III. Electronic Noise

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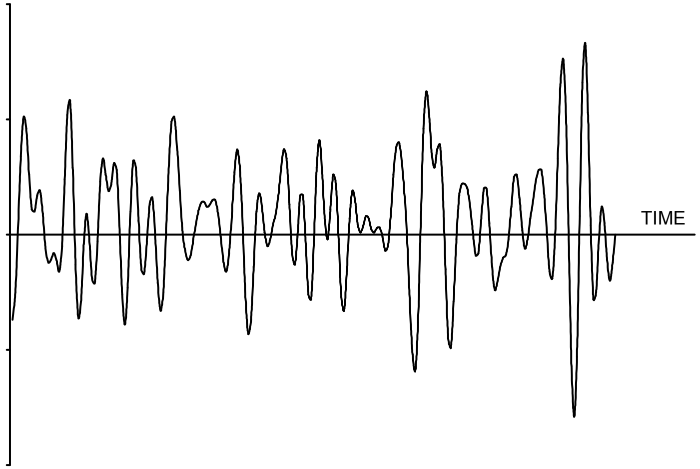
1. Signals and Noise

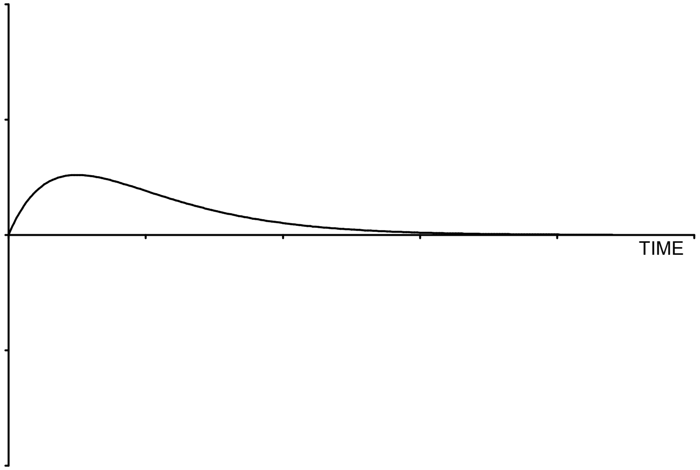
Choose a time when no signal is present.

Amplifier's quiescent output level (baseline):

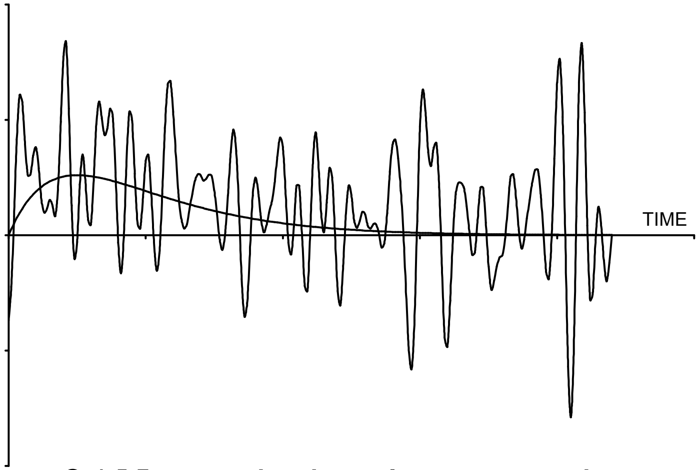


In the presence of a signal, noise + signal add.

Signal



Signal+Noise (S/N = 1)



$S/N \equiv$ peak signal to rms noise

Measurement of peak amplitude yields signal amplitude + noise fluctuation

The preceding example could imply that the fluctuations tend to increase the measured amplitude, since the noise fluctuations vary more rapidly than the signal.

In an optimized system, the time scale of the fluctuation is comparable to the signal peaking time.

Then the measured amplitude fluctuates positive and negative relative to the ideal signal.

