Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 6: Due on Tuesday, 9-Mar-99 at begin of lecture.
Discussion on Wednesday, 10-Mar-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. The signal at the input of a voltage sensitive amplifier is a 10 mV pulse with a rise time of 10 ns (10-90%). The equivalent input noise of the amplifier is 10 μV rms. The amplifier feeds a simple threshold comparator.

   a) Assume a comparator threshold of 5 mV. What is the time resolution?

   The time resolution

$$\sigma_t = \frac{\sigma_n}{dV/dt}$$

   The noise level σ_n = 10 mV and the rate of change

$$\frac{dV}{dt} = \frac{\Delta V}{t_c} \approx \frac{\Delta V}{1.25 \cdot t_r} = \frac{10 \cdot 10^{-3}}{12.5 \cdot 10^{-9}} = 8 \cdot 10^5 \text{ V/s}$$

   so

$$\sigma_t = \frac{10 \cdot 10^{-6}}{8 \cdot 10^5} = 12.5\,\text{ps}$$

   b) Still keeping the threshold at 5 mV, how much does the output of the comparator shift when the signal changes from 10 mV to 100 mV?

$$t(V_T) = \frac{V_T}{V_s} t_c$$

   For the 10 mV signal the threshold of 5 mV is at 50% of the rise time, so the comparator fires at 5 ns, whereas for the 100 mV signal the threshold is at 5% of the rise time, so the comparator fires at 0.5 ns. The time shift is 4.5 ns.

2. A timing system utilizes a silicon detector with an area of 100 mm^2 and 50 μm thickness. Alpha particles of 5 MeV impinge on the *p*-side of the detector. The detector is asymmetrically doped with a thin *p*-layer and a doping level in the *n*-bulk of 6.6 x 10^{12} cm^{-3}. The detector is operated at 100 V. The signal is sensed by a voltage amplifier, i.e. the input time constant is sufficiently large that the detector current pulse is integrated on the detector capacitance and the resulting voltage pulse is sensed by the amplifier. A simple threshold comparator provides the timing information.

a) What is the peak voltage of the signal developed at the amplifier input?

The peak signal voltage V_s is attained when all the charge is integrated on the detector capacitance

$$V_s = \frac{Q_s}{C}$$

The signal charge is determined by the number of electron-hole pairs formed by the incident energy, i.e. by the ratio of 5 MeV to the ionization energy, which is 3.6 eV in Si.

$$Q_s = N_{ep} q_e = \frac{E}{\varepsilon_i} q_e = \frac{5 \cdot 10^6}{3.6} 1.6 \cdot 10^{-19} = 223\,\text{fC}$$

The detector capacitance

$$C = \varepsilon_{Si} \varepsilon_0 \frac{A}{d} = 11.9 \cdot 8.85 \cdot 10^{-14} \frac{1}{50 \cdot 10^{-4}} = 211\,\text{pF}$$

so the signal is

$$V_s = \frac{Q_s}{C} = 1.06\,\text{mV}$$

b) What is the collection time? What is the 10 - 90% rise time of the voltage pulse? For simplicity assume a rectangular current pulse, i.e. that the detector current is constant during the collection time.

The depletion voltage

$$V_d = \frac{q_e N d^2}{2\varepsilon} = \frac{1.602 \cdot 10^{-19} \times 6.6 \cdot 10^{12} \times (50 \cdot 10^{-4})^2}{2 \times 11.9 \times 8.85 \cdot 10^{-14}} = 12.5\,\text{V}$$

The applied bias is 100 V, so the detector is operated well beyond full depletion. The average field $E = V/d = 2 \cdot 10^4$ V/cm. This is well into the regime of field-dependent mobility, so the drift velocity is $9 \cdot 10^6$ cm/s for the electrons and $5 \cdot 10^6$ cm/s for the holes.

