

*Lectures on Detector Techniques*  
*Stanford Linear Accelerator Center*  
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# Semiconductor Detectors

## Part 1

Helmuth Spieler

Physics Division  
Lawrence Berkeley National Laboratory

*for more details see UC Berkeley Physics 198 course notes  
at <http://www-physics.lbl.gov/~spieler>*

# 1. Principles

Important excitations when radiation is absorbed in solids

## 1. atomic electrons

⇒ mobile charge carriers

⇒ lattice excitations (phonons)

## 2. elastic scattering on nuclei

Recoil energy < order 10 eV

⇒ ionization

⇒ lattice excitations (phonons)

at higher recoil energies

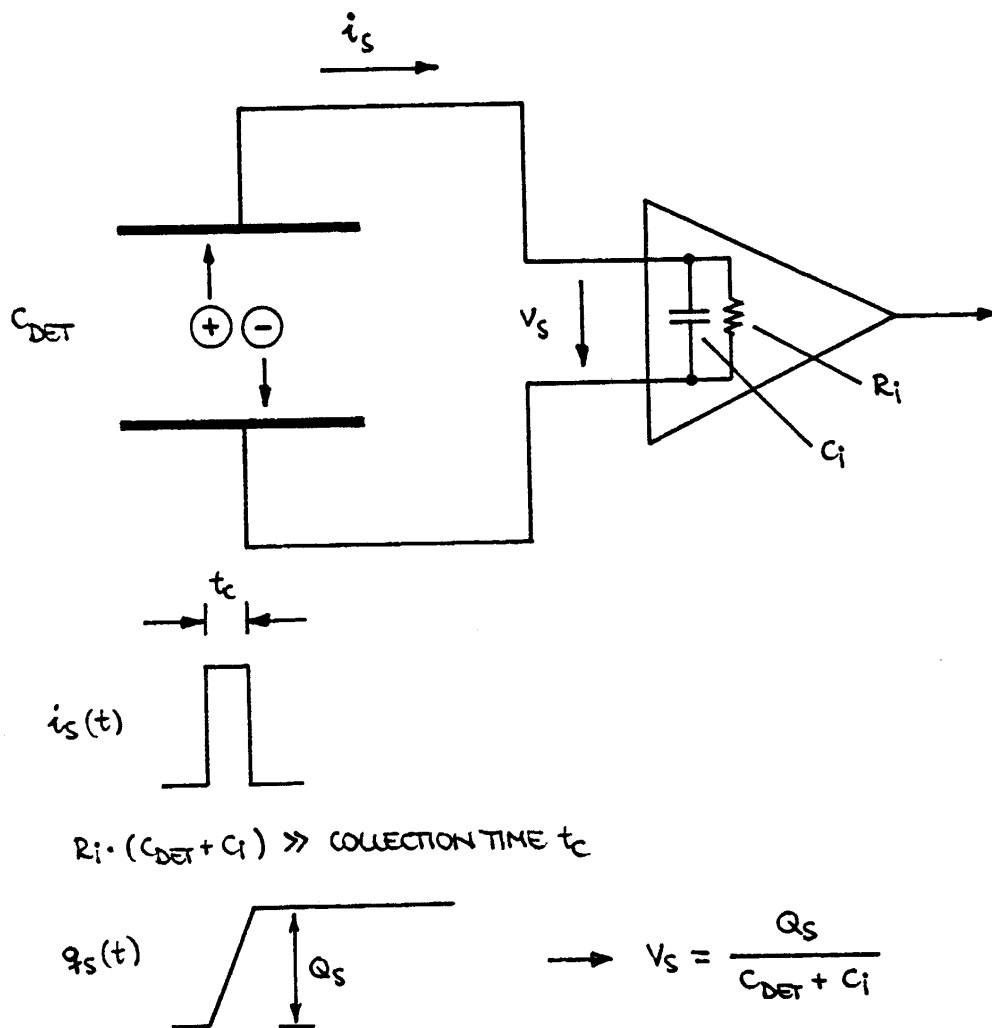
⇒ radiation damage  
(displacement of from lattice sites)

## Most Semiconductor Detectors are Ionization Chambers

Detection volume with electric field

Energy deposited  $\rightarrow$  positive and negative charge pairs

Charges move in field  $\rightarrow$  current in external circuit  
(continuity equation)





Ionization energy in solids is proportional to the band gap

small band gap  $\Rightarrow$  ~ conductor

electric field small

DC current  $\gg$  signal current

large band gap  $\Rightarrow$  insulator

high electric field

small signal charge  
+ small DC current

example: diamond

moderate band gap  $\Rightarrow$  semiconductor

high electric field

“large” signal charge  
small DC current, but  
“*pn*-junction” required.

examples: Si, Ge, GaAs

# Principle of Semiconductor Detector

## 1. Energy absorbed in semiconductor

⇒ formation of mobile electrons and holes

number of electron-hole pairs in Si:

$$N = \frac{E}{\epsilon_i}$$

$E$  = absorbed energy,  $\epsilon_i$  = 3.6 eV

## 2. Electric field in detection volume causes electrons and holes to move towards electrodes

typical transit times: 10 – 20 ns for 300  $\mu\text{m}$

## 3. Induced signal current measured

typical signal charge: 3.5 fC (22000 e) for  
minimum ionizing particles  
traversing 300  $\mu\text{m}$  of Si

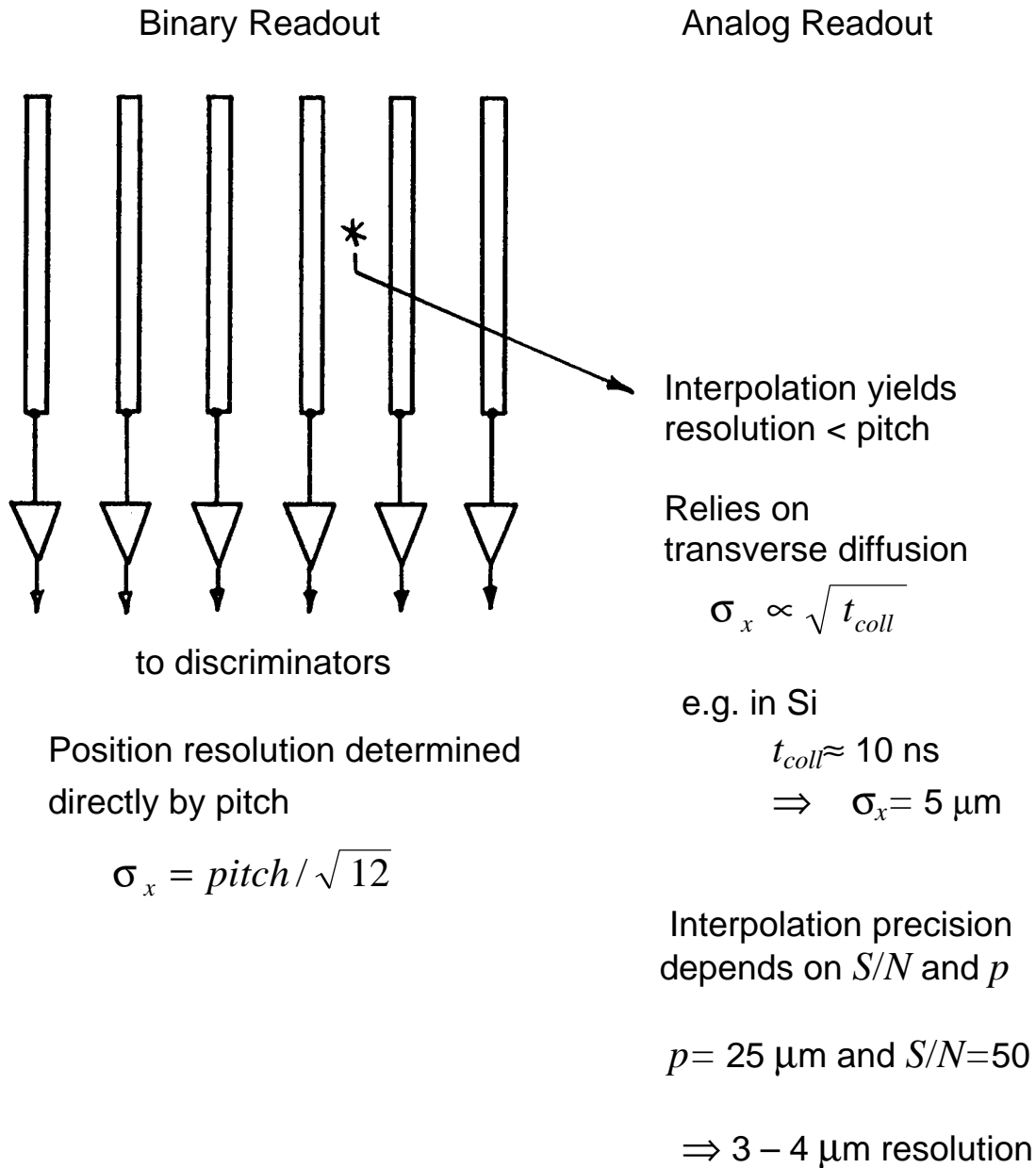
Semiconductor technology allows micron-scale patterning of electrodes

⇒ strip and pixel detectors

## Example: Strip Detectors

Resolution determined by precision of micron scale patterning  
(e.g. strips on 50  $\mu\text{m}$  pitch)

Two options:



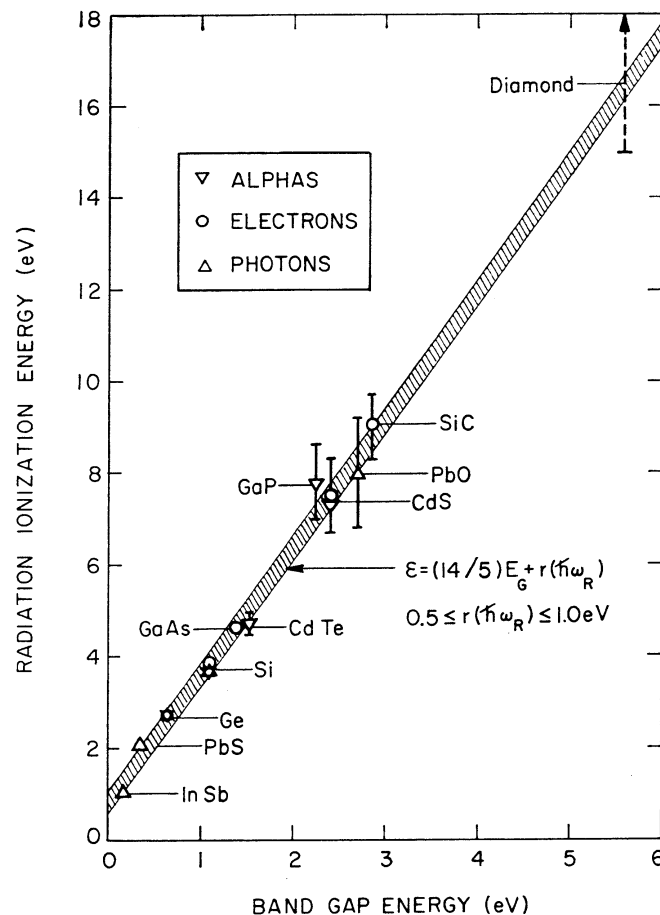
# Energy required for creation of an electron-hole pair

Ionization Energy > Band Gap

Formation of e-h pair requires both ...

1. Conservation of energy
  2. Conservation of momentum
- ⇒ additional energy excites phonons

Remarkably,  $\epsilon_i/E_g \approx \text{const}$  for all materials, types of radiation.



C.A. Klein, J. Applied Physics **39** (1968) 2029



## Fluctuations in the Signal Charge: the Fano Factor

The mean ionization energy exceeds the bandgap for two reasons

1. Conservation of momentum requires excitation of lattice vibrations
2. Many modes are available for the energy transfer with an excitation energy less than the bandgap.

Two types of collisions are possible:

- a) Lattice excitation, i.e. phonon production (with no formation of mobile charge).
- b) Ionization, i.e. formation of a mobile charge pair.

Assume that in the course of energy deposition

$N_x$  excitations produce  $N_P$  phonons and

$N_i$  ionization interactions form  $N_Q$  charge pairs.

On the average, the sum of the energies going into excitation and ionization is equal to the energy deposited by the incident radiation

$$E_0 = E_i N_i + E_x N_x$$

where  $E_i$  and  $E_x$  are the energies required for a single excitation or ionization.

Assuming gaussian statistics, the variance in the number of excitations

$$\sigma_x = \sqrt{N_x}$$

and the variance in the number of ionizations

$$\sigma_i = \sqrt{N_i}$$

For a single event, the energy  $E_0$  deposited in the detector is fixed (although this may vary from one event to the next).

If the energy required for excitation  $E_x$  is much smaller than required for ionization  $E_i$ , sufficient degrees of freedom will exist for some combination of ionization and excitation processes to dissipate precisely the total energy. Hence, for a given energy deposited in the sample a fluctuation in excitation must be balanced by an equivalent fluctuation in ionization.

$$E_x \Delta N_x + E_i \Delta N_i = 0$$

If for a given event more energy goes into charge formation, less energy will be available for excitation. Averaging over many events this means that the variances in the energy allocated to the two types of processes must be equal

$$E_i \sigma_i = E_x \sigma_x$$

$$\sigma_i = \frac{E_x}{E_i} \sqrt{N_x}$$

From the total energy  $E_i N_i + E_x N_x = E_0$

$$N_x = \frac{E_0 - E_i N_i}{E_x}$$

yielding

$$\sigma_i = \frac{E_x}{E_i} \sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x} N_i}$$

Since each ionization leads to a charge pair that contributes to the signal

$$N_i = N_Q = \frac{E_0}{\epsilon_i}$$

where  $\epsilon_i$  is the average energy loss required to produce a charge pair,

$$\sigma_i = \frac{E_x}{E_i} \sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x} \frac{E_0}{\epsilon_i}}$$

$$\sigma_i = \sqrt{\frac{E_0}{\epsilon_i}} \cdot \sqrt{\frac{E_x}{E_i} \left( \frac{\epsilon_i}{E_i} - 1 \right)}$$

The second factor on the right hand side is called the Fano factor  $F$ .

Since  $\sigma_i$  is the variance in signal charge  $Q$  and the number of charge pairs is  $N_Q = E_0/\epsilon_i$

$$\sigma_Q = \sqrt{F N_Q}$$

In Silicon  $E_x = 0.037$  eV  
 $E_i = E_g = 1.1$  eV  
 $\epsilon_i = 3.6$  eV

for which the above expression yields  $F = 0.08$ , in reasonable agreement with the measured value  $F = 0.1$ .

⇒ The variance of the signal charge is smaller than naively expected

$$\sigma_Q \approx 0.3 \sqrt{N_Q}$$

A similar treatment can be applied if the degrees of freedom are much more limited and Poisson statistics are necessary.

However, when applying Poisson statistics to the situation of a fixed energy deposition, which imposes an upper bound on the variance, one can not use the usual expression for the variance

$$\text{var } N = \overline{N}$$

Instead, the variance is

$$\overline{(N - \overline{N})^2} = F \overline{N}$$

as shown by Fano [1] in the original paper.

An accurate calculation of the Fano factor requires a detailed accounting of the energy dependent cross sections and the density of states of the phonon modes. This is discussed by Alkhazov [2] and van Roosbroeck [3].

#### References:

1. U. Fano, Phys. Rev. **72** ( 1947) 26
2. G.D. Alkhazov et al., NIM **48** (1967) 1
3. W. van Roosbroeck, Phys. Rev. **139** (1963) A1702

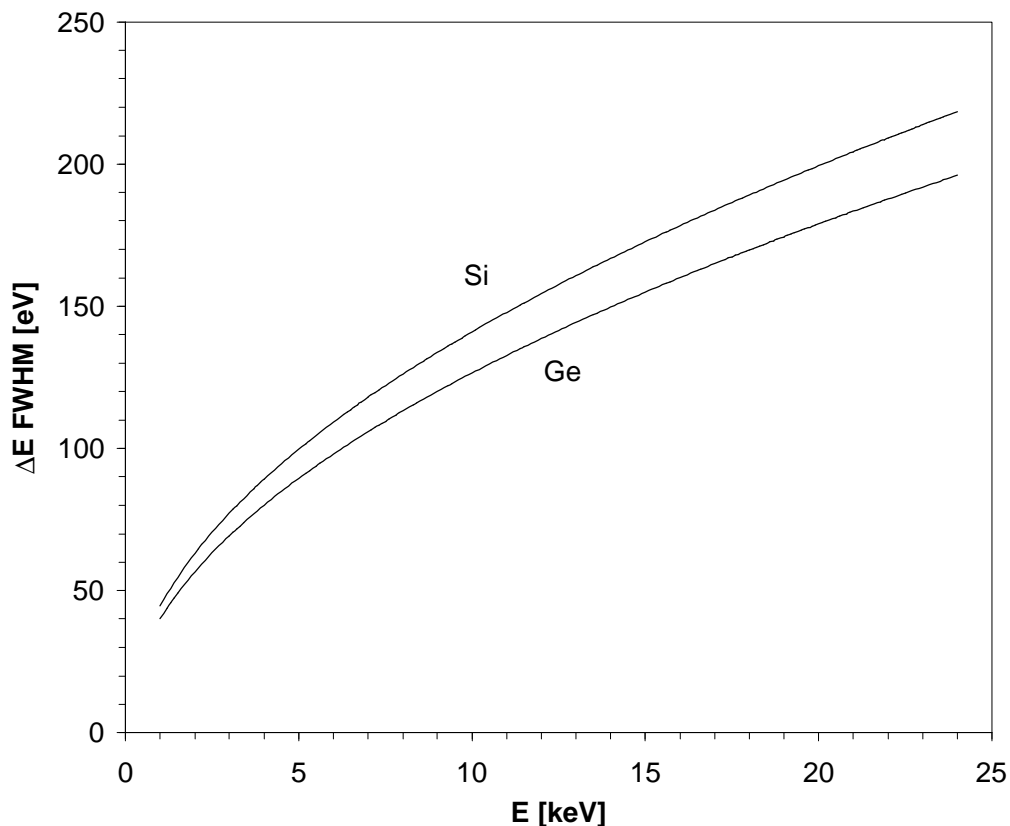
## Intrinsic Resolution of Semiconductor Detectors

$$\Delta E = 2.35 \cdot \epsilon_i \sqrt{FN_Q} = 2.35 \cdot \epsilon_i \sqrt{F \frac{E}{w}} = 2.35 \cdot \sqrt{FE\epsilon_i}$$

$$\text{Si:} \quad \epsilon_i = 3.6 \text{ eV} \quad F = 0.1$$

$$\text{Ge:} \quad \epsilon_i = 2.9 \text{ eV} \quad F = 0.1$$

**Intrinsic Resolution of Si and Ge Detectors**

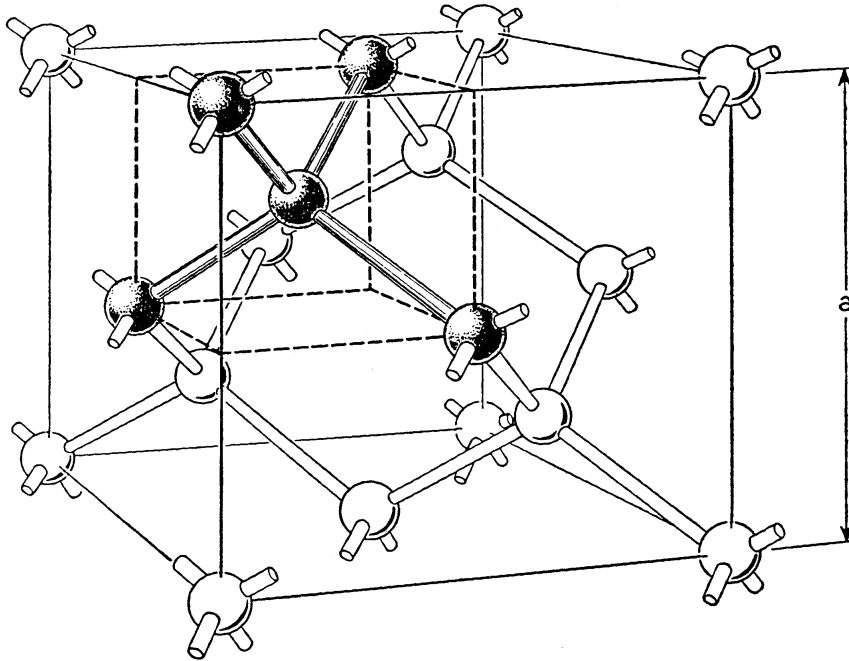


Detectors with good efficiency for this energy range have sufficiently small capacitance to allow electronic noise of  $\sim 100$  eV FWHM, so the variance of the detector signal is a significant contribution.

At energies  $>100$  keV the detector sizes required tend to increase the electronic noise to dominant levels.