Measurements taken at 4 different times:

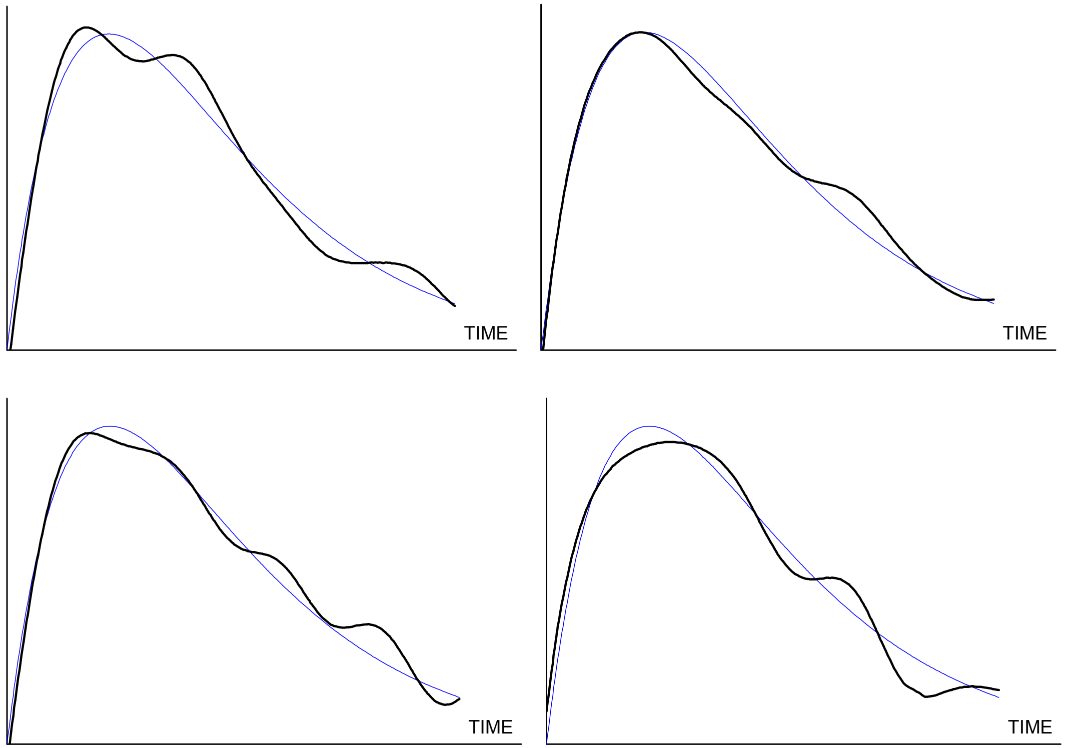
noiseless signal superimposed for comparison

S/N = 20

Noise affects

Peak signal

Time distribution



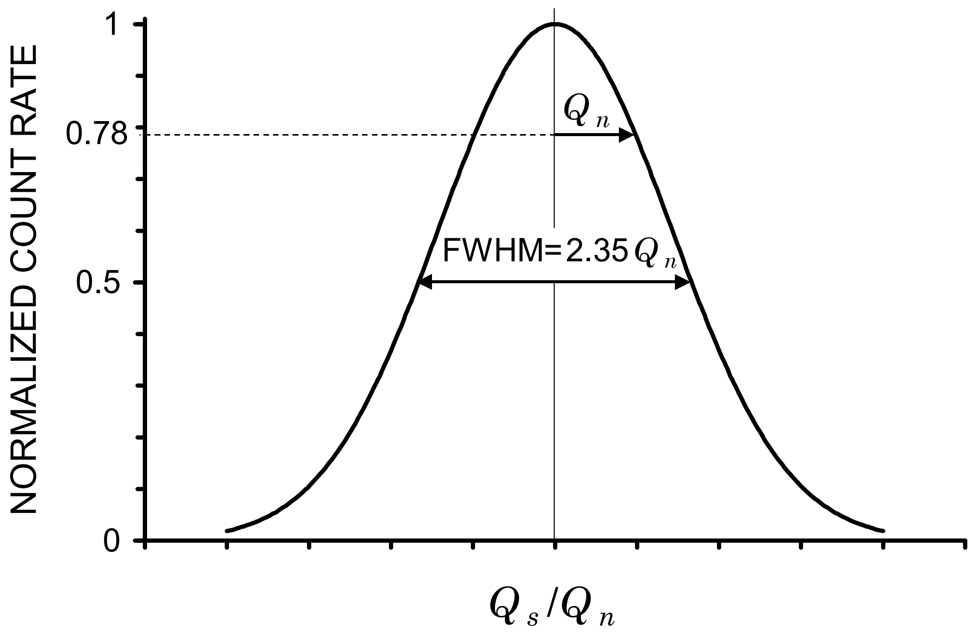
Electronic noise is purely random.

