

IV. Signal Processing

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1. The Problem

Radiation impinges on a sensor and creates an electrical signal.

The signal level is low and must be amplified to allow digitization and storage.

Both the sensor and amplifiers introduce signal fluctuations – noise.

1. Fluctuations in signal introduced by sensor
2. Noise from electronics superimposed on signal

The detection limit and measurement accuracy are determined by the signal-to-noise ratio.

Electronic noise affects all measurements:

1. Detect presence of hit: Noise level determines minimum threshold.
If threshold too low, output dominated by noise hits.
2. Energy measurement: Noise “smears” signal amplitude.
3. Time measurement: Noise alters time dependence of signal pulse.

How to optimize the signal-to-noise ratio?

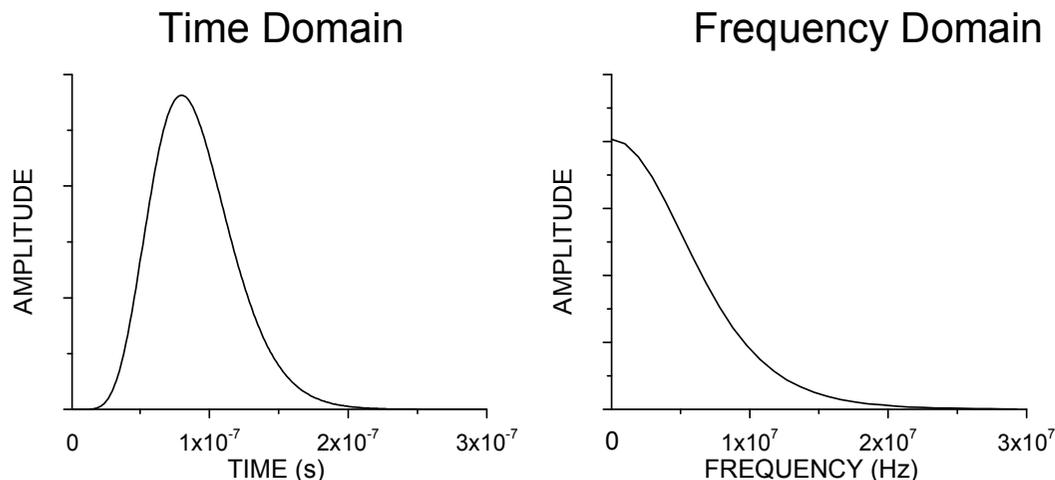
1. Increase signal and reduce noise
2. For a given sensor and signal: reduce electronic noise

Assume that the signal is a pulse.

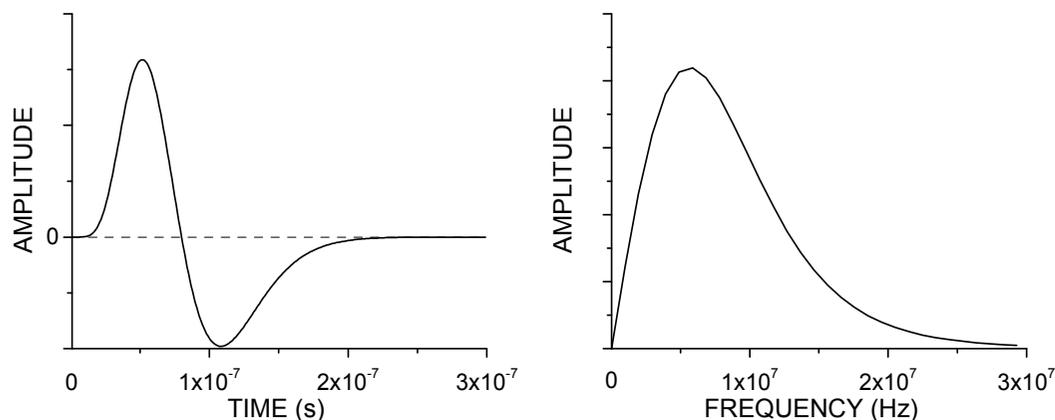
The time distribution of the signal corresponds to a frequency spectrum (Fourier transform).

Examples:

1. The pulse is unipolar, so it has a DC component and the frequency spectrum extends down to zero.

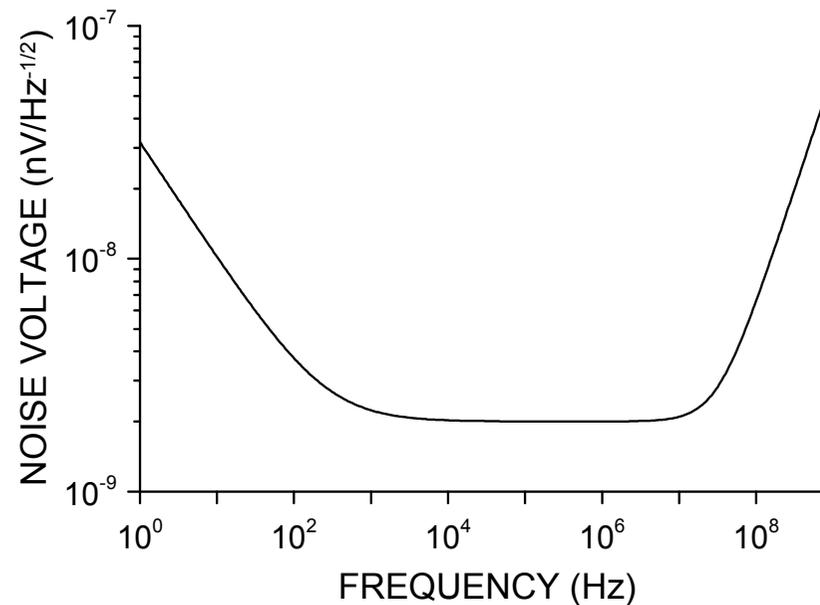


2. This bipolar pulse carries no net charge, so the frequency spectrum falls to zero at low frequencies, but extends to higher frequencies because of the faster slope.



The noise spectrum is generally not the same as the signal spectrum.

Typical Noise Spectrum:



⇒ Tailor frequency response of measurement system to optimize signal-to-noise ratio.

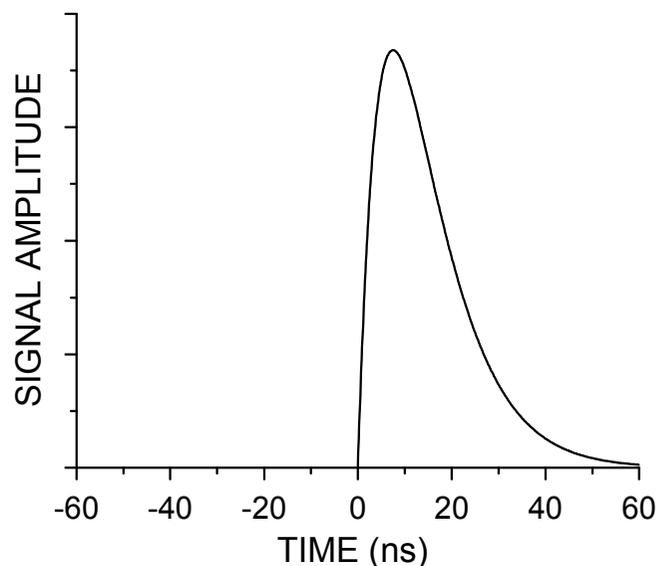
Frequency response of the measurement system affects both

- signal amplitude and
- noise.

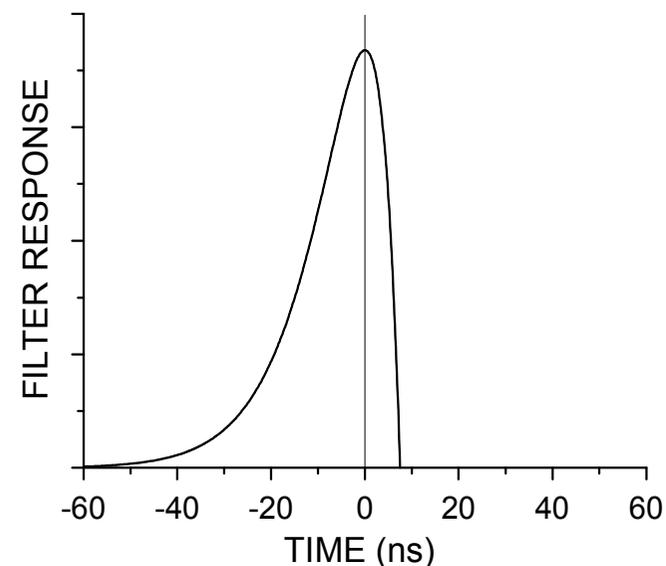
There is a general solution to this problem:

Apply a filter to make the noise spectrum white (constant over frequency). Then the optimum filter has an impulse response that is the signal pulse *mirrored in time* and shifted by the measurement time.

For example, if the signal pulse shape is:



⇒ The response of the optimum filter:



This is an “acausal” filter, i.e. it must act before the signal appears.

⇒ Only useful if the time of arrival is known in advance.

Not good for random events

Need time delay buffer memory

⇒ complexity!

Does that mean our problem is solved (and the lecture can end)?

1. The “optimum filter” preserves all information in signal, i.e. magnitude, timing, structure.

Usually, we need only subset of the information content,
i.e. area (charge) or time-of-arrival.

Then the raw detector signal is not of the optimum form for the information that is required.

For example, a short detector pulse would imply a fast filter function. This retains both amplitude and timing information.

If only charge information is required, a slower filter is better, as will be shown later.

2. The optimum filter is often difficult or impractical to implement

Digital signal processing would seem to remove this restriction, but this approach is not practical for very fast signals or systems that require low power.

4. Simpler filters often will do nearly as well

5. Even a digital system requires continuous (“analog”) pre-processing.

6. It’s often useful to understand what you’re doing, so we’ll spend some more time to bring out the physical background of signal formation and processing.

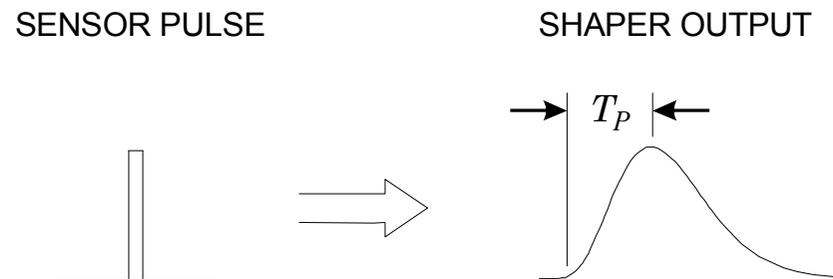
Signal Processing Objectives

Two conflicting objectives:

1. Improve Signal-to-Noise Ratio S/N

Restrict bandwidth to match measurement time \Rightarrow Increase pulse width

Typically, the pulse shaper transforms a narrow detector current pulse to a broader pulse (to reduce electronic noise), with a gradually rounded maximum at the peaking time T_P (to facilitate measurement of the peak amplitude)



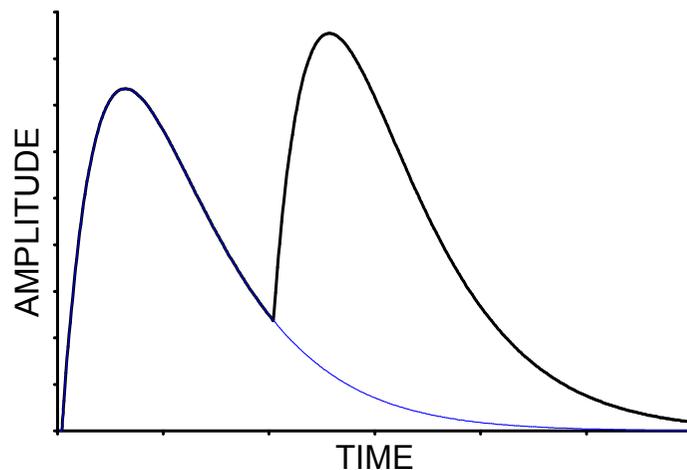
If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or pulse height analysis. The pulse height spectrum is the energy spectrum.

2. Improve Pulse Pair Resolution

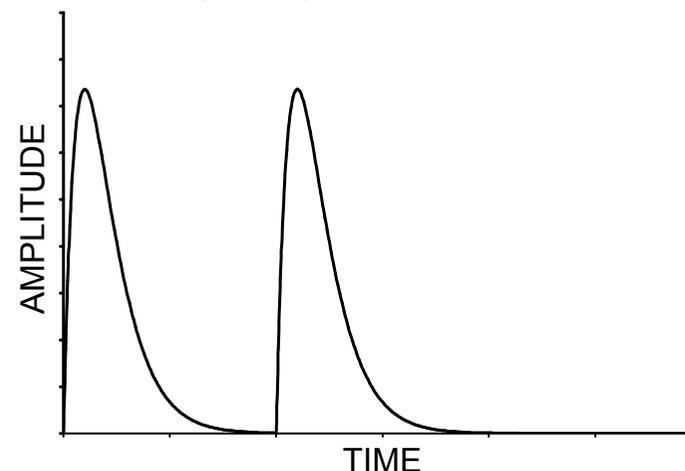


Decrease pulse width

Pulse pile-up distorts amplitude measurement.



Reducing pulse shaping time to 1/3 eliminates pile-up.



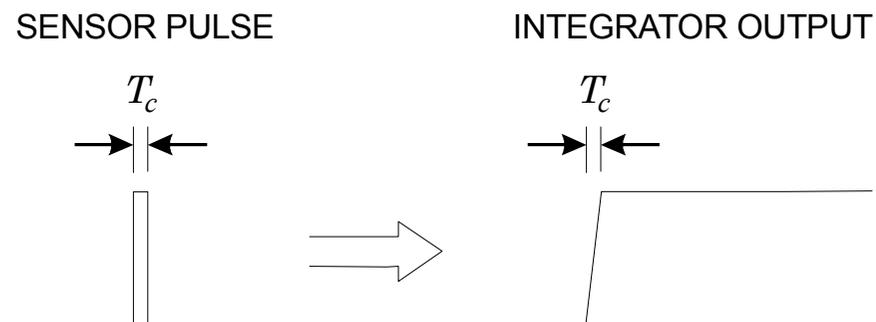
Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

Usually, many considerations combined lead to a “non-textbook” compromise.

- *“Optimum shaping” depends on the application!*
- Shapers need not be complicated – *Every amplifier is a pulse shaper!*

Goal: Improve energy resolution

Procedure: Integrate detector signal current \Rightarrow Step impulse



Commonly approximated as
“step” response (zero rise time).

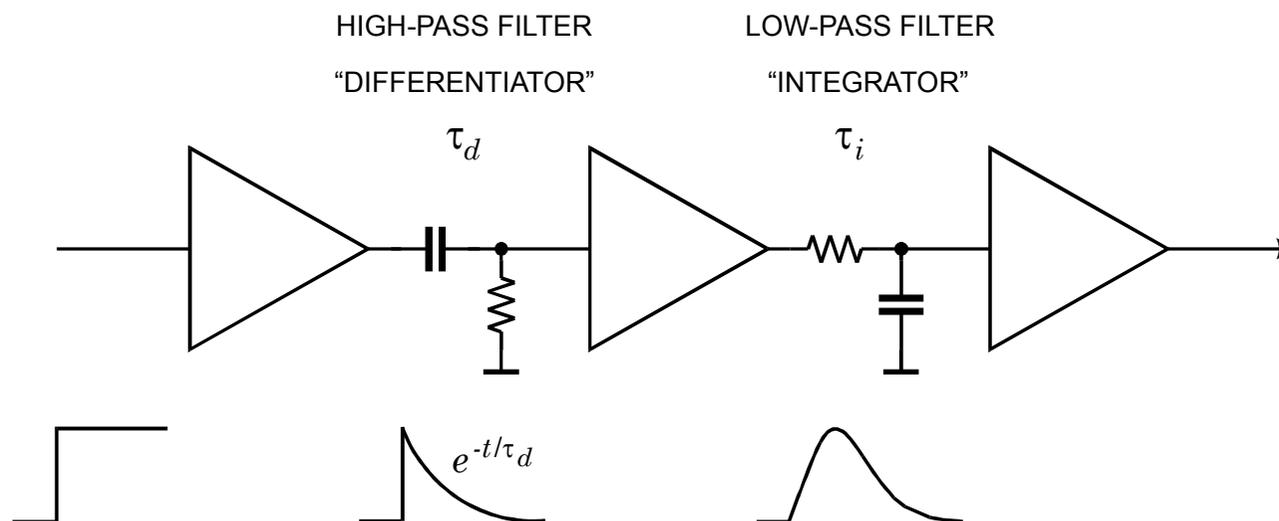
Long “flat top” allows measurements at times well beyond the collection time T_c .

\Rightarrow Allows reduced bandwidth and great flexibility in selecting shaper response.

Optimum for energy measurements, but not for fast timing!

“Fast-slow” systems utilize parallel processing chains to optimize both timing and energy resolution (see Timing Measurements).

Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

⇒ Useful for estimates, since simple to evaluate

Key elements:

- lower frequency bound ($\hat{=}$ pulse duration)
- upper frequency bound ($\hat{=}$ rise time)

are common to all shapers.

2. Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Equivalent Noise Charge

Inject known signal charge into preamp input
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge \equiv Input charge for which $S/N = 1$

Ballistic Deficit

When the rise time of the input pulse to the shaper extends beyond the nominal peaking time, the shaper output is both stretched in time and the amplitude decreases

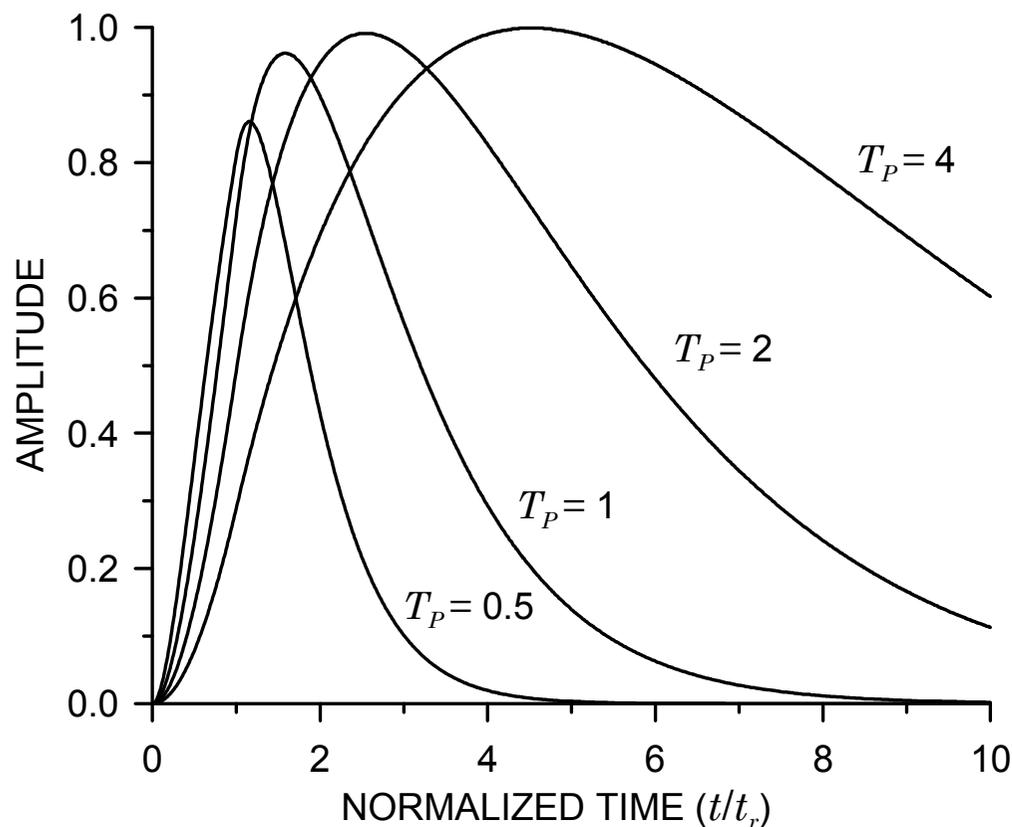
Shaper output for an input rise time

$$t_r = 1$$

for various values of nominal peaking time.

Note that the shaper with $T_P = 0.5$

- peaks at $t = 1.15t_r$
- and
- attains only 86% of the pulse height achieved at longer shaping times.



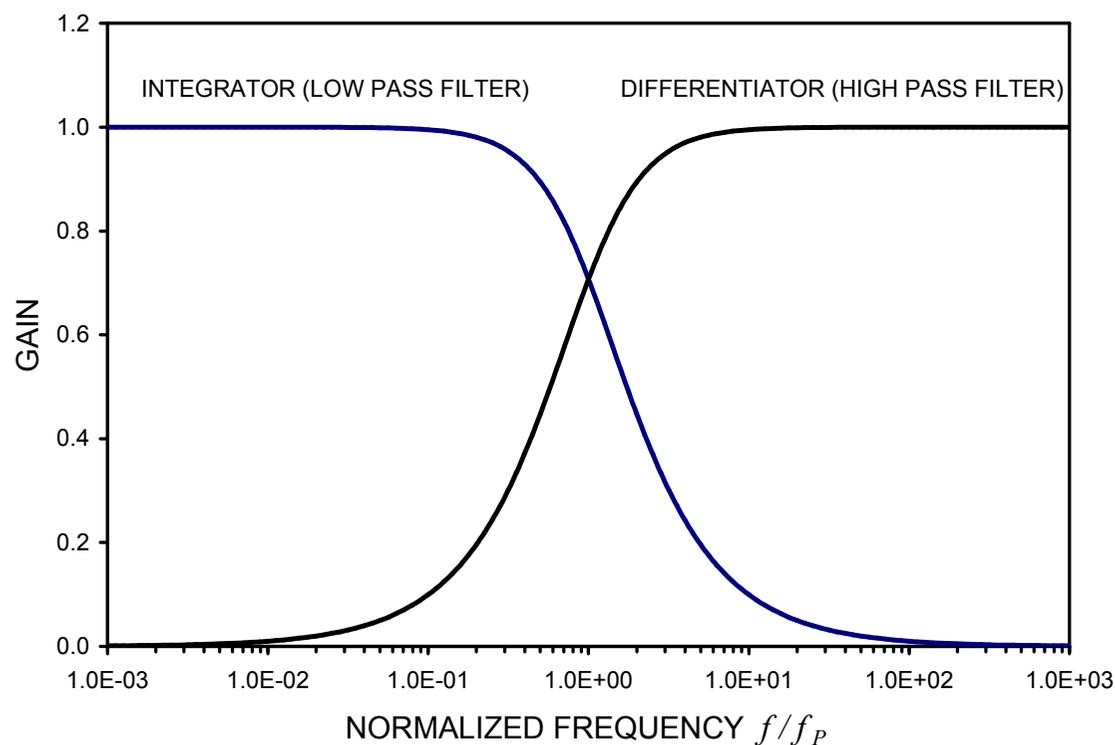
⇒ Increased equivalent noise charge

Noise Charge vs. Shaping Time

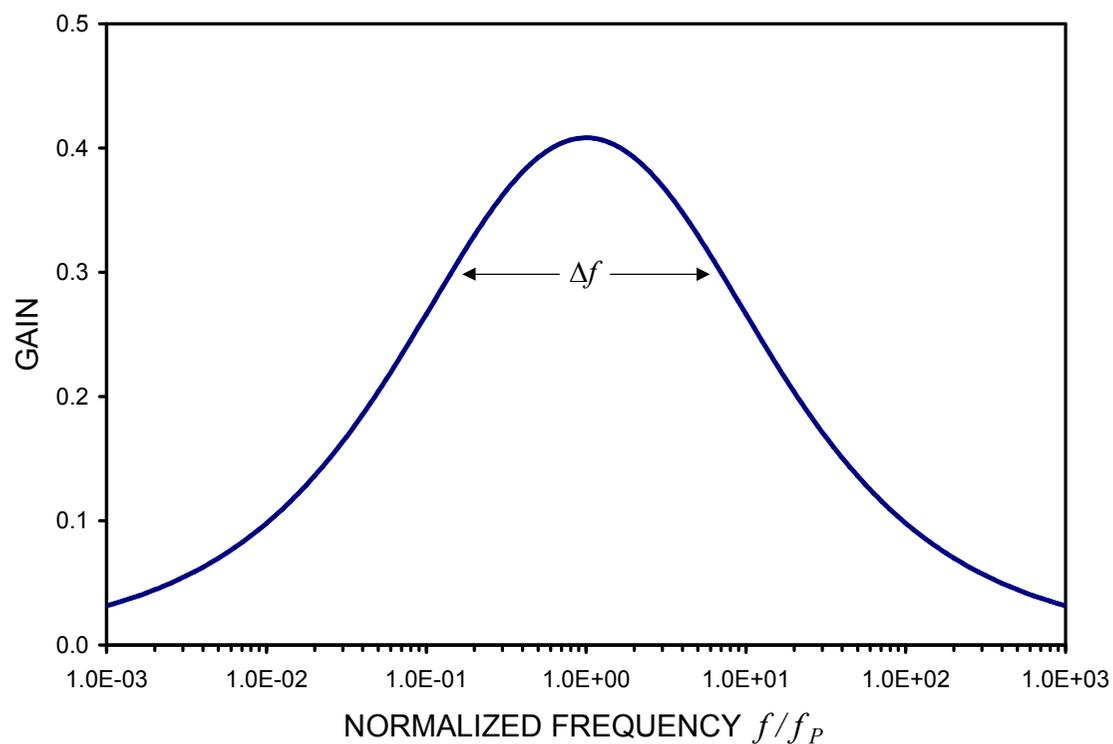
Assume that differentiator and integrator time constants are equal $\tau_i = \tau_d \equiv \tau$.

⇒ Both cutoff frequencies equal: $f_U = f_L \equiv f_P = 1/2\pi\tau$.