## Semiconductor Crystals

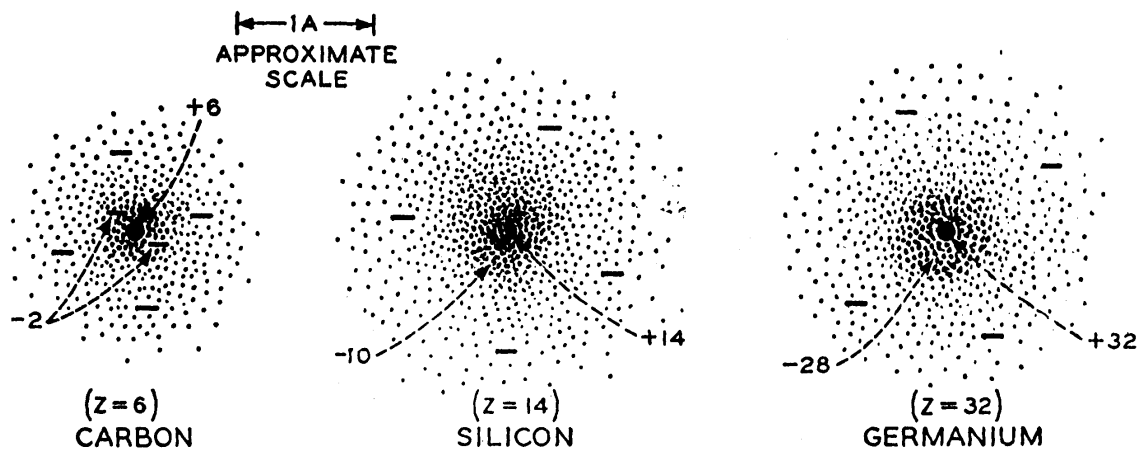
Lattice structure of diamond, Si, Ge ("diamond lattice")



(from Sockley)

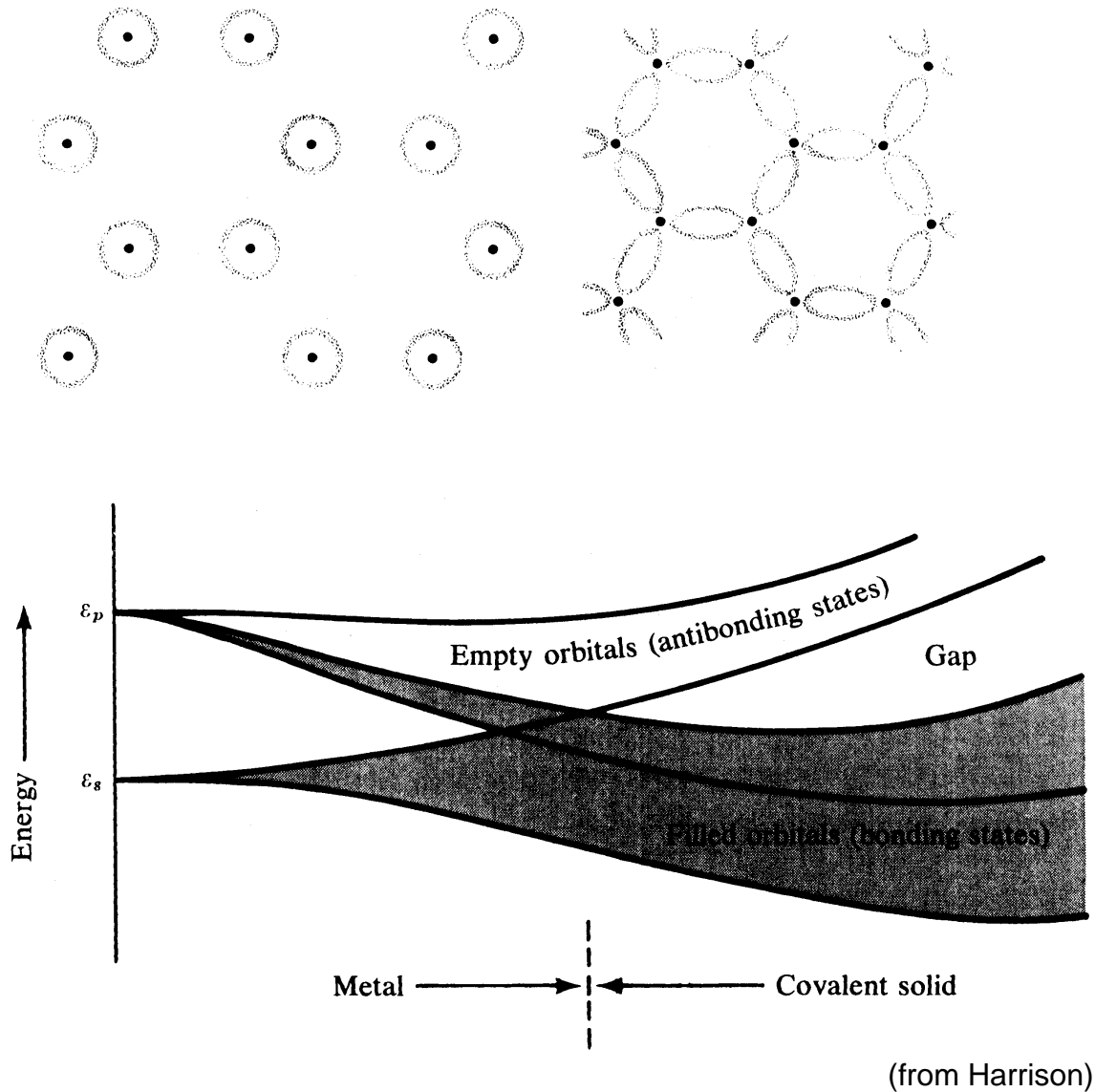
dimension a:	lattice constant	Diamond:	3.56 Å
		Ge:	5.65 Å
		Si:	5.43 Å

Extent of wavefunctions of typical constituent atoms:



(from Sockley)

When isolated atoms are brought together to form a lattice, the discrete atomic states shift to form energy bands:



Filled band formed by bonding states:  
 ( $\Psi_a$  = wavefunction of individual atom)

$$\Psi = \Psi_a + \Psi_a$$

Empty band formed by anti-bonding states:

$$\Psi = \Psi_a - \Psi_a$$

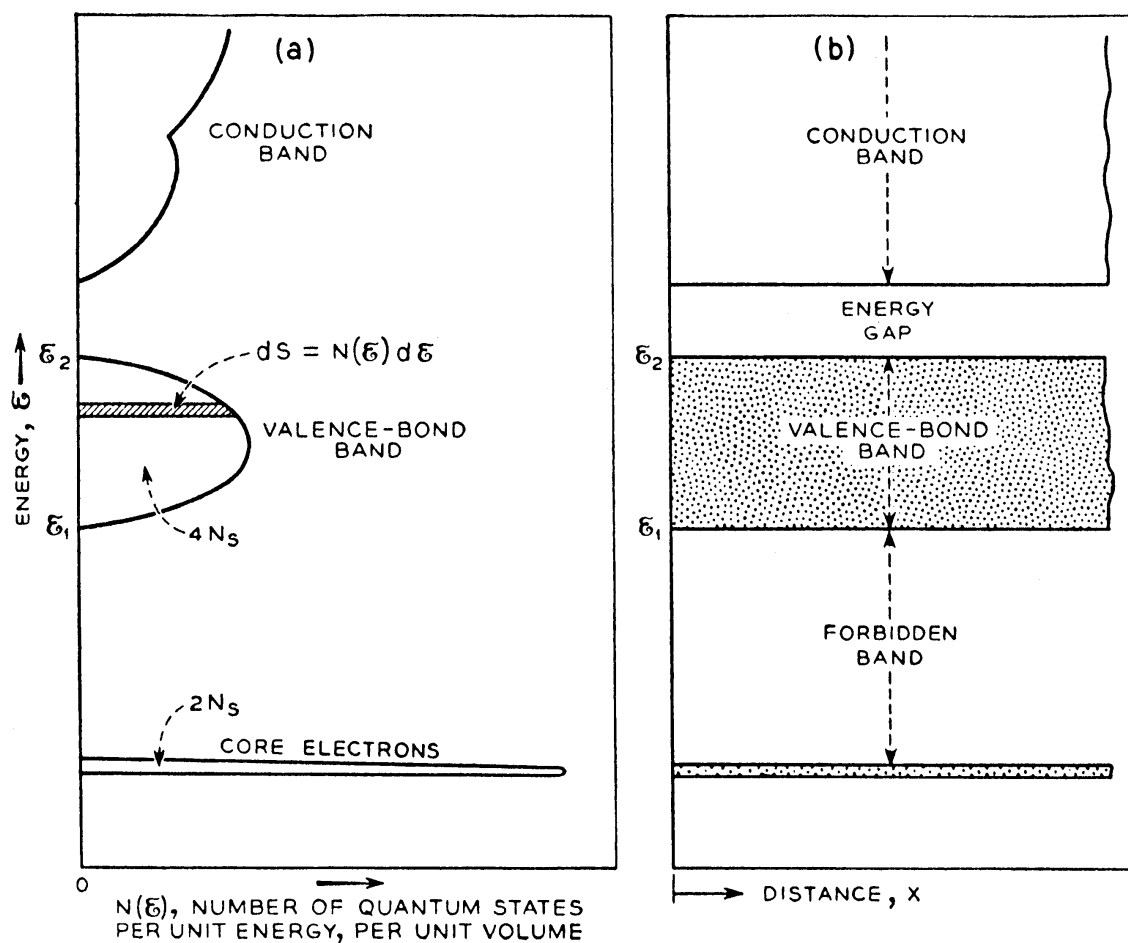
(vanishing occupancy at mid-point between atoms)

Each atom in the lattice contributes its quantum states to each band:

The number of quantum states in the band is equal to the number of states from which the band was formed.

The bands are extended states, i.e. the state contributed by an individual atom extends throughout the crystal.

## Energy band structure



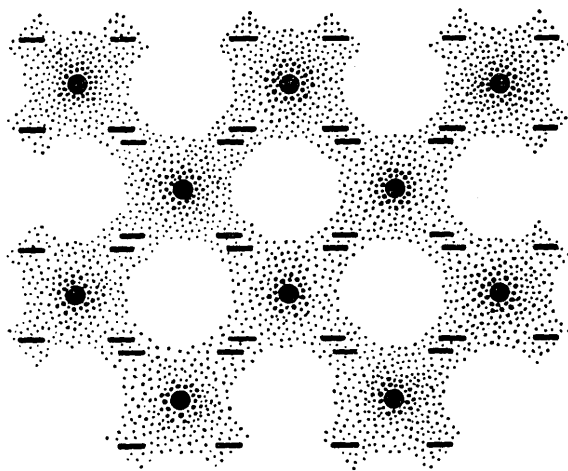
(from Shockley)

Typical band gaps (valence – conduction band)

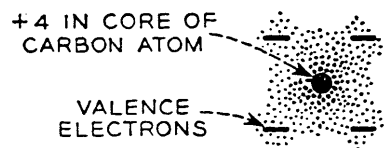
Ge	0.7 eV	GaAs	1.4 eV
Si	1.1 eV	Diamond	5.5 eV



At 0K all electrons occupy bonding states, completely filling the valence band.

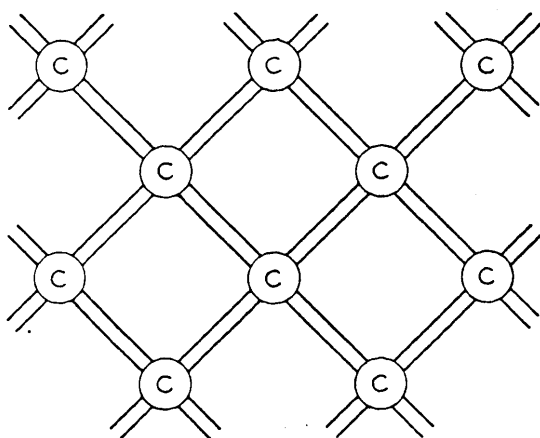


(a) ELECTRON PAIR BONDS

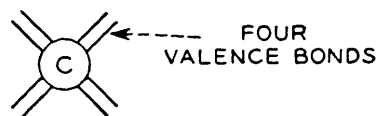


EACH ATOM, WITH THE CHARGE OF ITS SHARE OF VALENCE-BOND ELECTRONS, IS ELECTRICALLY NEUTRAL.

(b)



(c) PLANE DIAGRAM OF DIAMOND LATTICE WITH BONDS REPRESENTED BY LINES



(from Shockley)

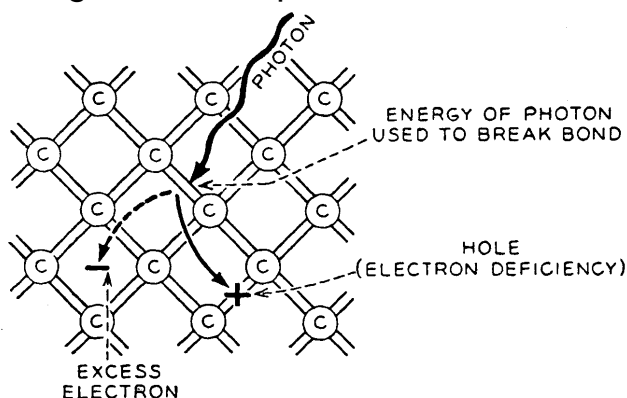
If an electric field is applied to the crystal, no current can flow, as this requires that the electrons acquire energy, which they can't, as no higher energy states are available in the valence band.

If energy is imparted to a bond by incident radiation, for example a photon, the bond can be broken,

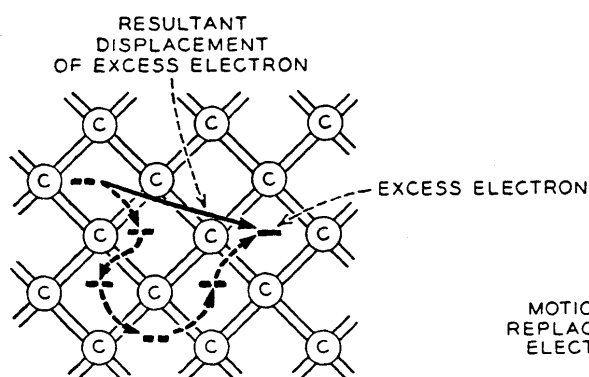
- exciting an electron into the conduction band and
- leaving back a vacant state in the valence band, a “hole”.

The electron can move freely in its extended state.

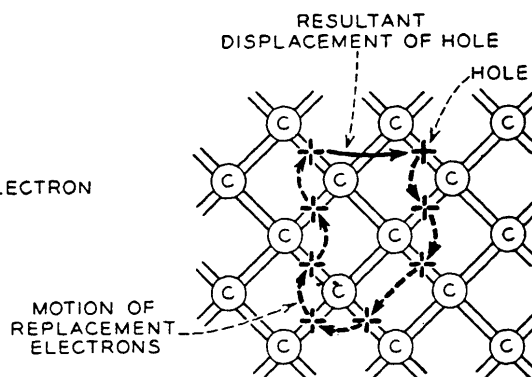
The hole can be filled by an electron from a nearby atom, thereby moving to another position.



(a) PRODUCTION OF A HOLE-ELECTRON PAIR BY A PHOTON



(c) RANDOM MOTION OF AN EXCESS ELECTRON



(d) RANDOM MOTION OF A HOLE

The motion of the electron and hole can be directed by an electric field.

Holes can be treated as positive charge carriers just like the electrons, although they tend to move more slowly as hole transport involves sequential transition probabilities (the wavefunction overlap of the hole and its replacement electron).

## Formation of a High-Field Region

The conduction band is only empty at 0K.

As the temperature is increased, thermal excitation can promote electrons across the band gap into the conduction band.

Pure Si: carrier concentration  $\sim 10^{10} \text{ cm}^{-3}$  at 300K  
(resistivity  $\approx 400 \text{ k}\Omega\cdot\text{cm}$ )

Since the Si lattice comprises  $5 \cdot 10^{22} \text{ atoms/cm}^3$ , many states are available in the conduction band to allow carrier motion.

In reality, crystal imperfections and minute impurity concentrations limit Si carrier concentrations to  $\sim 10^{11} \text{ cm}^{-3}$  at 300K.

This is too high for use in a simple crystal detector.

A crystal detector is feasible with diamond, but the charge yield is smaller due to the larger band gap.

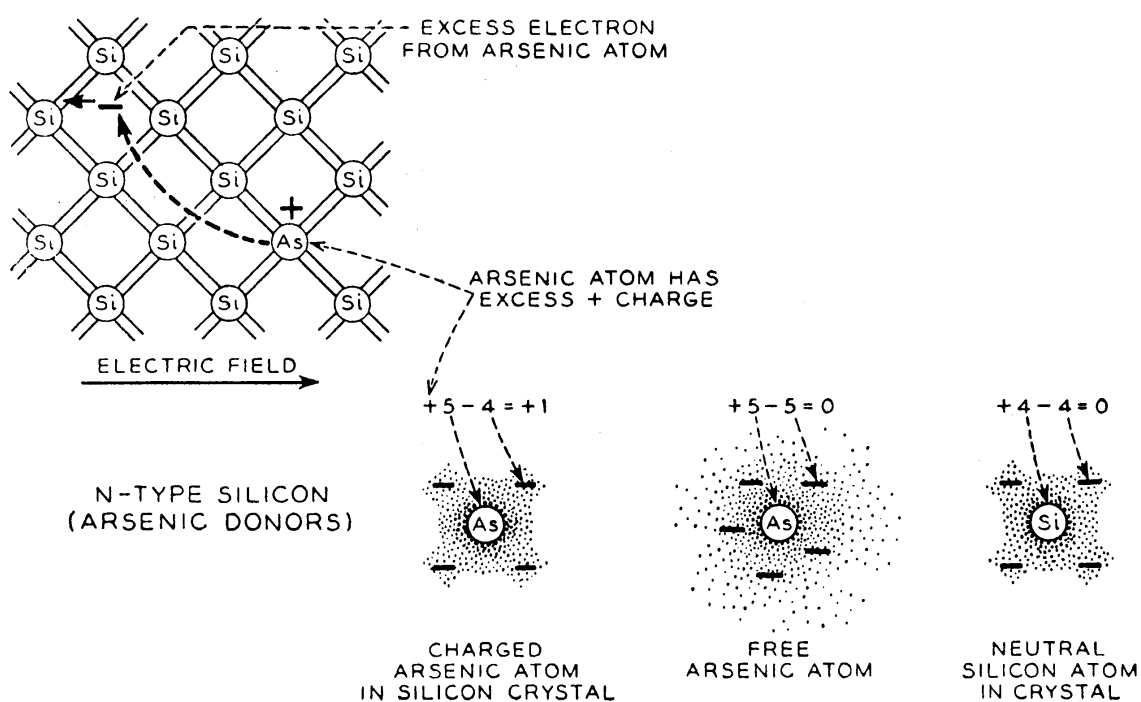
High-field region with low DC current in semiconductors is most easily achieved utilizing a  $p$ - $n$  junction.

$\Rightarrow$  Introduction of impurities to control conductivity.

The conductivity of semiconductors can be controlled by introducing special impurities.

required concentrations:  $\sim 10^{12} - 10^{18} \text{ cm}^{-3}$

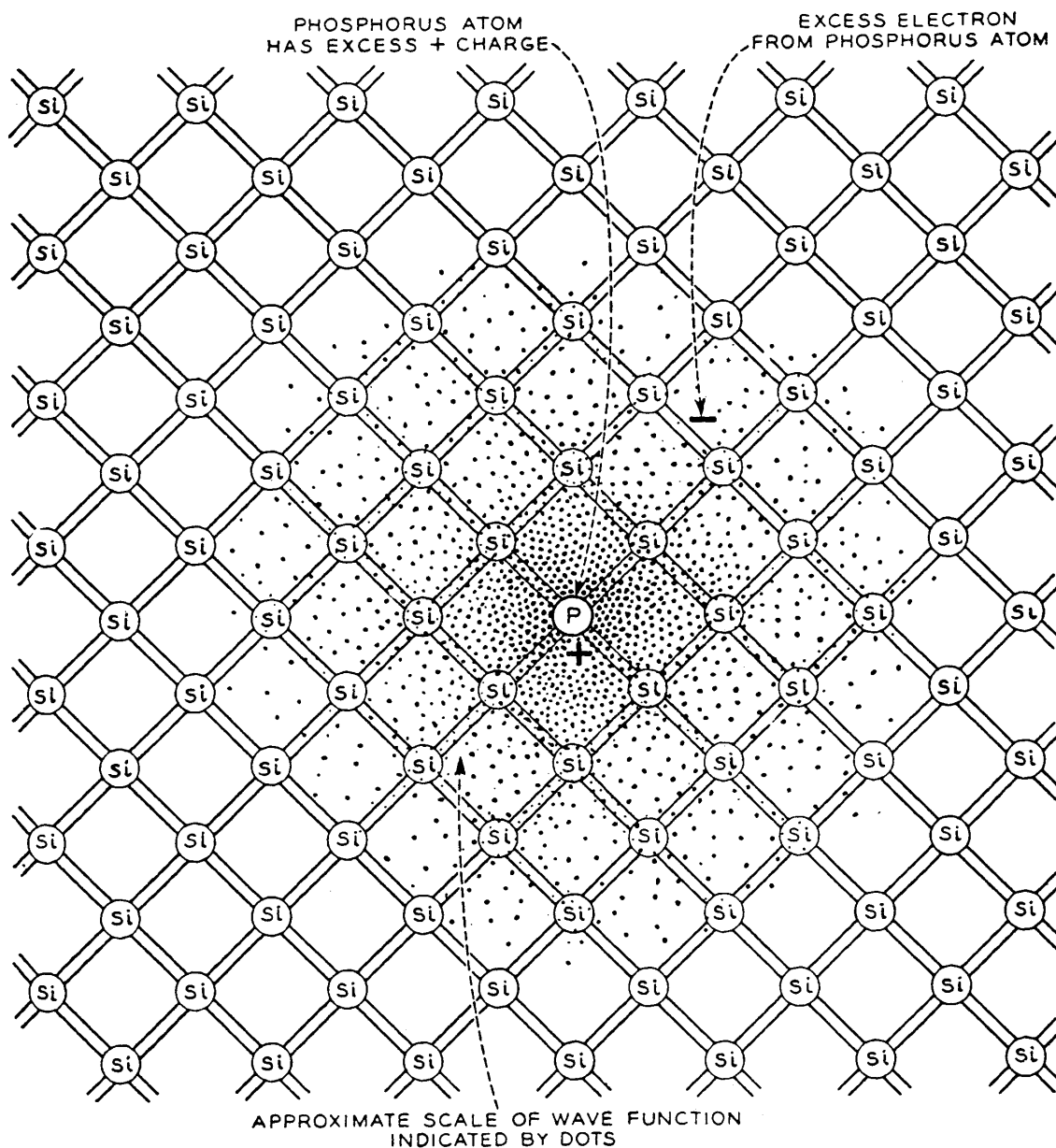
Replacing a silicon atom (group 4 in periodic table, i.e. 4 valence electrons) by an atom with 5 valence electrons, e.g. P, As, Sb, leaves one valence electron without a partner.



(from Schockley)

Since the impurity contributes an excess electron to the lattice, it is called a donor.

The wavefunction of the dopant atom extends over many neighbors.

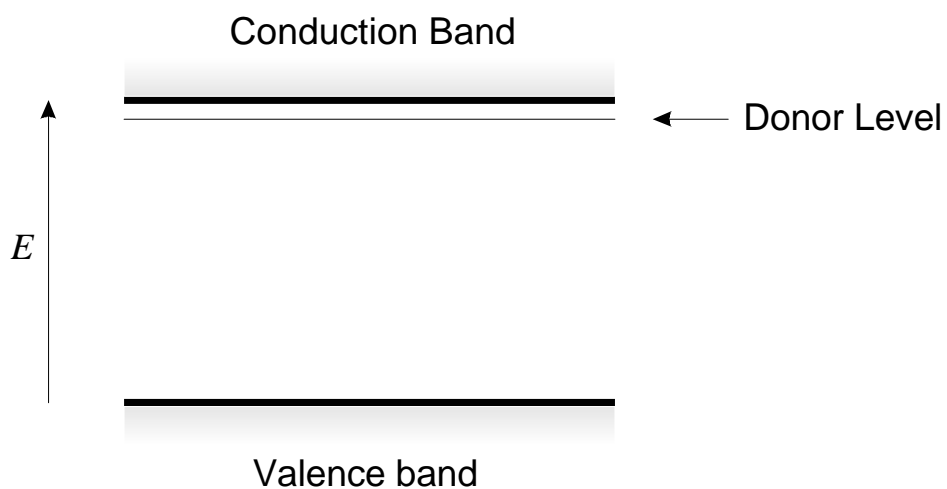


(from Shockley)

The excess electron is only loosely bound, as the coulomb force is reduced by the dielectric constant  $\epsilon$  of the medium ( $\epsilon = 12$  in Si).

