The PMT is often coupled to the scintillator through a light guide.

Match geometry of scintillator to photodetector.

Spatial separation of scintillator and detector.
Light Transmission Through Light Guides

In coupling a scintillator to a photodetector through a light guide, it is tempting to couple a large area crystal to a small area detector. This could save money and also, when using photodiodes, reduce the electronic noise.

What is the efficiency of light transmission?

The efficiency of light transmission through a light guide is limited by

- the angle of total reflection
- conservation of phase space (Liouville’s theorem)
1. Total reflection.

For rays to be reflected from the surface of the light guide the incident angle

\[ \sin \Theta \geq \frac{n_{ext}}{n} \]

where \( n \) is the refractive index of the light guide and \( n_{ext} \) that of the external medium. When the external medium is air (\( n_{ext} = 1 \))

\[ \sin \Theta \geq \frac{1}{n} \]

If the light guide is tapered with an angle \( \phi \), a ray at the limit of total reflection will impinge on the output face at an angle

\[ \frac{\pi}{2} + \phi - \Theta \]

This is the maximum angle at the light guide output. Since the maximum reflection angle in the light guide is \( \pi/2 \), the minimum angle of reflected rays at the exit is \( \phi \), whereas direct rays can impinge with zero angle.

2. Conservation of phase space

(see D. Marcuse, BSTJ 45 (1966) 743, Applied Optics 10/3 (1971) 494)

The trajectories of photons can be described analogously to particles whose position and slope are described as a point in phase space with the coordinates \( x \) and \( p \). For photons these canonically conjugate variables are the the transverse coordinates of the photon ray and its angle. In two dimensions (adopted here for simplicity) the variables are the transverse coordinate \( x \) and the quantity

\[ p = n \sin \alpha \]

where \( n \) is the refractive index of the medium and \( \alpha \) is the angular divergence of the photon beam.
At the entrance of the light guide the transverse dimension is $\Delta x_1$, so if the maximum angle of a light ray is $\alpha_1$, the volume element in phase space is

$$\Delta x_1 \Delta p_1 = 2\Delta x_1 n \sin \alpha_1$$

Correspondingly, at the output

$$\Delta x_2 \Delta p_2 = 2\Delta x_2 n \sin \alpha_2$$

Since the volume element must be conserved

$$\Delta x_1 \Delta p_1 = \Delta x_2 \Delta p_2 ,$$

a maximum acceptance angle $\alpha_2$ at the output means that at the input only rays within an entry angle

$$\sin \alpha_1 = \frac{\Delta x_2}{\Delta x_1} \sin \alpha_2$$

can propagate through the light guide.

- Note that even if total reflection obtained over all angles ($n = \infty$), a light guide with $\Delta x_1 >> \Delta x_2$ would incur substantial light loss because of limitation of the acceptance angle.

As shown above, total internal reflection allows a maximum angle of

$$\alpha_2 = \frac{\pi}{2} + \phi - \Theta$$

so

$$\sin \alpha_2 = \sin \left( \frac{\pi}{2} + \phi - \Theta \right) = \cos (\Theta - \phi) = \sqrt{1 - \sin^2 \Theta \sin^2 \phi}$$
For simplification, assume that the lightguide is only slightly tapered ($\phi << \Theta$). Then

$$\sin \alpha_2 \approx \sqrt{1 - \sin^2 \Theta} = \sqrt{1 - \frac{1}{n^2}}$$

Thus, the maximum acceptance angle imposed by phase space at the input of the light guide

$$\sin \alpha_1 = \frac{\Delta x_2}{\Delta x_1} \sin \alpha_2 = \frac{\Delta x_2}{\Delta x_1} \sqrt{1 - \frac{1}{n^2}}$$

Typical lightguide materials have a refractive index $n \approx 1.5$, so even for equal dimensions $\Delta x_1$ and $\Delta x_2$

$$\sin \alpha_1 = \sqrt{1 - \frac{1}{n^2}} = 0.75$$

Translated to three dimensions, conservation of phase space means that the flux of photons per unit area and per unit solid angle is constant throughout a given medium. Consequently, no optical coupling scheme relying on reflection or diffraction alone can transmit photons from a large source to a small detector with full efficiency.

This limitation can be overcome by wavelength shifters, that absorb the incident light and re-emit photons, thereby redefining the phase space element.
Implementation of Light Guides
ref: Kilvington et al., NIM 80 (1970) 177

Where the condition for total reflection is not met, an external reflector can help.

Experimental arrangement
Variation of pulse height with length of light guide

a) Total internal reflection only

b) Total internal reflection with reflective coating
   either... aluminum foil
   aluminized mylar
   transparent mylar painted with reflective paint

c) Surface of light guide coated with reflective paint

d) Specular reflector without light guide

e) Diffuse reflector without light guide

Although peak light output can be improved by reflective coatings, this only obtains with short light guides.

Critical that surface of light guide be smooth.
Operational aspects of using PMTs

Electron multiplication at dynodes depends on potential between successive dynodes.

Potential distribution commonly set by resistive divider.

Secondary electrons are emitted with low energy and accelerated by potential difference between dynodes.