Frequency response of individual pulse shaping stages



Combined frequency response

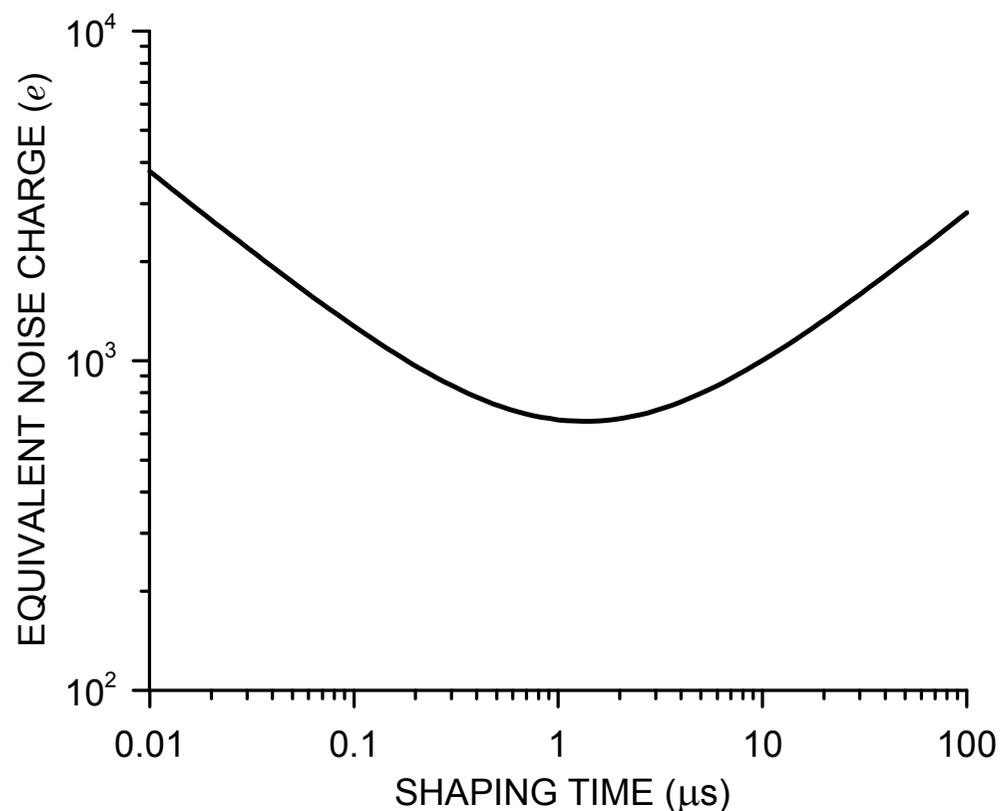


Logarithmic frequency scale \Rightarrow shape of response independent of τ .

However, bandwidth Δf decreases with increasing time constant τ .

\Rightarrow for white noise sources expect noise to decrease with bandwidth,
i.e. decrease with increasing time constant.

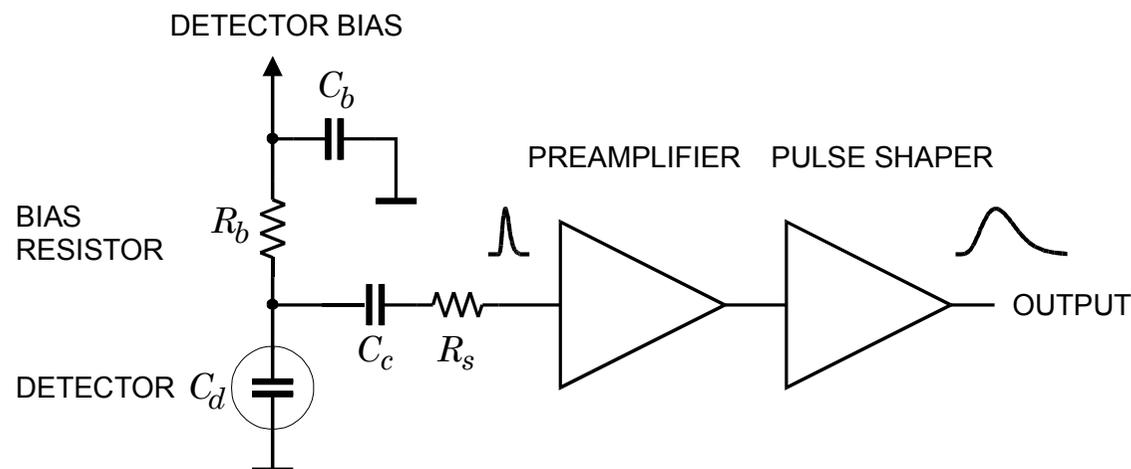
Result of typical noise measurement vs. shaping time



Noise sources (thermal and shot noise) have a flat (“white”) frequency distribution.

Why doesn't the noise decrease monotonically with increasing shaping time (decreasing bandwidth)?

Analytical Analysis of a Detector Front-End



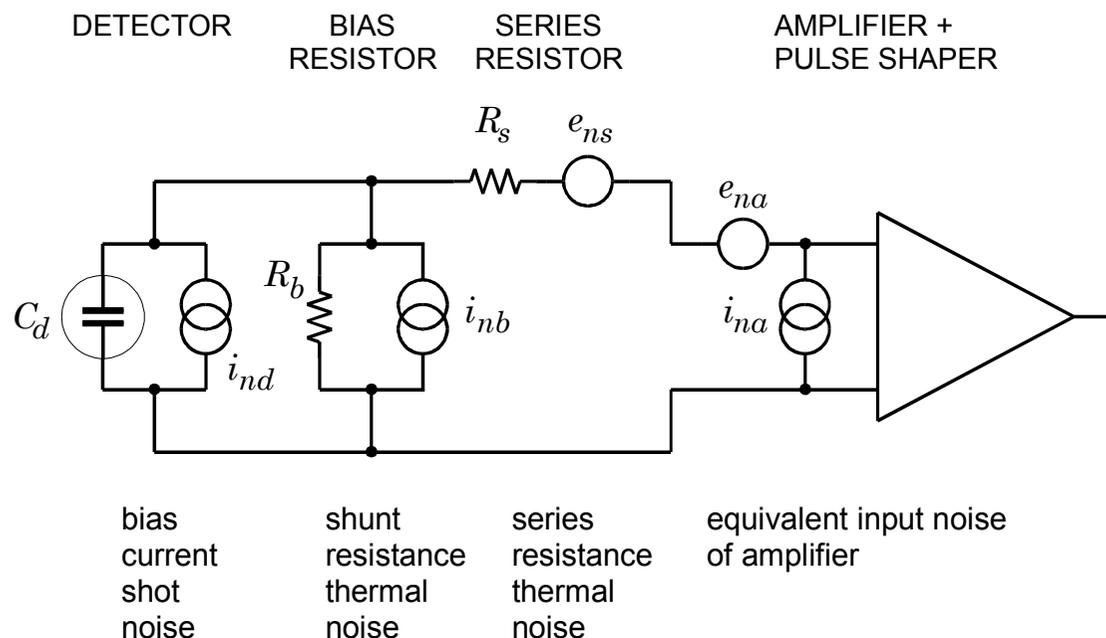
Detector bias voltage is applied through the resistor R_B . The bypass capacitor C_B serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the “far end” of the bias resistor to ground, so that R_B appears to be in parallel with the detector.

The coupling capacitor C_C in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why this capacitor is also called a “blocking capacitor”).

The series resistor R_S represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients (“input protection”)

Equivalent circuit for noise analysis



In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage appearing at the amplifier input.

Resistors can be modeled either as voltage or current generators.

- Resistors in parallel with the input act as current sources.
- Resistors in series with the input act as voltage sources.

Steps in the analysis:

1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources
2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.
3. Determine the output signal for a known signal charge and calculate equivalent noise charge (signal charge for $S/N= 1$)

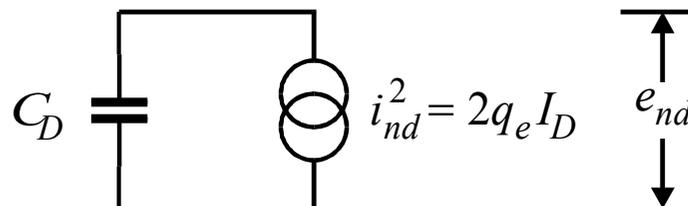
First, assume a simple *CR-RC* shaper with

equal differentiation and integration time constants $\tau_d = \tau_i = \tau$,

which in this special case is equal to the peaking time.

Noise Contributions

a) Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite
2. The shunt resistance R_p is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

Does this assumption make sense?

If R_p is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_p C_D \gg T_P \approx \frac{1}{\omega_P},$$

where ω_P is the midband frequency of the shaper.

Therefore, $R_p \gg \frac{1}{\omega_P C_D}$ as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega C_D)^2} = 2q_e I_D \frac{1}{(\omega C_D)^2}$$

⇒ The noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time T . The number of electron charges measured is

$$N_e = \frac{I_D T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

b) Parallel Resistance

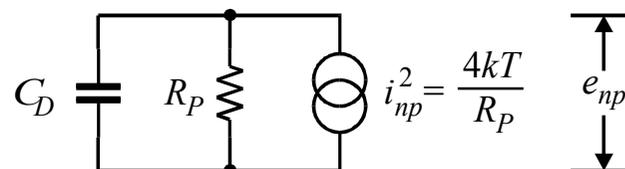
Any shunt resistance R_P acts as a noise current source. In the specific example shown above, the only shunt resistance is the bias resistor R_b .

Additional shunt components in the circuit:

1. bias noise current source (infinite resistance by definition)
2. detector capacitance

The noise current flows through both the resistance R_P and the detector capacitance C_D .

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$e_{np}^2 = \frac{4kT}{R_P} \left(\frac{R_P \cdot \frac{-i}{\omega C_D}}{R_P - \frac{i}{\omega C_D}} \right)^2$$

$$e_{np}^2 = 4kTR_P \frac{1}{1 + (\omega R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^{\infty} e_{np}^2(\omega) d\omega = \int_0^{\infty} \frac{4kTR_P}{1 + (\omega R_P C_D)^2} d\omega = \frac{kT}{C_D},$$

which is independent of R_P . Commonly referred to as “ kTC ” noise, this contribution is often erroneously interpreted as the “noise of the detector capacitance”.

An ideal capacitor has no thermal noise; all noise originates in the resistor.

So, why is the result independent of R_P ?

R_P determines the primary noise, but also the noise bandwidth of this subcircuit. As R_P increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of R_P .

However,

If one integrates e_{np} over a bandwidth-limited system (such as our shaper),

$$v_n^2 = \int_0^{\infty} 4kTR_P \left| \frac{G(i\omega)}{1 - i\omega R_P C_D} \right|^2 d\omega$$

the total noise decreases with increasing R_P .

c) Series Resistance

The noise voltage generator associated with the series resistance R_S is in series with the other noise sources, so it simply contributes

$$e_{nr}^2 = 4kTR_S$$

d) Amplifier input noise

The amplifier noise voltage sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain, where it appears as a noise voltage generator.

$$e_{na}^2 = e_{nw}^2 + \frac{A_f}{f}$$

\uparrow \uparrow
 “white noise” 1/f noise
 (can also originate in external components)

This noise voltage generator also adds in series with the other sources.

- Amplifiers generally also exhibit input current noise, which is physically present at the input. Its effect is the same as for the detector bias current, so the analysis given in 1. can be applied.
- In a well-designed amplifier the noise is dominated by the input transistor (fast, high-gain transistors generally best). Noise parameters of transistors are discussed in Chapter 6.

Transistor input noise decreases with transconductance \Rightarrow increased power

- Minimum device noise limited both by technology and fundamental physics.

Equivalent Noise Charge

$$Q_n^2 = \left(\frac{e^2}{8} \right) \left[\left(2q_e I_D + \frac{4kT}{R_p} + i_{na}^2 \right) \cdot \tau + \left(4kTR_S + e_{na}^2 \right) \cdot \frac{C_D^2}{\tau} + 4A_f C_D^2 \right]$$

$e = \exp(1)$	↑ current noise $\propto \tau$ independent of C_D	↑ voltage noise $\propto 1/\tau$ $\propto C_D^2$	↑ 1/f noise independent of τ $\propto C_D^2$
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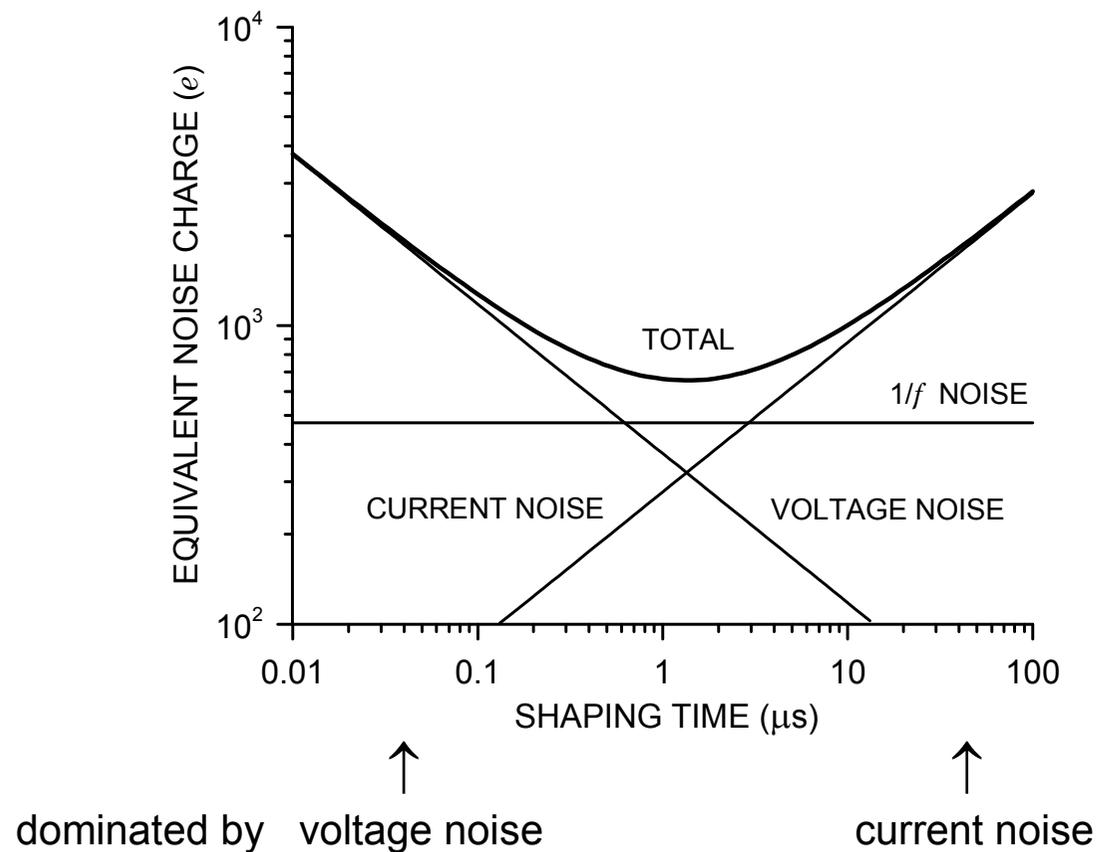
- Current noise is independent of detector capacitance, consistent with the notion of “counting electrons”.
- Voltage noise increases with detector capacitance (reduced signal voltage)
- 1/f noise is independent of shaping time.

In general, the total noise of a 1/f source depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If τ_d and τ_i are scaled by the same factor, this ratio remains constant.

- Detector leakage current and FET noise decrease with temperature

⇒ High resolution Si and Ge detectors for x-rays and gamma rays operate at cryogenic temperatures.

The equivalent noise charge Q_n assumes a minimum when the current and voltage noise contributions are equal. Typical result:



For a CR-RC shaper the noise minimum obtains for $\tau_d = \tau_i = \tau$.

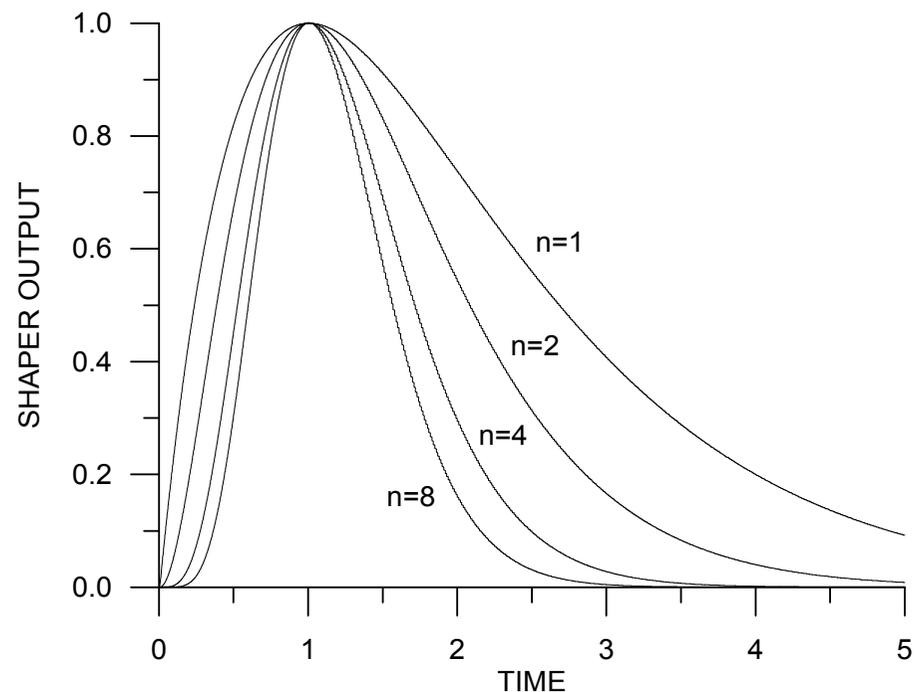
This criterion does not hold for more sophisticated shapers.

Other Types of Shapers

Shapers with Multiple Integrators

Start with simple *CR-RC* shaper and add additional integrators ($n=1$ to $n=2, \dots n=8$).

Change integrator time constants to preserve the peaking time $\tau_n = \tau_{n=1} / n$



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

⇒ improved rate capability at the same peaking time

Shapers with the equivalent of 8 *RC* integrators are common. Usually, this is achieved with active filters

(i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

Time-Variant Shapers

Time variant shaper change the filter parameters during the processing of individual pulses.

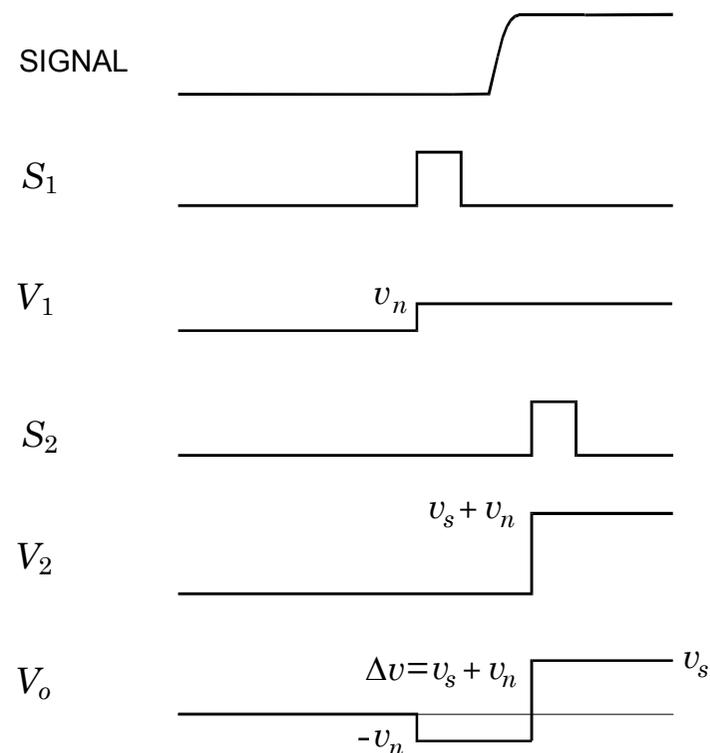
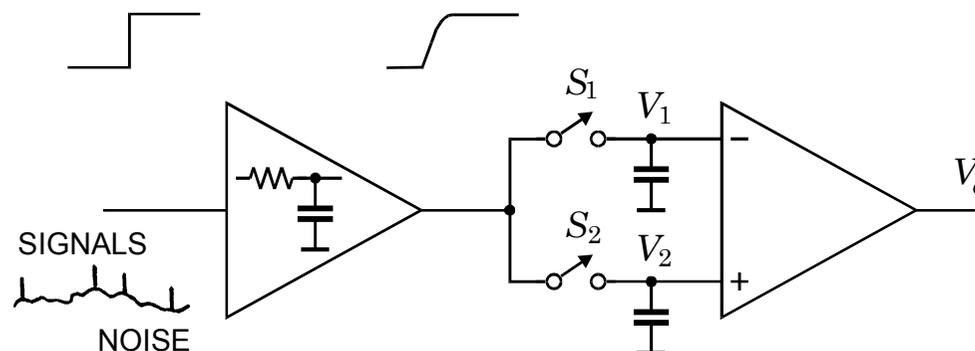
A commonly used time-variant filter is the correlated double-sampler.

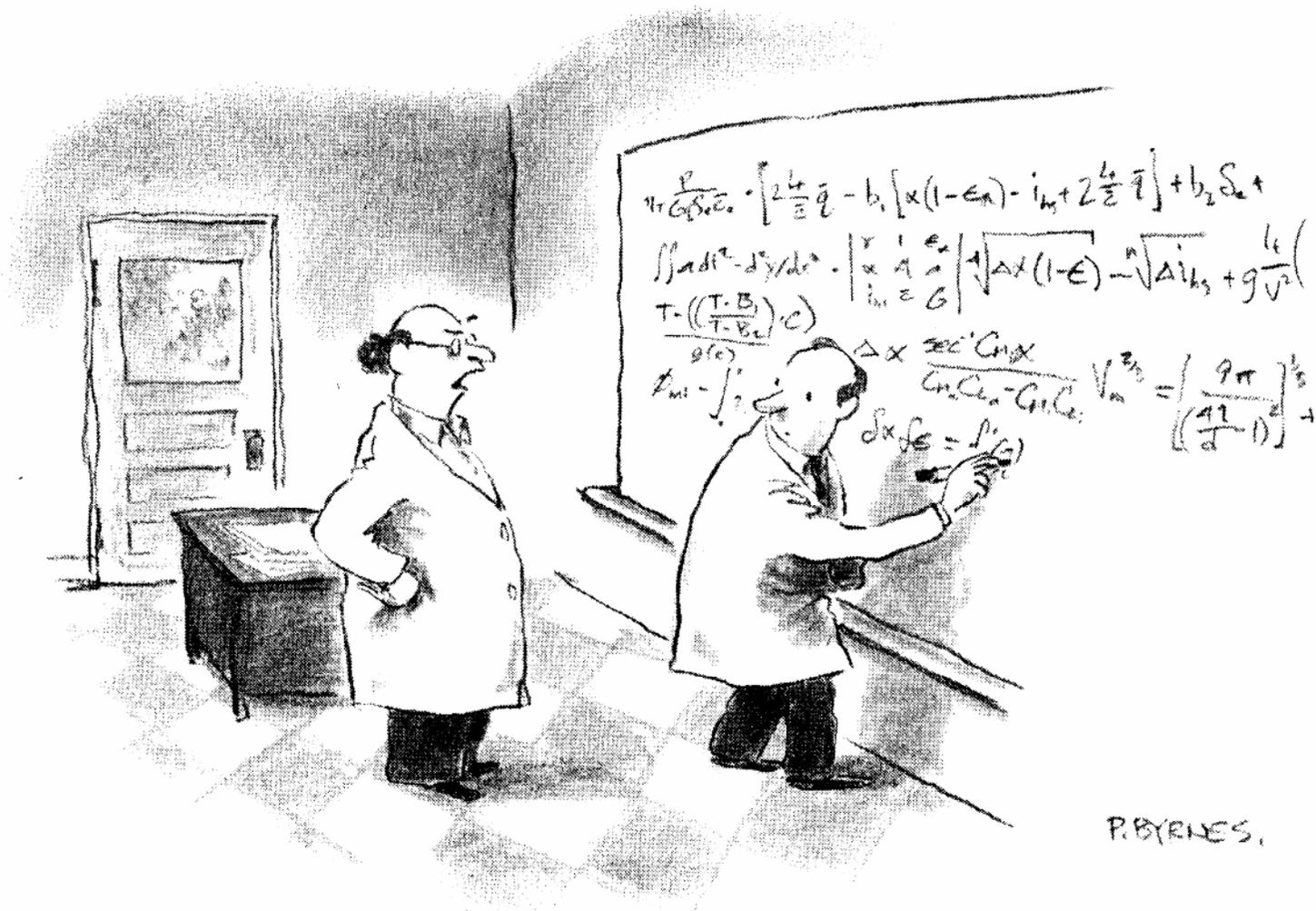
1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal

S/N depends on

1. time constant of prefilter
2. time difference between samples

See “Semiconductor Detector Systems”
for a detailed noise analysis.
(Chapter 4, pp 160-166)



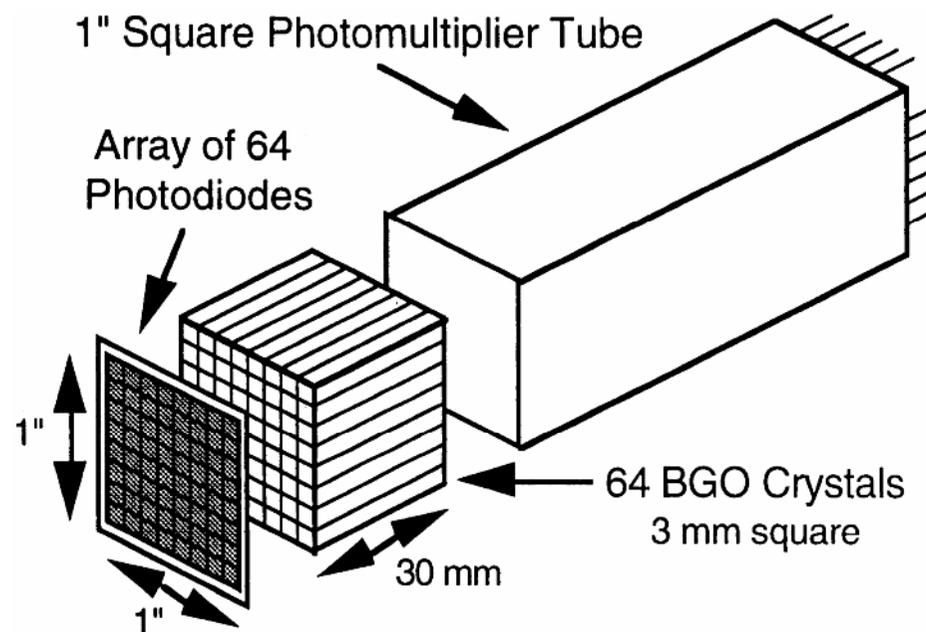


“Dub.”