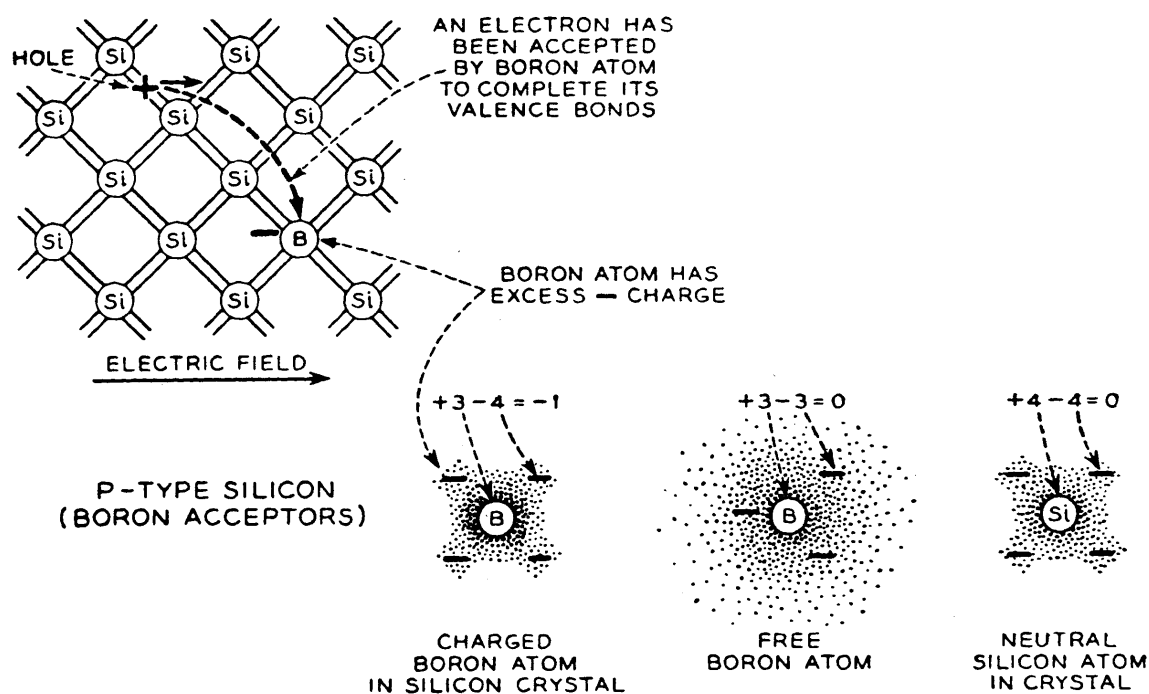
$$E_i(lattice) \propto \frac{E_i(atom)}{\epsilon^2}$$

The bound level of this unpaired electron is of order 0.01 eV below the conduction band (e.g. for P:  $E_c - 0.045$  eV).



- $\Rightarrow$  substantial ionization probability at room temperature ( $E = 0.026$  eV) – “donor”
- $\Rightarrow$  electrons in conduction band

Conversely, introducing a group 3 atom (B, Al, Ga, In) leaves a Si valence electron without a partner.



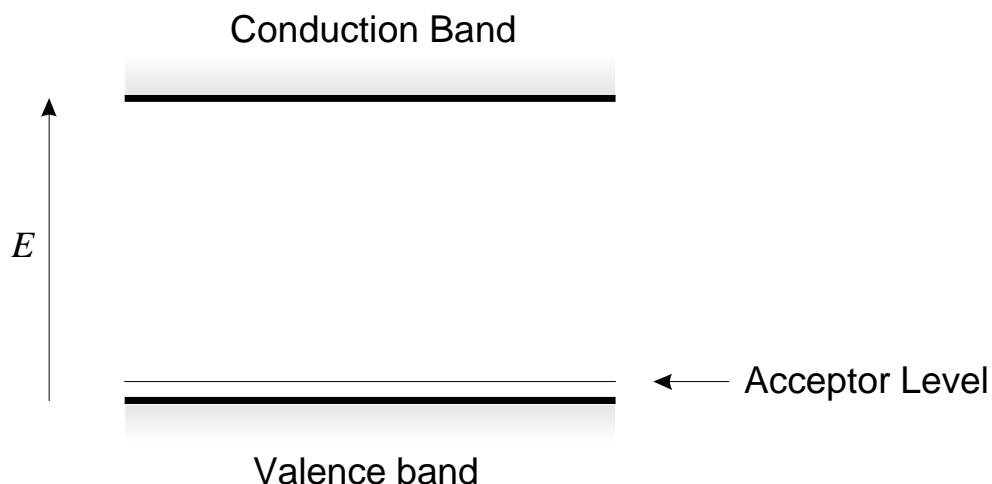
(from Shockley)

To close its shell the B atom “borrows” an electron from a lattice atom in the vicinity.

This type of dopant is called an “acceptor”.

The “borrowed” electron is bound, but somewhat less than other valence electrons since the B nucleus only has charge 3.

This introduces a bound state close to the valence band, also of order 0.01 eV from the band edge.



For example, a B atom in Si forms a state at  $E_v + 0.045$  eV.

Again, as this energy is comparable to  $kT$  at room temperature, electrons from the valence band can be excited to fill a substantial fraction of these states.

The electrons missing from the valence band form mobile charge states called “holes”, which behave similarly to an electron in the conduction band, i.e. they can move freely throughout the crystal.

Since the the charge carriers in the donor region are electrons, i.e. negative, it is called “ $n$ -type”.

Conversely, as the charge carriers in the acceptor region are holes, i.e. positive, it is called “ $p$ -type”.

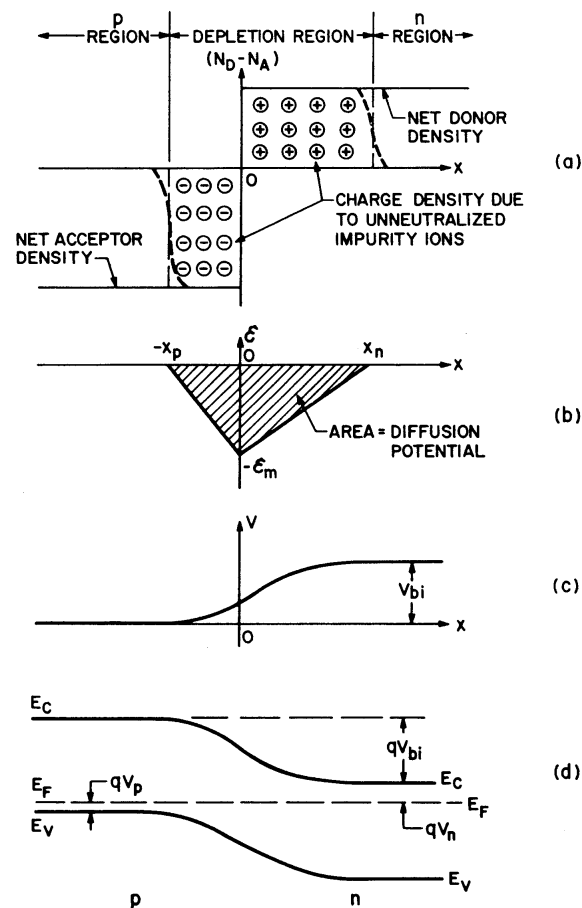


Consider a crystal suitably doped that a donor region and an acceptor adjoin each other, a “ $p$ - $n$  junction”.

Thermal diffusion will drive holes and electrons across the junction. Although the  $p$  and  $n$  regions were originally electrically neutral, as electrons diffuse from the  $n$  to the  $p$  region, they uncover their respective donor atoms, leaving a net positive charge in the  $n$  region.

This positive space charge exerts a restraining force on the electrons that diffused into the  $p$  region, i.e. diffusion of electrons into the  $p$  region builds up a potential. The diffusion depth is limited when the space charge potential exceeds the available energy for thermal diffusion.

The corresponding process also limits the diffusion of holes into the  $n$ -region.



(from Sze, *Physics of Semiconductor Devices*)

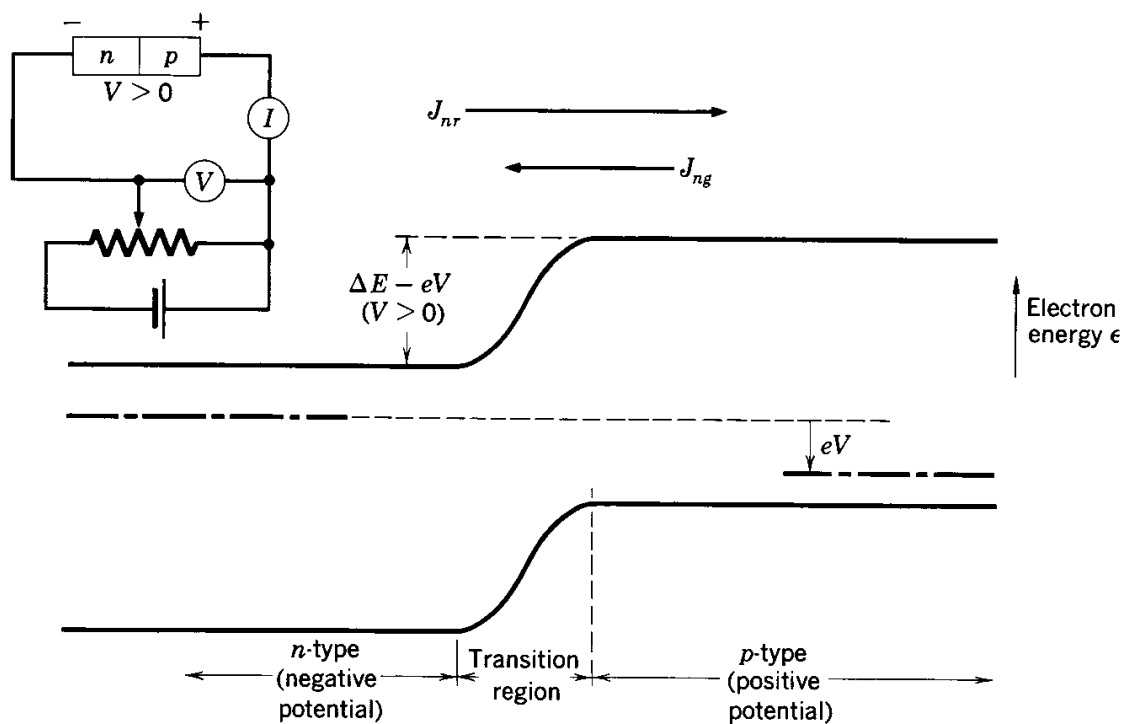
The diffusion of holes and electrons across the junction leads to a region with a reduced concentration of mobile carriers – the “depletion region”, bounded by conductive regions, which are  $n$ - and  $p$ -doped, respectively.

Furthermore, the formation of the two adjacent space charge regions builds up a potential barrier between the  $n$  and  $p$  regions, which impedes the further flow of charge.

*The magnitude of this potential barrier is typically 50 – 90% of the band-gap, depending on relative doping levels.*

This represents the situation in thermal equilibrium. By application of an external potential, two distinctly different non-equilibrium modes can be established.

- a) positive potential applied to the  $p$  region  
negative potential applied to the  $n$  region



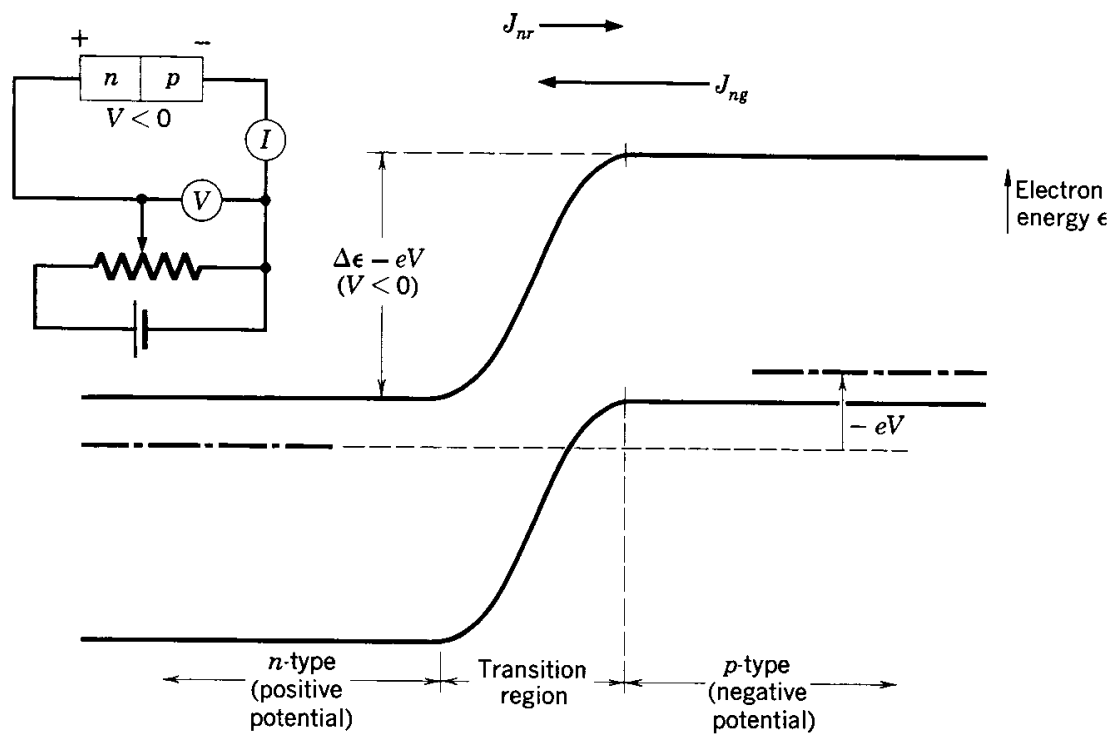
(from Kittel, *Introduction to Solid State Physics*)

The externally applied voltage reduces the potential barrier, allowing increased charge transfer across the junction.

⇒ “forward bias”

Electrons flowing from the  $n$ -region across the junction are replenished from the external voltage supply and large current flow is possible.

- b) negative potential applied to the  $p$  region  
positive potential applied to the  $n$  region



(from Kittel, *Introduction to Solid State Physics*)

This arrangement increases the potential barrier across the junction, impeding the flow of current.

⇒ “reverse bias”

The  $p$ - $n$  junction is asymmetric with respect to current flow (diode).

a) forward bias

positive supply connection  $\rightarrow p$  contact

negative supply connection  $\rightarrow n$  contact

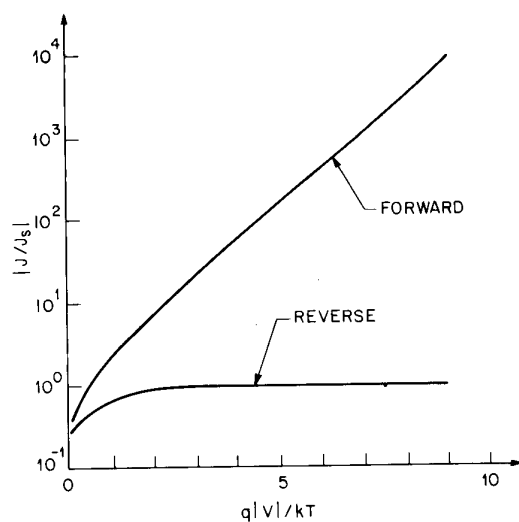
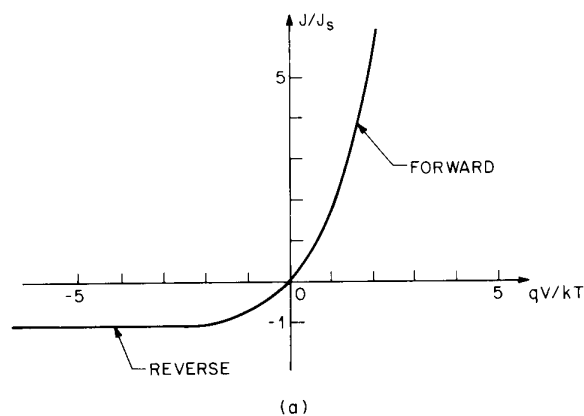
$\Rightarrow$  large current flow

b) reverse bias

positive supply connection  $\rightarrow n$  contact

negative supply connection  $\rightarrow p$  contact

$\Rightarrow$  small current flow



(from Sze, *Physics of Semiconductor Devices*)

Since the depletion region is a volume with an electric field, it by itself could be used as a radiation detector.

- The width of the depletion region is increased by reverse bias.

## 1.1 Depletion width and electric field in $p$ - $n$ junction

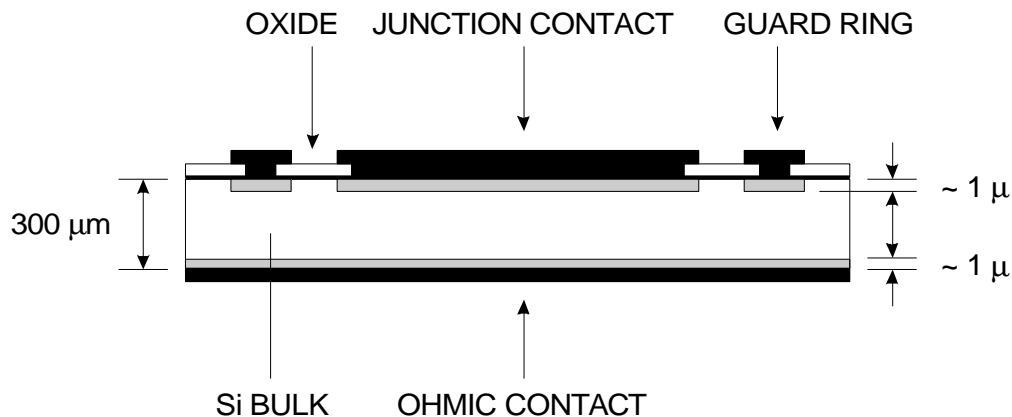
Assume a reverse bias voltage  $V_b$  and that the potential changes only in the direction perpendicular to the  $n$ - $p$  interface. Poisson's equation is then

$$\frac{d^2V}{dx^2} + \frac{Nq_e}{\epsilon} = 0$$

where  $N$  is the dopant concentration and  $q_e$  the electron charge.

Consider an abrupt junction where charge densities on the  $n$  and  $p$  sides are  $N_d q_e$  and  $N_a q_e$ , respectively.

Detector diodes are usually asymmetrically doped. The starting material (bulk) is lightly doped and the junction is formed by diffusing or ion-implanting a highly doped layer.



The external connection to the lightly doped bulk is made by an additional highly doped layer of the same type.

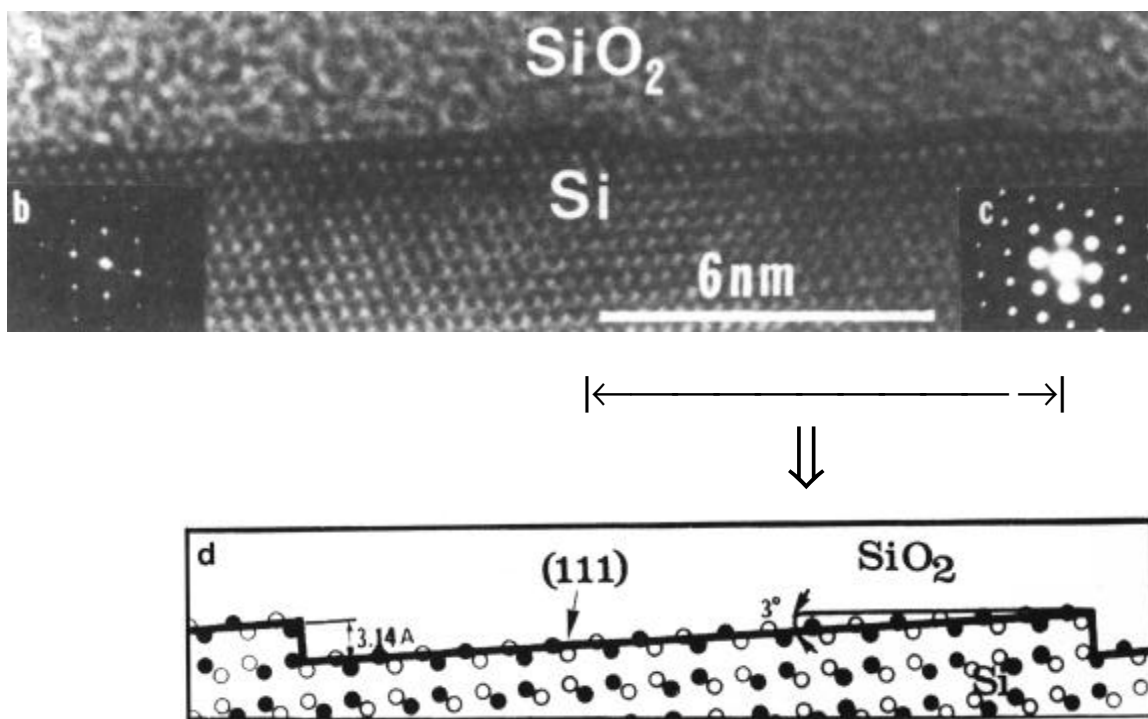
The guard ring isolates the wafer edge (saw cut) from the active region.

In the gap between the detector electrode and the guard ring it is critical to provide a neutral interface at the silicon surface to prevent formation of a conductive path.

This is best accomplished by oxide passivation ( $\text{SiO}_2$ ).

Silicon is unique in that the  $\text{SiO}_2$ -silicon interface can provide a well-controlled electrical interface.

Atomic resolution electron microscope image of  $\text{SiO}_2$ -silicon interface



(Gronsky et al., LBNL National Center for Electron Spectroscopy)

In a properly grown oxide the oxide – silicon transition is abrupt to one atomic layer.

In the asymmetrically doped diode the depletion region extends predominantly into the lightly doped bulk.