Secondary emission coefficients of commonly used dynode materials vs. incident electron energy:
The gain of GaP(Cs) NEA dynodes does not exhibit the gain saturation of conventional materials.

![Graph showing the energy response of GaP(Cs) NEA dynodes.](image)

(from *Photomultiplier Tubes*, Philips Photonics)

Advantageous especially at first dynode to improve gain distribution of multiplication chain.

Typically, PMTs are operated with total supply voltages of 2 kV.

8 to 14 stages (number of dynodes) are common, with 100 to 150 V between dynodes.

The potential between the photocathode and the first dynode is typically 4 times as large to improve the collection efficiency and the gain in the first stage.
Peak currents of anode pulses can be as high as 20 mA.

If the voltage divider is not capable of providing this current, the acceleration potential will “sag”, leading to non-linearity. (Note that total gain changes with $n$-th power of voltage!)

Necessary to provide capacitors as “charge reservoirs”:

![Diagram of resistive divider with capacitors](from Burle Photomultiplier Handbook)

DC current through resistive divider must be much greater (>10x, preferably more) than the average signal current.

The average current at a gamma rate of $n \text{ s}^{-1}$ is

$$\langle I \rangle = n \cdot N_{el,\text{anode}} \cdot q_e$$

Using the NaI(Tl) example used before, for which each 511 keV gamma produced $3 \cdot 10^9$ electrons at the anode, the average signal current at a rate of $10^5 \text{ s}^{-1}$ is 48 µA.

Thus, the standing current in the resistive divider should be 1 mA or more, leading to a power (heat) dissipation of 2 W at 2 kV total supply voltage.
Scintillators with higher light output or running at higher rates might require 10 mA, which becomes thermally problematic. In these cases, voltage dividers transistor current buffers are often used.

Although the polarity of the supply voltage is fixed, i.e. the anode must be more positive than the cathode, one can choose whether the anode or cathode is at ground potential.

Although grounding the cathode is widespread, operation with the anode at ground potential is advantageous in systems operating with fast output pulses and high counting rates. The only drawback is that the photocathode end of the tube must be well-insulated from ground to prevent corona discharge near the photocathode.

The voltage distribution in the dynode chain can be optimized for

- high gain
- time resolution
- good linearity up to high peak currents

Recommended voltage distributions can be found in the manufacturers data sheets.
Connections to the anode and adjacent dynodes must be made with low inductance to avoid parasitic resonances.

Upper waveform: correct pulse
Lower waveform: superimposed “ringing” due to parasitic resonances

(from *Burle Photomultiplier Handbook*)
Photomultiplier tubes are sensitive to magnetic fields

![Graph showing the relative anode current percentage against magnetic field intensity in oersteds.](Image)

(from Burle Photomultiplier Handbook)

Even in a laboratory environment, PMTs must be surrounded by magnetic shielding (“mu metal”) to avoid orientation-dependent gain changes due to stray magnetic fields.

**Typical:** 25% decrease in gain at 0.1 mT

Conventional PMTs will not function inside the magnet of a tracking detector!

**Alternatives:**
- MCP PMTs
- Semiconductor photodiodes
- special dynode structures
Time Response of Photomultiplier Tubes

For a typical fast 2” PMT (Philips XP2020) the transit time from the photocathode to the anode is about 30 ns at 2000V.

The intrinsic rise time is 1.6 ns, due to broadening of the initial electron packet in the course of the multiplication process.

The transit time varies by 0.25 ns between the center of the photocathode and a radius of 18 mm.

For two tubes operating in coincidence at a signal level of 1500 photoelectrons, a time resolution of 230 ps is possible.

Special dynode structures are used to reduce transit time spread.

Example: time compensating structure

(from Burle Photomultiplier Handbook)

More on timing with PMTs in chapter on timing electronics.
Various Dynode Structures

(a) Venetian blind
Allows simple input system with high collection efficiency.
Good gain stability, but mediocre timing

(b) Box and Grid:
Characteristics similar to a)

c) Linear focusing:
good timing characteristics

d) Circular cage:
compact

e) Mesh dynodes:
low gain, but usable up to $B \approx 1 \, \text{T}$

f) Foil dynodes:
perforated metal foils – particularly useful for multi-channel anodes
Example: Scintillation Detector using $dE/dx$ for Particle Identification

Plastic Ball Detector


Use two scintillators (“phoswich”):

- CaF$_2$(Eu) slow decay
- Plastic fast decay

Thin CaF$_2$(Eu) scintillator uses long plastic scintillator as a light guide.
Two integrating ADCs are used.

One with a short gate to measure the fast light output from the plastic, the second with a long gate to measure the light from the CaF$_2$.

$\pi^+$ tend to disappear in strong background, so additional identification applied.

$\pi^+$ lifetime 26 ns: $\pi^+ \rightarrow \mu^+ + \nu$

$\mu^+$ lifetime 2.2 $\mu$s : $\mu^+ \rightarrow e^+ + 2\nu$

e$^+$ emitted with energy up to 53 MeV and easily detected in plastic scintillator.
Particle ID Characteristics

![Graph showing energy vs. CaF2 for different particles]
Comparison between $\gamma$ spectra taken with NaI(Tl) scintillator and Ge semiconductor detector


Semiconductor detectors provide much better resolution than scintillators.