Examples: Photodiode Readout

(S. Holland, N. Wang, I. Kipnis, B. Krieger, W. Moses, LBNL)

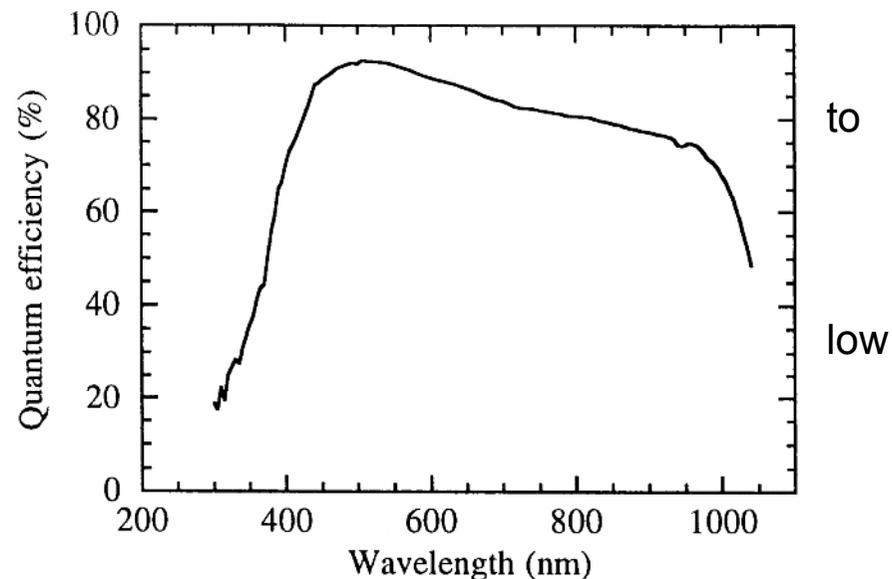
Medical Imaging (Positron Emission Tomography)



Read out 64 BGO crystals with one PMT (timing, energy) and tag crystal by segmented photodiode array.

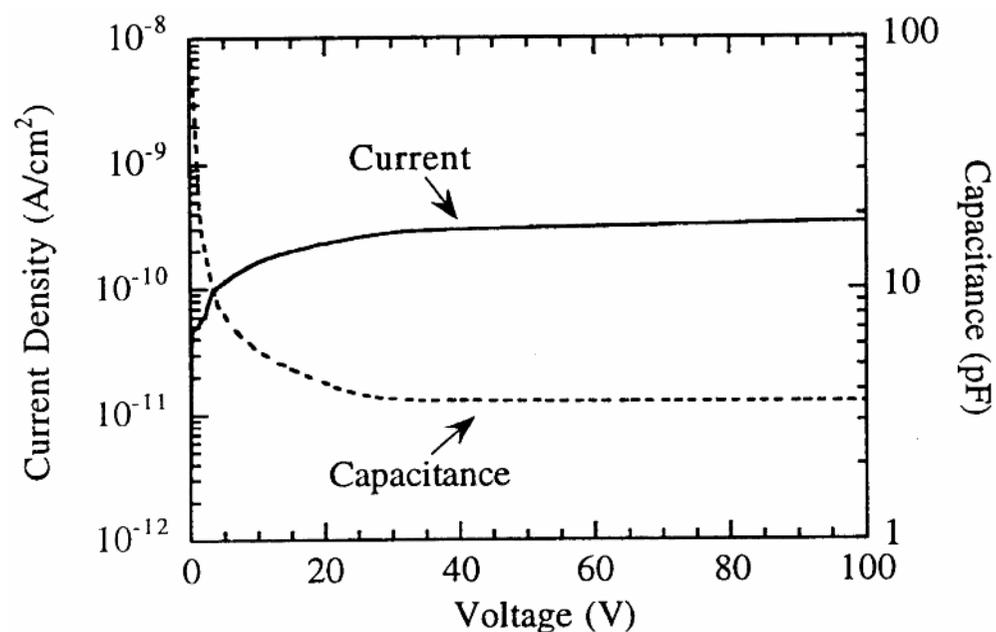
Requires thin dead layer on photodiode maximize quantum efficiency.

Thin electrode must be implemented with resistance to avoid significant degradation of electronic noise.



Furthermore, low reverse bias current critical to reduce noise.

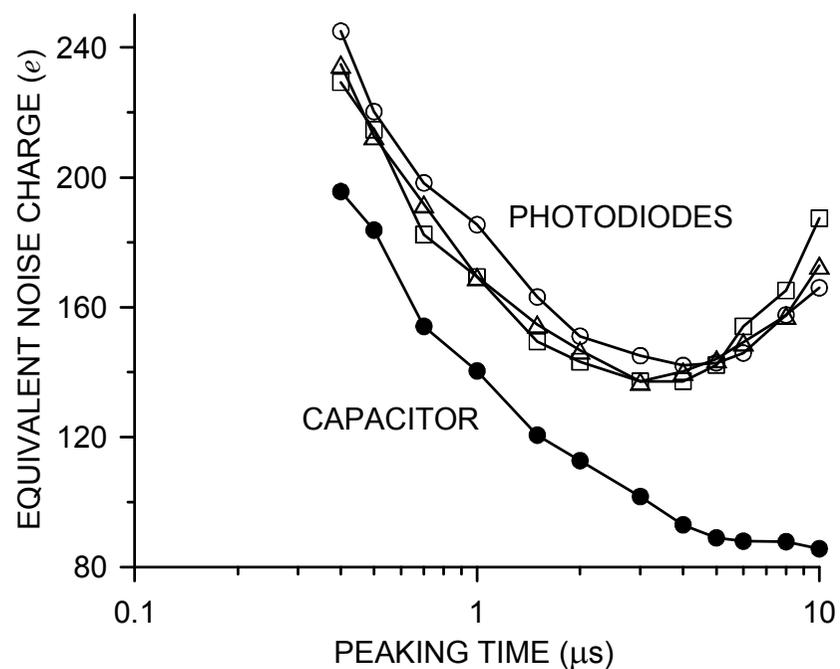
Photodiodes designed and fabricated in LBNL Microsystems Lab.



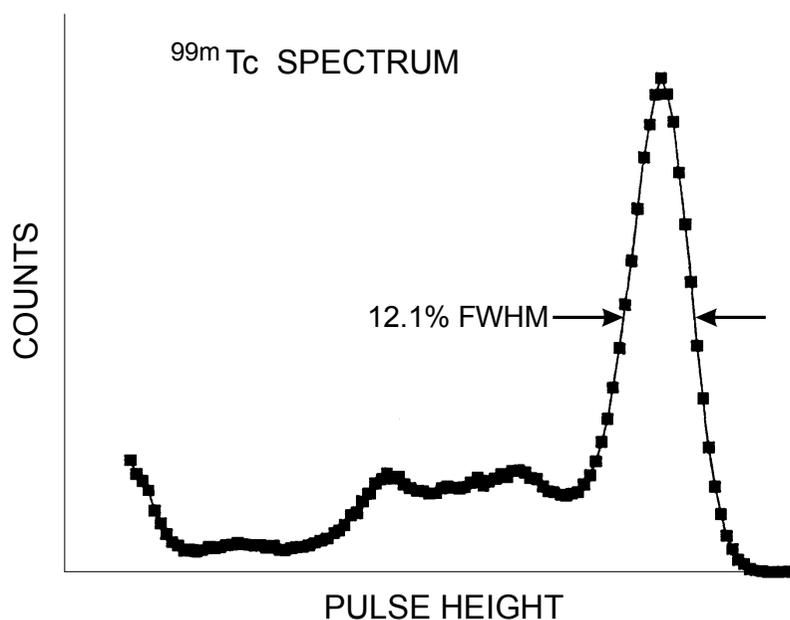
Front-end chip (preamplifier + shaper): 16 channels per chip, die size: 2 x 2 mm²,
1.2 μm CMOS

continuously adjustable shaping time (0.5 to 50 μs)

Noise vs. shaping time



Energy spectrum with BGO scintillator



Note increase in noise at long shaping times when photodiode is connected - shot noise contribution.

Examples: Short-Strip Si X-Ray Detector

(B. Ludewigt, C. Rossington, I. Kipnis, B. Krieger, LBNL)

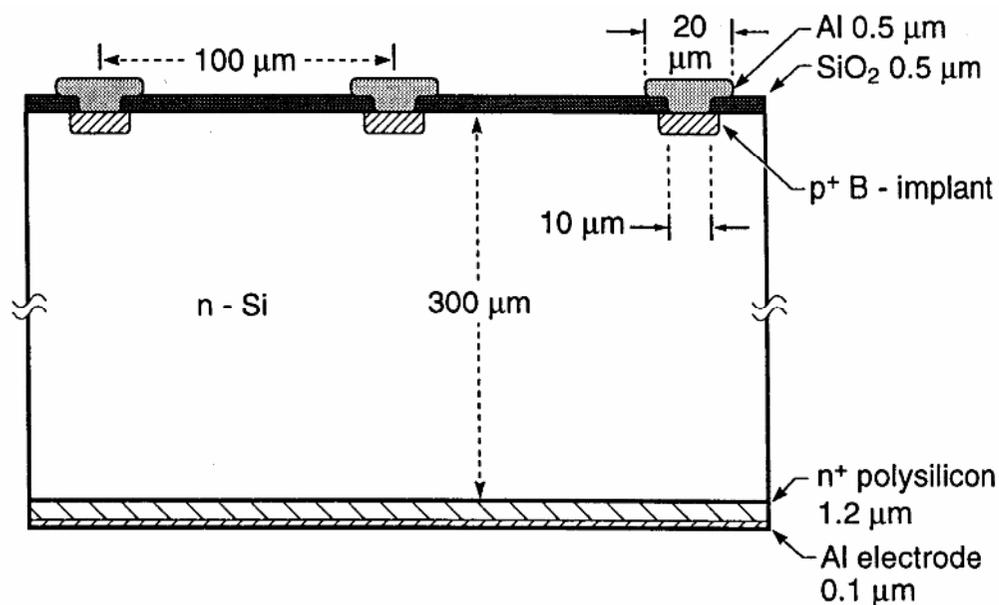
Use detector with multiple strip electrodes not for position resolution,

- but for segmentation
- ⇒ distribute rate over many channels
 - ⇒ reduced capacitance
 - ⇒ low noise at short shaping time
 - ⇒ higher rate per detector element

For x-ray energies 5 – 25 keV ⇒ photoelectric absorption dominates
(signal on 1 or 2 strips)

Strip pitch: 100 μm

Strip Length: 2 mm
(matched to ALS beam)

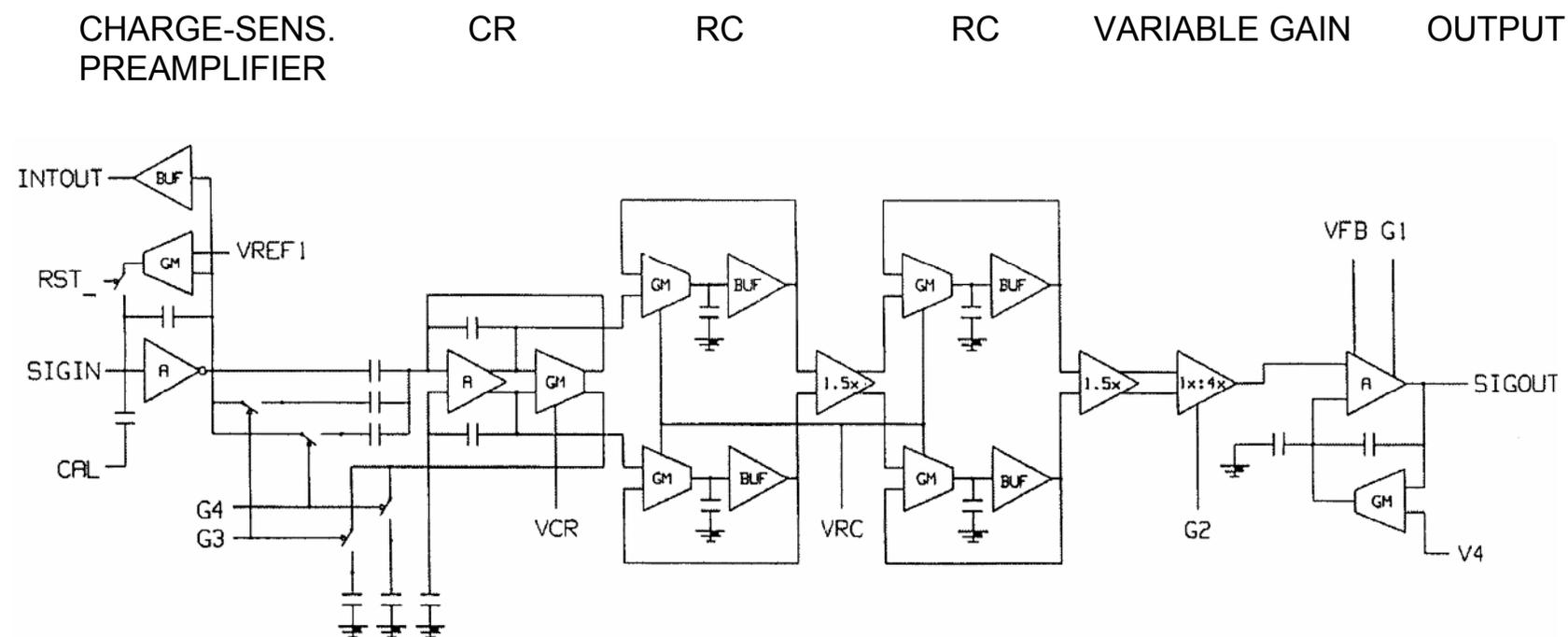


Readout IC tailored to detector

Preamplifier + CR-RC² shaper + cable driver to bank of parallel ADCs (M. Maier + H. Yaver)

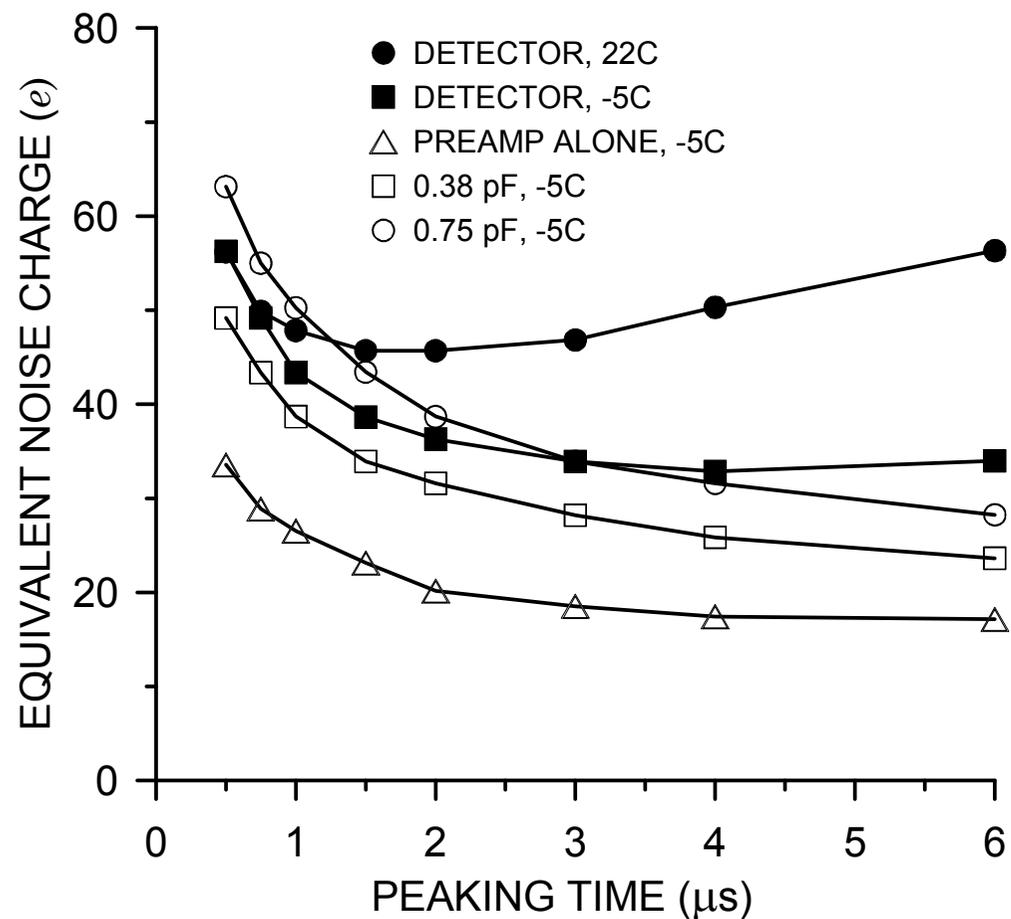
Preamplifier with pulsed reset.

Shaping time continuously variable 0.5 to 20 μ s.



Noise Charge vs. Peaking Time

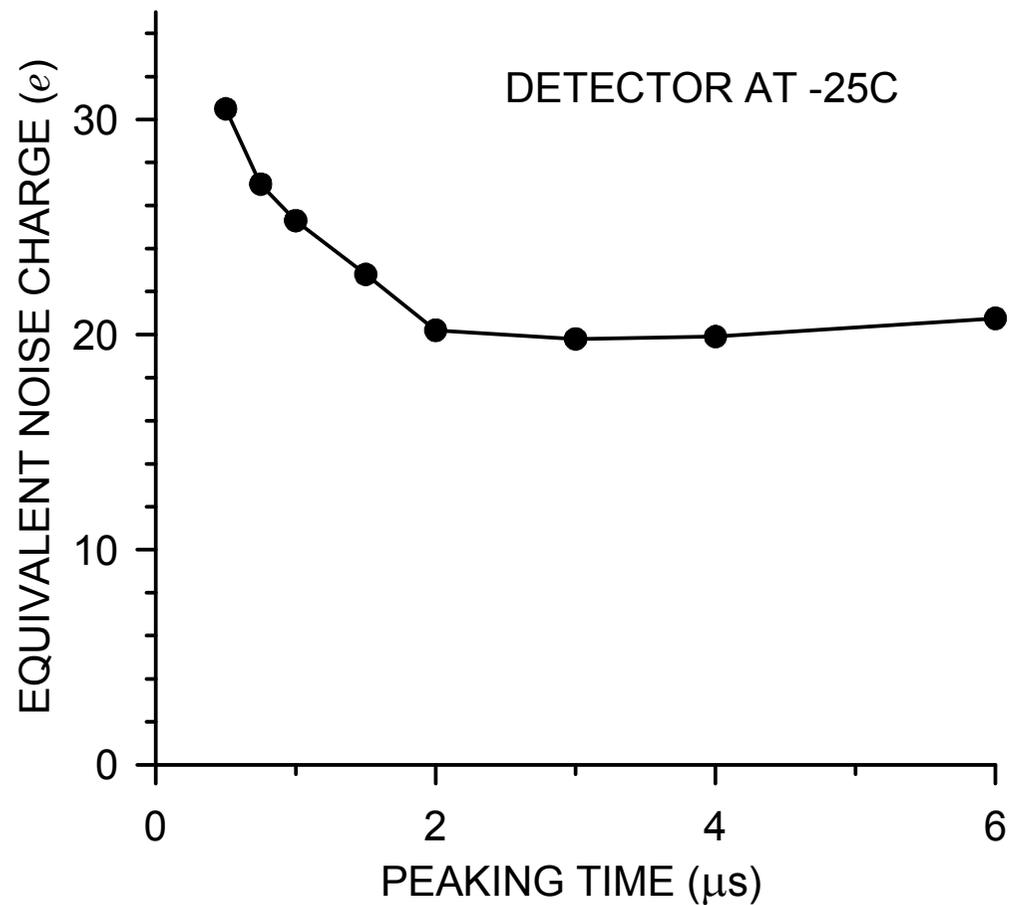
- Open symbols: preamplifier alone and with capacitors connected instead of a detector.
- Connecting the detector increases noise because of added capacitance and detector current (as indicated by increase of noise with peaking time).
- Cooling the detector reduces the current and noise improves.



Second prototype

Current noise negligible because of cooling.

“Flat” noise vs. shaping time indicates that $1/f$ noise dominates.



“Series” and “Parallel” Noise

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

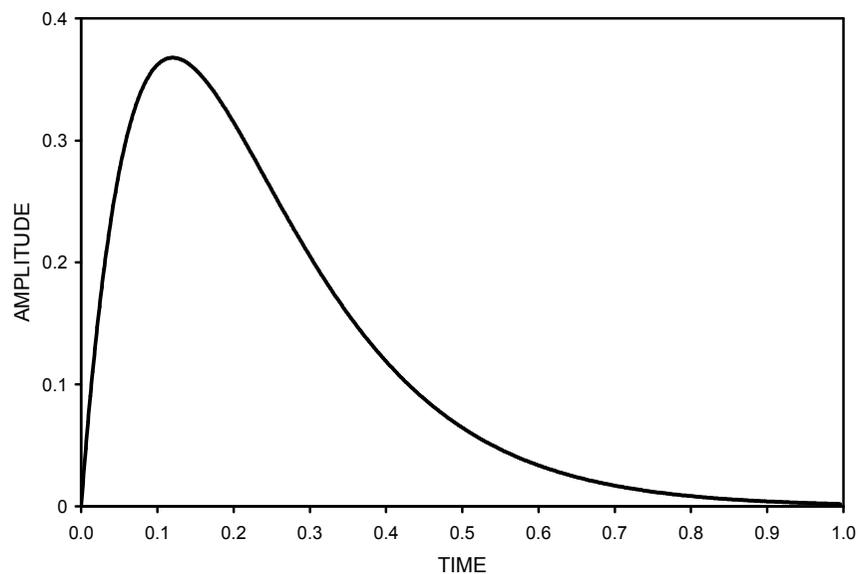
The rest of the world uses equivalent noise voltage and current.

Since they are physically meaningful, use of these widely understood terms is preferable.

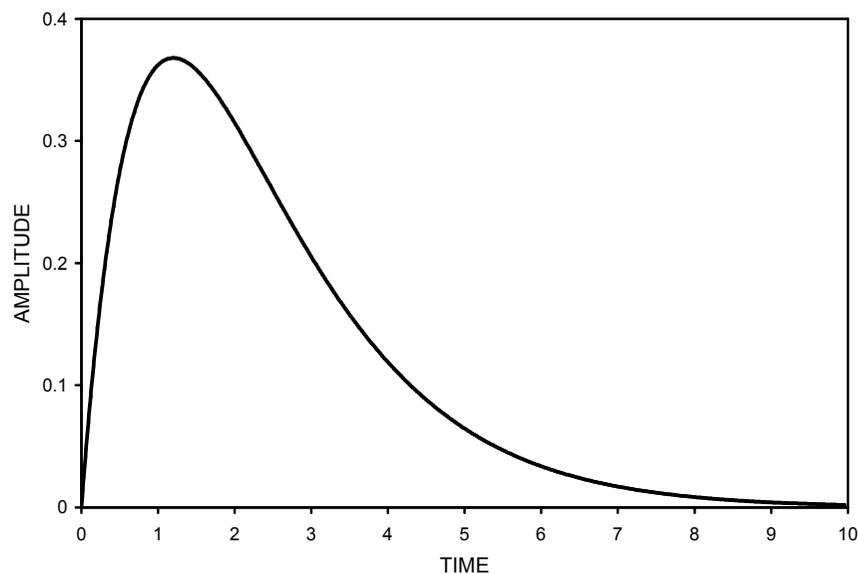
Scaling of Filter Noise Parameters

Pulse shape is the same when shaping time is changed.

shaping time = τ



shaping time = 10τ



Shaper can be characterized by a “shape factor” which multiplied by the shaping time sets the noise bandwidth.

The expression for the equivalent noise charge

$$Q_n^2 = \left(\frac{e^2}{8} \right) \left[\left(2q_e I_D + \frac{4kT}{R_p} + i_{na}^2 \right) \cdot \tau + \left(4kTR_s + e_{na}^2 \right) \cdot \frac{C_D^2}{\tau} + 4A_f C_D^2 \right]$$

	↑	↑	↑
$e = \exp(1)$	current noise	voltage noise	1/f noise
	$\propto \tau$	$\propto 1/\tau$	independent of τ
	independent of C_D	$\propto C_D^2$	$\propto C_D^2$

can be put in a more general form that applies to all type of pulse shapers:

$$Q_n^2 = i_n^2 T_s F_i + C^2 e_n^2 \frac{F_v}{T_s} + F_{vf} A_f C^2$$

- The current and voltage terms are combined and represented by i_n^2 and e_n^2 .
- The shaper is characterized by a shape and characteristic time (e.g. the peaking time).
- A specific shaper is described by the “shape factors” F_i , F_v , and F_{vf} .
- The effect of the shaping time is set by T_s .

Detector Noise Summary

Two basic noise mechanisms: input noise current i_n
 input noise voltage e_n

Equivalent Noise Charge:
$$Q_n^2 = i_n^2 T_s F_i + C^2 e_n^2 \frac{F_v}{T_s}$$

T_s Characteristic shaping time (*e.g.* peaking time)

F_i, F_v "Shape Factors" that are determined by the shape of the pulse.

C Total capacitance at the input (detector capacitance + input capacitance of preamplifier + stray capacitance + ...)

Typical values of F_i, F_v

CR-RC shaper $F_i = 0.924$ $F_v = 0.924$

CR-(RC)⁴ shaper $F_i = 0.45$ $F_v = 1.02$

CR-(RC)⁷ shaper $F_i = 0.34$ $F_v = 1.27$

CAFE chip $F_i = 0.4$ $F_v = 1.2$

Note that $F_i < F_v$ for higher order shapers.

Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).

Minimum noise obtains when the current and voltage noise contributions are equal.

Current noise

- detector bias current increases with detector size, strongly temperature dependent
- noise from resistors shunting the input increases as resistance is decreased
- input transistor – low for FET, higher for BJTs

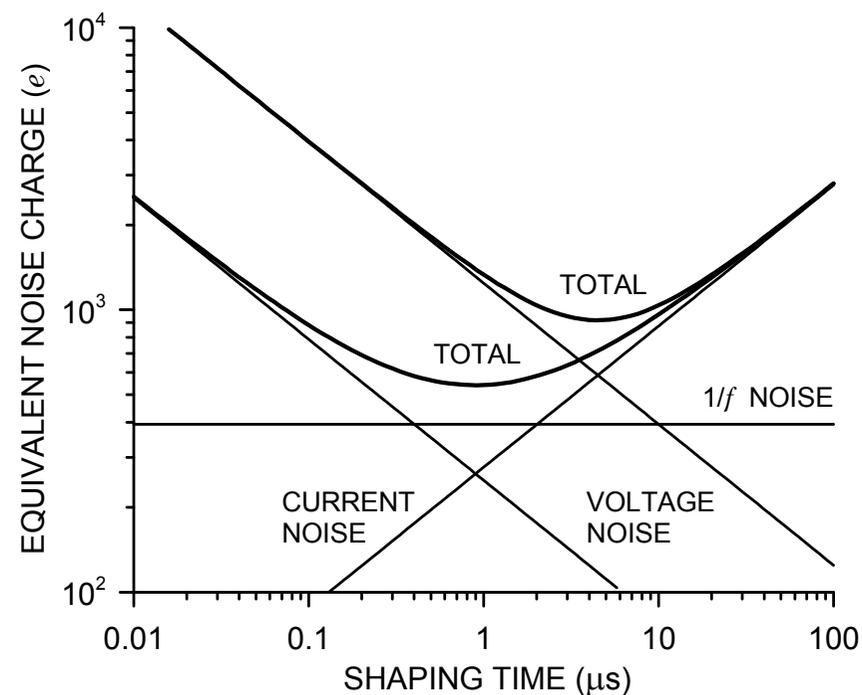
Voltage noise

- input transistor – noise decreases with increased current
- series resistance, e.g. detector electrode, protection circuits

FETs commonly used as input devices – improved noise performance when cooled ($T_{opt} \approx 130$ K)

Bipolar transistors advantageous at short shaping times (<100 ns).