If, for example,  $N_a \gg N_d$ , the depletion region extends predominantly into the  $n$ -side and the total depletion width is

$$W \approx x_n = \sqrt{\frac{2\epsilon V_b}{q_e N_d}}.$$

The doping concentration is commonly expressed in terms of resistivity

$$\rho = (\mu q_e N)^{-1},$$

because this is a readily measurable quantity. The parameter  $\mu$  describes the relationship between the applied field and carrier velocity  $v = \mu E$ .

Using resistivity the depletion width becomes

$$W = \sqrt{2\epsilon \mu_n \rho_n V_b}.$$

Note that this introduces an artificial distinction between the  $n$ - and  $p$ -regions, because the mobilities  $\mu$  for electrons and holes are different.

*Since the mobility of holes is approximately 1/3 that of electrons,  $p$ -type material of a given doping concentration will have 3 times the resistivity of  $n$ -type material of the same concentration.*

As discussed earlier, even in the absence of an external voltage electrons and holes to diffuse across the junction, establishing a "built-in" reverse bias voltage  $V_{bi}$ . If we take this inherent bias voltage into account,  $V_b \equiv V_b + V_{bi}$  and one obtains for the one-sided junction



$$W \approx x_1 = \sqrt{\frac{2\varepsilon(V_b + V_{bi})}{q_e N_d}} = \sqrt{2\varepsilon\mu_n\rho_n(V_b + V_{bi})}.$$

For example, in  $n$ -type silicon ( $V_b$  in volts and  $\rho$  in  $\Omega\cdot\text{cm}$ )

$$W = 0.5 \mu\text{m} \times \sqrt{\rho(V_b + V_{bi})}$$

and in  $p$ -type material

$$W = 0.3 \mu\text{m} \times \sqrt{\rho(V_b + V_{bi})}$$

The depleted junction volume is free of mobile charge and thus forms a capacitor, bounded by the conducting  $p$ - and  $n$ -type semiconductor on each side.

The capacitance is

$$C = \varepsilon \frac{A}{W} = A \sqrt{\frac{\varepsilon q_e N}{2(V_b + V_{bi})}}$$

For bias voltages  $V_b \gg V_{bi}$

$$C \propto \frac{1}{\sqrt{V_b}}$$

In technical units

$$\frac{C}{A} = \frac{\varepsilon}{W} \approx 1 [\text{pF/cm}] \frac{1}{W}$$

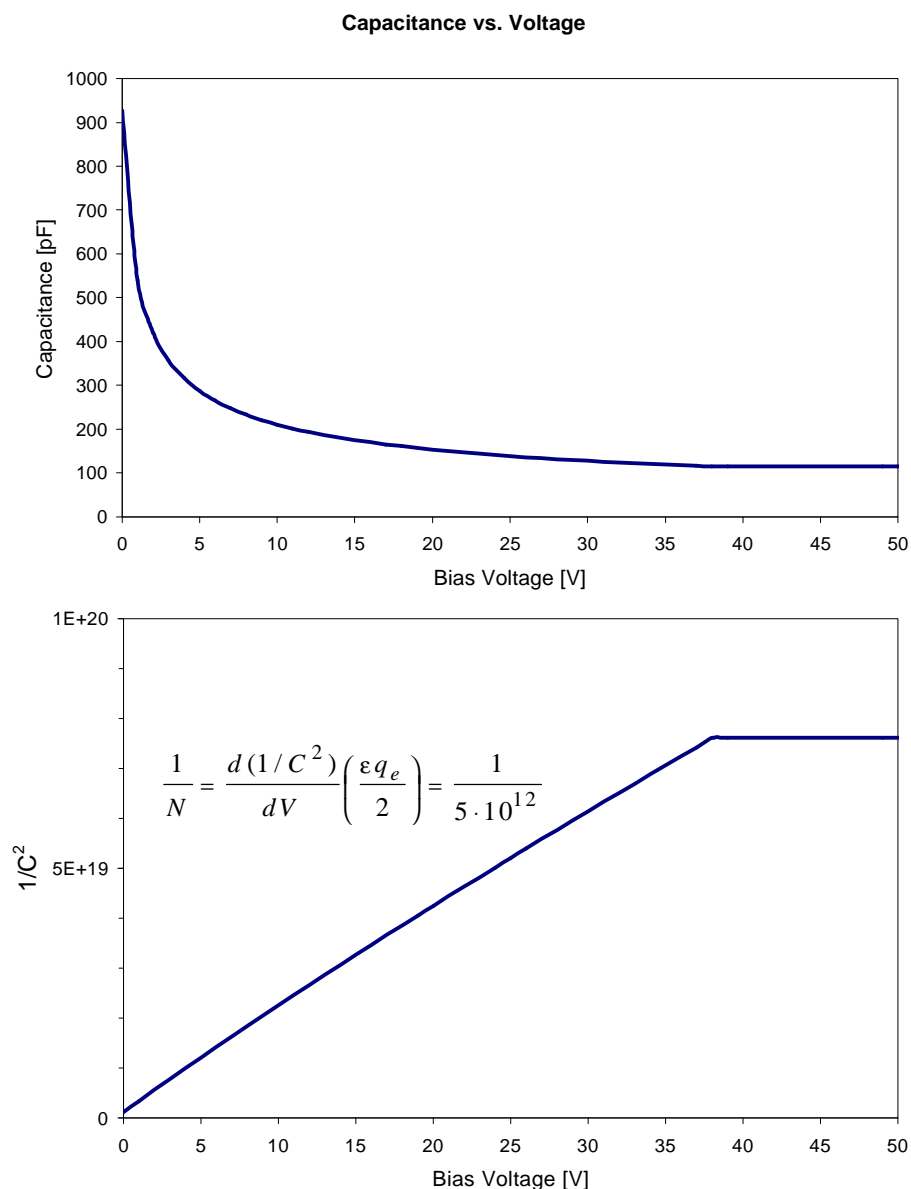
A diode with  $100 \mu\text{m}$  thickness has about  $1 \text{ pF/mm}^2$ .

The capacitance vs. voltage characteristic of a diode can be used to determine the doping concentration of the detector material.

$$\frac{C}{A} = \sqrt{\frac{\epsilon q_e N}{2(V_b + V_{bi})}}$$

In a plot of  $(A/C)^2$  vs. the detector bias voltage  $V_b$  the slope of the voltage dependent portion yields the doping concentration  $N$ .

Example: Si pad detector,  $A = 1 \text{ cm}^2$ ,  $100 \text{ } \mu\text{m}$  thick



## 1.2. Charge Collection

Mobile electrons and holes formed by radiation move under the influence of the electric field in the junction.

Although electrons and holes move in opposite directions, their contribution to the signal current is of the same polarity.

The time required for a charge carrier to traverse the sensitive volume is called the collection time.

The local velocity of a charge carrier

$$\vec{v}(x) = \mu \vec{E}(x)$$

Note that the velocity does not depend on the time during which the charge carrier is accelerated, as in normal ballistic motion, since the charge carrier also interacts with the crystal lattice, exciting lattice vibrations (phonons). Since the characteristic times for phonon excitation are much smaller than the transport times, the carrier is always in equilibrium with the lattice, so the velocity is only a function of the electric field, at every position in the depletion region.

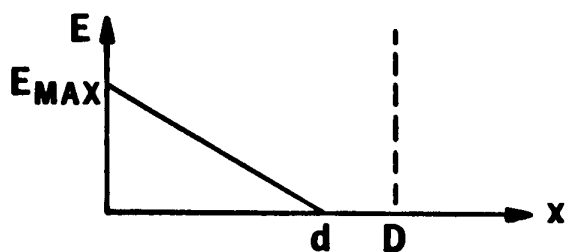
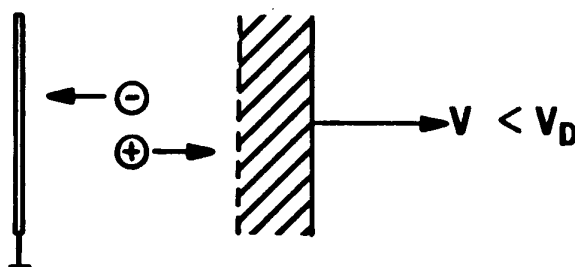
In Si the mobility at low fields is

$$\begin{aligned}\mu_n &= 1350 \text{ cm}^2/\text{Vs for electrons and} \\ \mu_p &= 480 \text{ cm}^2/\text{Vs for holes.}\end{aligned}$$

The mobility decreases at high fields  $\mu \propto 1/E$ , so that electrons attain a constant velocity of  $10^7 \text{ cm/s}$  at  $E > 5 \cdot 10^4 \text{ V/cm}$  (corresponding to a transit time of  $10 \text{ ps}/\mu\text{m}$ )

As the bias voltage is increased from  $V_{bi}$  the electric field

$$E(x) = \frac{2(V_b + V_{bi})}{W} \left( \frac{x}{W} - 1 \right)$$



The detector bulk is completely depleted of mobile charge when  $W = d$ , the thickness of the substrate. This occurs at the externally applied depletion voltage

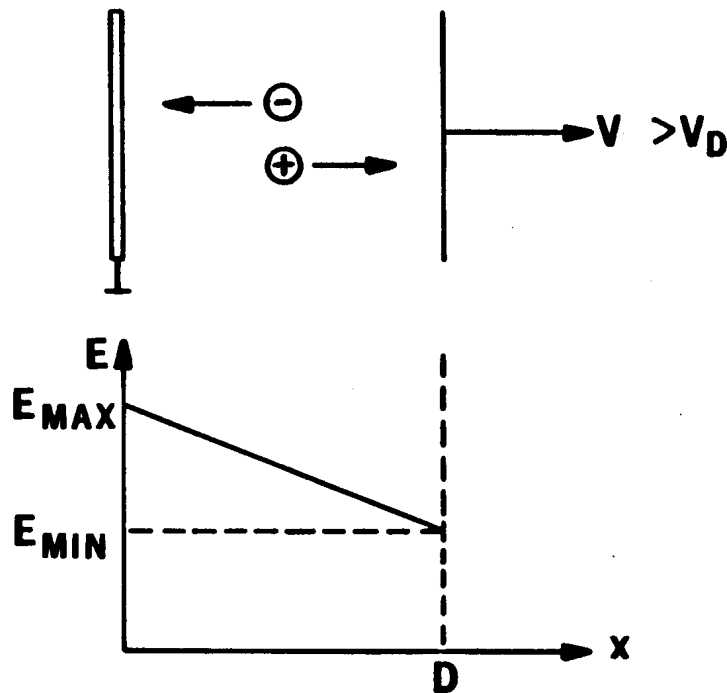
$$V_d = \frac{q_e N_d W^2}{2\epsilon} - V_{bi} .$$

The field drops linearly from its maximum value at the junction to zero at the opposite contact.

Increasing the bias voltage beyond this value adds a uniform field due to the voltage beyond depletion, yielding a distribution

$$E(x) = \frac{2V_{di}}{W} \left( 1 - \frac{x}{W} \right) + \frac{V_b - V_{di}}{W}$$

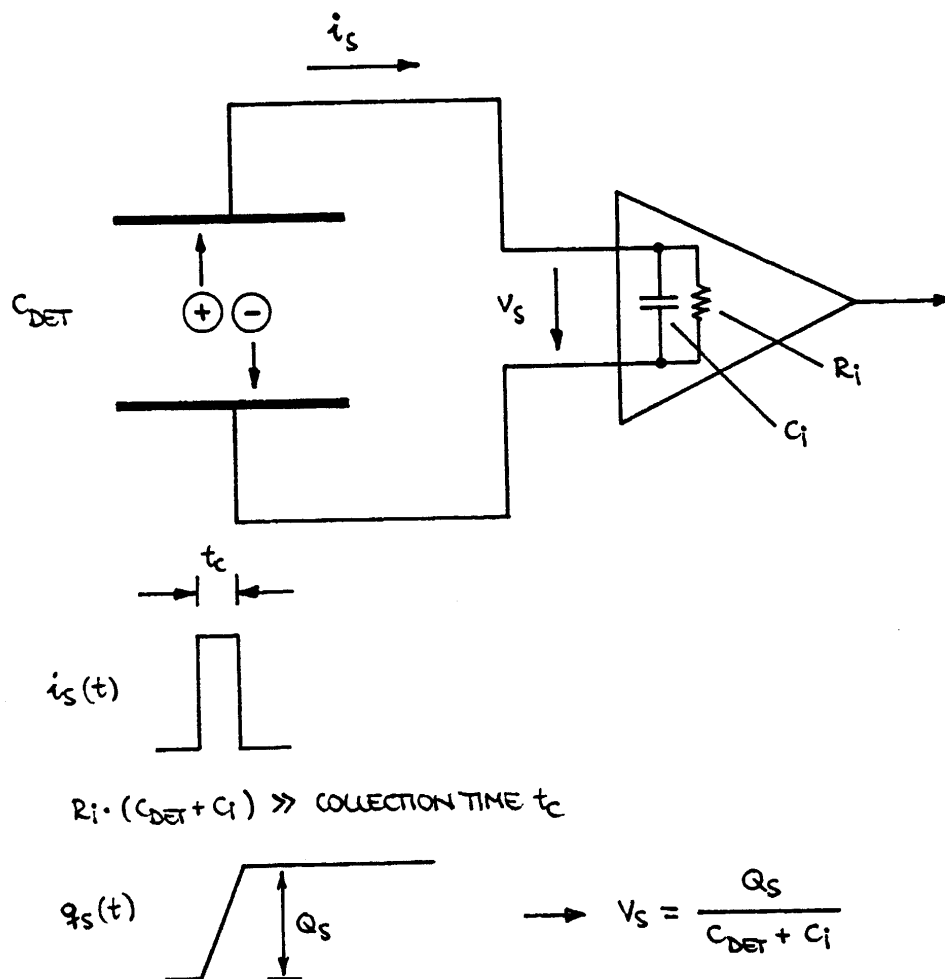
where  $V_{di} \equiv V_d + V_{bi}$  has been defined as the internal depletion voltage.



For  $n$ -type silicon of  $10 \text{ k}\Omega\cdot\text{cm}$  resistivity,  
 a detector thickness of  $300 \text{ }\mu\text{m}$ , and  
 a reverse bias voltage  $V_b = 60\text{V} = 2V_d$   
 (i.e.  $E_0 = 2 \cdot 10^3$  and  $E_I = 10^3 \text{ V/cm}$ )

the collection times for electrons and holes are 12 and 36 ns.

## 2. Induced Charge



When does the current pulse begin?

- a) when the charge reaches the electrode?  
 or  
 b) when the charge begins to move?

Although the first answer is quite popular (encouraged by the phrase “charge collection”), the second is correct.

When a charge pair is created, both the positive and negative charges couple to the electrodes and induce mirror charges of equal magnitude.

As the positive charge moves toward the negative electrode, it couples more strongly to it and less to the positive electrode.

Conversely, the negative charge couples more to the positive electrode and less to the negative electrode.

The net effect is a negative current at the positive electrode and a positive current at the negative electrode, due to both the positive and negative charges.

## Magnitude of the Induced Charge

(S. Ramo, Proc. IRE **27** (1939) 584)

Consider a mobile charge in the presence of any number of grounded electrodes.

Surround the charge  $q$  with a small equipotential sphere. Then, if  $V$  is the potential of the electrostatic field, in the region between conductors

$$\nabla^2 V = 0$$

Call  $V_q$  the potential of the small sphere and note that  $V = 0$  on the conductors. Applying Gauss' law yields

$$\int_{\text{sphere's surface}} \frac{\partial V}{\partial n} ds = 4\pi q$$

Next, consider the charge removed and one conductor A raised to unit potential.

Call the potential  $V_1$ , so that

$$\nabla^2 V_1 = 0$$

in the space between the conductors, including the site where the charge was situated. Call the new potential at this point  $V_{q1}$ .

Green's theorem states that

$$\int_{\text{volume between boundaries}} (V_1 \nabla^2 V - V \nabla^2 V_1) dv = - \int_{\text{boundary surfaces}} \left[ V_1 \frac{\partial V}{\partial n} - V \frac{\partial V_1}{\partial n} \right] ds$$



Choose the volume to be bounded by the conductors and the tiny sphere.

Then the left hand side is 0 and the right hand side may be divided into three integrals:

1. Over the surfaces of all conductors except A. This integral is 0 since on these surfaces  $V = V_1 = 0$ .
2. Over the surface of A. As  $V_1 = 1$  and  $V = 0$  this reduces to

$$- \int_{\text{surface A}} \frac{\partial V}{\partial n} ds$$

3. Over the surface of the sphere.

$$-V_{q1} \int_{\text{sphere's surface}} \frac{\partial V}{\partial n} ds + V_q \int_{\text{sphere's surface}} \frac{\partial V_1}{\partial n} ds$$

The second integral is 0 by Gauss' law, since in this case the charge is removed.

Combining these three integrals yields

$$0 = - \int_{\text{surface A}} \frac{\partial V}{\partial n} ds - V_{q1} \int_{\text{sphere's surface}} \frac{\partial V}{\partial n} ds = 4\pi Q_A - 4\pi q V_{q1}$$

or

$$Q_A = q V_{q1}$$

If the charge  $q$  moves in direction  $x$ , the current on electrode A is

$$i_A = \frac{dQ_A}{dt} = q \frac{dV_{q1}}{dt} = q \left( \frac{\partial V_{q1}}{\partial x} \frac{dx}{dt} \right)$$

Since the velocity of motion

$$\frac{dx}{dt} = v_x$$

the induced current on electrode A is

$$i_A = q v_x \frac{\partial V_{q1}}{\partial x}$$

where  $V_{q1}$  is the “weighting potential” that describes the coupling of a charge at any position to electrode A.

The weighting potential for a specific electrode is obtained by setting the potential of the electrode to 1 and setting all other electrodes to potential 0.

- If a charge  $q$  moves along any path  $s$  from position 1 to position 2, the net induced charge on electrode  $k$  is

$$\Delta Q_k = q(V_{q1}(2) - V_{q1}(1)) \equiv q(\Phi_k(2) - \Phi_k(1))$$

- The instantaneous current can be expressed in terms of a weighting field

$$i_k = -q \vec{v} \cdot \vec{F}_k$$

The weighting field is determined by applying unit potential to the measurement electrode and 0 to all others.

*Note that the electric field and the weighting field are distinctly different.*

- The electric field determines the charge trajectory and velocity
- The weighting field depends only on geometry and determines how charge motion couples to a specific electrode.
- Only in 2-electrode configurations are the electric field and the weighting field of the same form.

**Example 1: Parallel plate geometry, disregarding space charge**  
(semiconductor detector with very large overbias)

Assume a voltage  $V_b$  applied to the detector. The distance between the two parallel electrodes is  $d$ .

The electric field that determines the motion of charge in the detector is

$$E = \frac{V_b}{d}$$

so the velocity of the charge

$$v = \mu E = \mu \frac{V_b}{d}$$

The weighting field is obtained by applying unit potential to the collection electrode and grounding the other.

$$E_Q = \frac{1}{d}$$

so the induced current

$$i = qvE_Q = q\mu \frac{V_b}{d} \frac{1}{d} = q\mu \frac{V_b}{d^2}$$

since both the electric field and the weighting field are uniform throughout the detector, the current is constant until the charge reaches its terminal electrode.

Assume that the charge is created at the opposite electrode and traverses the detector thickness  $d$ .

The required collection time, i.e. the time required to traverse the detector thickness  $d$

$$t_c = \frac{d}{v} = \frac{d}{\mu \frac{V_b}{d}} = \frac{d^2}{\mu V_b}$$

The induced charge

$$Q = it_c = q\mu \frac{V_b}{d^2} \frac{d^2}{\mu V_b} = q$$

Next, assume an electron-hole pair formed at coordinate  $x$  from the positive electrode.

The collection time for the electron

$$t_{ce} = \frac{x}{v_e} = \frac{xd}{\mu_e V_b}$$

and the collection time for the hole

$$t_{ch} = \frac{d-x}{v_h} = \frac{(d-x)d}{\mu_h V_b}$$

Since electrons and holes move in opposite directions, they induce current of the same sign at a given electrode, despite their opposite charge.

The induced charge due to the motion of the electron

$$Q_e = q_e \mu_e \frac{V_b}{d^2} \frac{xd}{\mu_e V_b} = q_e \frac{x}{d}$$

whereas the hole contributes

$$Q_h = q_e \mu_h \frac{V_b}{d^2} \frac{(d-x)d}{\mu_h V_b} = q_e \left(1 - \frac{x}{d}\right)$$

Assume that  $x = d/2$ . After the collection time for the electron

$$t_{ce} = \frac{d^2}{2\mu_e V_b}$$

it has induced a charge  $q_e/2$ .

At this time the hole, due to its lower mobility  $\mu_h \approx \mu_e/3$ , has induced  $q_e/6$ , yielding a cumulative induced charge of  $2q_e/3$ .

After the additional time for the hole collection, the remaining charge  $q_e/3$  is induced, yielding the total charge  $q_e$ .

In this configuration

- Electrons and holes contribute equally to the currents on both electrodes
- The instantaneous current at any time is the same (although of opposite sign) on both electrodes

The continuity equation (Kirchhoff's law) must be satisfied

$$\sum_k i_k = 0$$

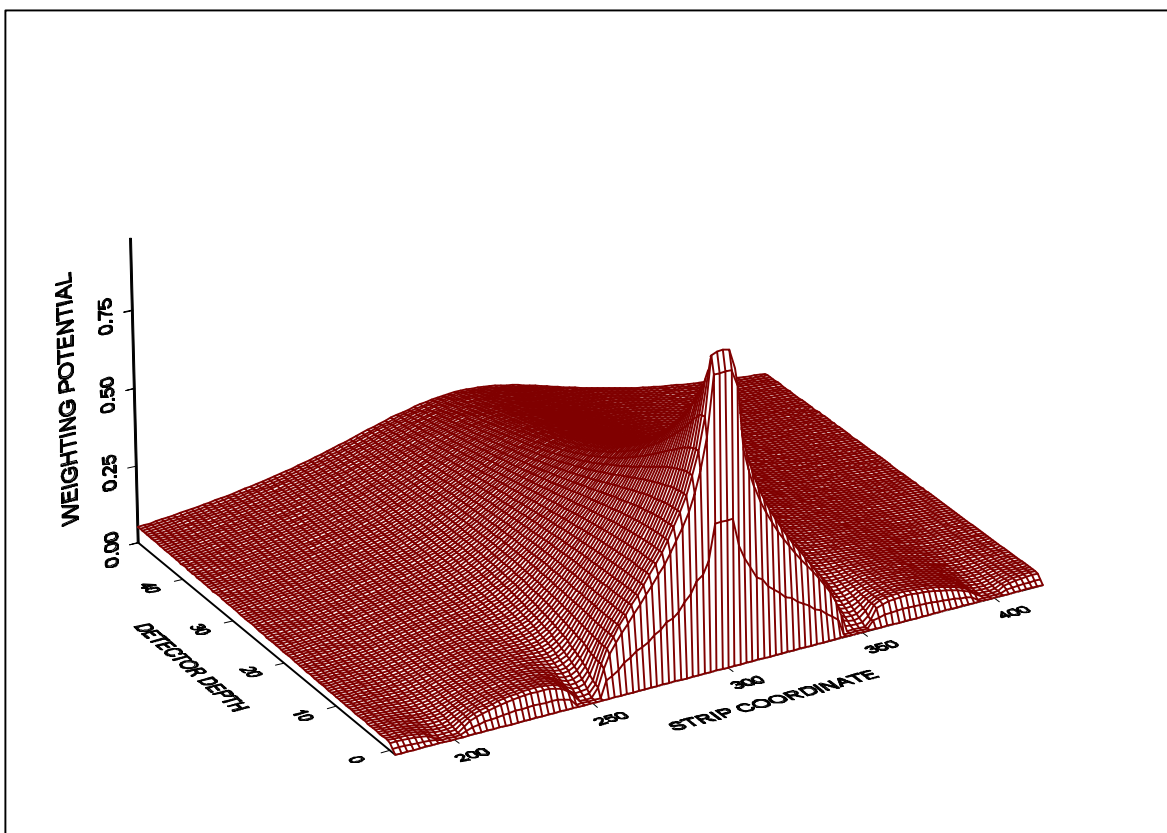
Since  $k=2$ :  $i_1 = -i_2$

## Example 2: Double-Sided Strip Detector

The strip pitch is assumed to be small compared to the detector thickness.