The range of 5 MeV a particles in Si is 25 μm, so the maximum distance that holes must traverse is from the end of range to the *p*-contact, only 25 μm, whereas electrons formed at the *p*-contact must traverse the full thickness of the detector to reach the *n*-contact. The collection time of the electrons

$$t_{ce} = \frac{d}{v} = \frac{50 \cdot 10^{-4}}{9 \cdot 10^{6}} = 556 \text{ ps}$$

The holes are drifting through half the detector thickness into a higher field, so the average field is slightly higher, but it is only 6% higher than the value calculated above, so we'll neglect the difference. The collection time for the holes

$$t_{cp} = \frac{d/2}{v} = \frac{25 \cdot 10^{-4}}{5 \cdot 10^{6}} = 500 \text{ ps},$$

so electrons and holes contribute about equally to the total rise time. The 10-90% rise time is 80% of the collection time, i.e. 0.8 x 556 ps = 445 ps.

c) What is the optimum rise time of the amplifier system?

The optimum rise time of the amplifier is equal to the 10-90% rise time of the detector voltage signal, i.e. 445 ps.

d) The input noise of the amplifier is 1.5 nV/Hz$^{1/2}$. What is the total noise voltage?

If the amplifier rise time is determined by a single RC time constant, the noise bandwidth

$$\Delta f_n = \frac{1}{4\tau} = \frac{1}{4 \cdot (t_r / 2.2)} = \frac{0.55}{t_r} = 1.24 \cdot 10^{9} = 1.24 \text{ GHz}$$

If multiple equal time constants determine the rise time, which is more realistic

$$\Delta f_n \approx f_u = \frac{350\,[\text{MHz} \cdot \text{ns}]}{t_r} = 790 \text{ MHz}$$

For Δf_n = 1.24 GHz the total noise voltage is

$$V_n = v_n \sqrt{\Delta f_n} = 1.5 \cdot 10^{-9} \sqrt{1.24 \cdot 10^{9}} = 53\,\mu\text{V}$$

and for Δf_n = 790 MHz it is 42 μV.

e) The reverse bias current of the detector is 1 nA. Is this a significant noise source for the timing measurement?

There are two ways of approaching this. The accurate approach is to integrate over the spectral noise density due to the detector shot noise current flowing through the detector capacitance

$$V_{no}^2 = \int_0^\infty i_{ni}^2 |A_V(\omega)|^2 d\omega = \int_0^\infty 2q_e I_b \cdot \left(\frac{\omega^2 \tau_d^2}{2\pi(1+\omega^2\tau_d^2)(1+\omega^2\tau_i^2)} \right) d\omega$$

A simpler estimate is to calculate the current noise contribution for a CR-RC shaper with $T \approx$ 500 ps, determine the signal-to-noise ratio for the current noise alone, and compare with signal-to-noise ratio calculated for the amplifier noise. Although the timing system will use a larger differentiation time constant, the change in noise bandwidth is relatively small.

The noise contribution from the detector bias current (using F_i = 1)

$$Q_{ni}^2 = i_n^2 F_i T = 2q_e I_b F_i T = 2 \times 1.6 \cdot 10^{-19} \times 10^{-9} \times 5 \cdot 10^{-10} = 1.6 \cdot 10^{-37}$$

The amplifier noise contributes

$$Q_{nv}^2 = v_n^2 F_v \frac{C^2}{T} = (1.5 \cdot 10^{-9})^2 \frac{(211 \cdot 10^{-12})^2}{5 \cdot 10^{-10}} = 2 \cdot 10^{-28}$$

so the ratio of amplifier to current noise

$$\frac{Q_{nv}}{Q_{ni}} = \sqrt{\frac{2 \cdot 10^{-28}}{1.6 \cdot 10^{-37}}} \approx 3 \cdot 10^4$$

f) What is the time resolution obtainable with this system?

$$\sigma_t = \frac{\sigma_n}{dV/dt}$$

The total rise time is the combined rise time of detector and amplifier

$$t_r = \sqrt{t_c^2(10-90\%) + t_{ra}^2} = 630\ \text{ps}$$

Using Δf_n = 790 MHz and ΔV equal 80% of the peak signal V_s= 1.06 mV, the time jitter

$$\sigma_t = \frac{\sigma_n}{dV/dt} = \frac{42 \cdot 10^{-6}}{0.8 \times 1.06 \cdot 10^{-3} / 630 \cdot 10^{-12}} = 31\ \text{ps}$$

g) For comparison, what is the optimum shaping time for an energy measurement? Assume a CR-RC shaper. Compare the signal-to-noise ratio with the timing system.