When collector current is optimized, bipolar transistor equivalent noise charge is independent of shaping time (see Chapter 6).



Equivalent Noise Charge vs. Detector Capacitance ($C = C_d + C_a$)

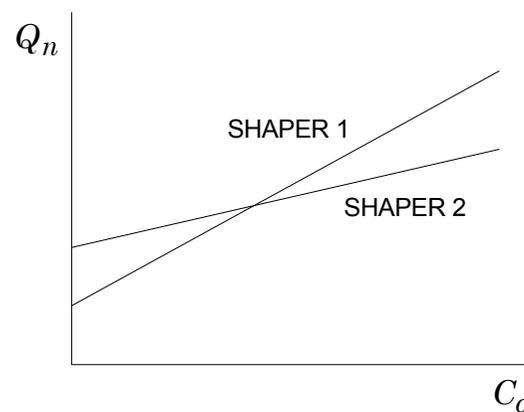
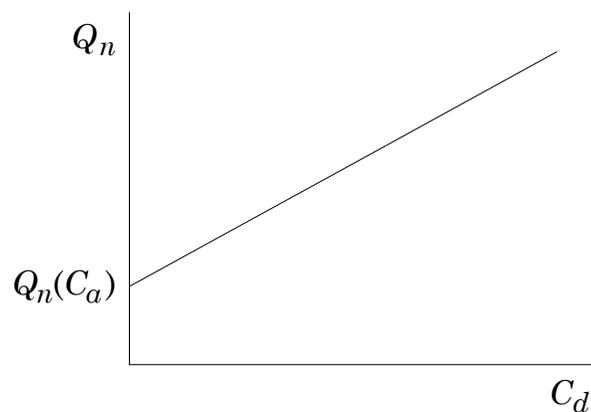
$$Q_n = \sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}$$

$$\frac{dQ_n}{dC_d} = \frac{2C_d e_n^2 F_v \frac{1}{T}}{\sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}}$$

If current noise $i_n^2 F_i T$ is negligible, i.e. **voltage noise dominates**: $\frac{dQ_n}{dC_d} \approx 2e_n \cdot \sqrt{\frac{F_v}{T}}$

Zero intercept: $Q_n|_{C_d=0} = C_a e_n \sqrt{F_v / T}$

↑ ↑
input shaper
stage



Noise vs. Power Dissipation

Analog front-end:

Equivalent Noise Charge:
$$Q_n^2 \approx i_n^2 T_S + e_n^2 C_d^2 \frac{1}{T_S}$$

T_S Shaping Time

i_n Spectral noise current density $i_n^2 = 2eI_{bias} \propto$ strip length

C_d Detector capacitance \propto strip length

e_n Amplifier spectral noise voltage density $e_n^2 \approx \frac{1}{g_m}$

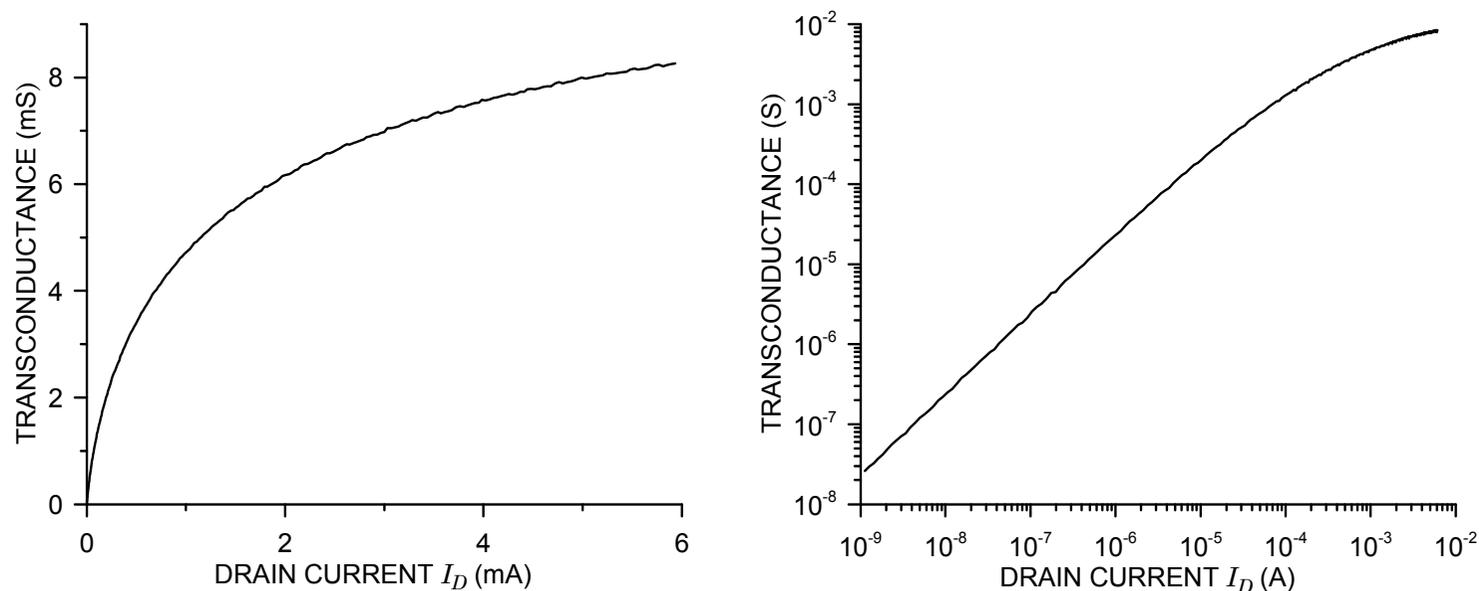
Amplifier spectral noise voltage density e_n depends on transconductance g_m of the input transistor.

How does transconductance depend on the current (power) of the input transistor?

In analog circuitry the current draw is driven by the requirements of noise and speed.

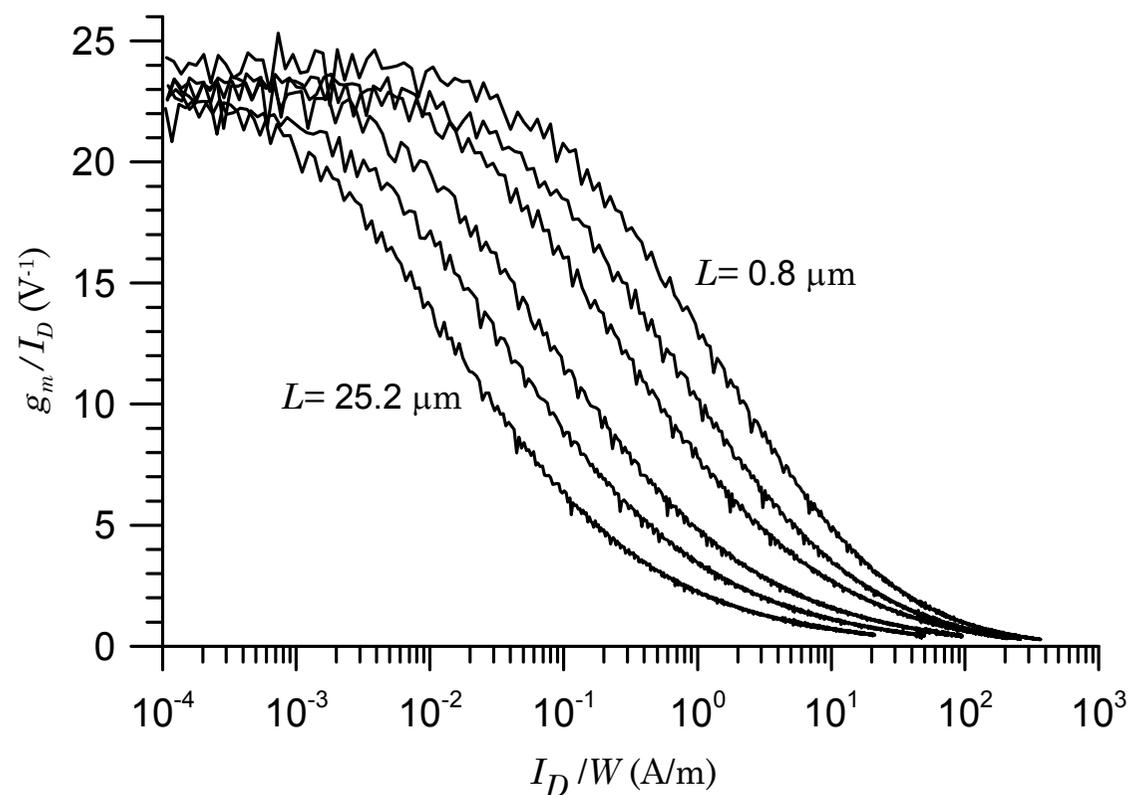
Both depend on transconductance $g_m = \frac{dI_C}{dV_{BE}}$ (BJT) or $g_m = \frac{dI_D}{dV_{GS}}$ (FET).

FET transconductance is a non-linear function of current ($W=100$, $L=0.8 \mu\text{m}$):



Power efficiency depends on transconductance per unit current g_m / I_D .

Measurements on 0.8 μm CMOS process



For a given device the x-values are proportional to device current.

e.g. for $W = 100 \mu\text{m}$, $I_D / W = 10$ corresponds to a current of 1 mA.

Traditional detector front-ends were designed to minimize noise, but accepting a 3 to 5-fold increase in noise reduces power by orders of magnitude!

Scaling of transistor size to optimize power

Procedure:

- For a small device select the current density for a given g_m / I_D from plot on previous page.
- Then increase device width while scaling the device current proportionally (maintain current density).

⇒ increase transconductance (reduce noise)

- Minimum noise when

$$C_{FET} = C_{det} \quad (\text{capacitive matching})$$

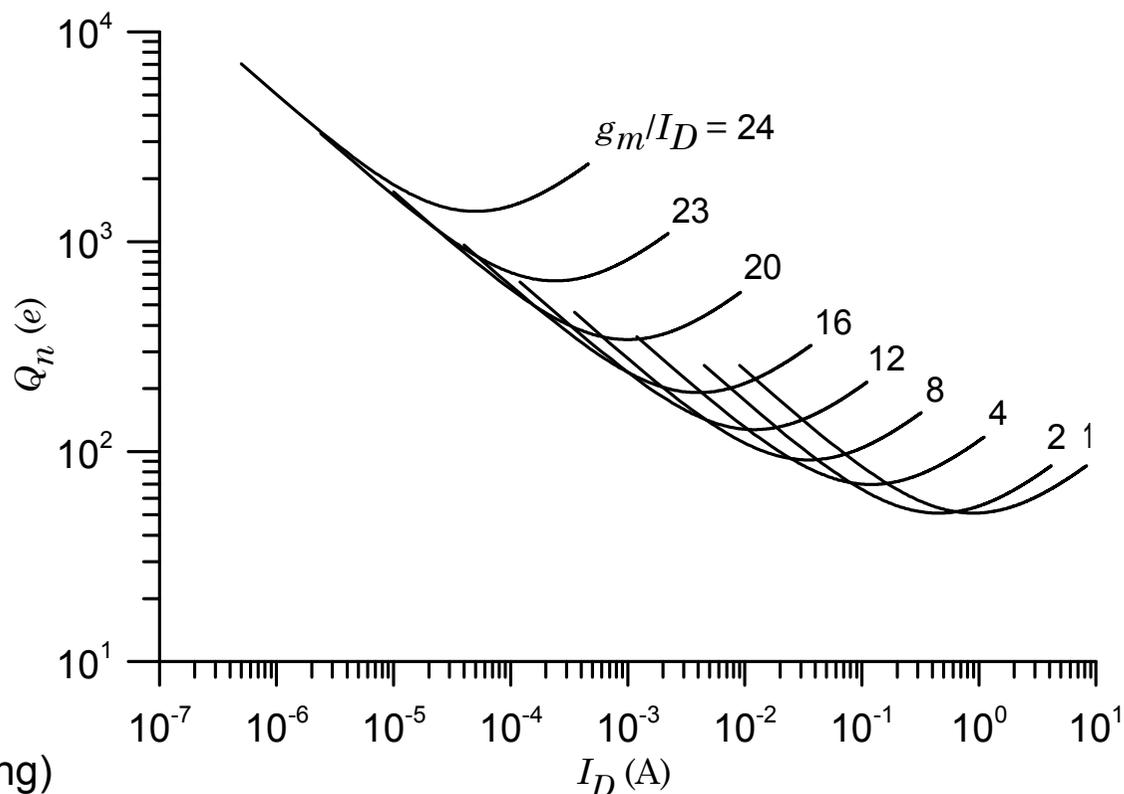
For larger device widths the increase in capacitance overrides the reduction in noise.

This yields minimum noise, but is not most power efficient!

For $g_m / I_D = 24$, minimum noise of 1400 e at 50 μA , but

for $g_m / I_D = 20$ a noise level of 1000 e is obtained at 30 μA .

Given noise level can be achieved at low and high current.



Under optimum scaling to maintain signal-to-noise ratio,
input transistor power (\approx preamp power) scales with $(S/N)^2$.

Power Reduction

1. Segmentation reduces detector capacitance
 - \Rightarrow lower noise for given power
2. Segmentation reduces the hit rate per channel
 - \Rightarrow longer shaping time, reduce voltage noise
3. Segmentation reduces the leakage current per channel (smaller detector volume)
 - \Rightarrow reduced shot noise, increased radiation resistance

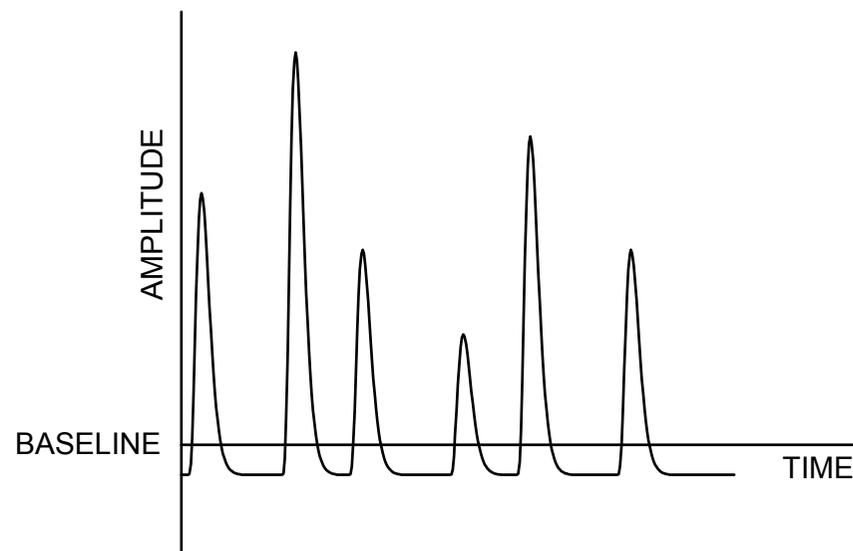
Segmentation key concept in large-scale detector systems.
(also to increase radiation resistance)

3. Some Other Aspects of Pulse Shaping

Baseline Restoration

Any series capacitor in a system prevents transmission of a DC component.

A sequence of unipolar pulses has a DC component that depends on the duty factor, i.e. the event rate.



⇒ The baseline shifts to make the overall transmitted charge equal zero.

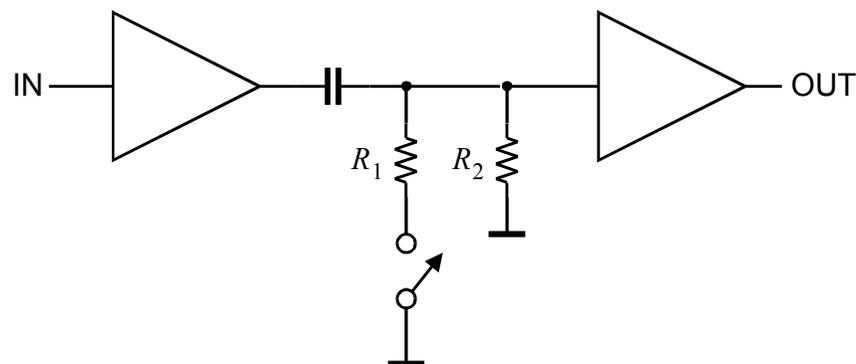
Random rates lead to random fluctuations of the baseline shift ⇒ spectral broadening

- These shifts occur whenever the DC gain is not equal to the midband gain.

The baseline shift can be mitigated by a baseline restorer (BLR).

Principle of a baseline restorer:

Connect signal line to ground during the absence of a signal to establish the baseline just prior to the arrival of a pulse.



R_1 and R_2 determine the charge and discharge time constants.

The discharge time constant (switch opened) must be much larger than the pulse width.

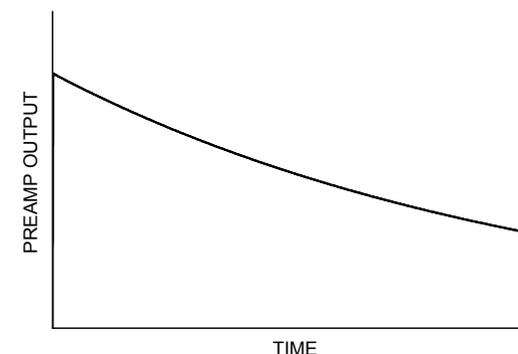
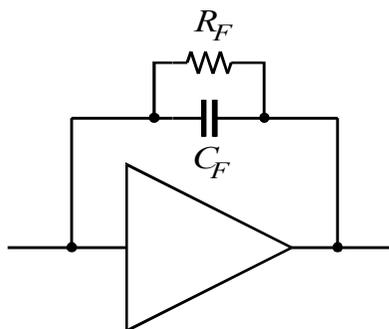
Originally performed with diodes (passive restorer), baseline restoration circuits now tend to include active loops with adjustable thresholds to sense the presence of a signal (gated restorer).

Asymmetric charge and discharge time constants improve performance at high count rates.

- This is a form of time-variant filtering. Care must be exercised to reduce noise and switching artifacts introduced by the BLR.
- Good pole-zero cancellation (next topic) is crucial for proper baseline restoration.

Tail (Pole Zero) Cancellation

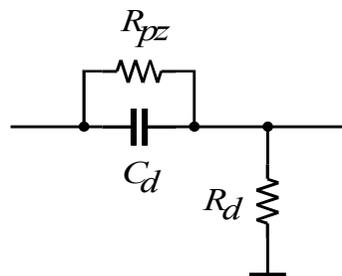
Feedback capacitor in charge sensitive preamplifier must be discharged. Commonly done with resistor.



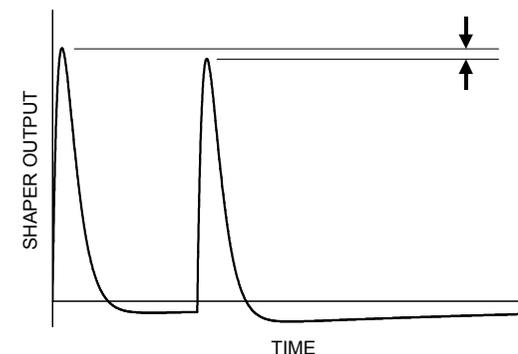
Output no longer a step, but decays exponentially
Exponential decay superimposed on shaper output.

⇒ undershoot

⇒ loss of resolution
due to baseline
variations

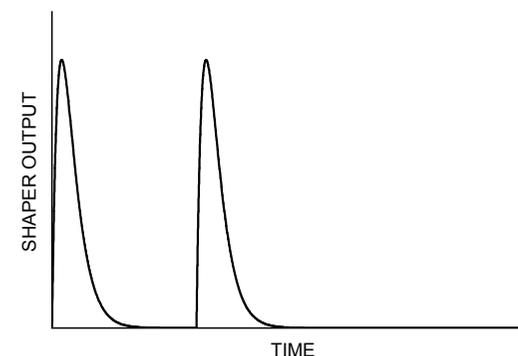


Add R_{pz} to differentiator:



“zero” cancels “pole” of preamp when $R_F C_F = R_{pz} C_d$

Technique also used to compensate for “tails” of detector pulses: “tail cancellation”



Bipolar vs. Unipolar Shaping

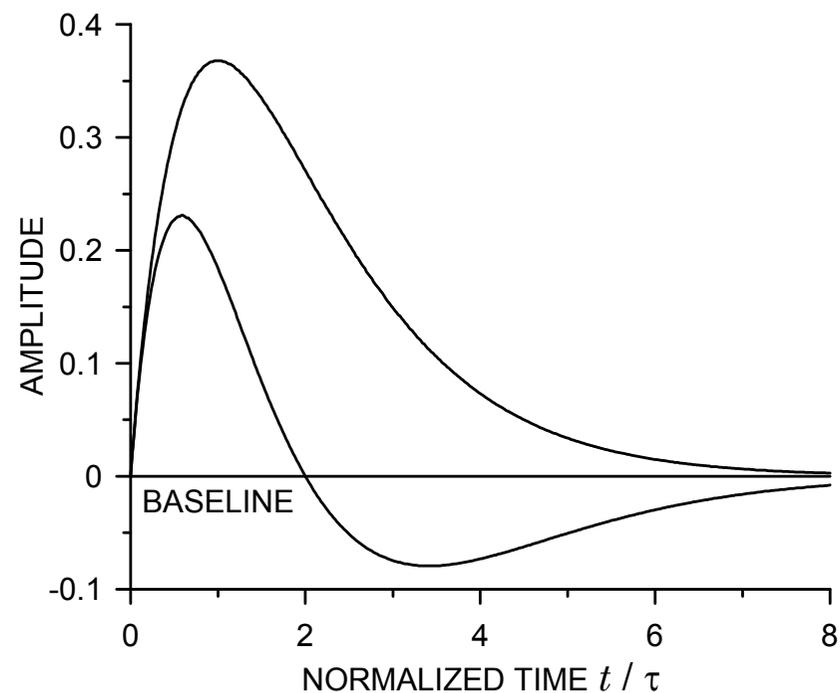
Unipolar pulse + 2nd differentiator

→ Bipolar pulse

Electronic resolution with bipolar shaping
typ. 25 – 50% worse than for
corresponding unipolar shaper.

However ...

- Bipolar shaping eliminates baseline shift
(as the DC component is zero).
- Pole-zero adjustment less critical
- Added suppression of low-frequency noise.
- Not all measurements require optimum noise performance.
Bipolar shaping is much more convenient for the user (important in large systems!)
– often the method of choice.

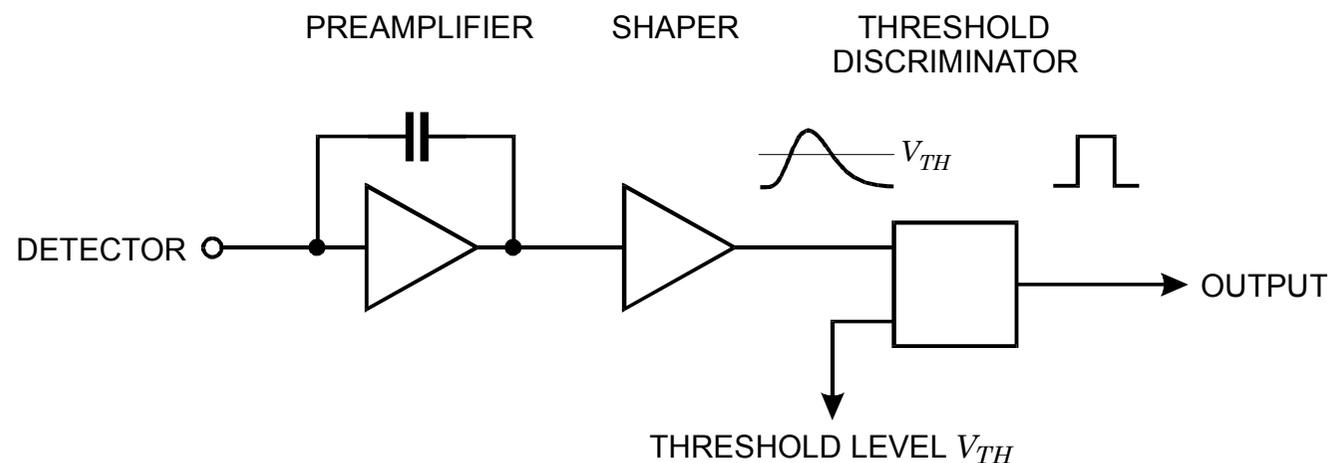


4. Threshold Discriminator Systems

The simplest form of a digitized readout is a threshold discriminator system, which produces a normalized (digital) output pulse when the input signal exceeds a certain level.

Noise affects not only the resolution of amplitude measurements, but also the determines the minimum detectable signal threshold.

Consider a system that only records the presence of a signal if it exceeds a fixed threshold.



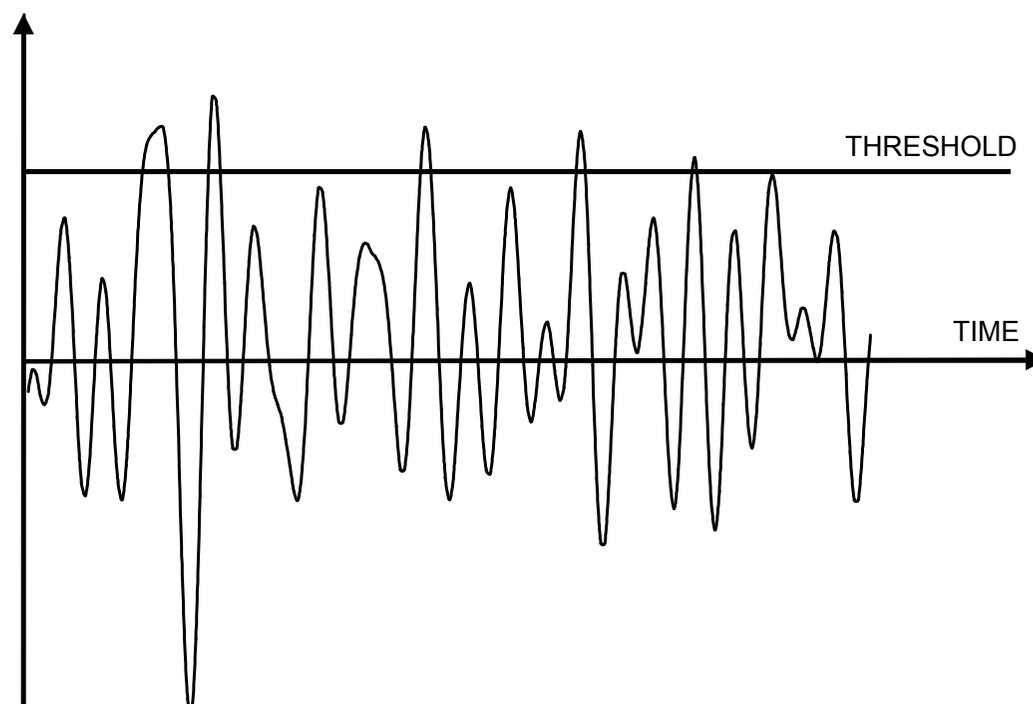
How small a detector pulse can still be detected reliably?