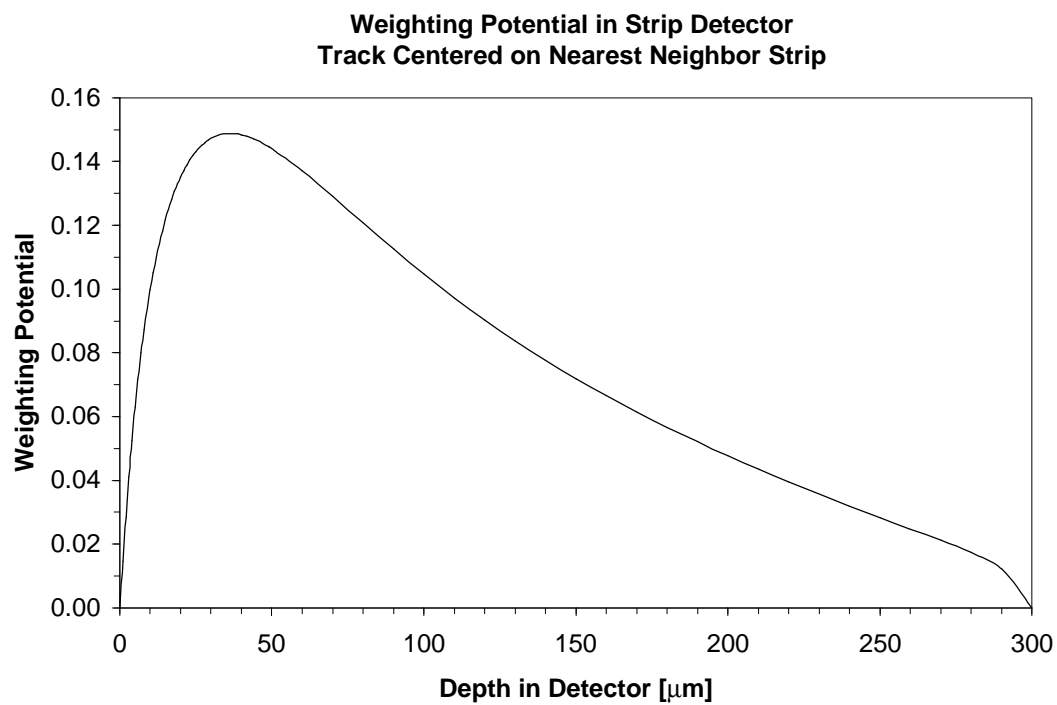
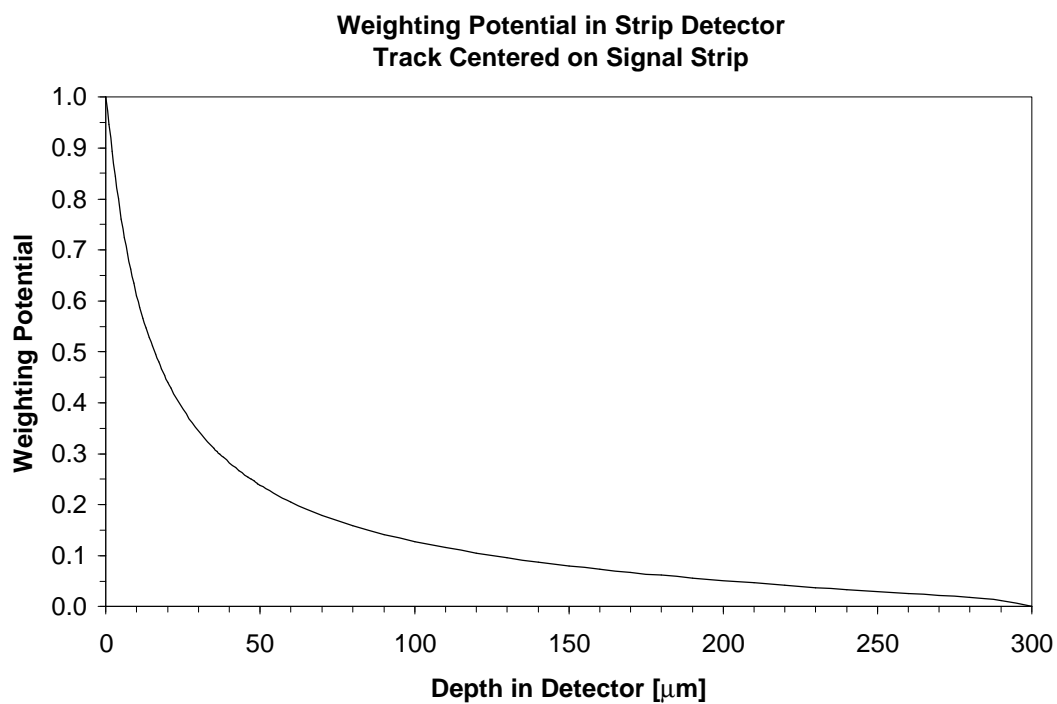
The electric field is similar to a parallel-plate geometry, except in the immediate vicinity of the strips.

The signal weighting potential, however is very different.



Weighting potential for a 300  $\mu\text{m}$  thick strip detector with strips on a pitch of 50  $\mu\text{m}$ . Only 50  $\mu\text{m}$  of depth are shown.

## Cuts through the weighting potential





Consider an electron-hole pair  $q_n, q_p$  originating on a point  $x_0$  on the center-line of two opposite strips of a double-sided strip detector. The motion of the electron towards the  $n$ -electrode  $x_n$  is equivalent to the motion of a hole in the opposite direction to the  $p$ -electrode  $x_p$ . The total induced charge on electrode  $k$  after the charges have traversed the detector is

$$Q_k = q_p [\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] + q_n [\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$

since the hole charge  $q_p = q_e$  and  $q_n = -q_e$

$$Q_k = q_e [\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] - q_e [\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$

$$Q_k = q_e [\Phi_{Qk}(x_p) - \Phi_{Qk}(x_n)]$$

If the signal is measured on the  $p$ -electrode, collecting the holes,

$$\Phi_{Qk}(x_p) = 1,$$

$$\Phi_{Qk}(x_n) = 0$$

and  $Q_k = q_e$ .

If, however, the charge is collected on the neighboring strip  $k+1$ , then

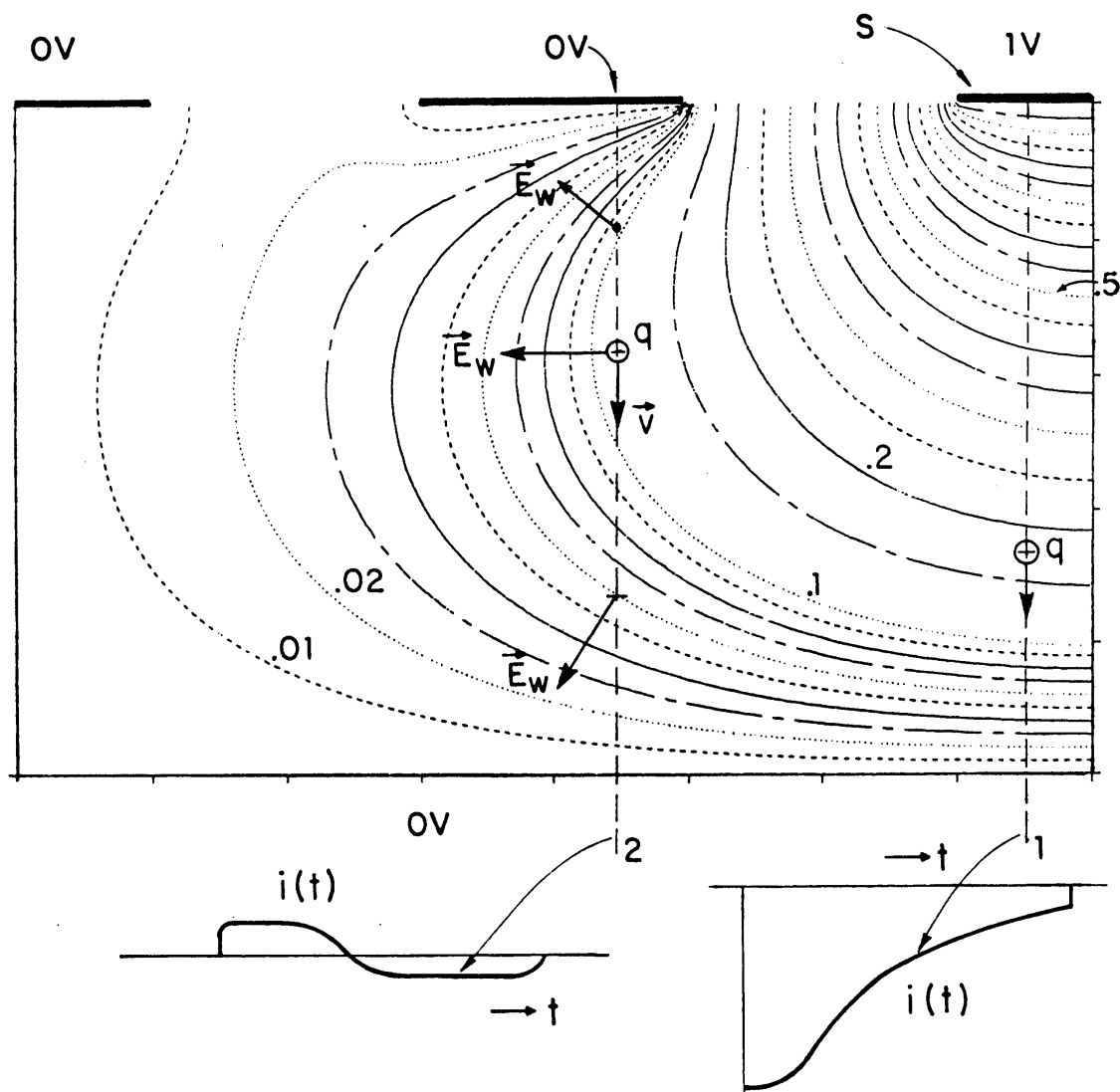
$$\Phi_{Qk+1}(x_p) = 0,$$

$$\Phi_{Qk+1}(x_n) = 0$$

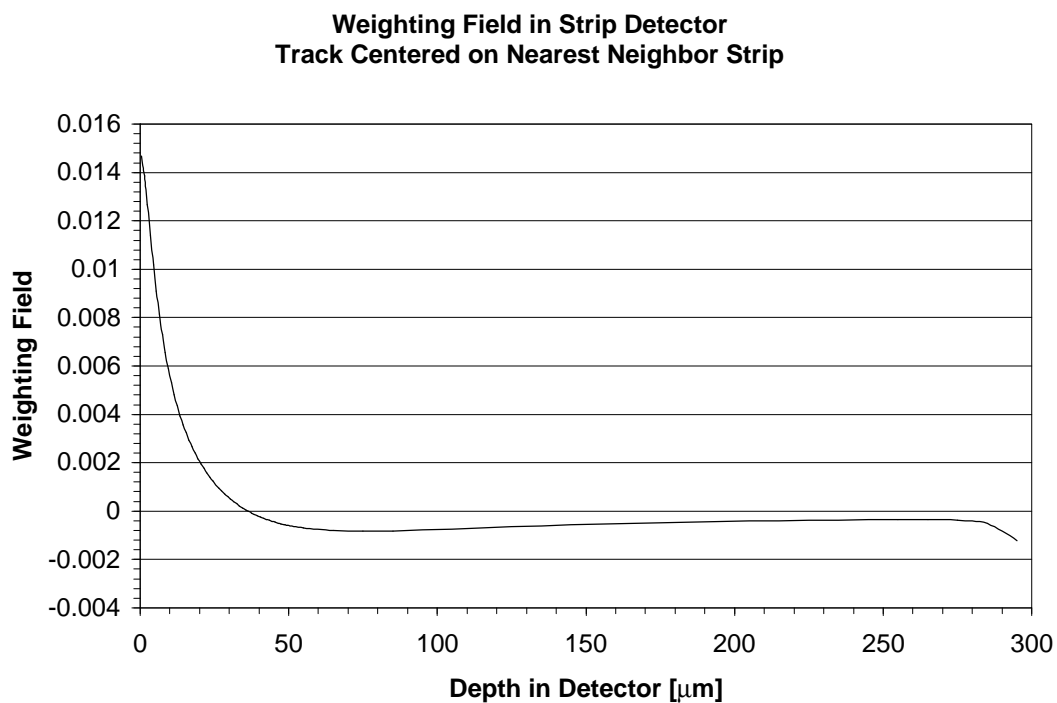
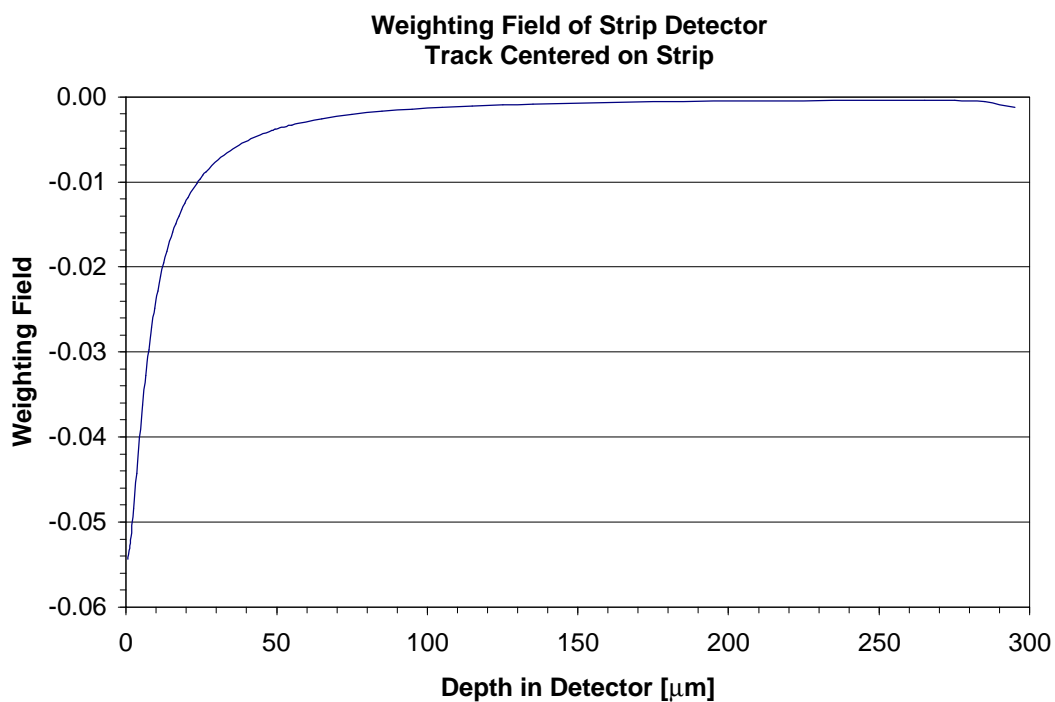
and  $Q_{k+1} = 0$ .

In general, if moving charge does not terminate on the measurement electrode, signal current will be induced, but the current changes sign and integrates to zero.

This is illustrated in the following schematic plot of the weighting field in a strip detector (from Radeka)



# Cuts through the Weighting Field in a Strip Detector ( $d = 300\ \mu\text{m}$ , $p = 50\ \mu\text{m}$ )



Note, however that this charge cancellation on “non-collecting” electrodes relies on the motion of both electrons and holes.

Assume, for example, that the holes are stationary, so they don't induce a signal. Then the first term of the first equation above vanishes, which leaves a residual charge

$$Q_k = q_e [\Phi_{Qk}(x_0) - \Phi_{Qk}(x_n)]$$

since for any coordinate not on an electrode

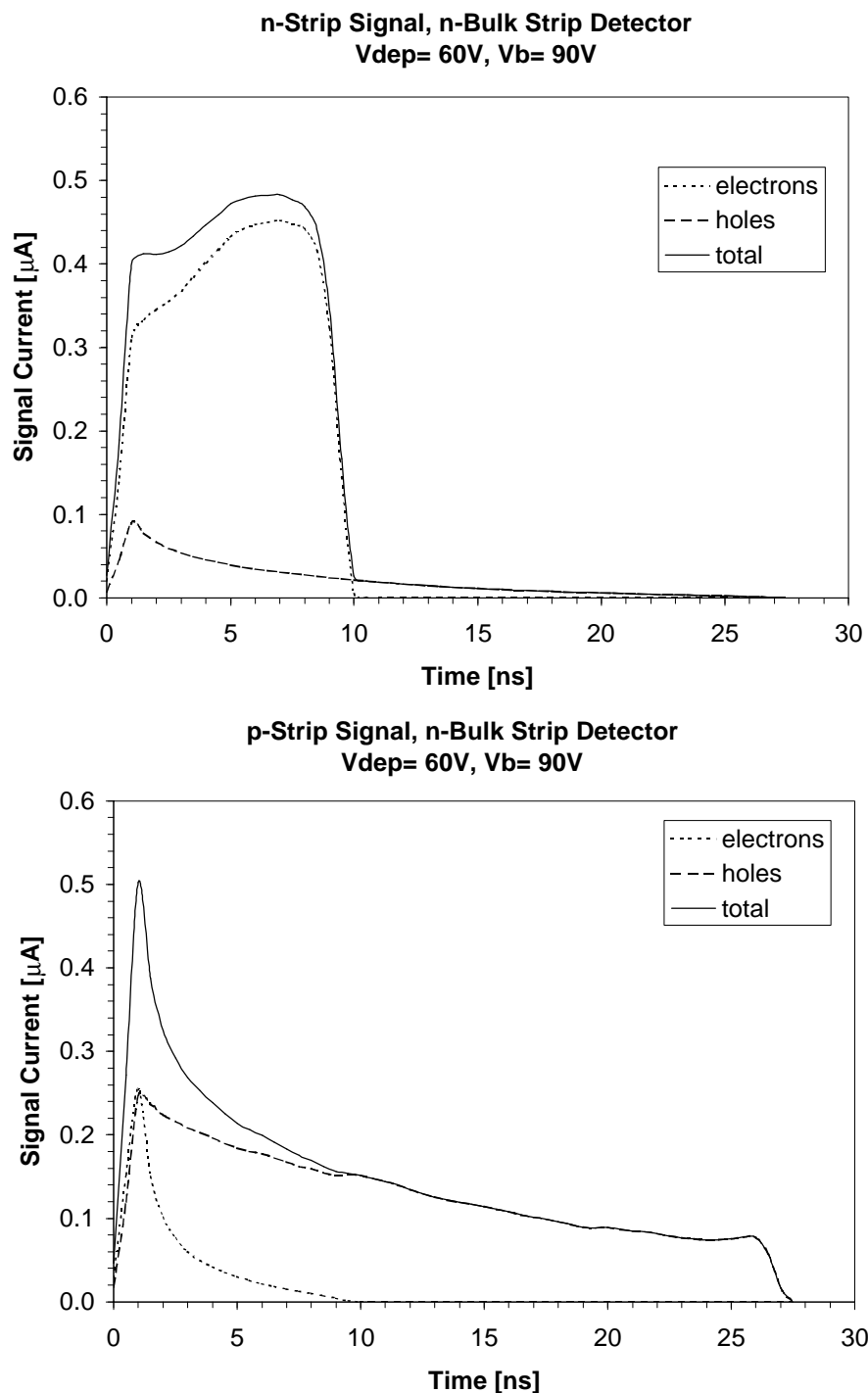
$$Q_k(x_0) \neq 0,$$

although it may be very small.

