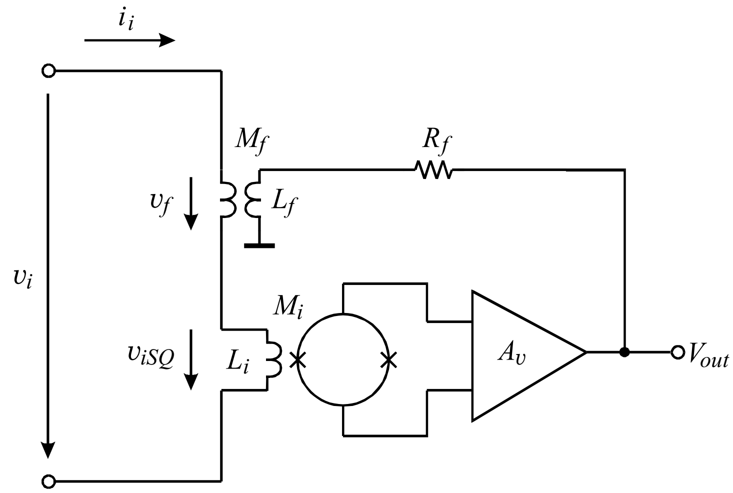
Input Impedance of a Series Feedback SQUID Amplifier

The feedback is introduced by a transformer in series with the SQUID input, as shown below.



Let the voltage at the SQUID input coil be v_{iSQ}. Then the voltage at the SQUID output is

$$v_{oSQ} = \frac{v_{iSQ}}{\mathbf{i}\omega L_i} M_i \frac{dV}{d\Phi} \quad ,$$

where M_i is the mutual inductance between the input coil and the SQUID loop and $dV/d\Phi$ is the voltage sensitivity of the SQUID.

The signal voltage at the secondary of the feedback coil

$$v_f = -\mathbf{i}\omega M_f \frac{v_{oSQ} A_v}{R_f} = -\omega M_f \frac{v_{iSQ}}{\omega L_i} M_i \frac{dV}{d\Phi} \frac{A_v}{R_f} \quad ,$$

so the net voltage at the input of the complete fedback amplifier

$$v_i = v_{iSQ} + v_f = v_{iSQ} \left(1 - \frac{M_i M_f}{L_i} \frac{dV}{d\Phi} \frac{A_v}{R_f} \right) \; .$$

If the second term is made positive, *e.g.* by using an inverting amplifier, the input voltage for a given input current i_i is increased, so the net input impedance is increased over the SQUID's input impedance by the factor

$$\frac{Z_i}{Z_{iSQ}} = \frac{v_i}{v_{iSQ}} = \left(1 - \frac{M_i M_f}{L_i} \frac{dV}{d\Phi} \frac{A_v}{R_f} \right) \; .$$

For zero source impedance the loop gain

$$A_L \equiv -\frac{v_f}{v_{iSQ}} = \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{A_v}{R_f} \ ,$$

and the input impedance can be written as

$$Z_i = \left(1 - A_L\right) Z_{iSQ} \ .$$

If $|A_L| >> 1$, the input impedance $Z_i \approx -A_L Z_{iSQ} = -A_L(\mathbf{i}\omega L_i)$.

Assume a single-pole inverting amplifier, i.e. $A_v < 0$ and $A_L < 0$. In the low frequency regime where the additional phase shift of the amplifier is zero, the input impedance

$$Z_i \approx |A_L|(\mathbf{i}\omega L_i)$$

is set by the input inductance multiplied by the magnitude of the loop gain.

Beyond the amplifier's cutoff frequency the gain $A_v = \mathbf{i}\omega_0 / \omega$, where ω_0 is the unity gain frequency, and the input impedance

$$Z_i \approx -\mathbf{i} A_L \omega L_i = -\mathbf{i} \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{1}{R_f} \left(\mathbf{i} \frac{\omega_0}{\omega} \right) \omega L_i = \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{1}{R_f} \omega_0 L_i$$

is real and independent of frequency.

Extension to non-zero source impedance

In any feedback amplifier where the input and feedback circuits share a common signal path, the loop gain depends on the source impedance. The assumption of zero source impedance made above simplifies the results and demonstrates the effect of feedback on the input impedance, but it is not generally valid. For a finite source impedance Z_S, the fraction of the feedback voltage that appears at the SQUID input is

$$\frac{\mathbf{i}\omega L_i}{Z_S + \mathbf{i}\omega(L_f + L_i)} \ ,$$

so the loop gain

$$A_L = \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{A_v}{R_f} \frac{\mathbf{i}\omega L_i}{Z_S + \mathbf{i}\omega(L_f + L_i)} \ .$$

For an inductive source, $Z_S = \mathbf{i}\omega L_S$ and

$$A_L = \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{A_v}{R_f} \frac{L_i}{L_S + L_f + L_i} \ .$$

Now the phase of the loop gain is only determined by the amplifier and the above results for zero source impedance are only modified by the reduced magnitude of the loop gain.

For a single-pole inverting amplifier in the constant gain regime, *i.e.* at low frequencies,

$$Z_i = (1 - A_L)\,\mathbf{i}\omega L_i = \mathbf{i}\omega L_i \left(1 + \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{|A_v|}{R_f} \frac{L_i}{L_S + L_f + L_i} \right) .$$

The input impedance is purely inductive and increases linearly with frequency, as in the simplified case.

Beyond the amplifier's cutoff frequency, where $A_v = \mathbf{i}\omega_0 / \omega$,

$$Z_i = \mathbf{i}\omega L_i \left(1 - \mathbf{i}\frac{\omega_0}{\omega} \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{1}{R_f} \frac{L_i}{L_S + L_f + L_i} \right) = \omega_0 L_i \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{1}{R_f} \frac{L_i}{L_S + L_f + L_i} + \mathbf{i}\omega L_i \ ,$$

so the input impedance is complex and frequency-dependent. However, at frequencies sufficiently below the unity-gain frequency such that $|A_L| >> 1$, the first term dominates and

$$Z_i \approx \omega_0 L_i \frac{M_f M_i}{L_i} \frac{dV}{d\Phi} \frac{1}{R_f} \frac{L_i}{L_S + L_f + L_i} \ .$$

Then the input impedance is real and independent of frequency.

Summary

Series feedback increases the SQUID's input impedance by the loop gain. A resistive or complex source impedance introduces an additional phase shift into the loop gain and leads to a complex input impedance.

A purely inductive source simplifies the results. At frequencies where the amplifier gain is constant (no excess phase shift), the input impedance is inductive and increases with frequency. Above the amplifier's cutoff frequency, where the gain falls linearly with frequency and the phase shift is 90°, the input impedance is real and independent of frequency, but as one approaches the feedback loop's unity gain frequency, the input impedance becomes complex and frequency-dependent.