Consider the system at times when no detector signal is present.

Noise will be superimposed on the baseline.

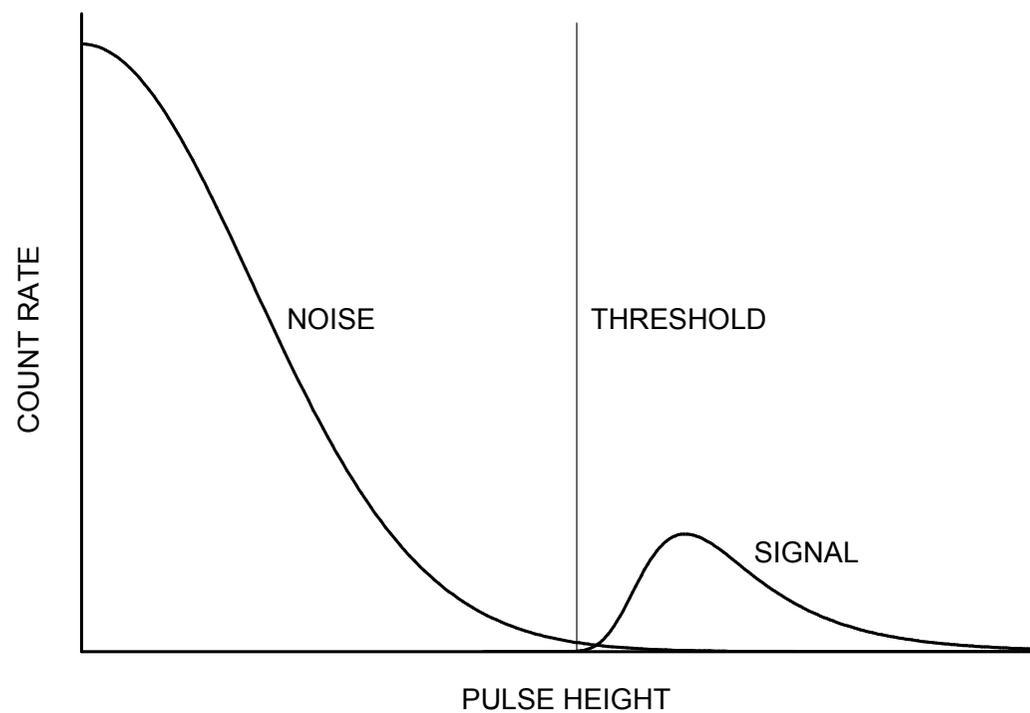
Some noise pulses will exceed the threshold.

This is always true since the amplitude spectrum of Gaussian noise extends to infinity



The threshold must be set

1. high enough to suppress noise hits
2. low enough to capture the signal



With the threshold level set to 0 relative to the baseline, all of the positive excursions will be recorded.

Assume that the desired signals are occurring at a certain rate.

If the detection reliability is to be >99%, for example, then the rate of noise hits must be less than 1% of the signal rate.

The rate of noise hits can be reduced by increasing the threshold.

If the system were sensitive to pulse magnitude alone, the integral over the Gaussian distribution (the error function) would determine the factor by which the noise rate f_{n0} is reduced.

$$\frac{f_n}{f_{n0}} = \frac{1}{Q_n \sqrt{2\pi}} \int_{Q_T}^{\infty} e^{-(Q/2Q_n)^2} dQ ,$$

where Q is the equivalent signal charge,

Q_n the equivalent noise charge and

Q_T the threshold level.

However, since the pulse shaper broadens each noise impulse, **the time dependence is equally important**. For example, after a noise pulse has crossed the threshold, a subsequent pulse will not be recorded if it occurs before the trailing edge of the first pulse has dropped below threshold.

Combined probability function

Both the amplitude and time distribution are Gaussian.

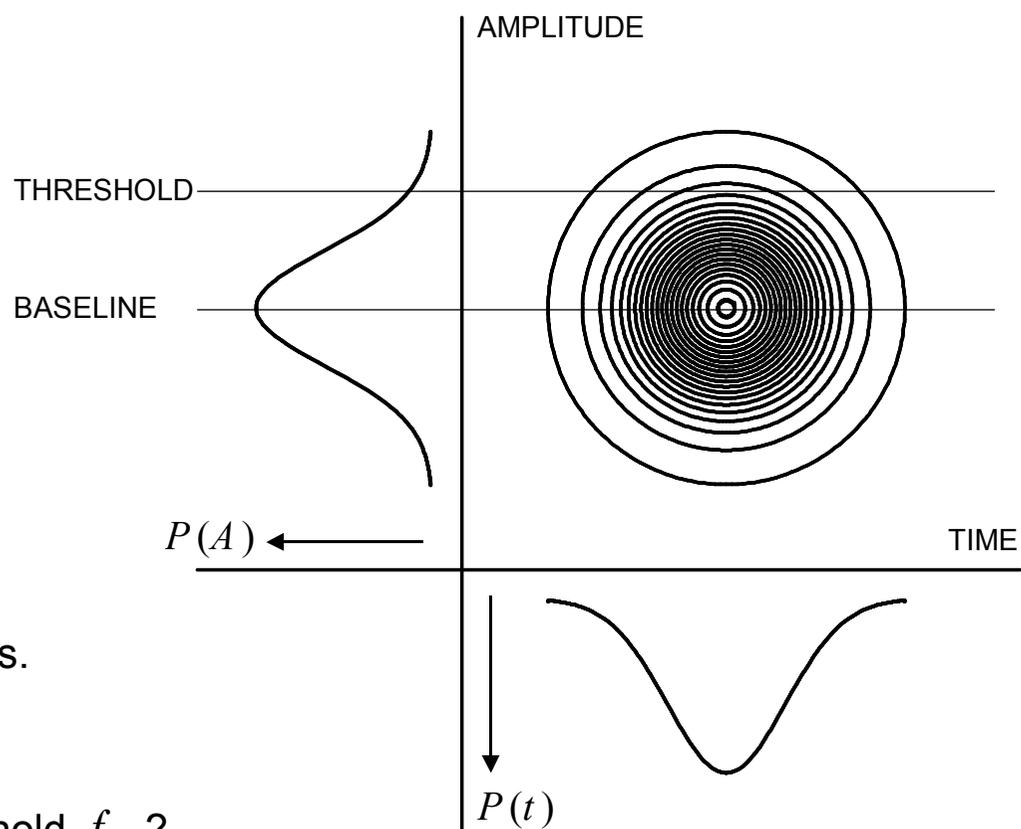
The rate of noise hits is determined by integrating the combined probability density function in the regime that exceeds the threshold.

This yields

$$f_n = f_{n0} \cdot e^{-Q_T^2 / 2Q_n^2}$$

Of course, one can just as well use the corresponding voltage levels.

What is the noise rate at zero threshold f_{n0} ?



For a system with the frequency response $A(f)$ the frequency of zeros

$$f_0^2 = 4 \cdot \frac{\int_0^{\infty} f^2 A^2(f) df}{\int_0^{\infty} A^2(f) df}$$

(Rice, Bell System Technical Journal, **23** (1944) 282 and **24** (1945) 46)

Since we are interested in the number of positive excursions exceeding the threshold, f_{n0} is $\frac{1}{2}$ the frequency of zero-crossings.

For an ideal band-pass filter with lower and upper cutoff frequencies f_1 and f_2 the noise rate

$$f_0 = 2 \sqrt{\frac{1}{3} \frac{f_2^3 - f_1^3}{f_2 - f_1}}$$

For a $CR-RC$ filter with $\tau_i = \tau_d$ the ratio of cutoff frequencies of the noise bandwidth is

$$\frac{f_2}{f_1} = 4.5$$

so to a good approximation one can neglect the lower cutoff frequency and treat the shaper as a low-pass filter, *i.e.* $f_1 = 0$.

Then

$$f_0 = \frac{2}{\sqrt{3}} f_2$$

An ideal bandpass filter has infinitely steep slopes, so the upper cutoff frequency f_2 must be replaced by the noise bandwidth.

The noise bandwidth of an RC low-pass filter with time constant τ is $\Delta f_n = \frac{1}{4\tau}$

Setting $f_2 = \Delta f_n$ yields the frequency of zeros $f_0 = \frac{1}{2\sqrt{3} \tau}$

and the frequency of noise hits vs. threshold

$$f_n = f_{n0} \cdot e^{-Q_{th}^2/2Q_n^2} = \frac{f_0}{2} \cdot e^{-Q_{th}^2/2Q_n^2} = \frac{1}{4\sqrt{3} \tau} \cdot e^{-Q_{th}^2/2Q_n^2}$$

Thus, the required threshold-to-noise ratio for a given frequency of noise hits f_n is

$$\frac{Q_T}{Q_n} = \sqrt{-2 \log(4\sqrt{3} f_n \tau)} \approx \sqrt{-2 \log\left(\frac{f_n}{f_P}\right)},$$

where f_P is the peaking frequency of the shaper

Note that product of noise rate and shaping time $f_n \tau$ determines the required threshold-to-noise ratio, i.e. for a given threshold-to-noise ratio the noise rate is higher at short shaping times

⇒ The noise rate for a given threshold-to-noise ratio is proportional to bandwidth.

⇒ To obtain the same noise rate, a fast system requires a larger threshold-to-noise ratio than a slow system with the same noise level.

Efficiency vs. Occupancy

Frequently a threshold discriminator system is used in conjunction with other detectors that provide additional information, for example the time of a desired event.

In a collider detector the time of beam crossings is known, so the output of the discriminator is sampled at specific times.

The number of recorded noise hits then depends on

1. the sampling frequency (e.g. bunch crossing frequency) f_S
2. the width of the sampling interval Δt , which is determined by the time resolution of the system.

The product $f_S \Delta t$ determines the fraction of time the system is open to recording noise hits, so the rate of recorded noise hits is $f_S \Delta t f_n$.

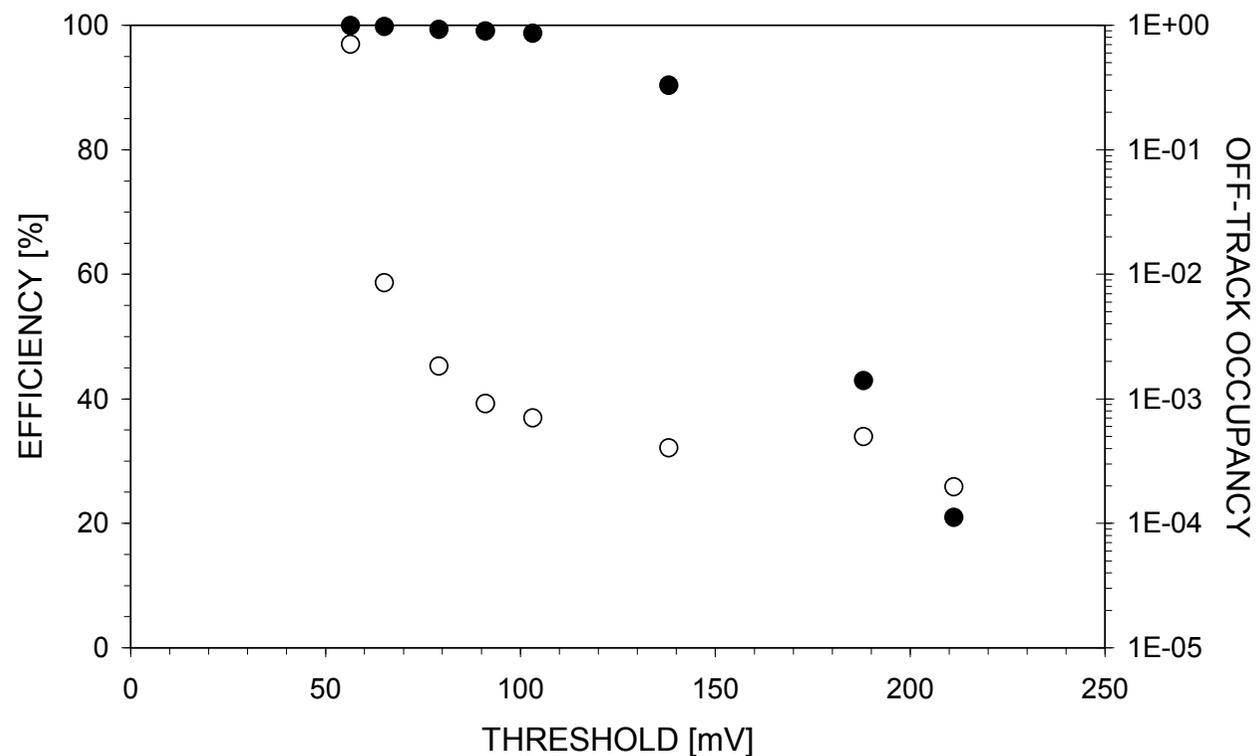
Often it is more interesting to know the probability of finding a noise hit in a given interval, i.e. the occupancy of noise hits, which can be compared to the occupancy of signal hits in the same interval.

This is the situation in a storage pipeline, where a specific time interval is read out after a certain delay time (e.g. trigger latency)

The occupancy of noise hits in a time interval Δt :
$$P_n = \Delta t \cdot f_n = \frac{\Delta t}{2\sqrt{3} \tau} \cdot e^{-Q_T^2 / 2Q_n^2}$$

i.e. the occupancy falls exponentially with the square of the threshold-to-noise ratio.

Example of noise occupancy (open circles) and efficiency (solid circles) vs. threshold in a practical detector module:



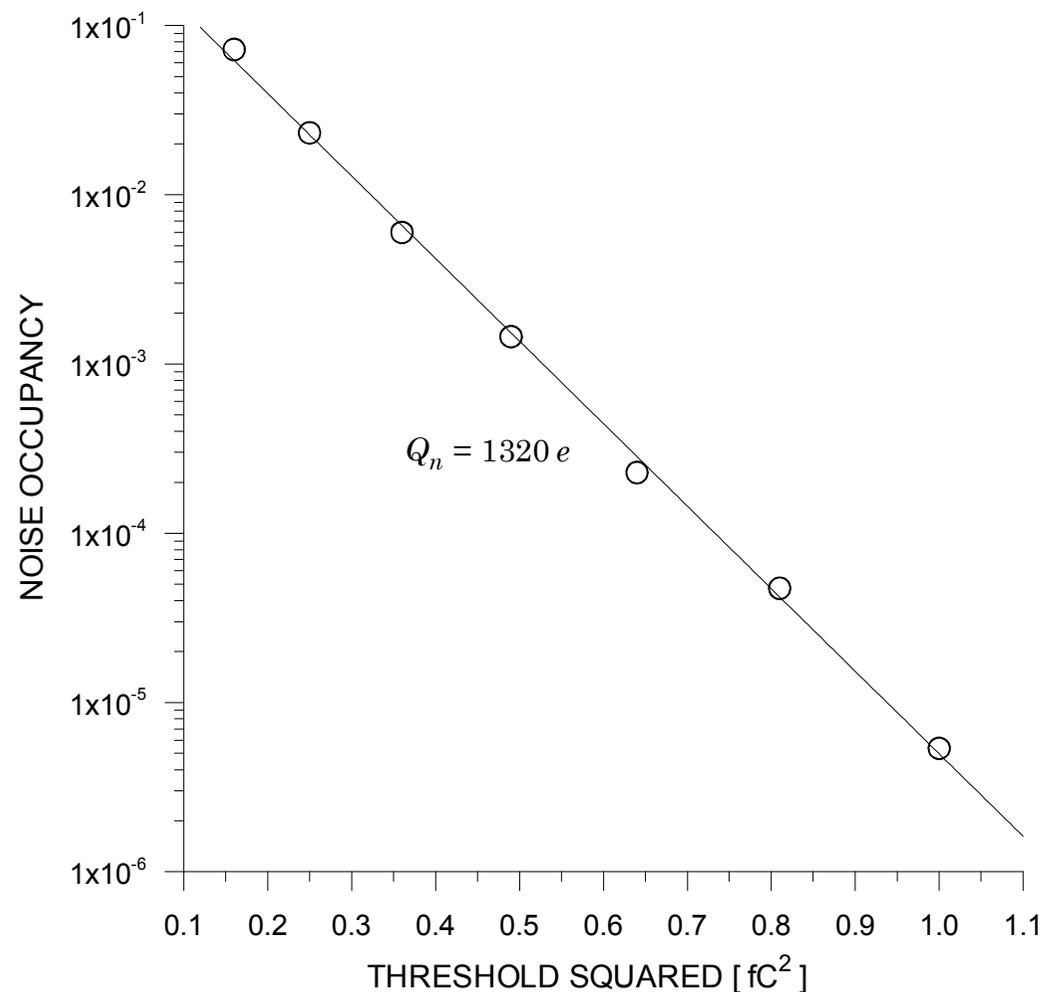
Note that an extended overlap region of high efficiency and low noise occupancy is desired.

The dependence of occupancy on threshold can be used to measure the noise level.

$$\log P_n = \log \left(\frac{\Delta t}{2\sqrt{3} \tau} \right) - \frac{1}{2} \left(\frac{Q_T}{Q_n} \right)^2,$$

so the *slope* of $\log P_n$ vs. Q_T^2 yields the noise level.

This analysis is *independent of the details of the shaper*, which affect only the offset.



5. Timing Measurements

Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

- Timing measurements optimize determination of time of occurrence.
- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

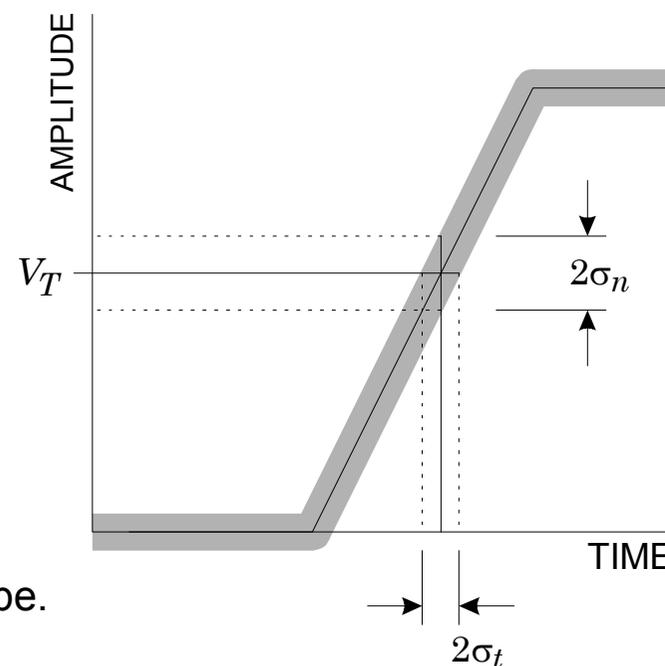
The instantaneous signal level is modulated by noise.

⇒ time of threshold crossing fluctuates

$$\sigma_t = \frac{\sigma_n}{\left. \frac{dV}{dt} \right|_{V_T}} \approx \frac{t_r}{S/N}$$

t_r = rise time

Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.



Pulse Shaping

Consider a system whose bandwidth is determined by a single RC integrator.

The time constant of the RC low-pass filter determines the

- rise time (and hence dV/dt)
- amplifier bandwidth (and hence the noise)

Time dependence: $V_o(t) = V_0(1 - e^{-t/\tau})$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2 \tau$$

In terms of bandwidth

$$t_r = 2.2 \tau = \frac{2.2}{2\pi f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

For a cascade of amplifiers: $t_r \approx \sqrt{t_{r1}^2 + t_{r2}^2 + \dots + t_{rn}^2}$

Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude V_0 and a rise time t_c passing through an amplifier chain with a rise time t_{ra} .

1. amplifier rise time \gg signal rise time:

$$\text{Noise} \propto \sqrt{f_u} \propto \sqrt{\frac{1}{t_{ra}}}$$

$$\frac{dV}{dt} \propto \frac{1}{t_{ra}} \propto f_u$$

increase in bandwidth \Rightarrow improvement in dV/dt outweighs increase in noise.

2. amplifier rise time \ll signal rise time

increase in noise without increase in dV/dt

Optimum: The amplifier rise time should be chosen to match the signal rise time.

Differentiation time constant: choose greater than rise time constant

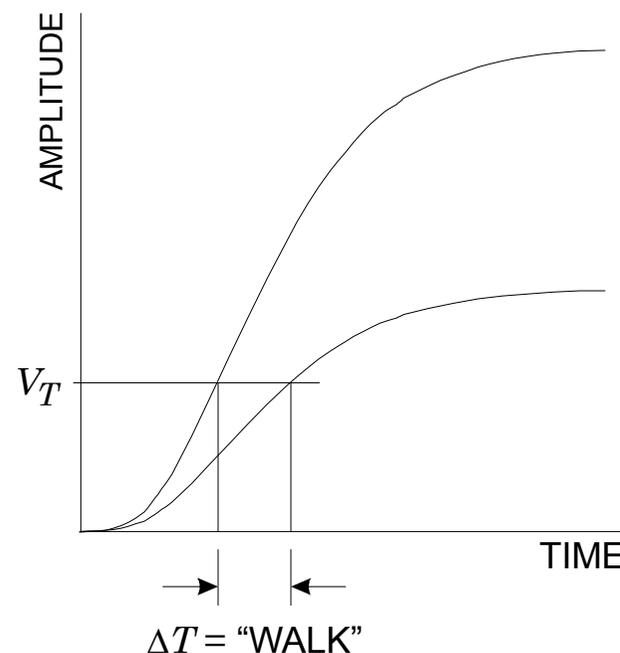
$$(\tau_{diff} = 10\tau_{int} \text{ incurs 20\% loss in pulse height})$$

Time Walk

For a fixed trigger level the time of threshold crossing depends on pulse amplitude.

⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)



If the rise time is known, “time walk” can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

Lowest Practical Threshold

Single RC integrator has maximum slope at $t=0$: $\frac{d}{dt}(1 - e^{-t/\tau}) = \frac{1}{\tau} e^{-t/\tau}$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small t the slope at the output of a single RC integrator is linear, so initially the pulse can be approximated by a ramp αt .

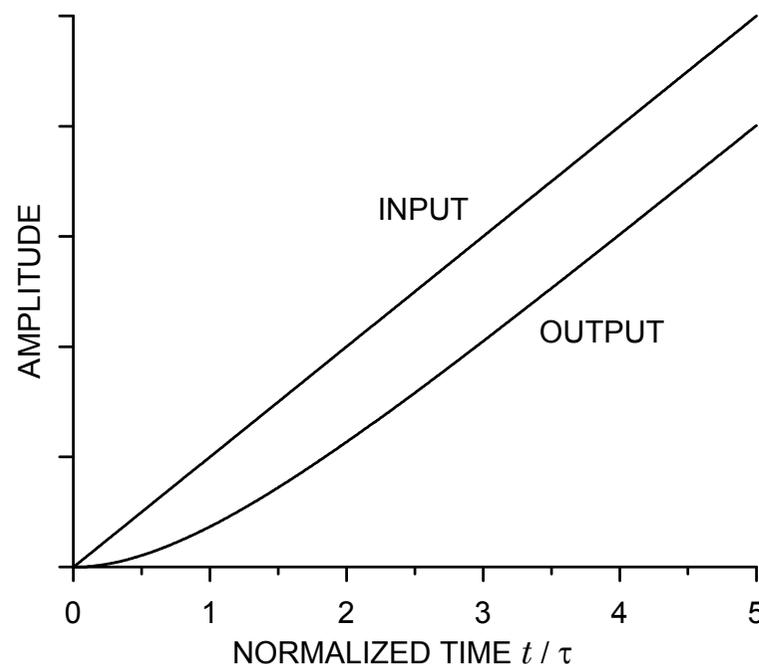
Response of the following integrator

$$V_i = \alpha t \rightarrow V_o = \alpha(t - \tau) - \alpha \tau e^{-t/\tau}$$

\Rightarrow The output is delayed by τ and curvature is introduced at small t .

Output attains 90% of input slope after $t = 2.3\tau$.

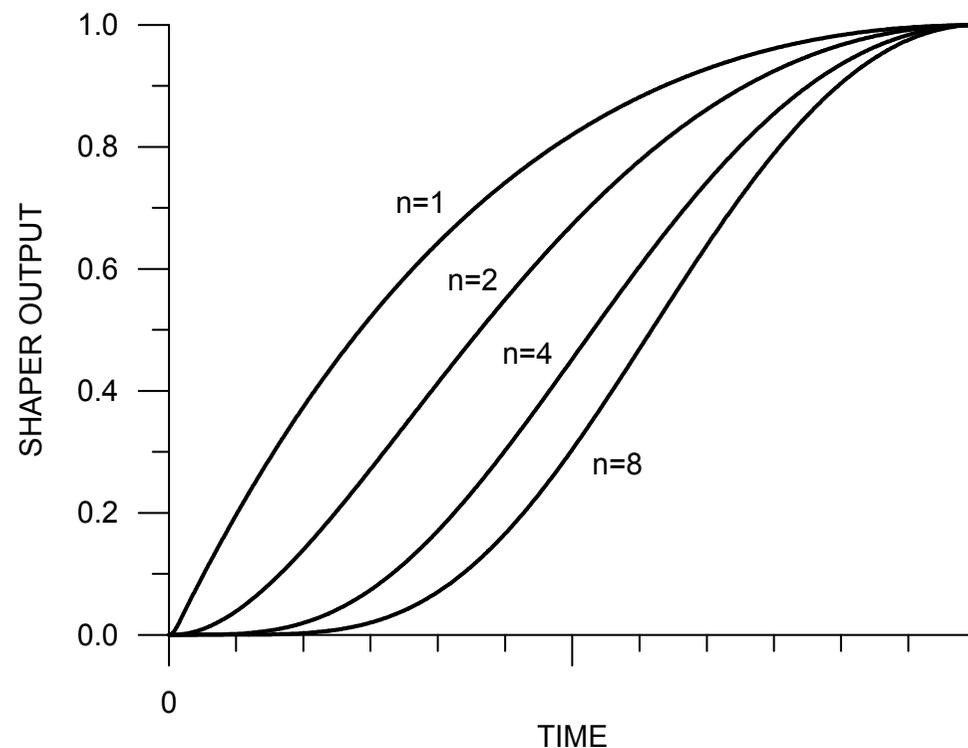
Delay for n integrators = $n\tau$



Additional RC integrators introduce more curvature at the beginning of the pulse.