An important consequence of this analysis is that one cannot simply derive pulse shapes by analogy with a detector with contiguous electrodes (i.e. a parallel plate detector of the same overall dimensions as a strip detector). Specifically,

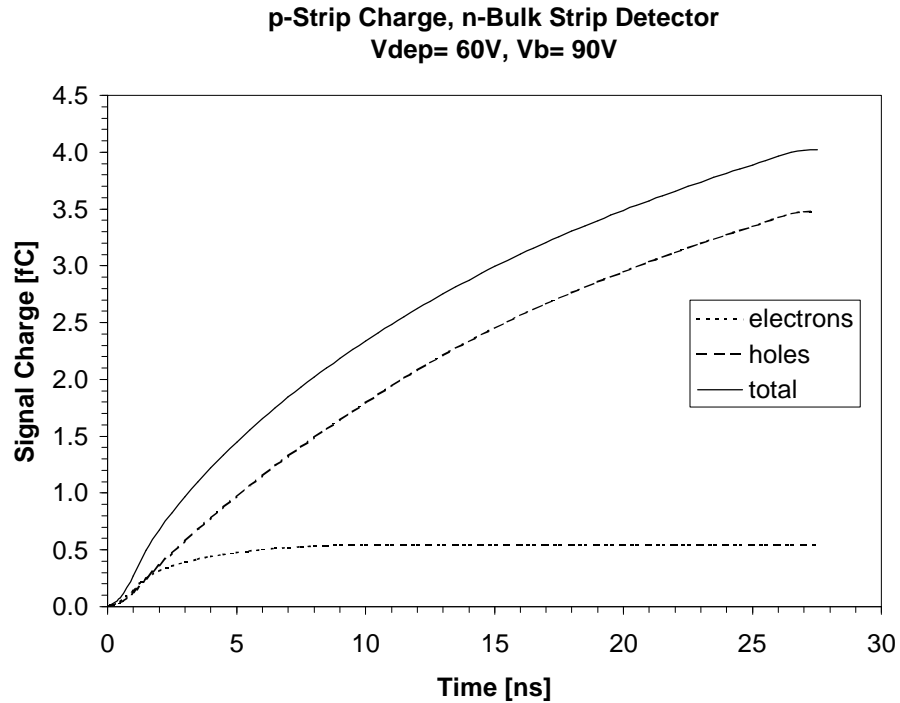
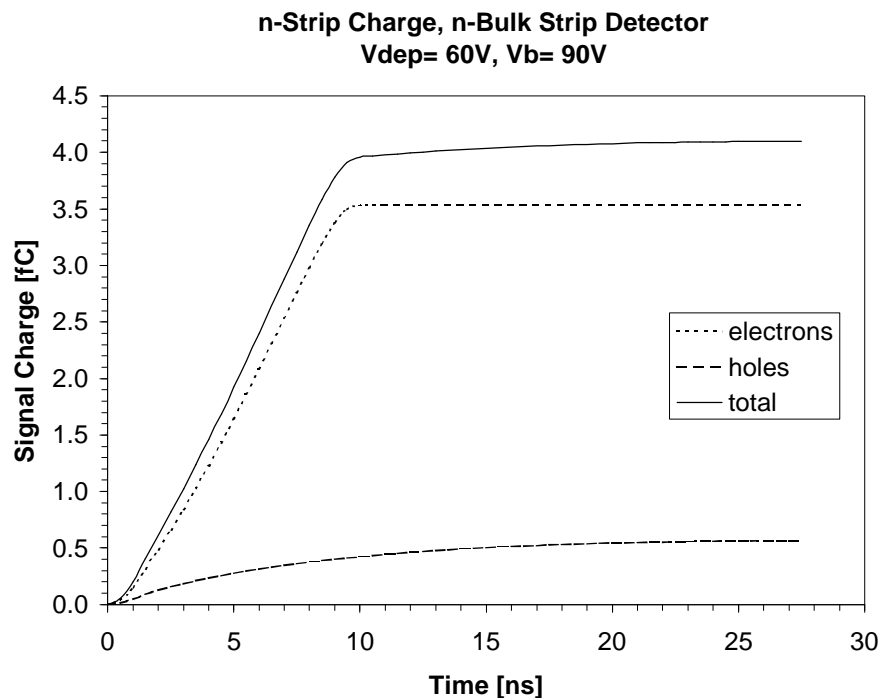
1. the shape of the current pulses can be quite different,
2. the signals seen on opposite strips of a double-sided detector are not the same (although opposite in sign), and
3. the net induced charge on the  $p$ - or  $n$ -side is not split evenly between electrons and holes.
  - Because the weighting potential is strongly peaked near the signal electrode, most of the charge is induced when the moving charge is near the signal electrode.
  - As a result, most of the signal charge is due to the charge terminating on the signal electrode.

## Current pulses in strip detectors



The duration of the electron and hole pulses is determined by the time required to traverse the detector as in a parallel-plate detector, but the shapes are very different.

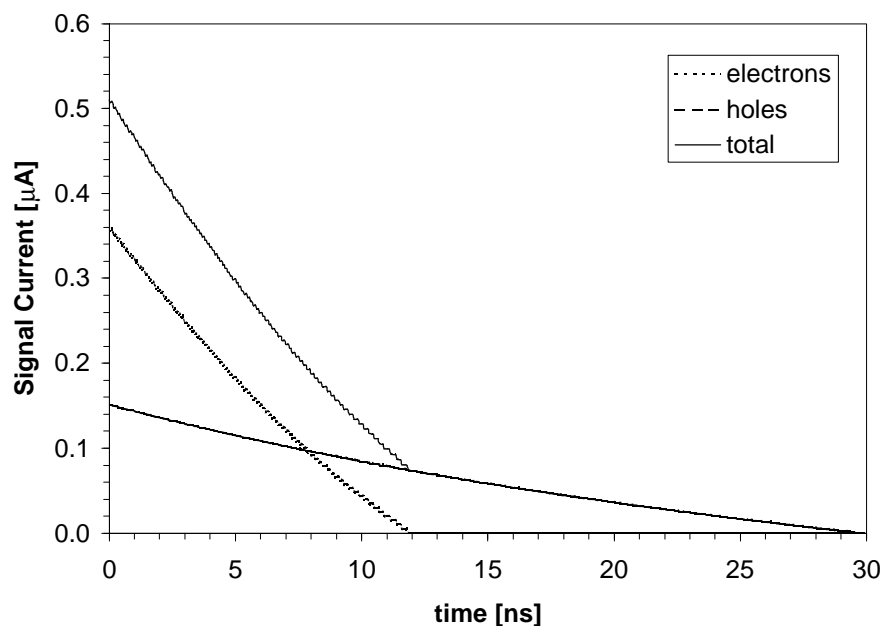
## Strip Detector Signal Charge Pulses



For comparison:

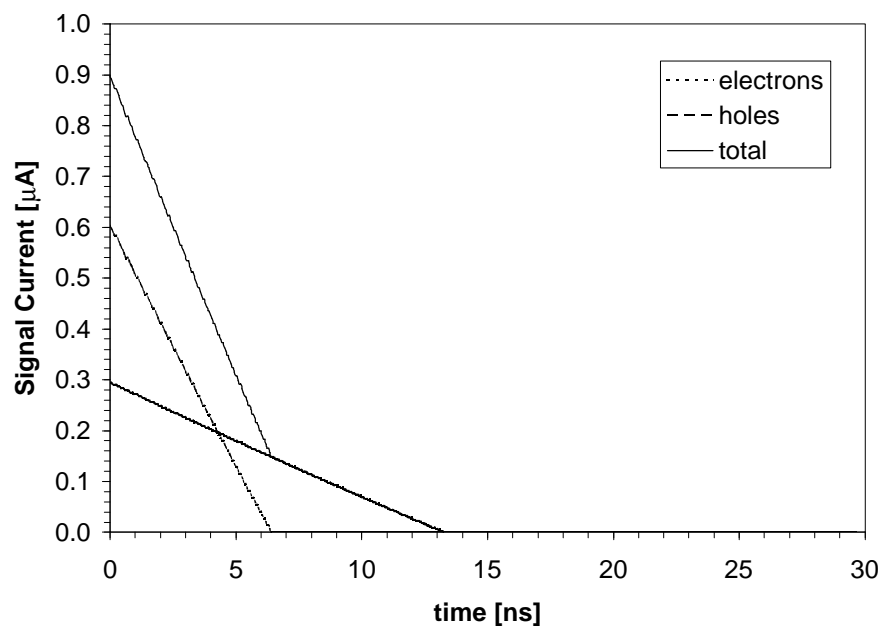
Current pulses in pad detectors

Pad Detector,  $V_{dep}=60V$ ,  $V_b=90V$



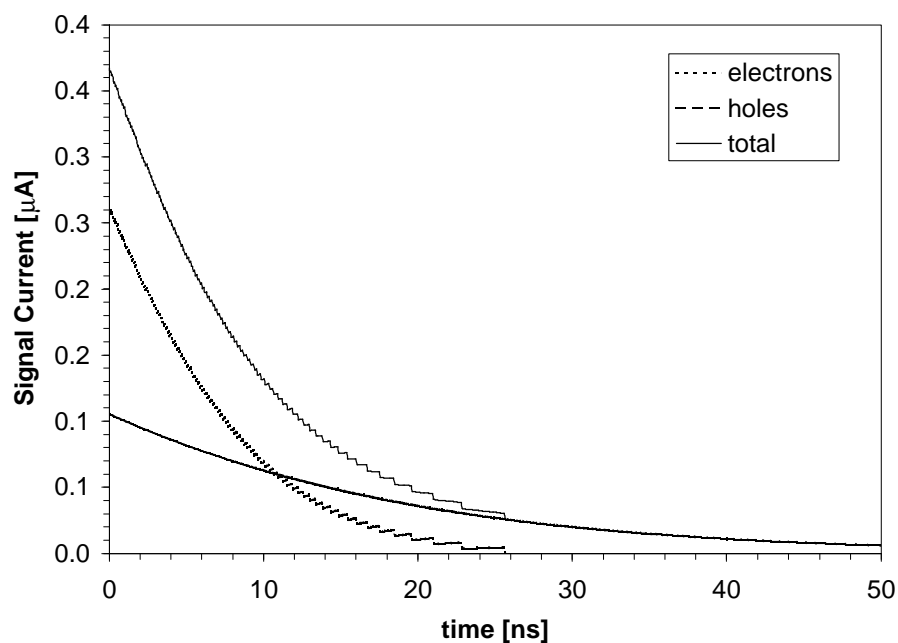
For the same depletion and bias voltages the pulse durations are the same as in strip detectors. Overbias decreases the collection time.

Pad Detector,  $V_{dep}=60V$ ,  $V_b=200V$

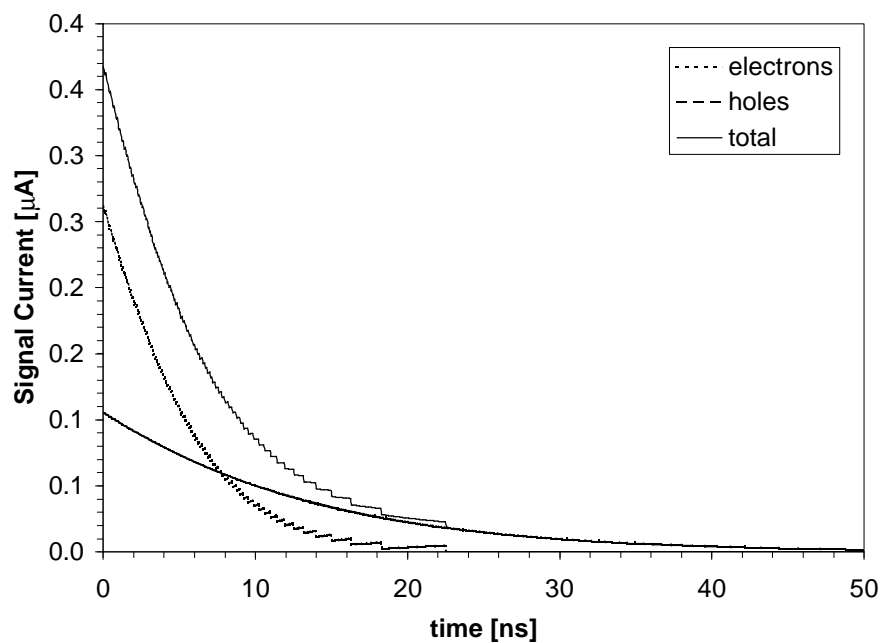


Operation at or under full depletion leads to long “tails” from the low-field region.

Pad Detector,  $V_{dep}=60V$ ,  $V_b=60V$



Pad Detector,  $V_{dep}=60V$ ,  $V_b=30V$

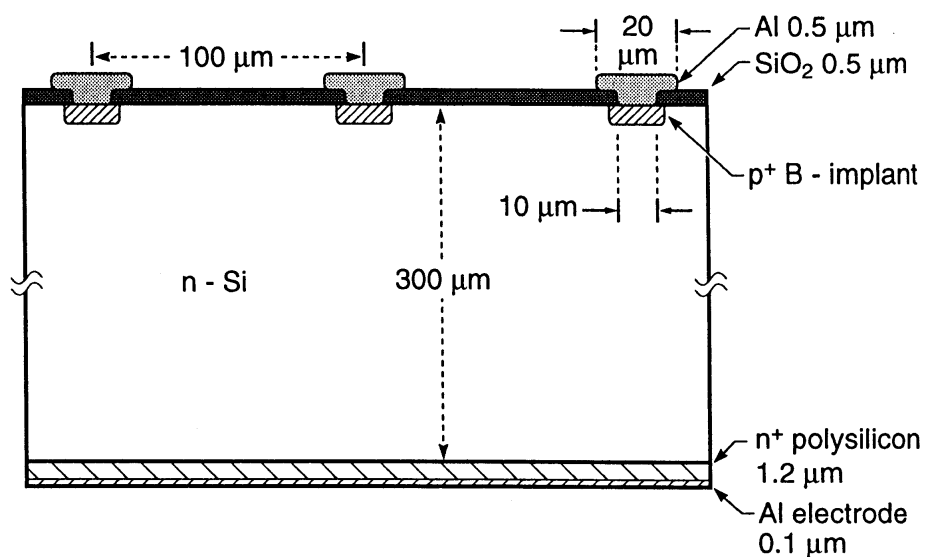


Note: The “steps” in the curves are artifacts of the calculation resolution.



## Implementation of strip detectors

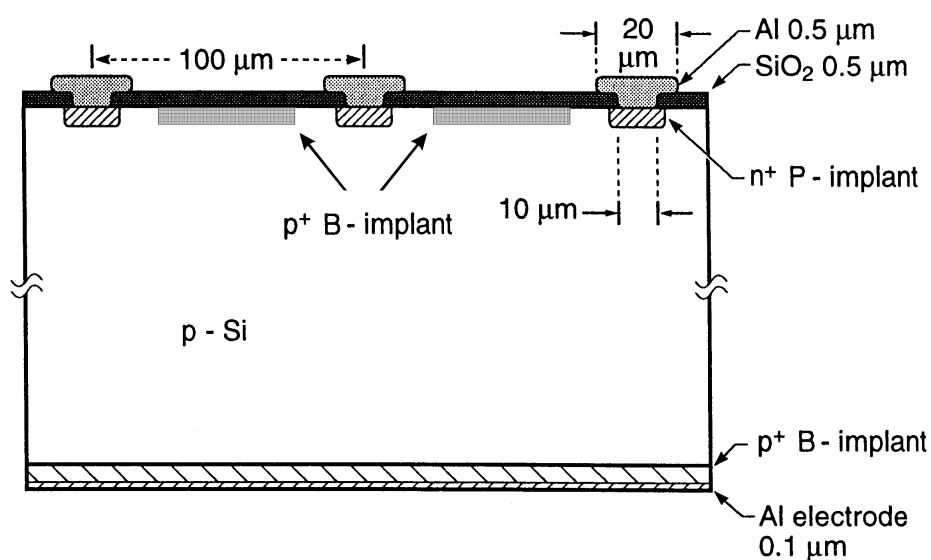
### $p$ -strips on $n$ -bulk



### $n$ -strips on $p$ -bulk

silicon surface is naturally  $n$ -type  
(dangling bonds at lattice boundary)

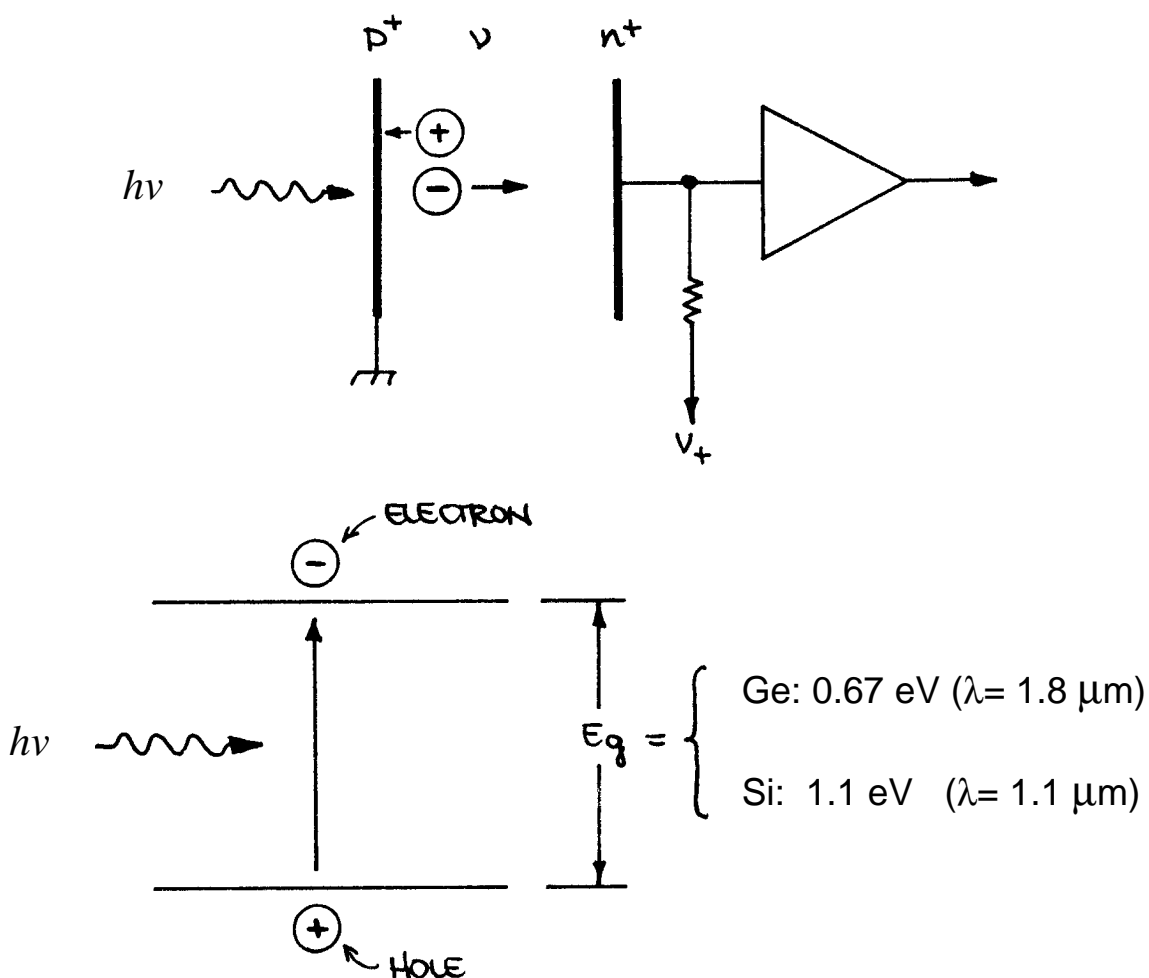
⇒ isolation structures required to block conducting path  
between  $n$ -strips (for example, intermediate  $p$ -strips)



Strip detectors can be fabricated with integrated coupling capacitors and bias resistors (e.g. polysilicon).

## Photodiodes

Although photomultiplier tubes still dominate in scintillation detectors, silicon photodiodes are becoming increasingly popular.

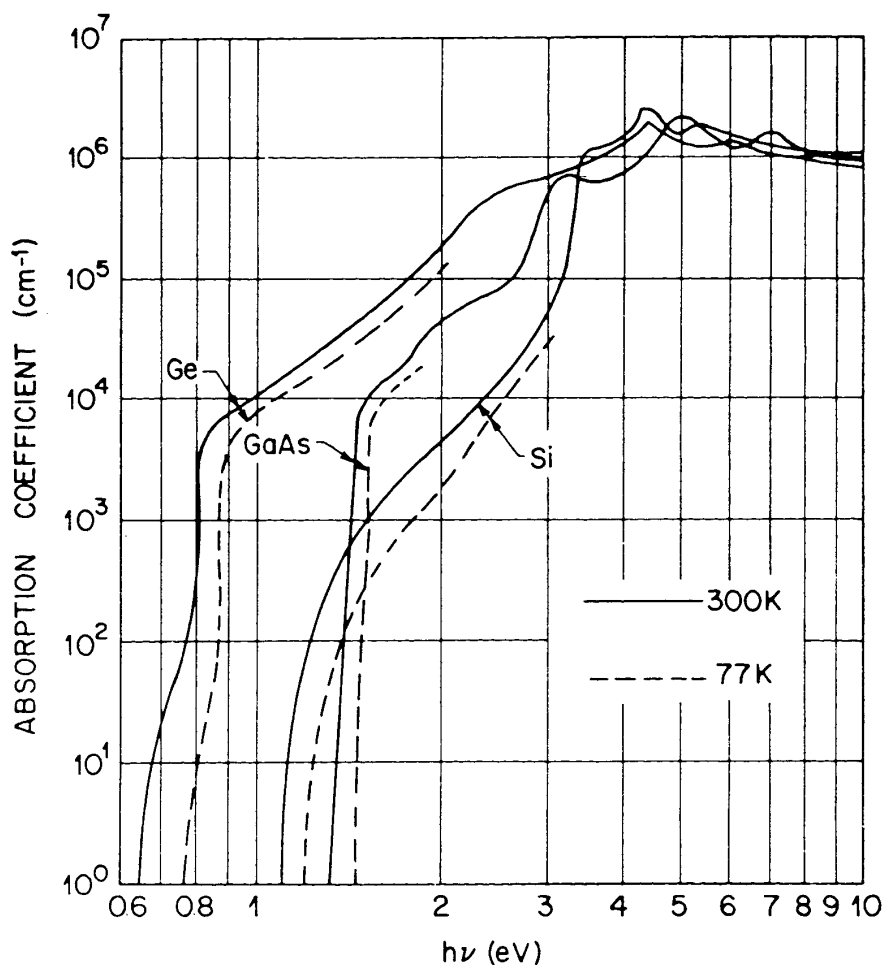


Scintillation light:  $\lambda = 200 - 500 \text{ nm}$  ( $E = 6.2 - 2.5 \text{ eV}$ )

Semiconductor photodiodes offer

- a) high quantum efficiency  
(70 - 90% instead of 30% for PMTs)
- b) insensitivity to magnetic fields
- c) small size

Although all semiconductor diodes are light sensitive, for high quantum efficiency they must be designed to avoid significant dead layers at the surface, as most of the photons in the visible range are absorbed within about 1  $\mu\text{m}$  of the surface.



