where would the peaks from a ^{22}Na source appear?

   The 511 keV line is at channel 343 and 1275 keV is at 867.

   c) A second system shows the ^{60}Co and ^{137}Cs lines at channels 869, 677 and 299. What do you conclude from this result?

   In this case the slope k in keV/ch for pairs of adjacent energies is

$$(1173 - 662) / (677 - 299) = 1.35$$
$$(1332 - 1173) / (869 - 677) = 0.83$$

Obviously, the slope is decreasing with increasing energy; the system is non-linear. Nevertheless, it is still usable, but more calibration points - especially at lower energies - are needed to obtain a reliable calibration.

Note: Simply applying a straight line fit would be wrong, yielding large errors at low energies. On the other hand, one can easily find a function that will provide an excellent fit to the three data points, but it is not at all guaranteed that a function that works well over the upper half of the range will be as good in the lower half .

3. The LBNL supernova search group has determined that in a ground-based 4 m telescope their system has a sensitivity of 1 photon/s for a magnitude 26.7 object. The magnitude describes the faintness of a star and is defined as

$$\Delta magnitude = -2.5 \cdot {}_{10}\log \frac{\Phi_1}{\Phi_2},$$

i.e. the magnitude increases with decreasing photon flux Φ. Five units of magnitude correspond to a flux ratio of 100.

a) Humans can recognize stars of magnitude 5 to 6 with the naked eye (assume 6 for young folks). What is the photon flux captured by the eye?

The photon flux for a magnitude 6 object in a 4 m telescope is

$$\Phi_1 = \Phi_2 \cdot 10^{(M_1 - M_2/(-2.5))} = 10^{(6-26.7)/(-2.5)} = 1.91 \cdot 10^8 \text{ s}^{-1}$$

The flux captured by the eye is smaller by the ratio of the capture areas,

$$\Phi_{eye} = \Phi_1 \cdot \frac{A_{eye}}{A_{telescope}} = \Phi_1 \cdot \left(\frac{d_{eye}}{d_{telescope}} \right)^2$$

Assume that the diameter of the eye is 4 mm. Then the flux captured by the eye is 10^{-6} smaller than in the 4 m telescope, so Φ_{eye}= 191 s^{-1}.

b) The eye + brain system integrates over about 0.1 s. What is the statistical fluctuation of photons sensed during this integration time?

In 0.1 s the eye will register 19 photons, with a statistical fluctuation of

$$\sigma_{\Phi} = \sqrt{\Phi_{eye}} = 4$$