Output pulse shape for multiple RC integrators

(normalized to preserve the peaking time, $\tau_n = \tau_{n-1} / n$)



Increased curvature at beginning of pulse limits the minimum threshold for good timing.

⇒ One dominant time constant best for timing measurements

Unlike amplitude measurements, where multiple integrators are desirable to improve pulse symmetry and count rate performance.

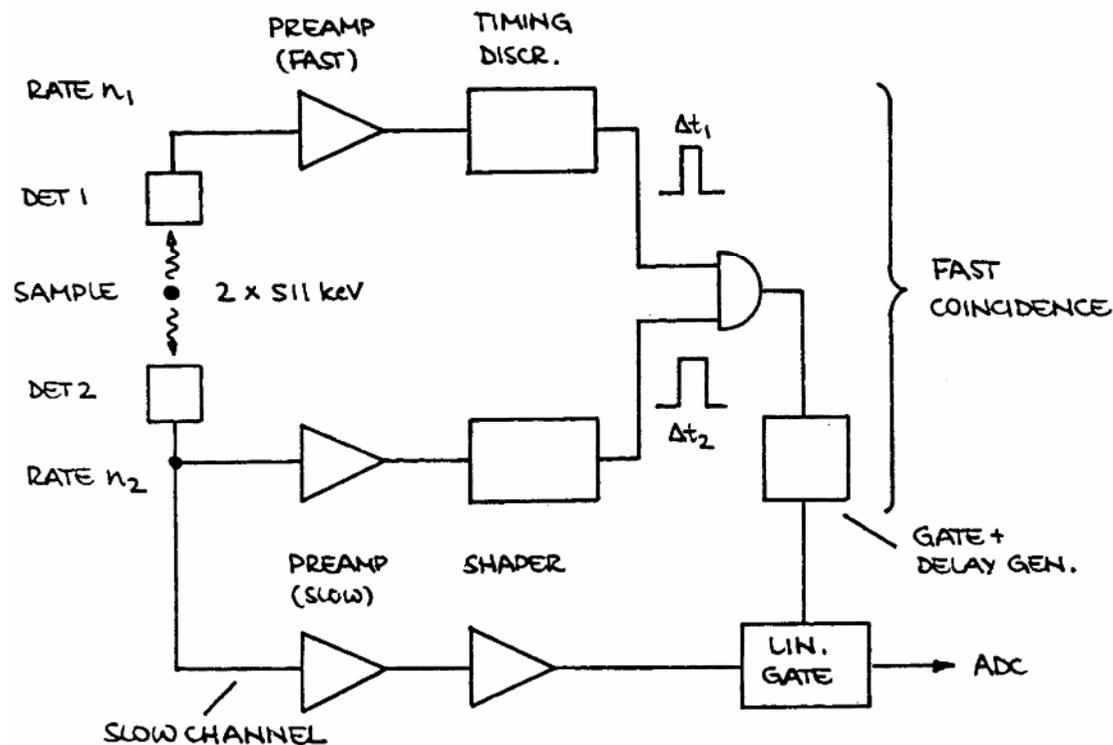
Example: $\gamma - \gamma$ coincidence (as used in positron emission tomography)

Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.

- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output if the two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.



This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are n_1 and n_2 , and the timing pulse widths are Δt_1 and Δt_2 , the probability of a pulse from the first source occurring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate n_2 , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$

$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

i.e. in the arrangement shown above, the chance coincidence rate increases with the *square* of the source strength.

Example:

$$n_1 = n_2 = 10^6 \text{ s}^{-1}$$

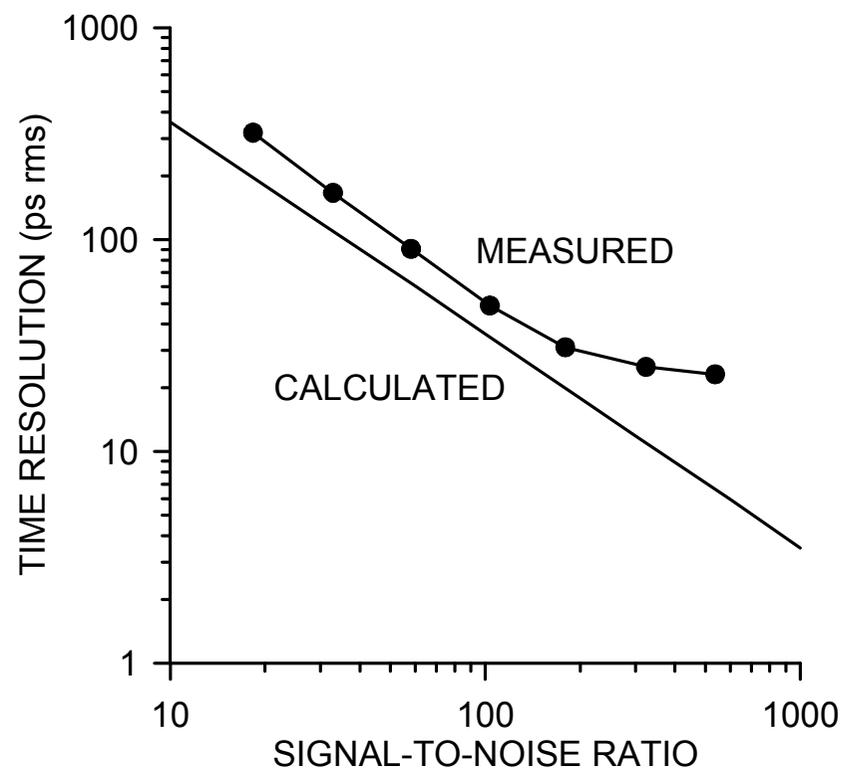
$$\Delta t_1 = \Delta t_2 = 5 \text{ ns} \Rightarrow n_c = 10^4 \text{ s}^{-1}$$

Fast Timing: Comparison between theory and experiment

Time resolution $\propto 1/(S/N)$

At $S/N < 100$ the measured curve lies above the calculation because the timing discriminator limited the rise time.

At high S/N the residual jitter of the time digitizer limits the resolution.



For more details on fast timing with semiconductor detectors, see

H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.

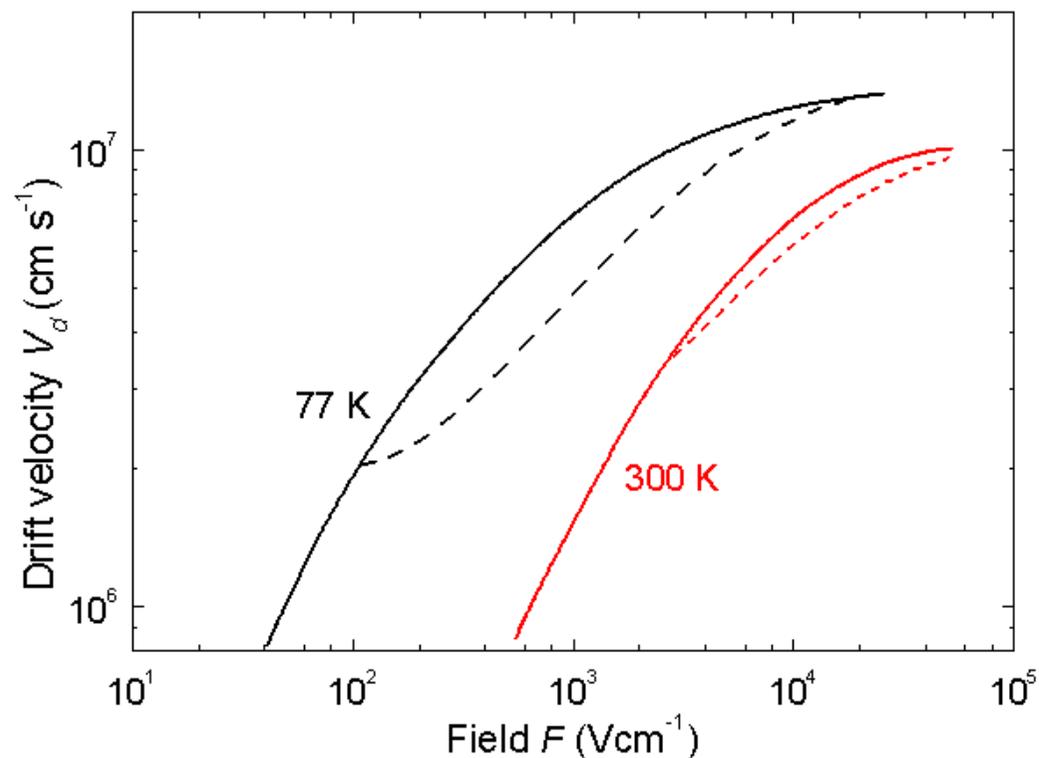
Improved Timing at Low Temperatures?

Carrier mobility increases at low temperatures.

Electron drift velocity in Si:
(low doping levels)

solid: $E \parallel (111)$

dashed: $E \parallel (100)$



Jacobini et al. Solid State Elec 20/2 (1977) 77-89

At low fields ~10-fold increase, but saturation velocity at 77K only increases 30%, so at the high fields optimal for timing only modest improvement.

Ionization coefficient $\alpha(77\text{K})/\alpha(300\text{K}) \approx 2$, so maximum bias voltage reduced (breakdown).

6. Digitization

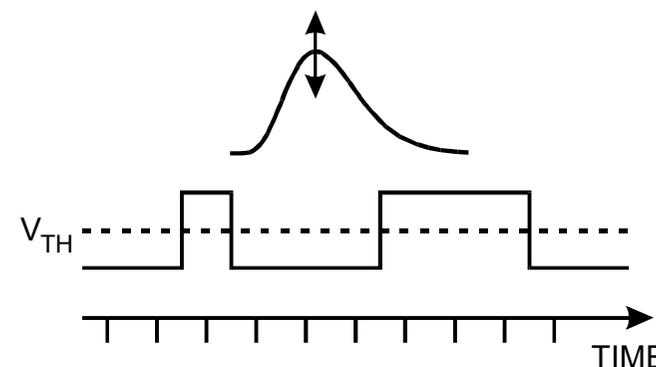
Elements of Digital Electronics

Basic differences

Analog signals have variable amplitude

Digital have constant amplitude, but variable timing

Presence of signal at specific times is evaluated:
(does the signal level exceed threshold?)



Transmission capacity of a digital link (bits per second)

Shannon's theorem:

$$C = B \cdot \log_2 \left(1 + \frac{S}{N} \right)$$

B = Bandwidth

S = Signal (pulse amplitude)

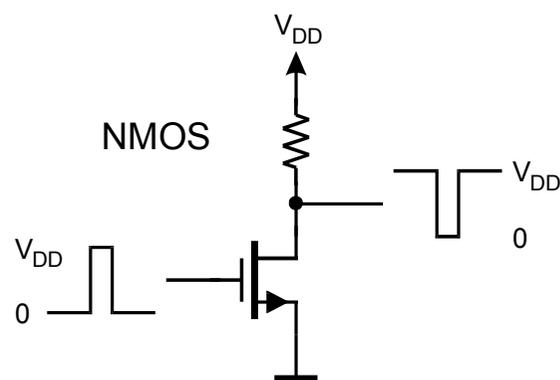
N = Noise

Noise enters, because near the switching threshold, digital elements are amplifiers.

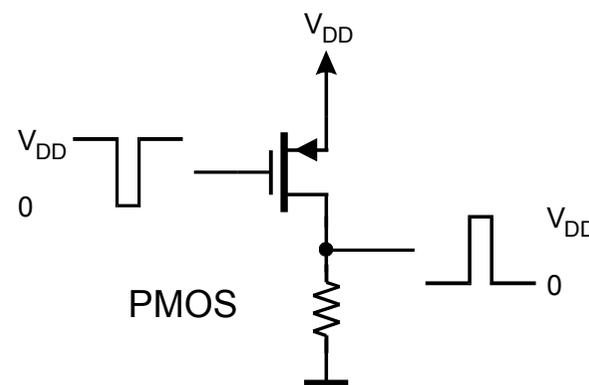
If the noise is due to cross-talk from other digital signals,
increasing the pulse amplitude will not improve S/N

Digital electronics not just a matter of “yes” or “no” – real systems must also deal with “maybe”.

LOGIC CIRCUITRY - Modern logic uses MOS technology.



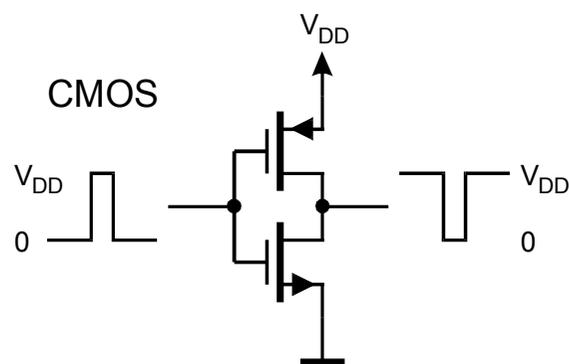
Transistor conducts when input is high.



PMOS: Transistor conducts when input is low.

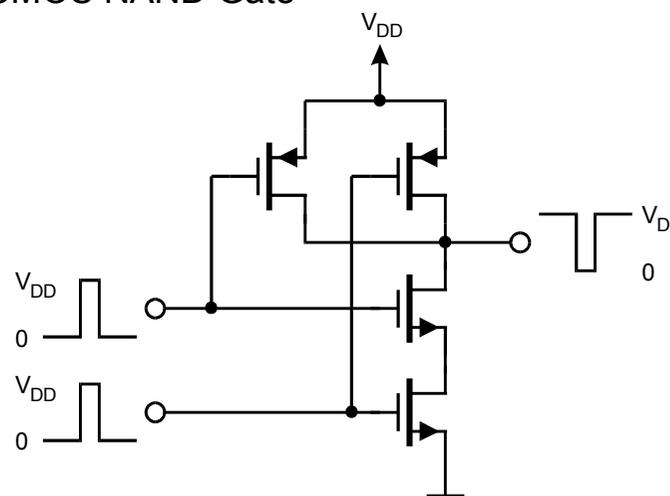
CMOS – combine NMOS and PMOS \Rightarrow significant power reduction

CMOS Inverter



Current flows only during transition.

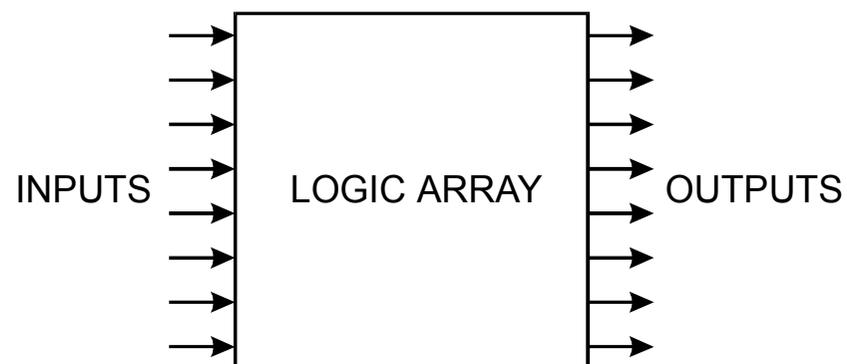
CMOS NAND Gate



LOGIC ARRAYS

Complex logic systems are not designed using individual gates.

Instead, logic functions are described in a high-level language (e.g. VHDL) and synthesized using design libraries (in custom ICs, “ASICs”) or programmable logic arrays.



Typical: 512 pads usable for inputs and outputs, $\sim 10^6$ gates, $\sim 100\text{K}$ memory

Software also generates “test vectors” that can be used to test finished parts.

POWER DISSIPATION AND PROPAGATION DELAYS

Energy dissipated in wiring resistance R :

$$E = \int i^2(t)R dt$$

$$i(t) = \frac{V}{R} \exp\left(-\frac{t}{RC}\right)$$

$$E = \frac{V^2}{R} \int_0^{\infty} \exp(-2t/RC) dt = \frac{1}{2} CV^2$$

If pulses (rising + falling edge transitions) occur at frequency f ,

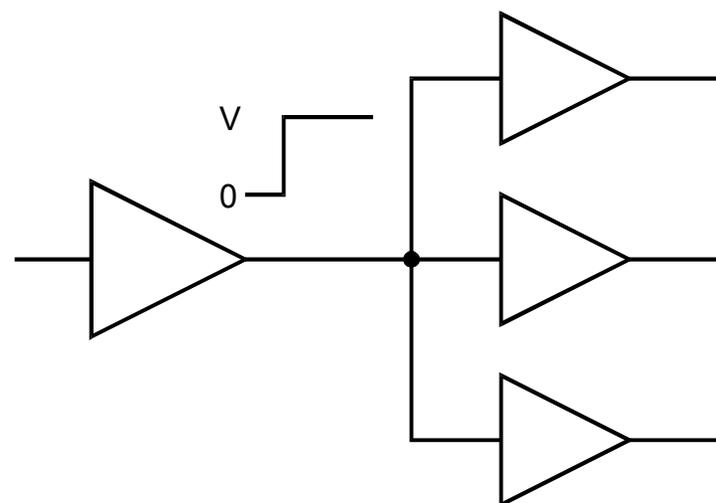
$$P = fCV^2$$

Power dissipation increases with clock frequency and (logic swing)².

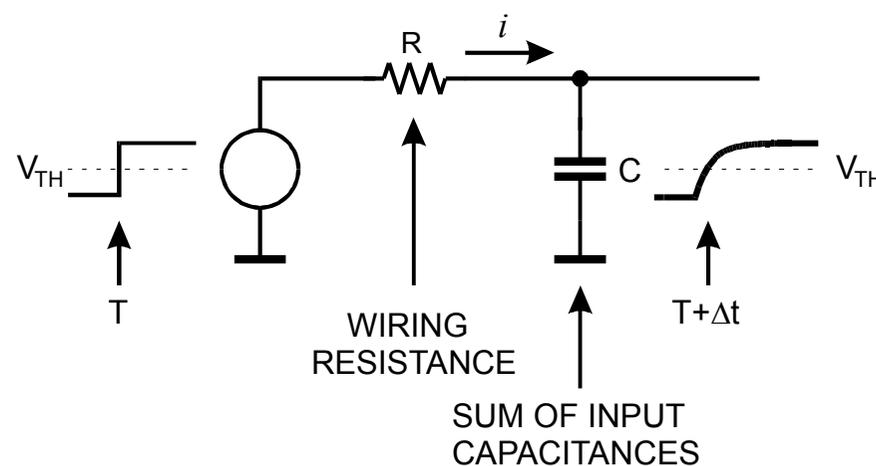
RC time constant also introduces time delay in addition to propagation delay of gates/buffers!

Depends on wiring resistance and total load capacitance.

CASCADED CMOS STAGES



EQUIVALENT CIRCUIT



Digitization of Pulse Height and Time – Analog to Digital Conversion

For data storage and subsequent analysis the analog signal at the shaper output must be digitized.

Important parameters for ADCs used in detector systems:

1. Resolution
The “granularity” of the digitized output
2. Differential Non-Linearity
How uniform are the digitization increments?
3. Integral Non-Linearity
Is the digital output proportional to the analog input?
4. Conversion Time
How much time is required to convert an analog signal to a digital output?
5. Count-Rate Performance
How quickly can a new conversion commence after completion of a prior one without introducing deleterious artifacts?
6. Stability
Do the conversion parameters change with time?

Instrumentation ADCs used in industrial data acquisition and control systems share most of these requirements. However, detector systems place greater emphasis on differential non-linearity and count-rate performance. The latter is important, as detector signals often occur randomly, in contrast to measurement systems where signals are sampled at regular intervals.

1. Resolution

Digitization incurs approximation, as a continuous signal distribution is transformed into a discrete set of values. To reduce the additional errors (noise) introduced by digitization, the discrete digital steps must correspond to a sufficiently small analog increment.

Simplistic assumption:

Resolution is defined by the number of output bits, e.g. 13 bits $\rightarrow \frac{\Delta V}{V} = \frac{1}{8192} = 1.2 \cdot 10^{-4}$

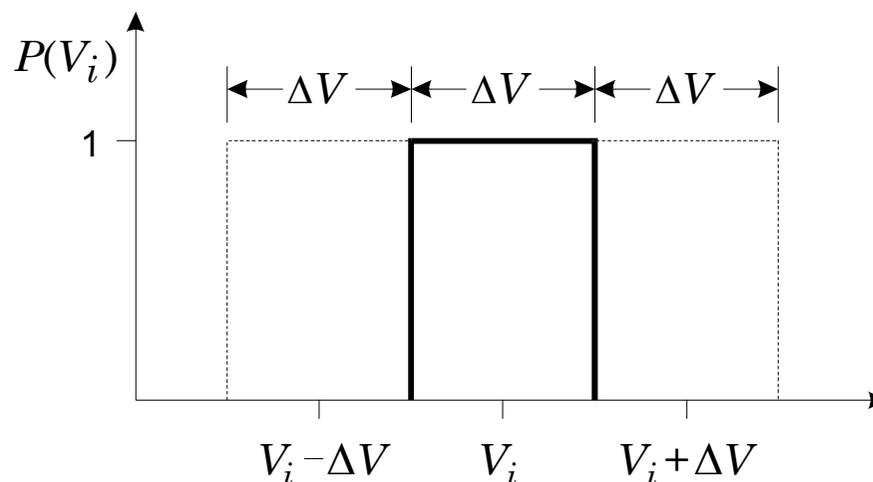
True Measure: Channel Profile

Plot probability vs. pulse amplitude that a pulse height corresponding to a specific output bin is actually converted to that address.

Ideal ADC:

Measurement accuracy:

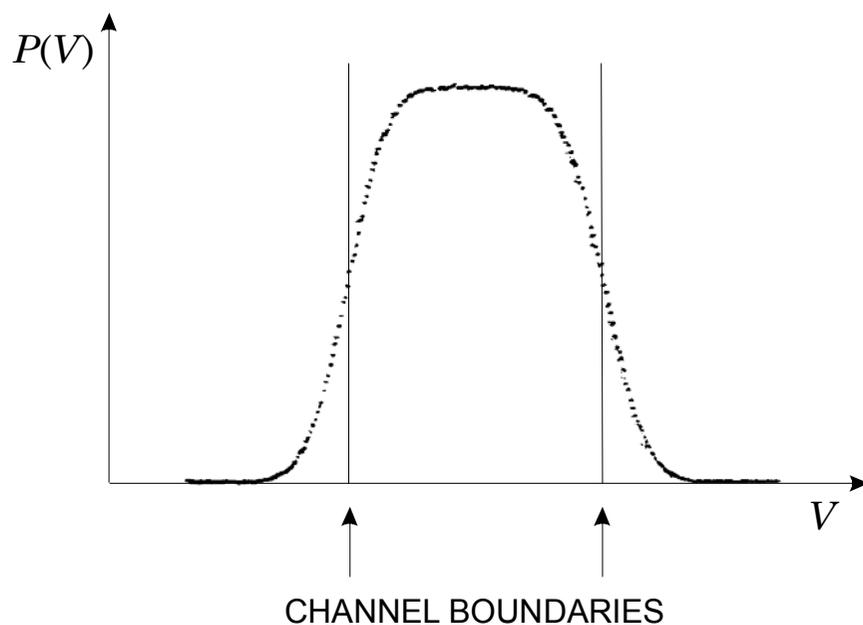
- If all counts of a peak fall in one bin, the resolution is ΔV .
- If the counts are distributed over several bins, peak fitting can yield a resolution of $10^{-1} - 10^{-2} \Delta V$, *if the distribution is known and reproducible* (not necessarily a valid assumption for an ADC).



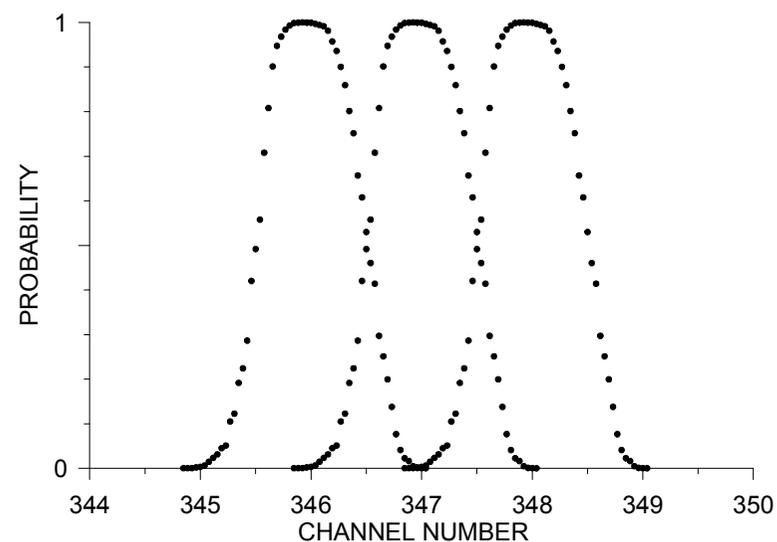
In reality, the channel profile is not rectangular as sketched above.

Electronic noise in the threshold discrimination process that determines the channel boundaries “smears” the transition from one bin to the next.

Measured channel profile (13 bit ADC)



The profiles of adjacent channels overlap.

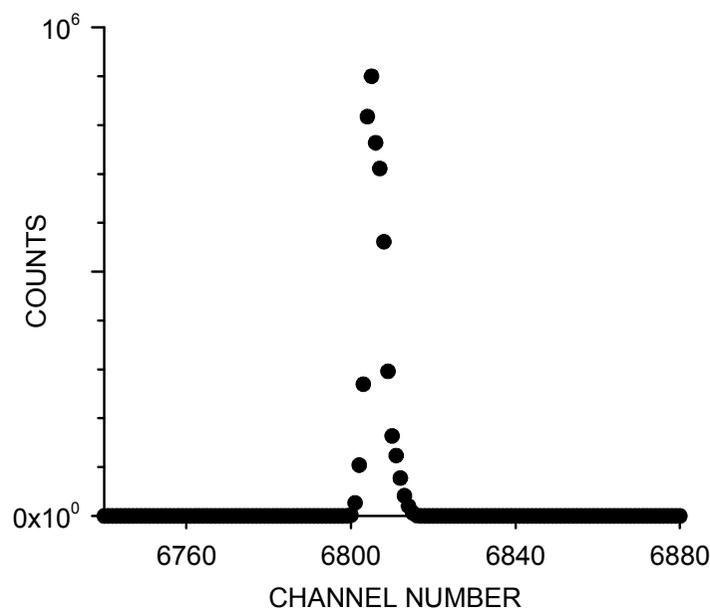


Channel profile can be checked quickly by applying the output of a precision pulser to the ADC.