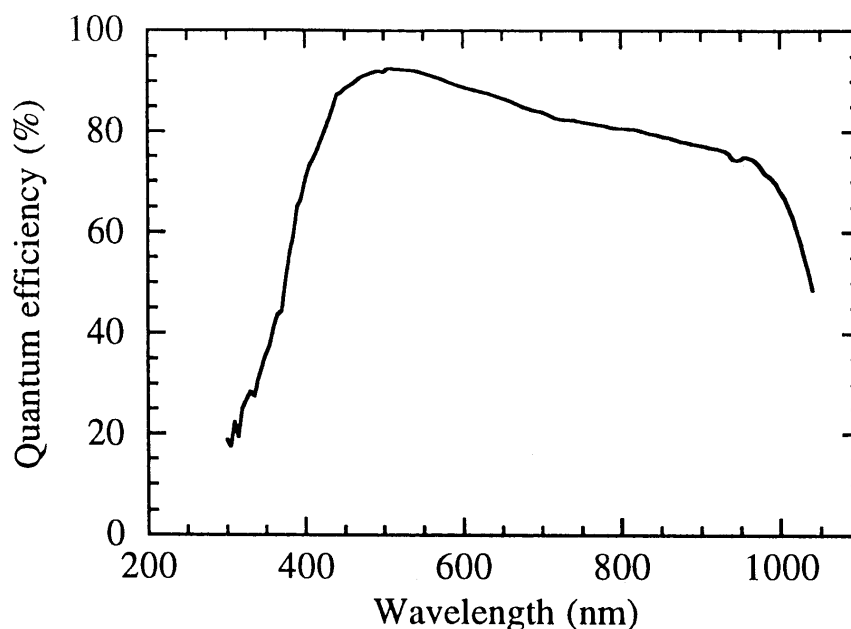
The number of absorbed photons

$$N_{abs} = N_0 \int e^{-\alpha x} dx$$

If the absorption coefficient  $\alpha = 10^4 \text{ cm}^{-1}$ , dead layers must be  $< 0.1 \mu\text{m}$  to avoid significant losses ( $< 10\%$ ).

Quantum efficiency of well-designed photodiodes is 2 – 3 times better than of PMTs.

Measured data of photodiodes fabricated in LBNL Microsystems Lab (used for medical imaging, N. Wang + S. Holland)



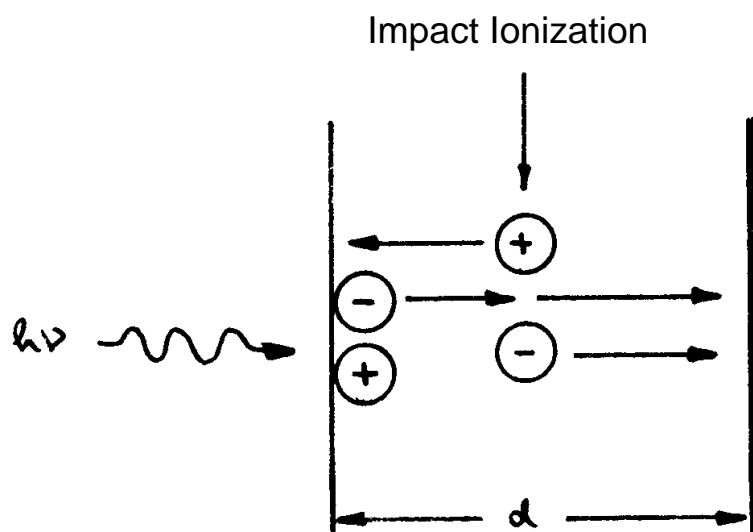
Photomultiplier tubes provide high gain without introducing significant electronic noise, whereas photodiode systems depend critically on low noise.

Unlike PMT systems, photodiode readouts must be very carefully optimized.

⇒ Reduce demands on electronics by developing photodiodes with internal gain, avalanche photodiodes (APDs).

## Principle of an Avalanche Photodiode

Charge carriers are accelerated sufficiently to form additional electron-hole pairs.



An electron-hole pair is created at the left-most electrode by incident light.

Under the influence of the electric field the electron drifts towards the right, gaining sufficient energy for ionization, i.e. formation of an additional electron-hole pair.

The gain of this process

$$G_n = e^{\alpha_n d}$$

where the electron ionization coefficient

$$\alpha_n = \alpha_{n0} e^{-E_n / |E|}$$

is a function of the electric field. The parameters  $\alpha_{n0}$  and  $E_n$  are material constants.

The ionization coefficient is also strongly temperature dependent.

The secondary hole can also ionize and form additional electron-hole pairs. Since the hole mobility is less than the electron mobility, higher fields are required than for same electron ionization.

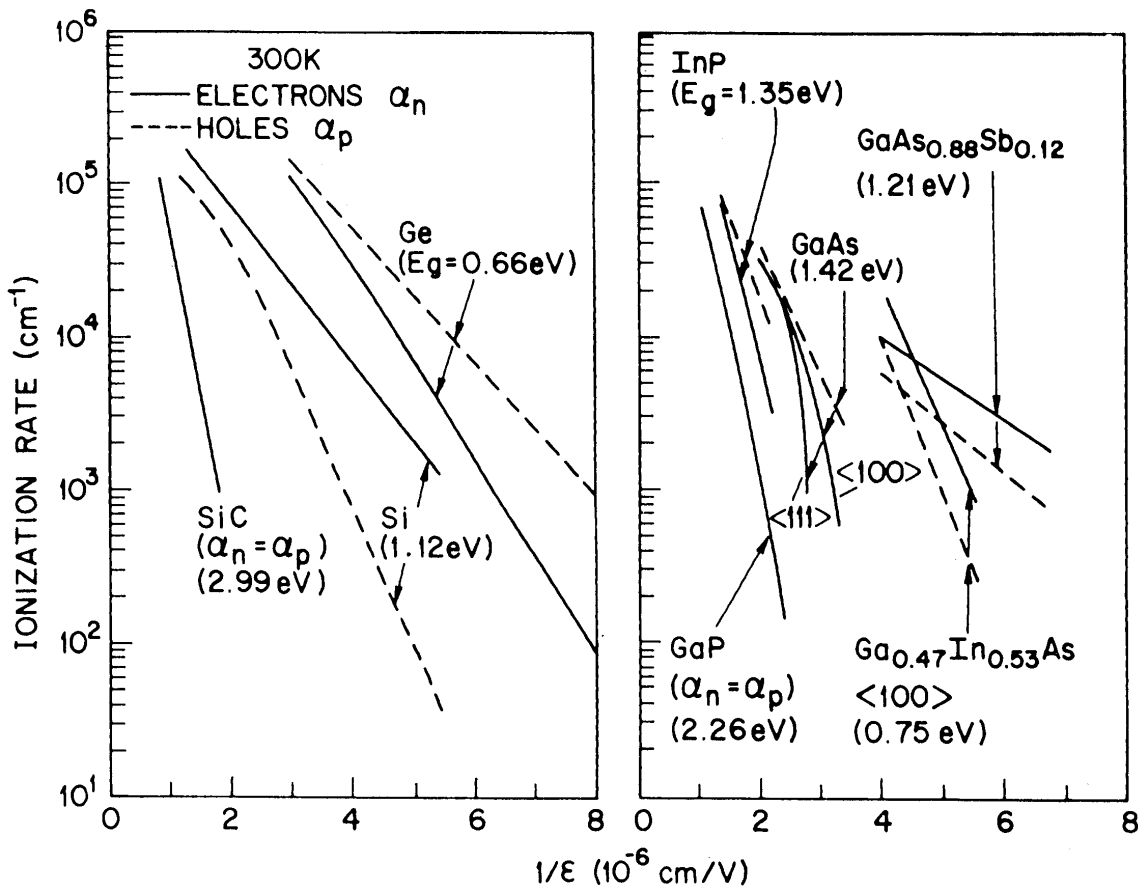
This is fortunate, since the formation of secondary holes is a positive feedback process, i.e. when the partial gain due to holes

$$G_p \geq 2$$

the combined multiplication of electrons and holes leads to a sustained avalanche, i.e. breakdown.

In silicon the ratio of electron to hole ionization coefficients is field dependent, so the sensitivity to breakdown is reduced at low fields.

$$\frac{\alpha_n}{\alpha_p} = 0.15 \cdot \exp\left(\frac{1.15 \cdot 10^6}{|E|}\right)$$



This leads to the following limits of gain and detector thickness vs. electric field

$E = 2 \cdot 10^5 \text{ V/cm}$	$G_n = 2.2 \cdot 10^3$	$d = 520 \text{ } \mu\text{m}$	$V_b = 10 \text{ kV}$
$E = 3 \cdot 10^5 \text{ V/cm}$	$G_n = 50$	$d = 5 \text{ } \mu\text{m}$	$V_b = 150 \text{ V}$
$E = 4 \cdot 10^5 \text{ V/cm}$	$G_n = 6.5$	$d = 0.5 \text{ } \mu\text{m}$	$V_b = 20 \text{ V}$
$E = 5 \cdot 10^5 \text{ V/cm}$	$G_n = 2.8$	$d = 0.1 \text{ } \mu\text{m}$	$V_b = 5 \text{ V}$

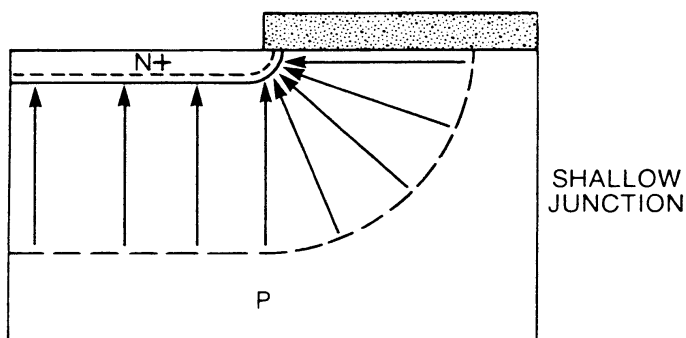
To achieve gains in the range 100 – 1000 requires

a depletion region of several hundred microns thick

bias voltages in the range 500 – 1000 V

excellent control of the field distribution

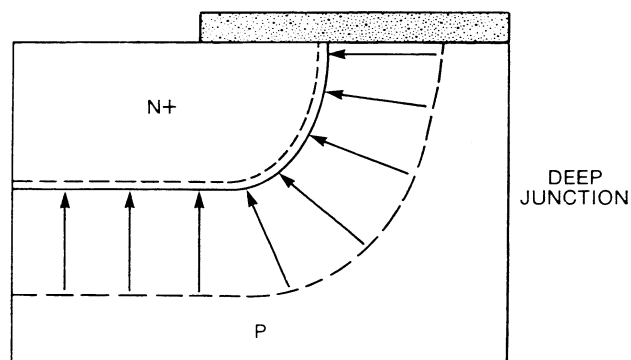
Operation at high voltages is limited by local high-field regions



(from Baliga, Modern Power Devices)

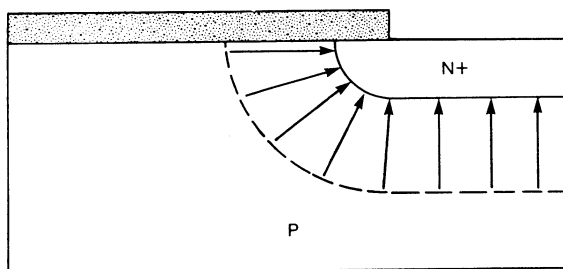
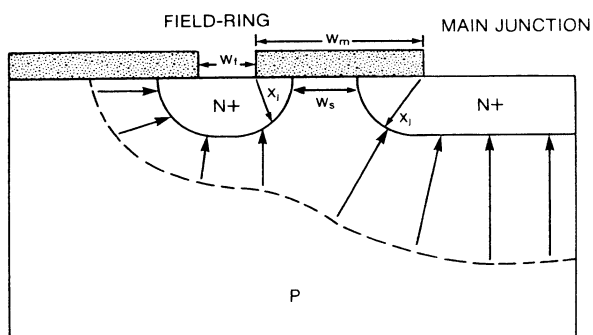
Breakdown will occur at the edge of the junction before the full operating field in the bulk is attained.

This problem can be alleviated by increasing the depth of the  $n^+$  region, i.e. increasing the radius of curvature at the edge.



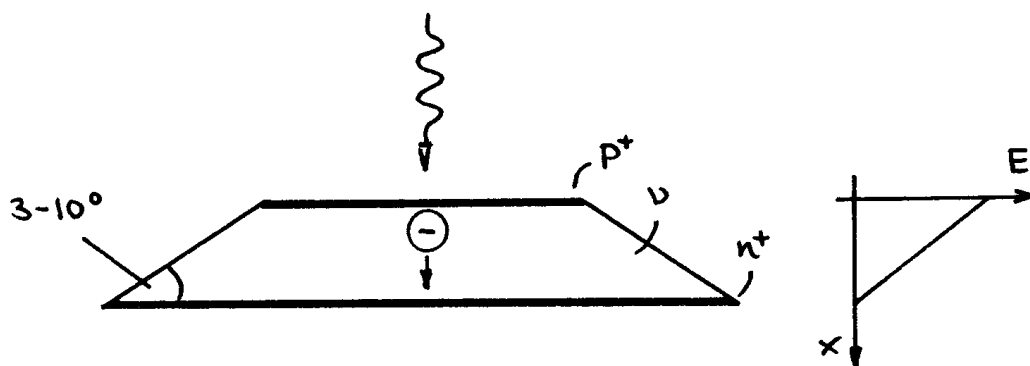
The field can also be reduced by adding a guard ring, which also reduces the field at the edge of the wafer.

(from Baliga)

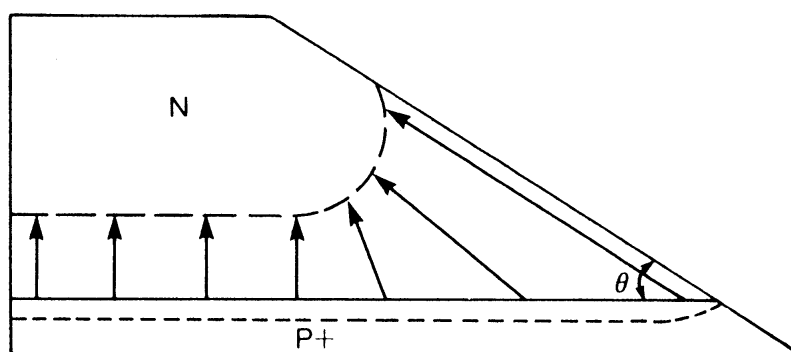




Another solution is the beveled diode:



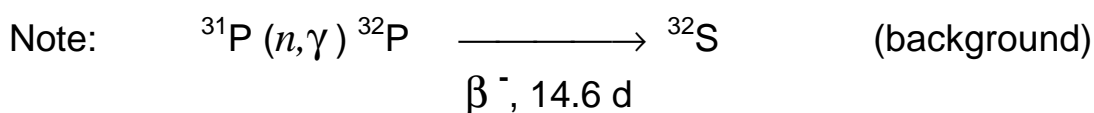
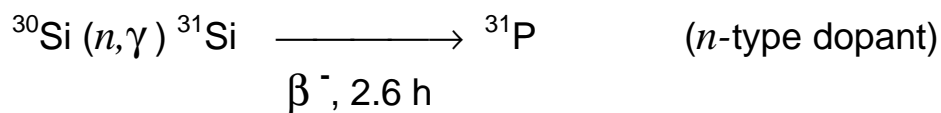
The bevel reduces the field both at the junction edge and at the edge of the wafer.



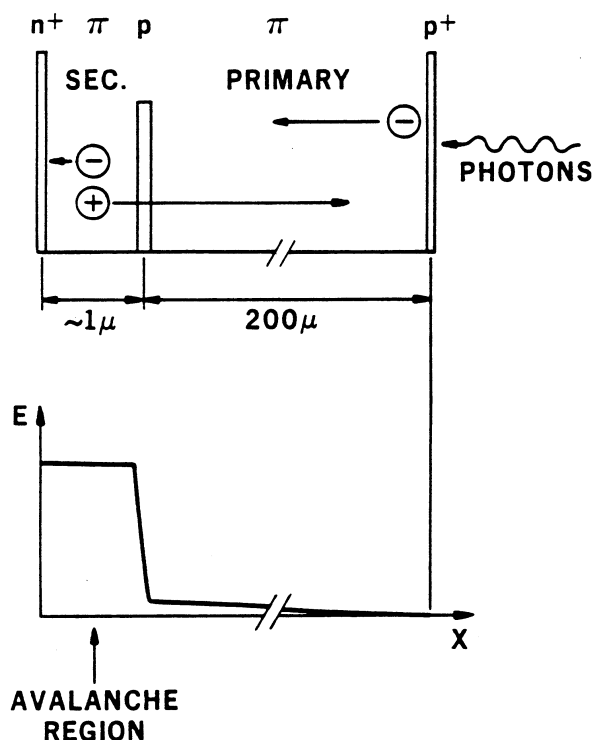
(from Baliga)

All of these structures require very uniform doping of the bulk material. Typical doping variations are 20% across wafer.

Solution: neutron transmutation doping



An alternative APD structure is the “Reach-Through” APD:



Lightly doped  $p$ -type material is used for the bulk.

A local high-field region is created by introducing an intermediate  $p$ -layer through deep diffusion.

When a depletion voltage is applied, the diode depletes from the left side. Initially the depletion region progresses with voltage until the intermediate  $p$ -layer is reached. Since this layer is more highly doped, the voltage required to deplete the intermediate layer is rather high.

As a result, a high field is set up in the region between the junction and the  $p$ -layer.

Depletion beyond the  $p$ -layer requires less voltage, due to low doping.

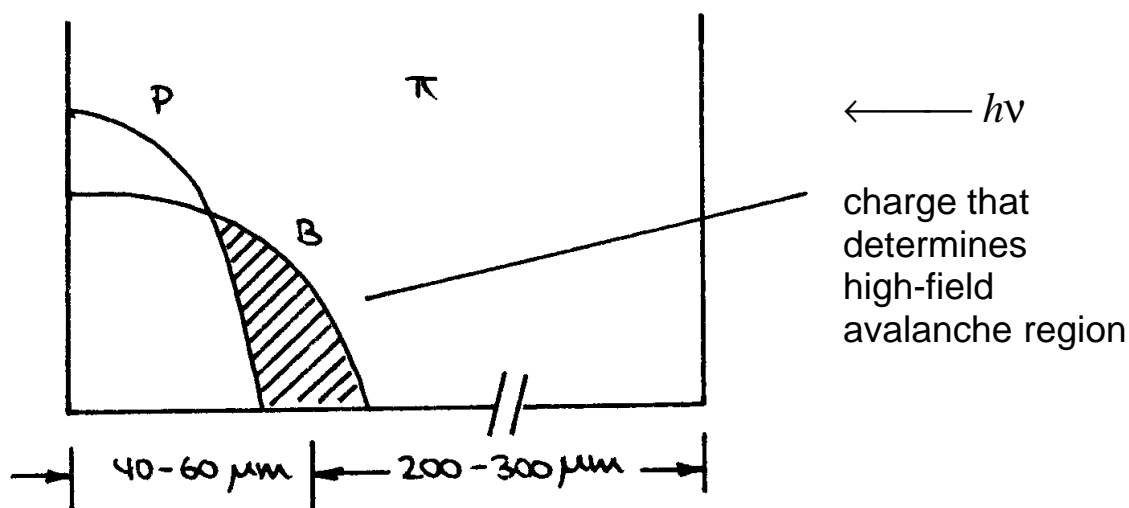
Photons impinge on the right surface. Electrons drift towards the high field region, where they avalanche.

Secondary holes drift through the low-field region, contributing most of the induced signal

The advantage of this structure is that the primary holes remain in the low-field region.

Furthermore, the secondary holes drift into the low-field region, thus reducing the hole partial gain and the risk of breakdown.

### Common Implementation of Reach Through Structure



Lightly doped  $p$ -type material is doped with two deep diffusions to create a local high-field region.

The deep B diffusion introduces acceptors.

The overlapping P diffusion (shaded) forms a lightly doped  $p$ -region due to compensation ( $N_A - N_D$ ) with a high field determined by the local  $p^+$  doping.

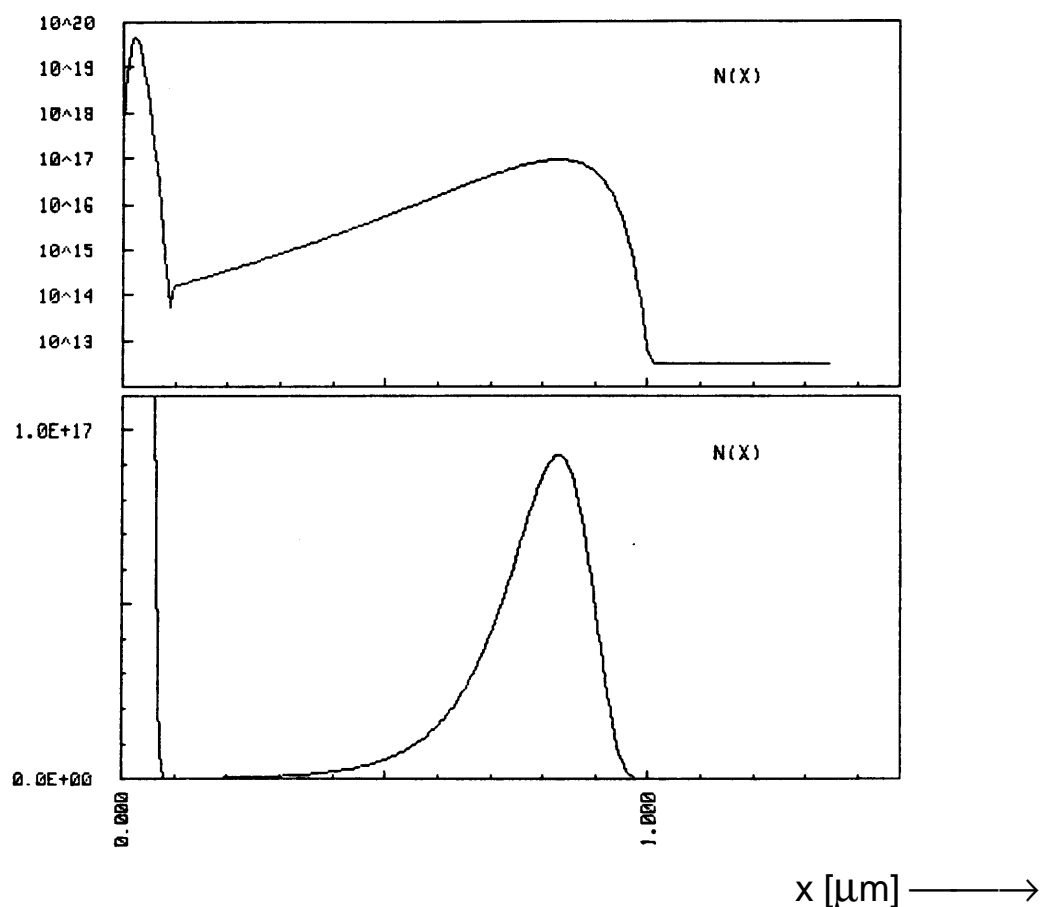
Problem: deep diffusions are difficult to control – low yield

Nevertheless, these devices are commercially available with gains of  $10^3$  to  $10^6$ .