The optimum shaping time

$$T_{s.opt} = C_i \frac{v_n}{i_n} \sqrt{\frac{F_v}{F_i}}$$

yields the minimum noise level

$$Q_n^2 = 2C_i v_n i_n \sqrt{F_i F_v}$$

The input noise current

$$i_n = \sqrt{2q_e I_b} = 1.79 \cdot 10^{-14} \text{ [A/}\sqrt{\text{Hz}}\text{]}$$

The optimum shaping time for a CR-RC shaper (F_i= F_v= 0.924)

$$T_{s.opt} = C_i \frac{v_n}{i_n} \sqrt{\frac{F_v}{F_i}} = 211 \cdot 10^{-12} \frac{1.5 \cdot 10^{-9}}{1.8 \cdot 10^{-14}} = 18 \ \mu\text{s}$$

The corresponding equivalent noise charge

$$Q_n^2 = 2C_i v_n i_n \sqrt{F_i F_v} = 2 \times 211 \cdot 10^{-12} \times 1.5 \cdot 10^{-9} \times 1.8 \cdot 10^{-14} \times 0.924$$
$$Q_n^2 = 1.1 \cdot 10^{-32} \text{ [C}^2\text{]}$$

or 640 el. The absorbed energy of 5 MeV yields 5 MeV/3.6 eV = $1.4 \cdot 10^6$ el, so *S/N*= 2170.

In the timing system

$$\frac{S}{N} = \frac{1.05 \text{ mV}}{42 \ \mu\text{V}} = 25$$

The optimized timing system has a signal-to-noise ratio that is roughly 100 times worse than for the optimized energy measurement.

3. Compare the timing performance of two detectors similar to the one in problem 2, but with thicknesses of 100 and 1000 μm. Both have an area of 200 mm^2 and are fully depleted with the same average electric field of 10^3 V/cm in the detector. Again, assume 5 MeV alpha particles incident on the *p*-side. For the two detectors, calculate

   a) the bias voltage,

   The bias voltage is the average field times the thickness, so the 100 μm thick detector requires 10 V and the 1000 μm device requires 100 V.

   b) the collection times,

   The collection time is dominated by the electrons (see problem 2b). At a field of 10^3 V/cm the electron velocity $v = \mu E = 1500 \times 10^3 = 1.5 \cdot 10^6$ cm/s, so the collection times are 6.7 ns for 100 μm and 67 ns for 1000 μm.

   c) the magnitude of the voltage pulse.

   The capacitances are 211 and 21 pF (the 100 μm detector has the same capacitance as the one in problem 2, as it has twice the thickness and twice the area), so the voltage signals

$$V_s = \frac{Q_s}{C} = q_e \frac{E}{\varepsilon_i} \frac{1}{C}$$

   are (using the result from problem 2) 1.05 and 10.5 mV for the thick and thin detector.

   d) The same amplifier is used with both detectors. Its bandwidth is 500 MHz and the integrated noise voltage, referred to the input, is 30 μV. What are the time resolutions obtainable with the two detectors?

   The rise time of the 500 MHz amplifier is 700 ps. This degrades the 6.7 ns rise time from the 100 μm detector by 0.5%, so we can neglect it for both the 6.7 ns and 67 ns rise times.

   The time resolution

$$\sigma_t = \frac{\sigma_n}{dV/dt}$$

   Since the same amplifier is used with both detectors, σ_n is the same for both, so let's consider the slope. For the 100 μm detector

$$\frac{dV}{dt} \approx \frac{Q_s / C}{t_c} = \frac{1.05 \cdot 10^{-3}}{6.7 \cdot 10^{-9}} = 1.6 \cdot 10^5 \ \frac{\mathrm{V}}{\mathrm{s}}$$

   and the time resolution is about 200 ps. Since the thick detector has 10 times the rise time, but 1/10 the capacitance, it yields the same time resolution of 200 ps, which is about 0.3% of the 67 ns rise time.