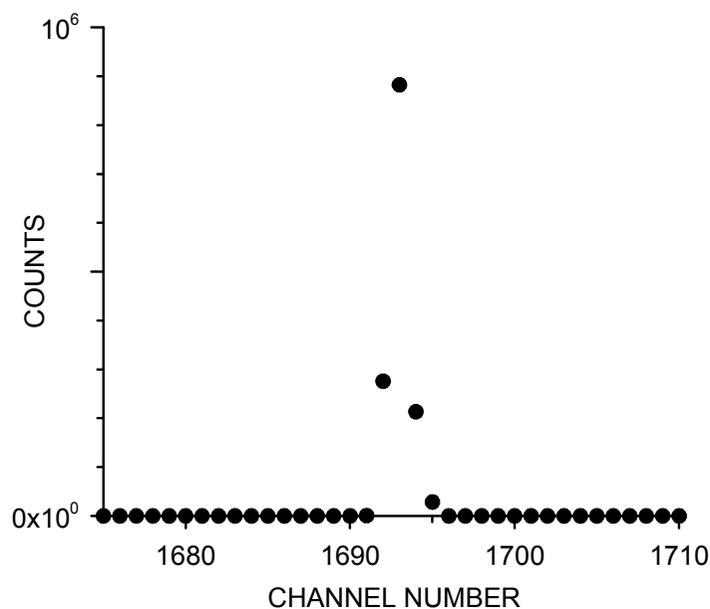
If the pulser output has very low noise, i.e. the amplitude jitter is much smaller than the voltage increment corresponding to one ADC channel or bin, all pulses will be converted to a single channel, with only a small fraction appearing in the neighbor channels.

Example of an ADC whose digital resolution is greater than its analog resolution:

8192 ch conversion range (13 bits)



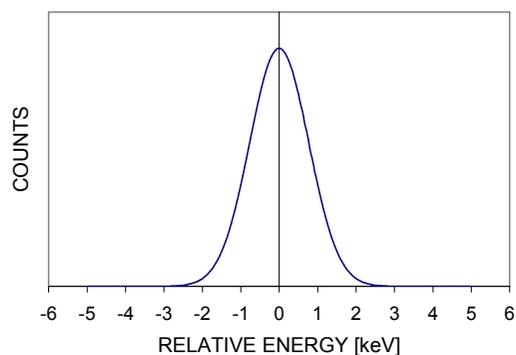
2048 ch conversion range (11 bits)



2K range provides maximum resolution – higher ranges superfluous.

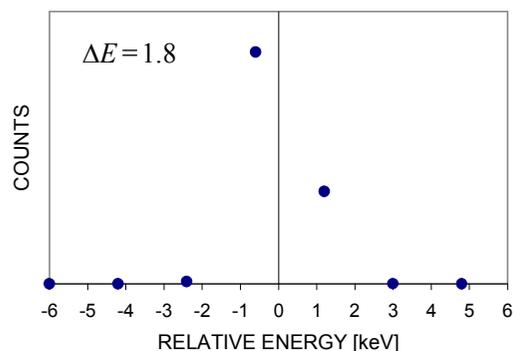
How much ADC Resolution is Required?

Example: Detector resolution ΔE
1.8 keV FWHM

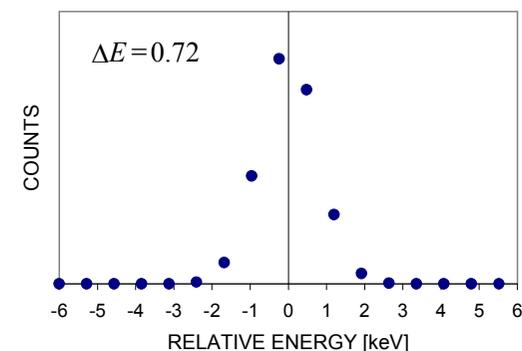


Digitized spectra for various ADC resolutions (bin widths)

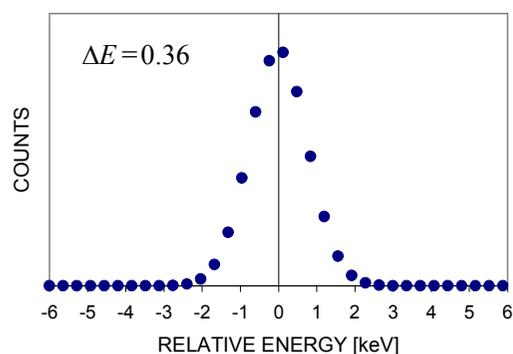
$\Delta E = 1.8 \text{ keV} = 1 \times \text{FWHM}$



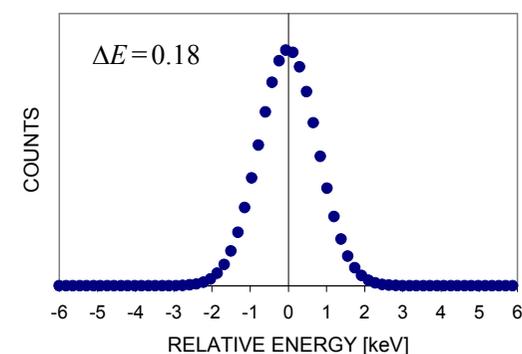
$\Delta E = 0.72 \text{ keV} = 0.4 \times \text{FWHM}$



$\Delta E = 0.36 \text{ keV} = 0.2 \times \text{FWHM}$



$\Delta E = 0.18 \text{ keV} = 0.1 \times \text{FWHM}$



Fitting can determine centroid position to fraction of bin width even with coarse digitization,
if only a single peak is present and the line shape is known.

2. Differential Non-Linearity

Differential non-linearity is a measure of the non-uniformity of channel profiles over the range of the ADC.

Depending on the nature of the distribution, either a peak or an rms specification may be appropriate.

$$DNL = \max \left\{ \frac{\Delta V(i)}{\langle \Delta V \rangle} - 1 \right\} \quad \text{or} \quad DNL = \text{r.m.s.} \left\{ \frac{\Delta V(i)}{\langle \Delta V \rangle} - 1 \right\}$$

where $\langle \Delta V \rangle$ is the average channel width and

$\Delta V(i)$ is the width of an individual channel.

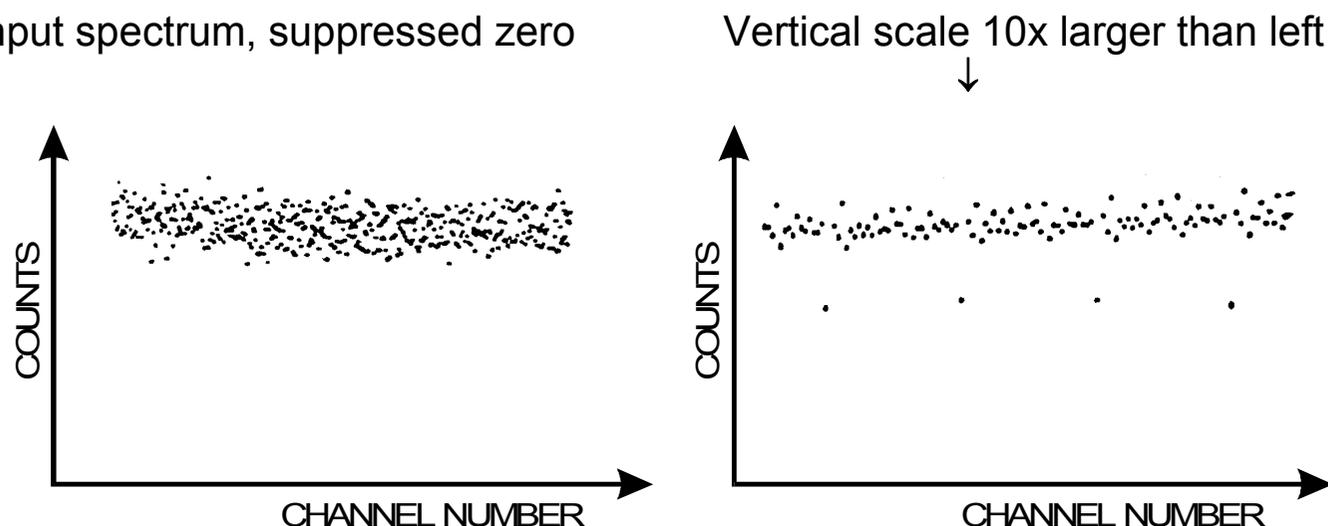
Differential non-linearity of $< \pm 1\%$ max. is typical,

but state-of-the-art ADCs can achieve 10^{-3} rms,

i.e. the variation is comparable to the statistical fluctuation for 10^6 random counts.

Typical differential non-linearity patterns

“white” input spectrum, suppressed zero



An ideal ADC would show an equal number of counts in each bin.

The spectrum to the left shows a random pattern, but
note the multiple periodicities visible in the right hand spectrum.

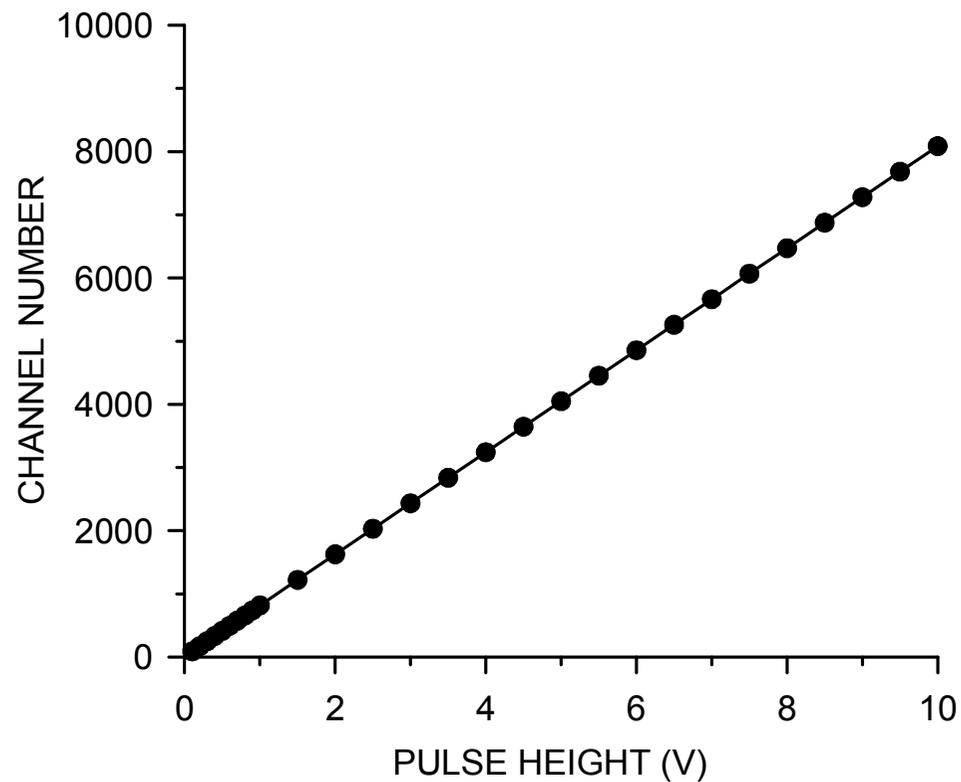
Note: Instrumentation ADCs are often specified with an accuracy of ± 0.5 LSB (least significant bit) or more, so

1. the differential non-linearity may be 50% or more,
2. the response may be non-monotonic

⇒ output may decrease when input rises.

3. Integral Non-Linearity

Integral non-linearity measures the deviation from proportionality of the measured amplitude to the input signal level.

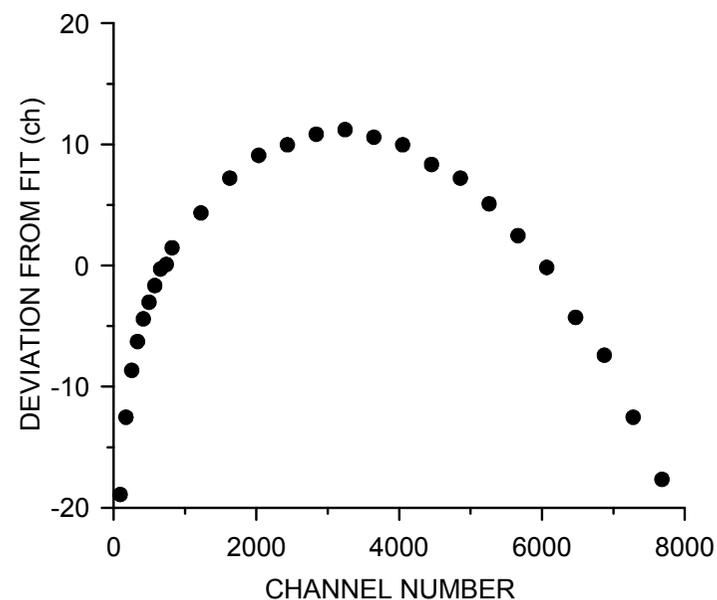


The dots are measured values and the line is a fit to the data.

This plot is not very useful if the deviations from linearity are small.

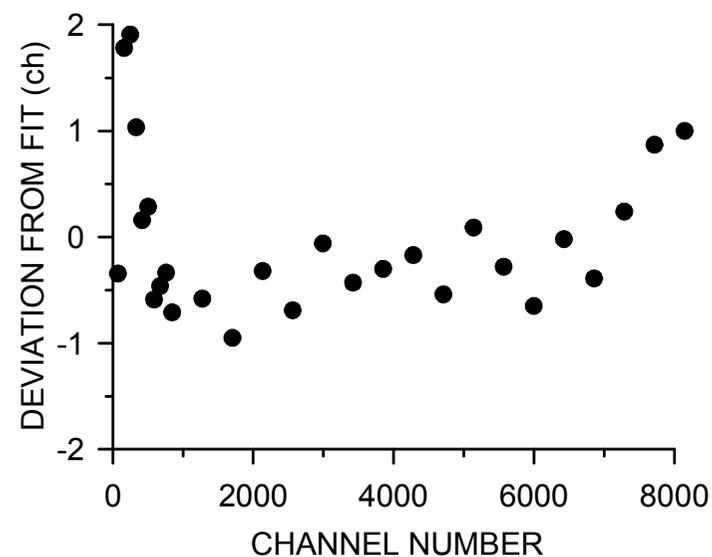
Plotting the deviations of the measured points from the fit yields a more useful result.

Integral non-linearity measured with a 400 ns wide input pulse



The linearity of an ADC can depend on the input pulse shape and duration, due to bandwidth limitations in the circuitry.

Increasing the pulse width to 3 μ s improved the linearity significantly:



4. Conversion Time

During the acquisition of a signal the system cannot accept a subsequent signal (“dead time”)

Dead Time =

- signal acquisition time → time-to-peak + const.
- + conversion time → can depend on pulse height
- + readout time to memory → depends on speed of data transmission and buffer memory access

Dead time affects measurements of yields or reaction cross-sections. Unless the event rate $\ll 1/(\text{dead time})$, it is necessary to measure the dead time, e.g. with a reference pulser fed simultaneously into the spectrum.

The total number of reference pulses issued during the measurement is determined by a scaler and compared with the number of pulses recorded in the spectrum.

Does a pulse-height dependent dead time mean that the correction is a function of pulse height?

Usually not. If events in different part of the spectrum are not correlated in time, i.e. random, they are all subject to the same average dead time (although this average will depend on the spectral distribution).

- Caution with correlated events!
 Example: Decay chains, where lifetime is $<$ dead time.
 The daughter decay will be lost systematically.

5. Count Rate Effects

Problems are usually due to internal baseline shifts with event rate or undershoots following a pulse.

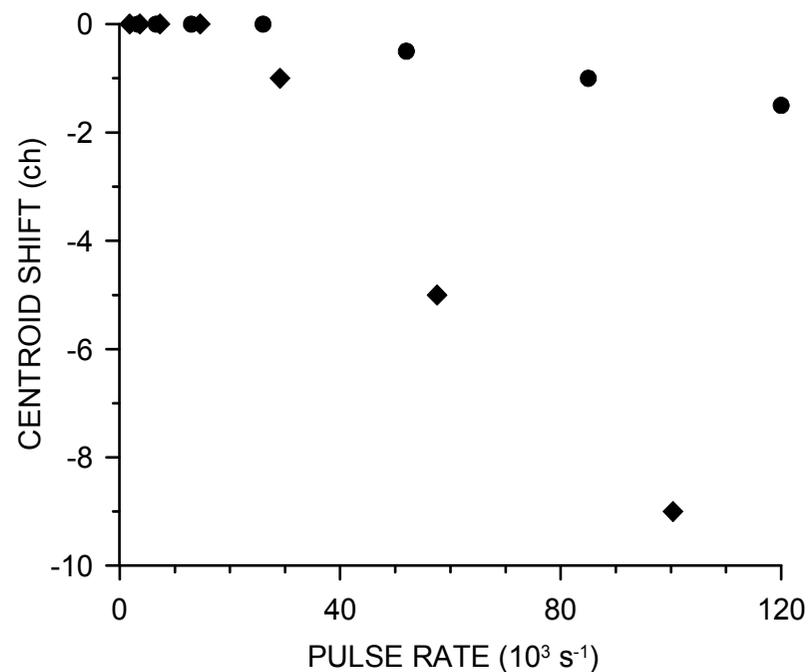
If signals occur at constant intervals, the effect of an undershoot will always be the same.

However, in a random sequence of pulses, the effect will vary from pulse to pulse.

⇒ spectral broadening

Baseline shifts tend to manifest themselves as a systematic shift in centroid position with event rate.

Centroid shifts for two 13 bit ADCs vs. random rate:



6. Stability

Stability vs. temperature is usually adequate with modern electronics in a laboratory environment.

- Note that temperature changes within a module are typically much smaller than ambient.

However: Highly precise or long-term measurements require spectrum stabilization to compensate for changes in gain and baseline of the overall system.

Technique: Using precision pulsers place a reference peak at both the low and high end of the spectrum.

(Pk. Pos. 2) – (Pk. Pos. 1) → Gain, ...

then

(Pk. Pos. 1) or (Pk. Pos. 2) → Offset

Traditional Implementation: Hardware, spectrum stabilizer module

Today, it is more convenient to determine the corrections in software.

These can be applied to calibration corrections or used to derive an electrical signal that is applied to the hardware (simplest and best in the ADC).

Analog to Digital Conversion Techniques

1. Flash ADC

The input signal is applied to n comparators in parallel. The switching thresholds are set by a resistor chain, such that the voltage difference between individual taps is equal to the desired measurement resolution.

2^n comparators for n bits (8 bit resolution requires 256 comparators)

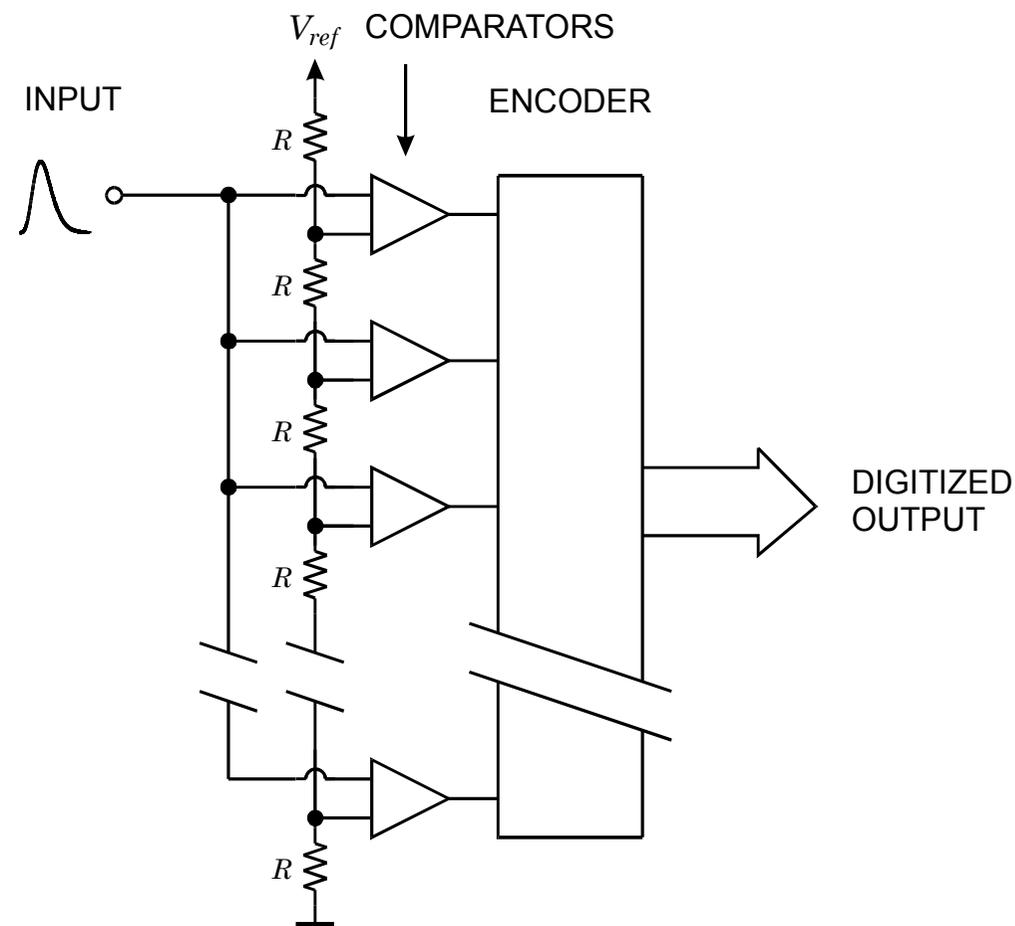
Feasible in monolithic ICs since the absolute value of the resistors in the reference divider chain is not critical, only the relative matching.

Advantage: short conversion time
(<10 ns available)

Drawbacks:

- limited accuracy
- (many comparators)
- power consumption
- Differential non-linearity $\sim 1\%$
- High input capacitance

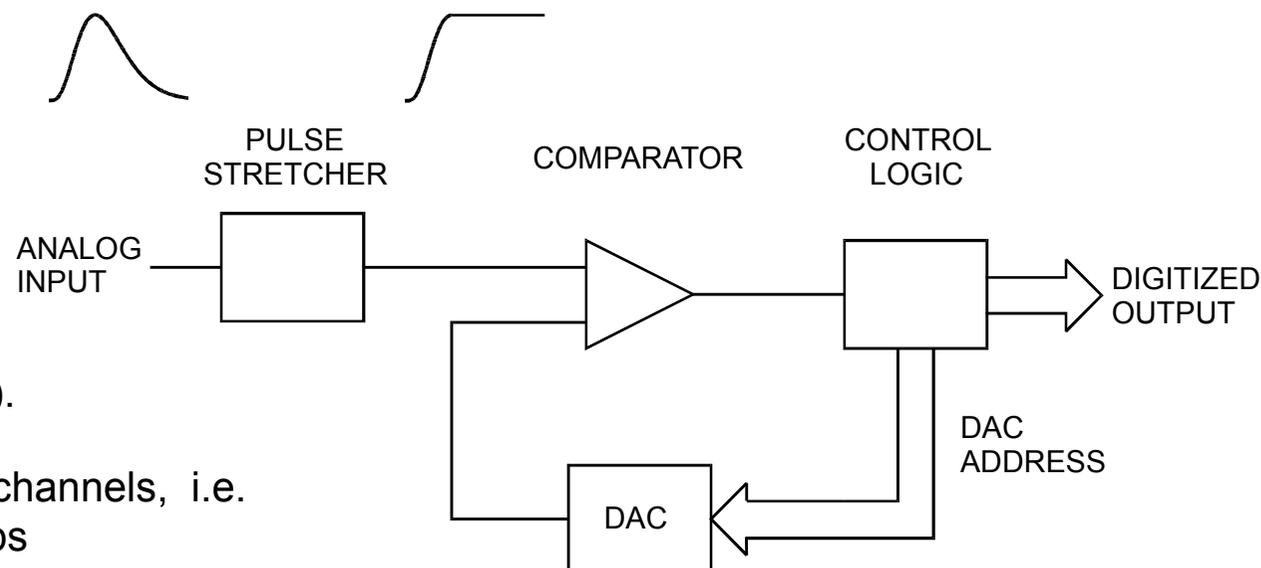
Speed is often limited by the analog driver feeding the input.



2. Successive Approximation ADC

Sequentially raise comparator threshold proportional to $2^n, 2^{n-1}, \dots, 2^0$ and set corresponding bit if the comparator output is high (DAC output < pulse height).

n conversion steps yield 2^n channels, i.e. 8K channels require 13 steps



Advantages: speed ($\sim \mu\text{s}$)
 high resolution
 ICs (monolithic + hybrid) available

Drawback: Differential non-linearity (typ. 10 – 20%)

Reason: Resistors that set DAC output must be extremely accurate. For $\text{DNL} < 1\%$ the resistor determining the 2^{12} level in an 8K ADC must be accurate to $< 2.4 \cdot 10^{-6}$.