Often moderate gains ( $\sim 10$ ) are adequate.

A modified reach-through structure using shallower implants can be utilized. Again, the high field region is formed by two successive diffusions, but only about  $1\text{ }\mu\text{m}$ , rather than  $50\text{ }\mu\text{m}$  deep.

### Dopant Concentration vs. Depth



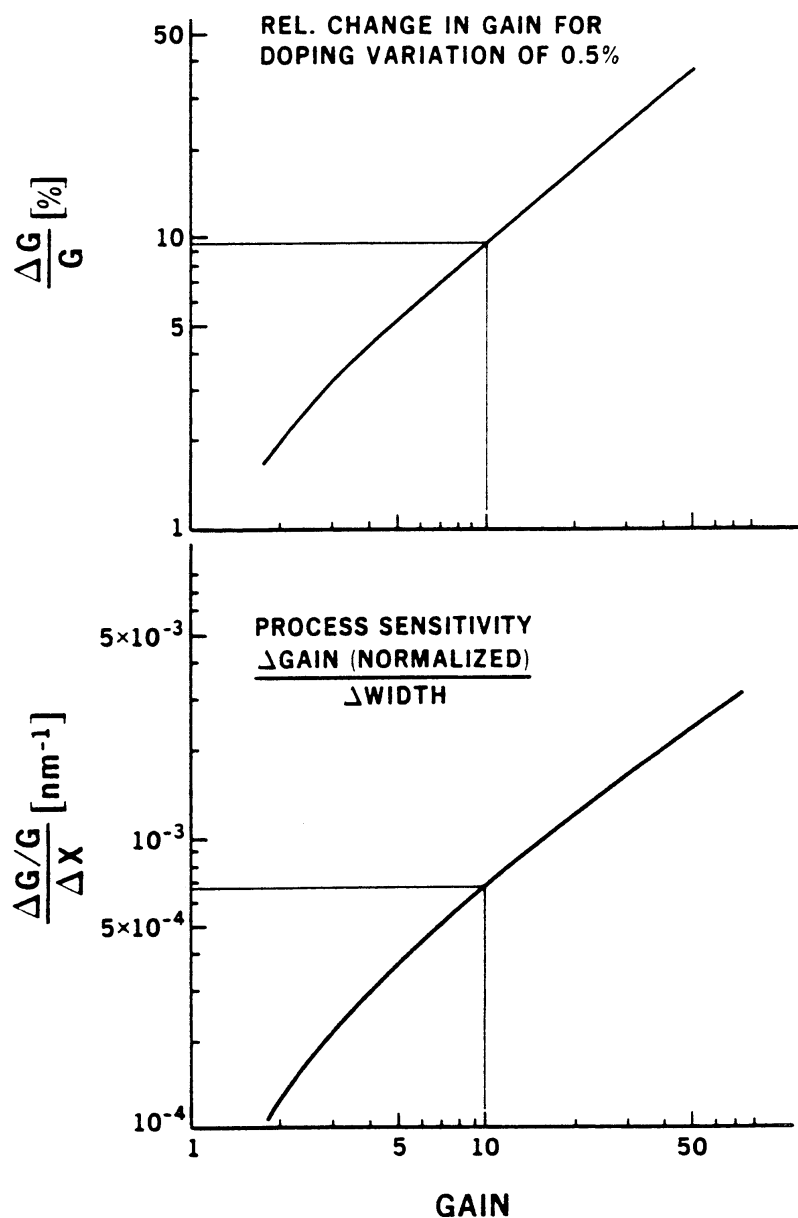
In the upper plot (log scale) the two dopants are clearly visible. The deeper diffusion is B, the shallow diffusion As.

The bottom plot shows the net dopant distribution on a linear scale, showing the lightly doped avalanche region.

Both dopants are introduced by ion-implantation and then diffused, making use of the higher diffusivity of B.

## Gain Sensitivity for Realistic Implant Uniformity

The upper plot shows the gain variation for a non-uniformity of the implanted ion dose over the diode area of 0.5%. For a gain of 10 the gain variation would be 10 %, increasing to 50% at a gain of 50.

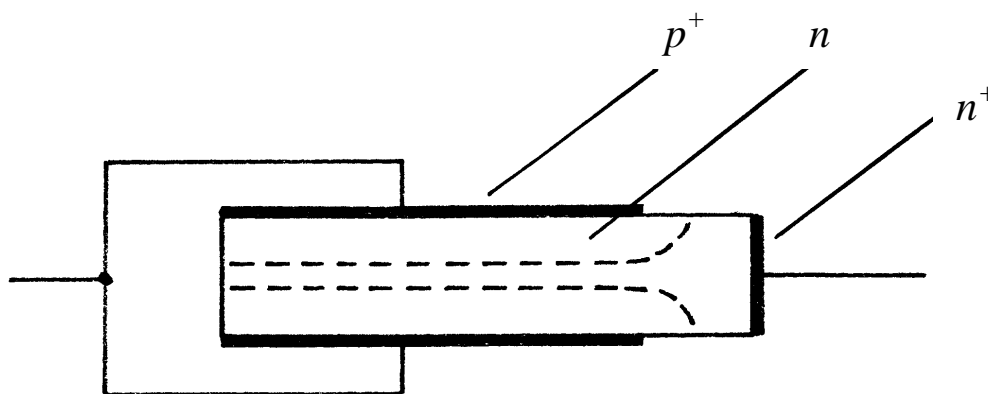


The lower plot shows the variation in gain vs. variations in diffusion depth, e.g. due to material imperfections. Even if the depth of the B diffusion varies by 100 nm (0.1  $\mu\text{m}$ ), the gain only changes by 7%.

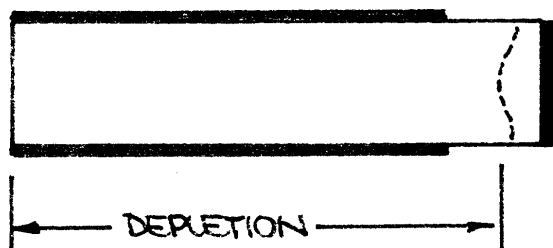
## Some Other Types of Semiconductor Detectors

### Semiconductor Drift Chamber

1<sup>st</sup> Ingredient: depletion from edge of detector

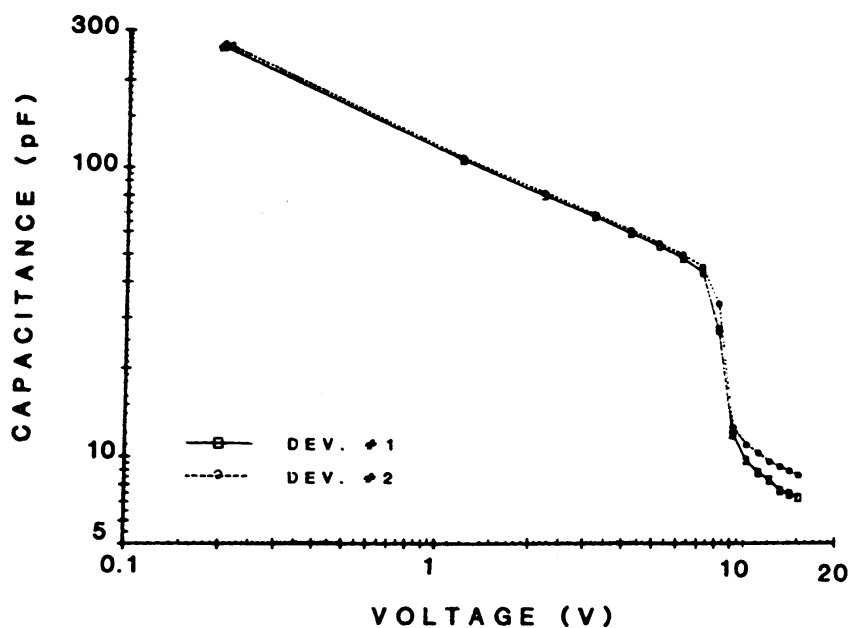
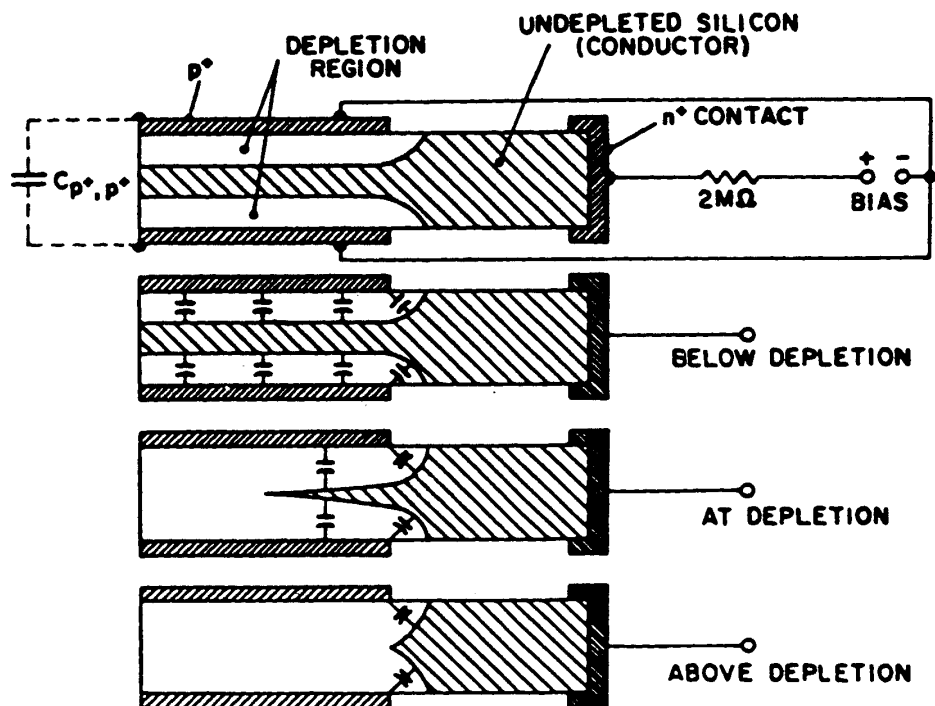


INCREASE REVERSE BIAS :



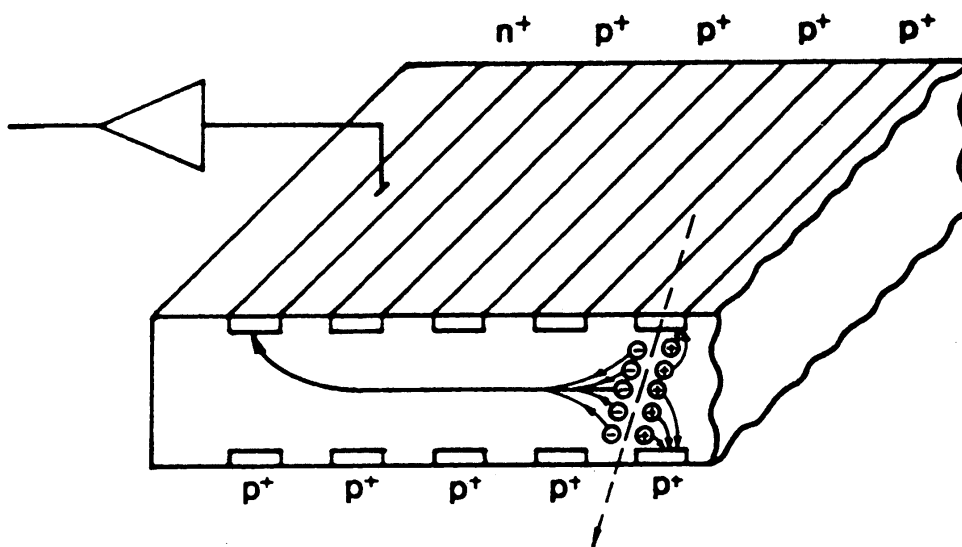
## Depletion vs. Reverse Bias Voltage

(from Gatti et al. IEEE Trans. Nucl. Sci. **NS-32** (1985) 1204)

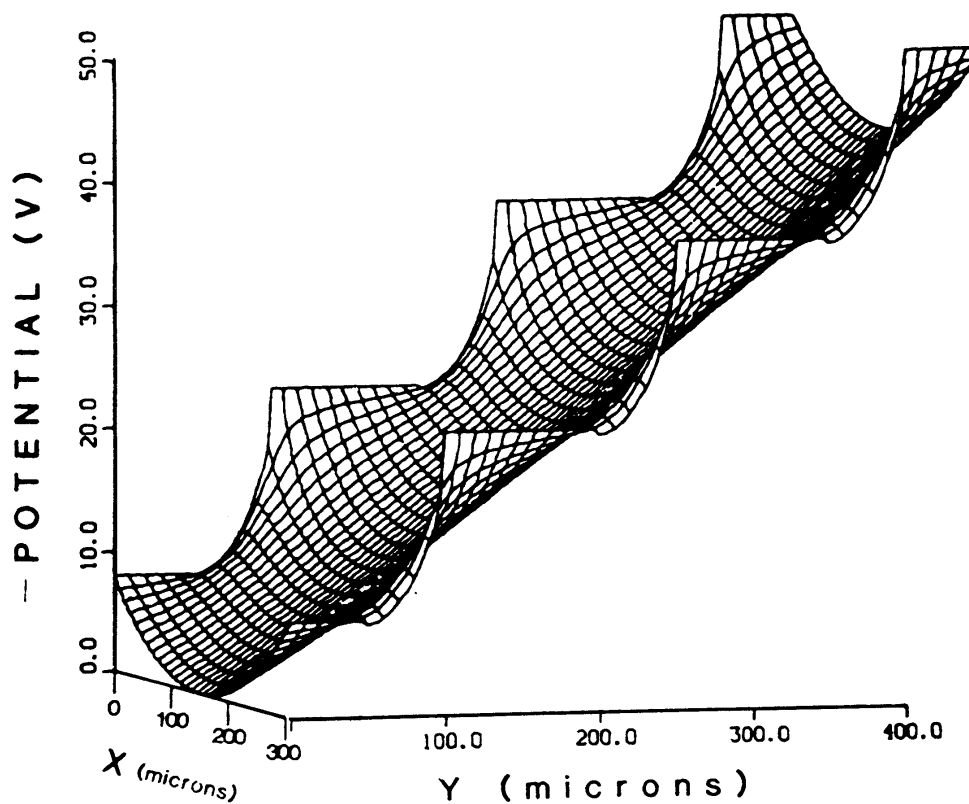


## 2<sup>nd</sup> Ingredient:

Add additional electrodes to form drift field parallel to surface

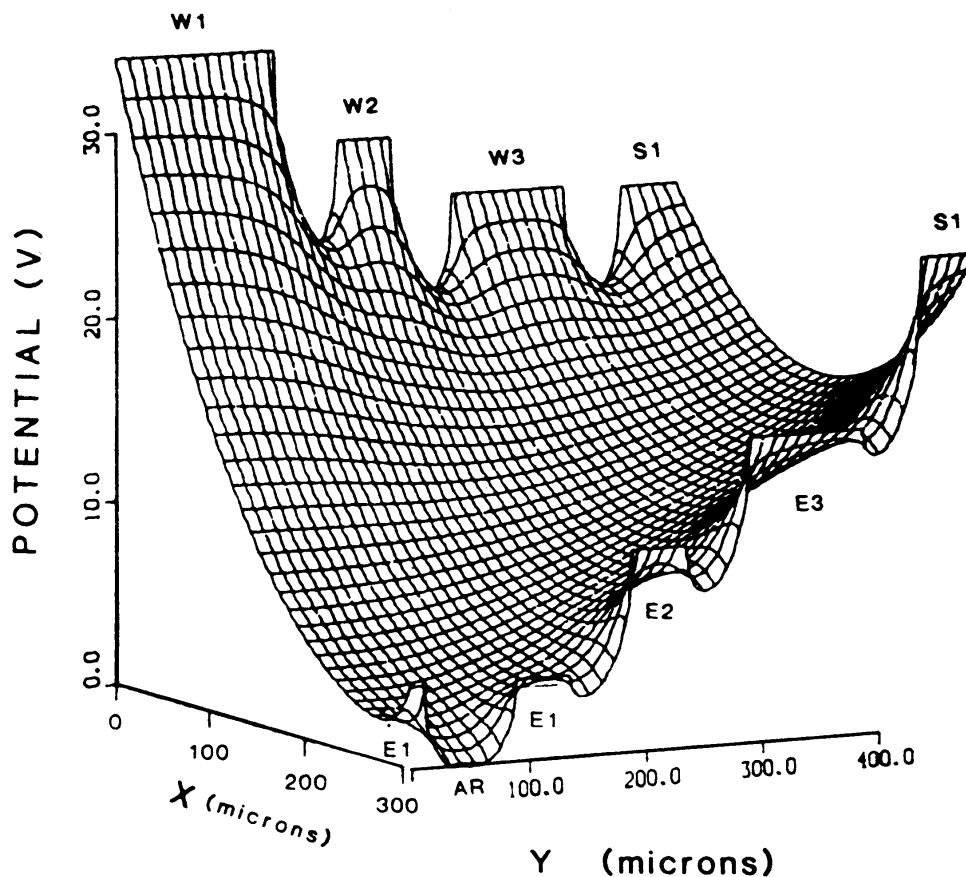


## Potential Distribution





Potential trough can be skewed to direct charge to readout electrode on surface.



Silicon drift chamber has advantage that the collection electrode is decoupled from the large track-acceptance area.

⇒ capacitance can be very small, even on a large area detector  
( $C \sim 50 - 100$  fF for  $A = 10$  cm<sup>2</sup>)

⇒  $\sim 10$   $\mu$ m resolution over 5 – 10 cm drift distance

Drift velocity must be predictable.

Trapping must be low for long drift distances ( $\sim$  cm)

⇒ problem with radiation damage.

Electronics optimized for timing – multi-hit capability requires fast time digitization.

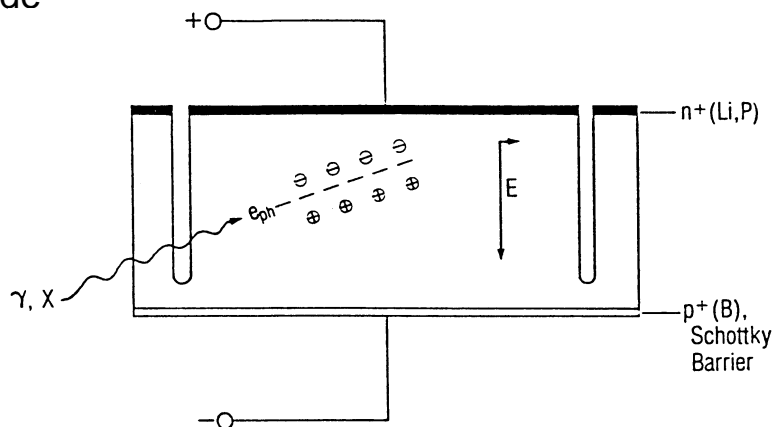
## Large Volume Ge Detectors for $\gamma$ - Spectroscopy

Typical configurations

5 – 10 cm diameter, up to 400 cm<sup>3</sup> volume

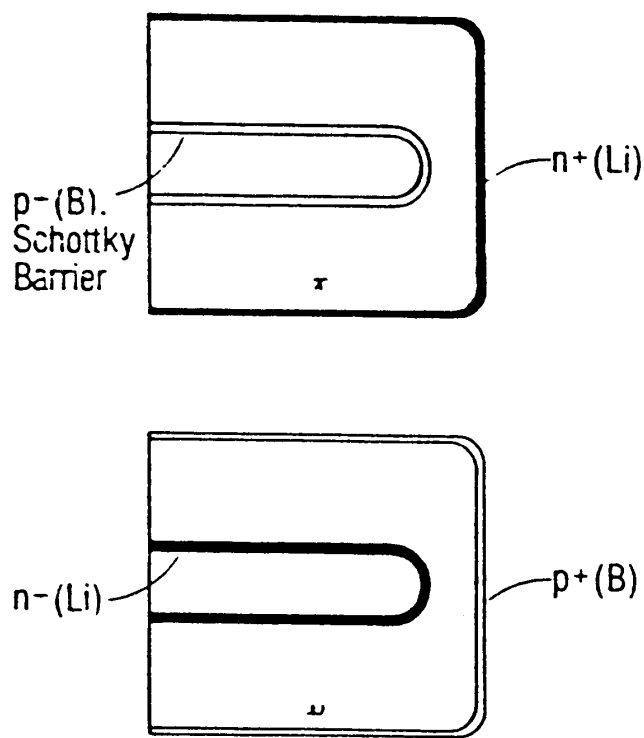
Use *p-i-n* structure, where bulk is very lightly doped.

Planar Diode



Grooves at edge define active area and isolate it from outer surface.

Coaxial Diodes



(from E. Haller)

## Semiconductor Detector Materials

Si and Ge are the traditional materials.

Si is typically used for x-ray detectors (up to 50 keV or so), whereas Ge by virtue of its greater  $Z$  is used for  $\gamma$  ray spectroscopy. Ge detectors must be operated at low temperatures (liquid nitrogen) to achieve low leakage currents.

High energy physics tracking detectors use silicon.

Diamond strip and pixel detectors using have been fabricated and tested for use in experiments with extreme radiation damage.

Most alternative materials have been studied as replacements for Ge in  $\gamma$  ray detectors, with a goal of room temperature operation.

Material	Z	Bandgap [eV]	Mobility [ $\text{cm}^2/\text{Vs}$ ]		Density $\text{g/cm}^3$
			electrons	holes	
Si	14	1.1	1350	480	2.3
Ge	32	0.7	3800	1800	5.3
Diamond	6	5.5	1800	1200	3.5
GaAs	31-33	1.5	8600	400	5.4
AlSb	13-51	1.6	200	700	4.3
GaSe	31-34	2.0	60	250	4.6
CdSe	48-34	1.7	50	50	
CdS	48-16	2.4	300	15	4.8
InP	49-15	1.4	4800	150	
ZnTe	30-52	2.3	350	110	
WSe <sub>2</sub>	74-34	1.4	100	80	
BiI <sub>3</sub>	83-53	1.7	680	20	
Bi <sub>2</sub> S <sub>3</sub>	83-16	1.3	1100	200	6.7
Cs <sub>3</sub> Sb	55-51	1.6	500	10	
PbI <sub>2</sub>	82-53	2.6	8	2	6.2
HgI <sub>2</sub>	89-53	2.1	100	4	6.3
CdTe	48-52	1.5	1100	100	6.1
CdZnTe	48-30-52	1.5-2.4			

Alternative materials tend to be limited by trapping, difficulty in growing single crystals, and processing limitations.