DNL can be corrected by various techniques:

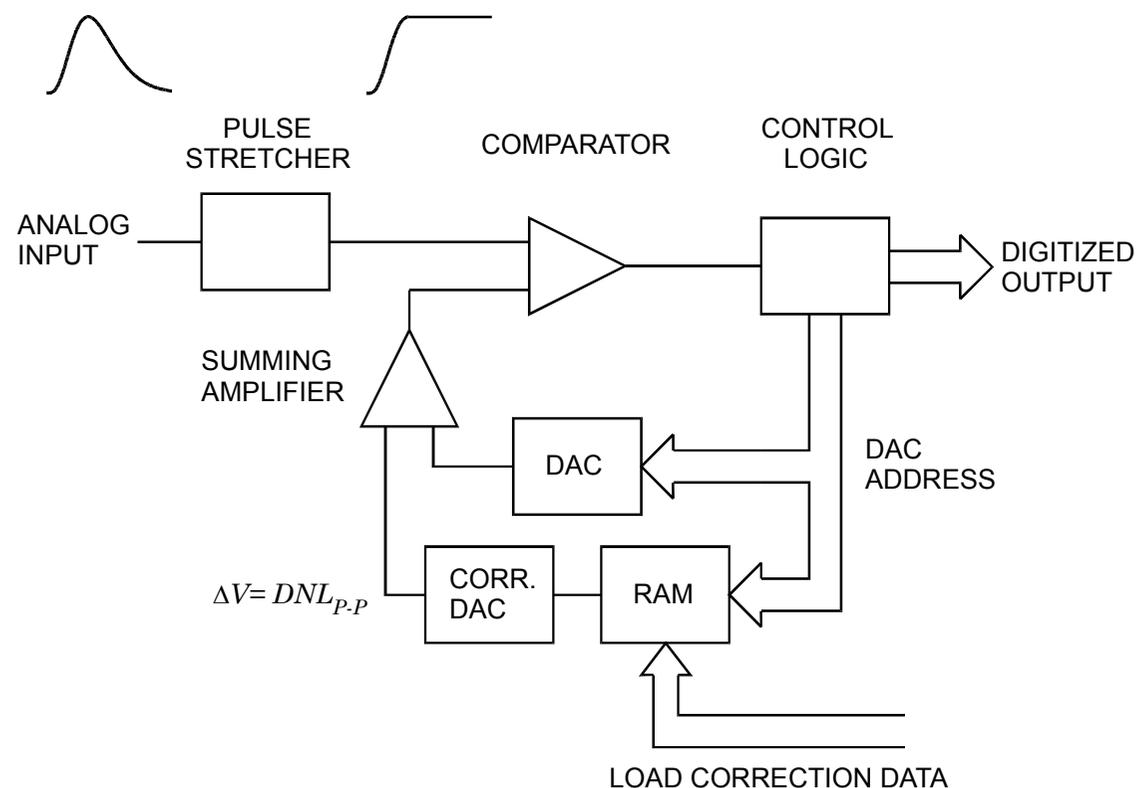
- averaging over many channel profiles for a given pulse amplitude (“sliding scale” or “Gatti principle”)
- correction DAC (“brute force” application of IC technology)

The primary DAC output is adjusted by the output of a correction DAC to reduce differential non-linearity.

Correction data are derived from a measurement of DNL. Corrections for each bit are loaded into the RAM, which acts as a look-up table to provide the appropriate value to the correction DAC for each bit of the main DAC.

The range of the correction DAC must exceed the peak-to-peak differential non-linearity.

If the correction DAC has N bits, the maximum DNL is reduced by $1/2^{(N-1)}$ (if deviations are symmetrical).

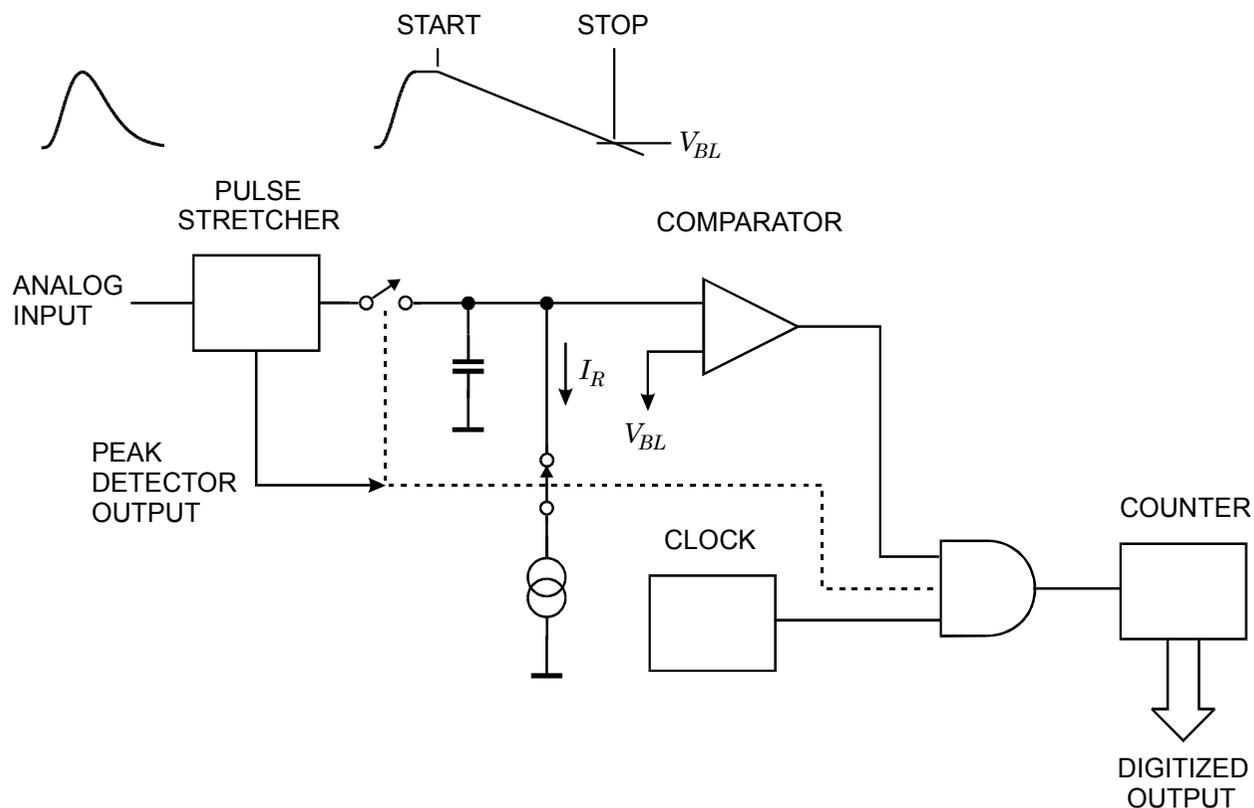


3. Wilkinson ADC

The peak signal amplitude is acquired by a pulse stretcher and transferred to a memory capacitor.

Then, simultaneously,

1. the capacitor is disconnected from the stretcher,
2. a current source is switched on to linearly discharge the capacitor,
3. a counter is enabled to determine the number of clock pulses until the voltage on the capacitor reaches the baseline.



Advantage: excellent differential linearity (continuous conversion process)

Drawbacks: slow – conversion time = $n \cdot T_{clock}$ (n = channel number \propto pulse height)
 $T_{clock} = 10 \text{ ns} \rightarrow T_{conv} = 82 \mu\text{s}$ for 13 bits

Clock frequencies of 100 MHz typical, >400 MHz possible with excellent performance

“Standard” technique for high-resolution spectroscopy.

4. Pipelined ADCs

Most common architecture for high-speed high-resolution ADCs

Input to each stage is fed both to a sample and hold (S&H) and a 3-bit flash ADC.

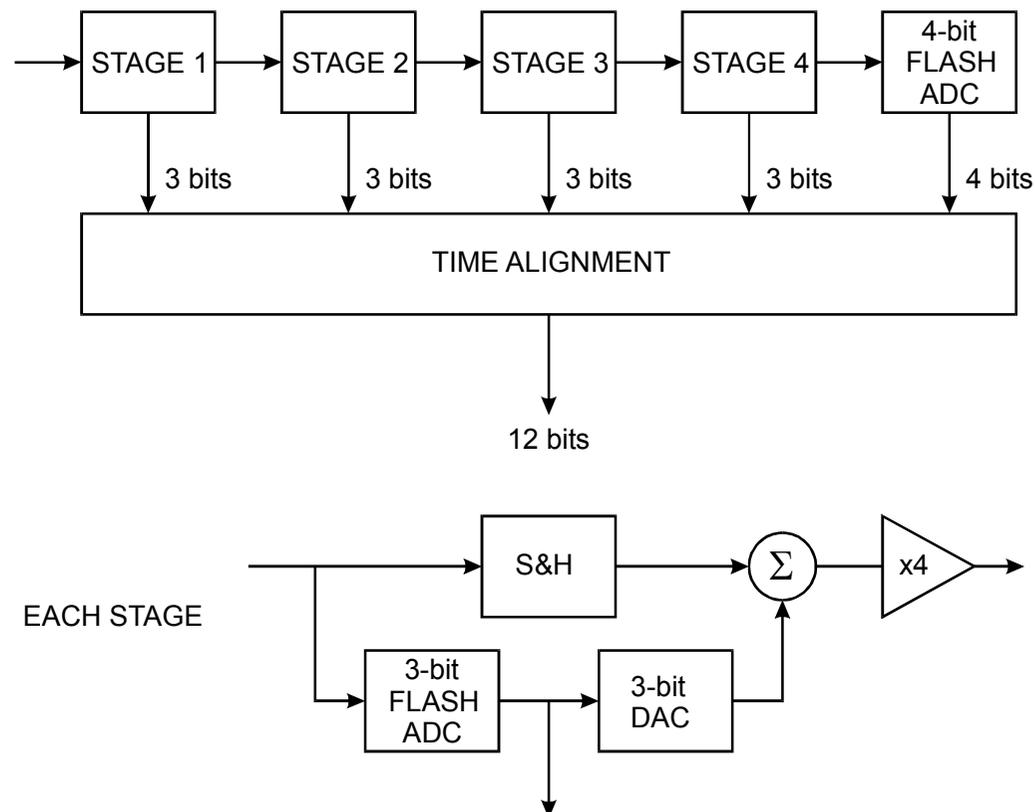
The S&H maintains the signal level during conversion. The flash ADC quantizes its input to 3 bit accuracy. This output is fed to a DAC with 12 bit accuracy. The DAC's analog output is subtracted from the original signal and the difference signal is passed on to the next stage.

The last 4 bits are resolved by a 4-bit flash ADC.

As soon as a stage has passed its result to the next stage it can begin processing the next signal, so throughput is not determined by the total conversion time, but by the time per stage.

Since the interstage gain is only 4 (rather than 8 corresponding to 3 bits), each stage only contributes 2 bits of resolution. The extra bit is used for error correction.

Commercially available: 1 GS/s conversion rates with 8-bit resolution and a power dissipation of about 1.5 W.



Hybrid Analog-to-Digital Converters

Conversion techniques can be combined to obtain high resolution and short conversion time.

1. Flash + Successive Approximation or Flash + Wilkinson (Ramp Run-Down)

Utilize fast flash ADC for coarse conversion (e.g. 8 out of 13 bits)

Successive approximation or Wilkinson converter to provide fine resolution.

Limited range, so short conversion time: 256 ch with 100 MHz clock \Rightarrow 2.6 μ s

Results: 13 bit conversion in $< 4 \mu$ s with excellent integral and differential linearity

2. Flash ADCs with Sub-Ranging

Not all applications require constant absolute resolution over the full range.

Sometimes only *relative* resolution must be maintained, especially in systems with a very large dynamic range.

Precision binary divider at input to determine coarse range + fast flash ADC for fine digitization.

Example: Fast digitizer that fits in phototube base (FNAL)

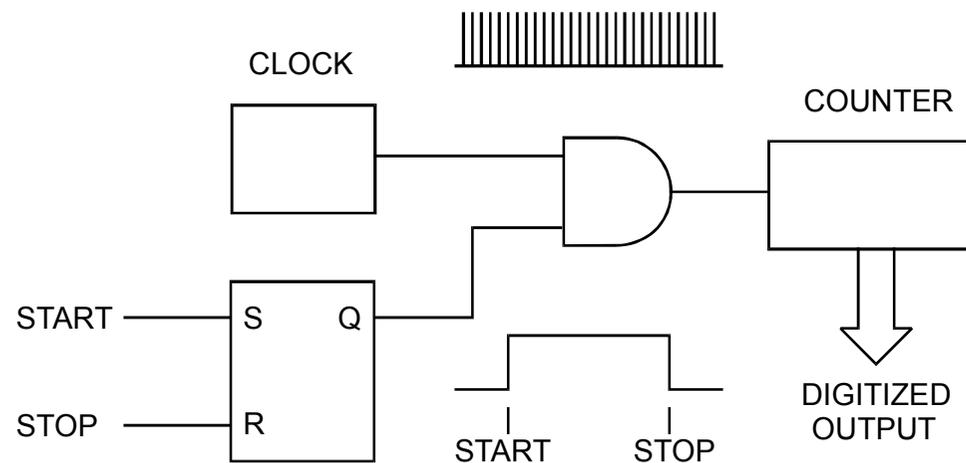
17 to 18 bit dynamic range

Digital floating point output (4 bit exponent, 8+1 bit mantissa)

16 ns conversion time

Time Digitizers

1. Counter



Simplest arrangement: Count clock pulses between start and stop.

Limitation: Speed of counter
 Current technology limits speed of counter system to about 1 GHz

$$\Rightarrow \Delta t = 1 \text{ ns}$$

Advantages: Simplicity
 Multi-hit capability

2. Analog Ramp

Commonly used in high-resolution digitizers ($\Delta t = 10$ ps)

Principle:

Charge capacitor through switchable current source

Start pulse: turn on current source

Stop pulse: turn off current source

⇒ Voltage on storage capacitor

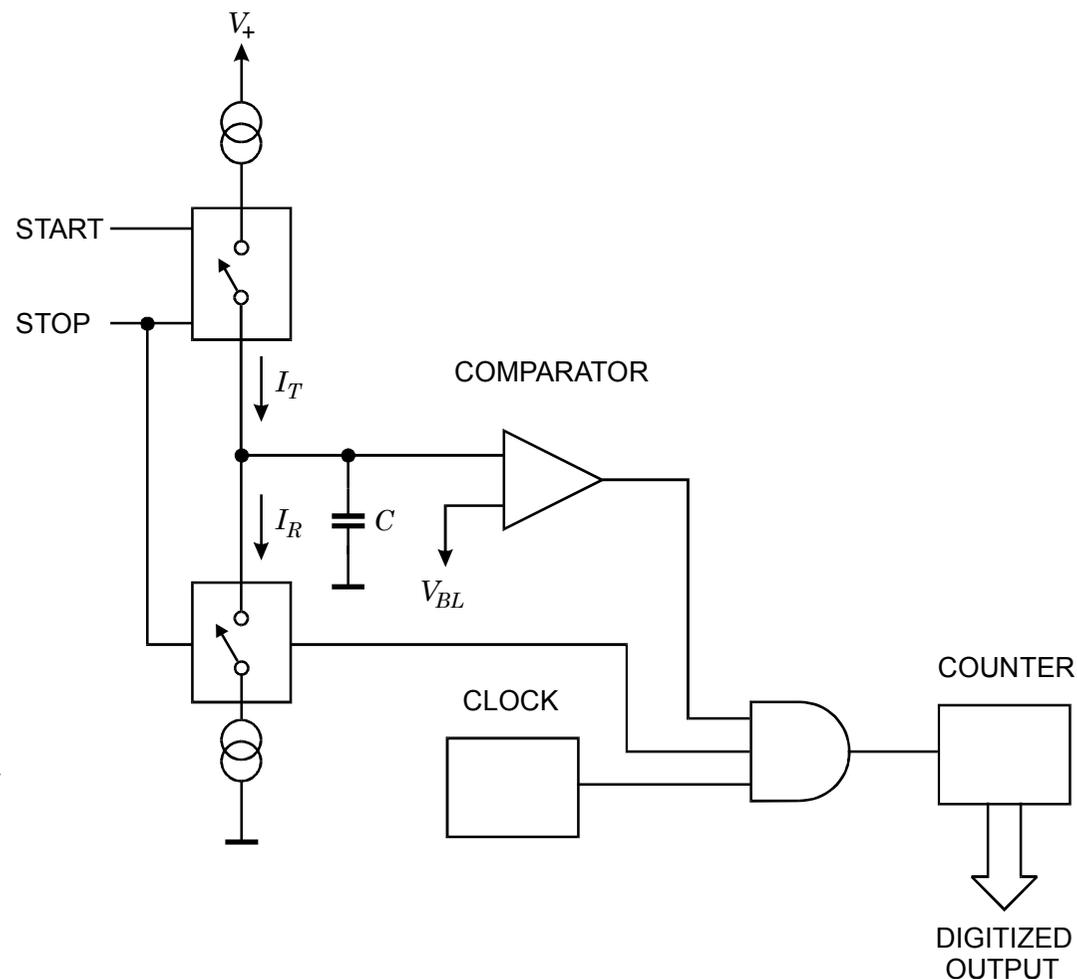
Use Wilkinson ADC with smaller discharge current to digitize voltage.

Drawbacks: No multi-hit capability

Deadtime

Advantages: High resolution (\sim ps)

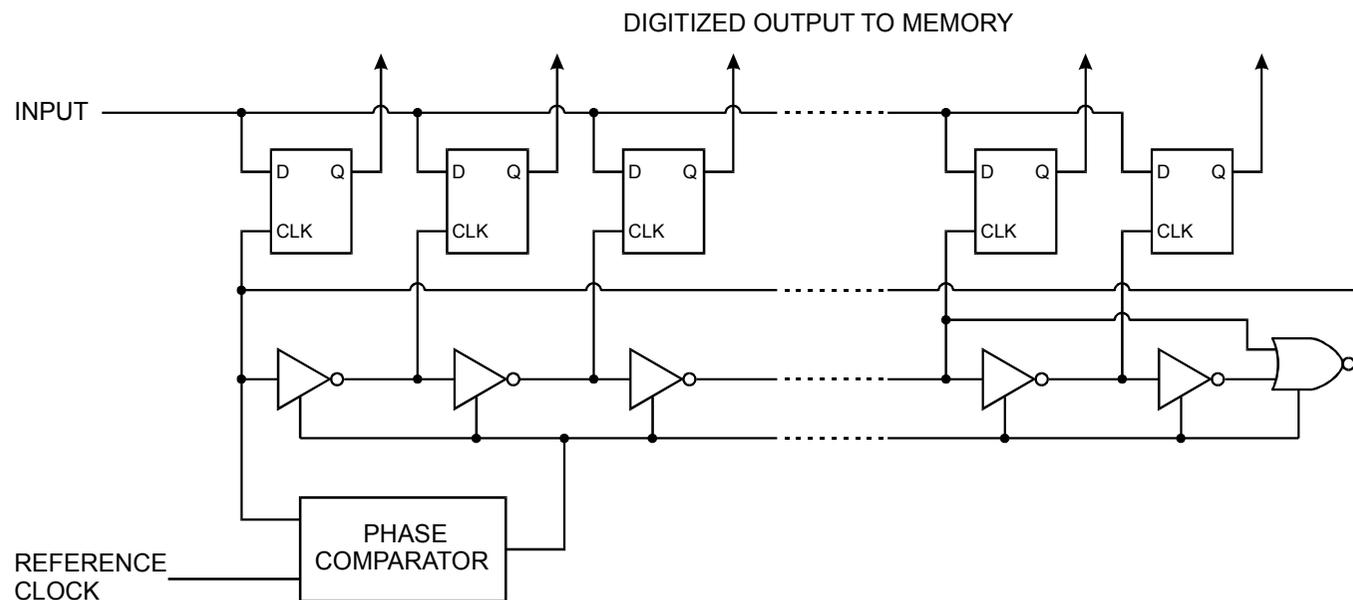
Excellent differential linearity



3. Digitizers with Clock Interpolation

Most experiments in HEP require multi-hit capability, no deadtime

Commonly used in HEP ICs for time digitization (Y. Arai, KEK)



Clock period interpolated by inverter delays (U1, U2, ...).

Delay can be fine-tuned by adjusting operating point of inverters. Stabilized by delay locked loop.

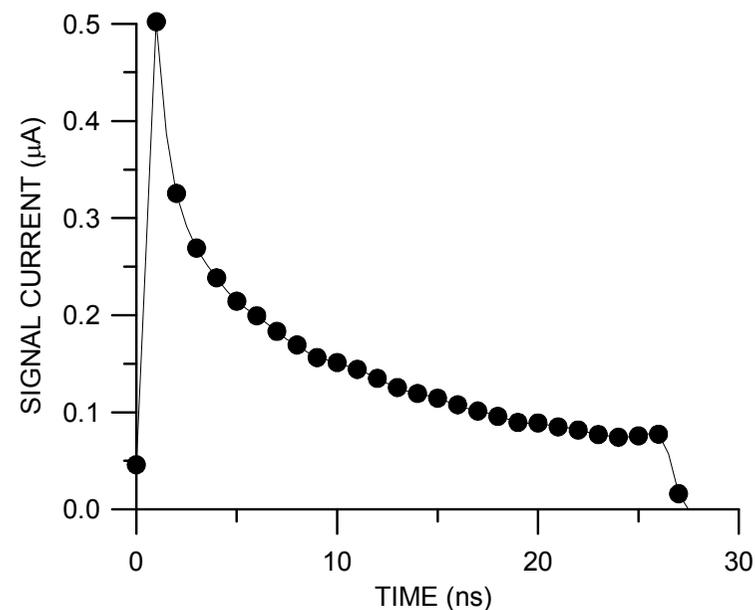
Devices with 250 ps resolution fabricated and tested.

see Y. Arai et al., IEEE Trans. Nucl. Sci. **NS-45/3** (1998) 735-739 and references therein.

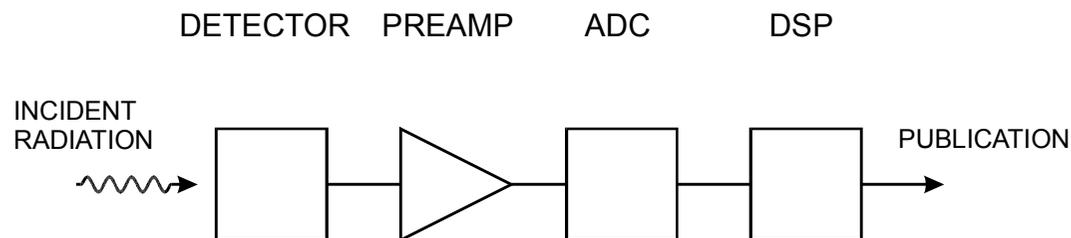
7. Digital Signal Processing

Sample detector signal with fast digitizer to reconstruct pulse:

Then use digital signal processor to perform mathematical operations for desired pulse shaping.



Block Diagram



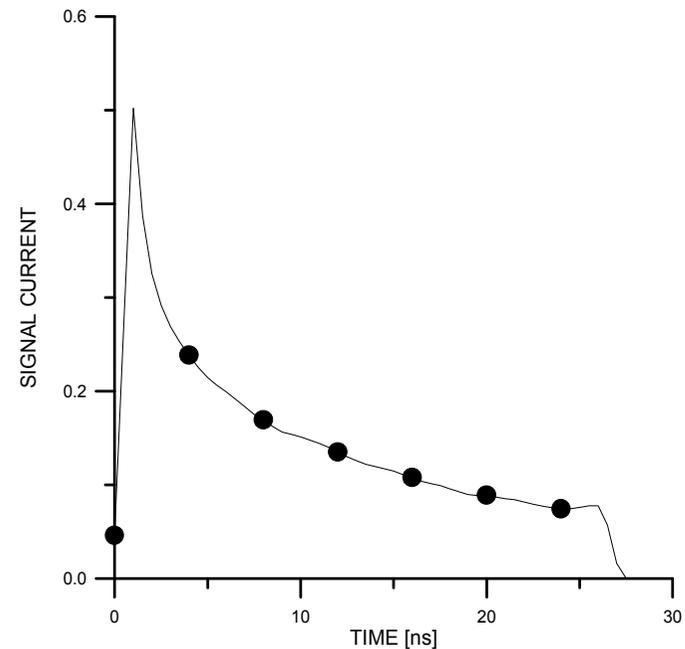
DSP allows great flexibility in implementing filtering functions

However: increased circuit complexity

increased demands on ADC, compared to traditional shaping.

Important to choose sample interval sufficiently small to capture pulse structure.

Sampling interval of 4 ns misses initial peak.

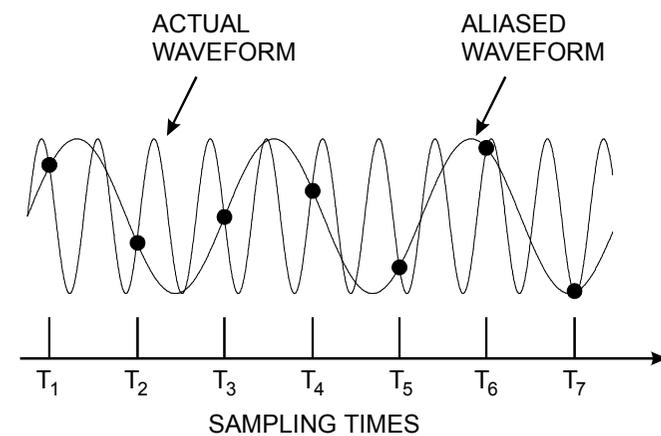


With too low a sampling rate high frequency components will be “aliased” to lower frequencies:

Applies to any form of sampling
(time waveform, image, ...)

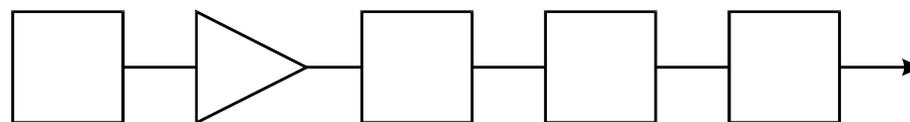
Nyquist condition:

Sampling frequency > 2x highest signal frequency



⇒ Fast ADC required + Pre-Filter to limit signal bandwidth

DETECTOR PREAMP PRE-FILTER ADC DSP



- Dynamic range requirements for ADC may be more severe than in analog filtered system (depending on pulse shape and pre-filter).
- Digitization introduces additional noise (“quantization noise”)

If one bit corresponds to an amplitude interval Δ , the quantization noise

$$\sigma_v^2 = \int_{-\Delta/2}^{\Delta/2} \frac{v^2}{\Delta} dv = \frac{\Delta^2}{12}.$$

(differential non-linearity introduces quasi-random noise)

- Electronics preceding ADC and front-end of ADC must exhibit same precision as analog system, i.e. baseline and other pulse-to-pulse amplitude fluctuations less than order $Q_n/10$, i.e. typically 10^{-4} in high-resolution systems. For 10 V FS at the ADC input this corresponds to < 1 mV.

⇒ ADC must provide high performance at short conversion times. Today this is technically feasible for some applications, e.g. detectors with moderate to long collection times (γ and x-ray detectors).

Digital Filtering

Filtering is performed by convolution:
$$S_o(n) = \sum_{k=0}^{N-1} W(k) \cdot S_i(n-k)$$

$W(k)$ is a set of coefficients that describes the weighting function yielding the desired pulse shape.

A filter performing this function is called a Finite Impulse Response (FIR) filter.

This is analogous to filtering in the frequency domain:

In the frequency domain the result of filtering is determined by multiplying the responses of the individual stages:

$$G(f) = G_1(f) \cdot G_2(f)$$

where $G_1(f)$ and $G_2(f)$ are complex numbers.

The theory of Fourier transforms states that the equivalent result in the time domain is formed by convolution of the individual time responses:

$$g(t) = g_1(t) * g_2(t) \equiv \int_{-\infty}^{+\infty} g_1(\tau) \cdot g_2(t-\tau) d\tau,$$

analogously to the discrete sum shown above.