⇒ amplitude distribution is Gaussian

⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has Gaussian distribution



Measuring Resolution

Inject an input signal with known charge using a pulse generator set to approximate the detector signal shape.

Measure the pulse height spectrum.

peak centroid ⇒ signal magnitude
peak width ⇒ noise (FWHM= 2.35 Q_n)

2. Basic Noise Mechanisms and Characteristics

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\,e\,v}{l}$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2 ,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or "1/f" noise

Thermal noise and shot noise are both "white" noise sources, i.e.

power per unit bandwidth (≡ spectral density) is constant: $$\frac{dP_{noise}}{df} = const.$$

Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Noise power density vs. frequency f: $$\frac{dP_{noise}}{df} = 4kT$$

k = Boltzmann constant

T = absolute temperature

since $P = \dfrac{V^2}{R} = I^2 R$

R = DC resistance

the spectral noise voltage density $$\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$$

and the spectral noise current density $$\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$$

The total noise depends on the bandwidth of the system.
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \, A_v^2(f) \, df$$

Note: Since spectral noise components are not correlated, one must integrate over the noise power (proportional to voltage or current squared).

Spectral Density of Thermal Noise (Johnson Noise)

Two approaches can be used to derive the spectral distribution of thermal noise.

1. The thermal velocity distribution of the charge carriers is used to calculate the time dependence of the induced current, which is then transformed into the frequency domain.

2. Application of Planck's theory of black body radiation.

The first approach clearly shows the underlying physics, whereas the second "hides" the physics by applying a general result of statistical mechnics. However, the first requires some advanced concepts that go well beyond the standard curriculum, so the "black body" approach will be used.

In Planck's theory of black body radiation the energy per mode

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

and the spectral density of the radiated power

$$\frac{dP}{d\nu} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

i.e. this is the power that can be extracted in equilibrium.

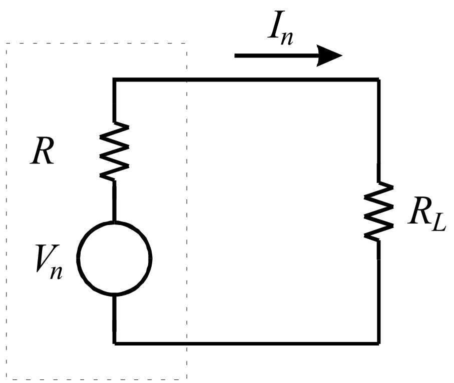
At low frequencies $h\nu \ll kT$:

$$\frac{dP}{d\nu} \approx \frac{h\nu}{\left(1+\frac{h\nu}{kT}\right)-1} = kT \ ,$$

so at low frequencies the spectral density is independent of frequency and for a total bandwidth B the noise power that can be transferred to an external device

$$P_n = kTB \, .$$

To apply this result to the noise of a resistor, consider a resistor R whose thermal noise gives rise to a noise voltage V_n . To determine the power transferred to an external device consider the circuit



The dotted box encloses the equivalent circuit of the resistive noise source.

The power dissipated in the load resistor R_L

$$\frac{V_{nL}^2}{R_L} = I_n^2 R_L = \frac{V_n^2 R_L}{(R+R_L)^2}$$

The maximum power transfer occurs when the load resistance equals the source resistance $R_L = R$, so

$$V_{nL}^2 = \frac{V_n^2}{4} \; .$$

Since the maximum power that can be transferred to R_L is kTB ,

$$\frac{V_{nL}^2}{R} = \frac{V_n^2}{4R} = kTB$$

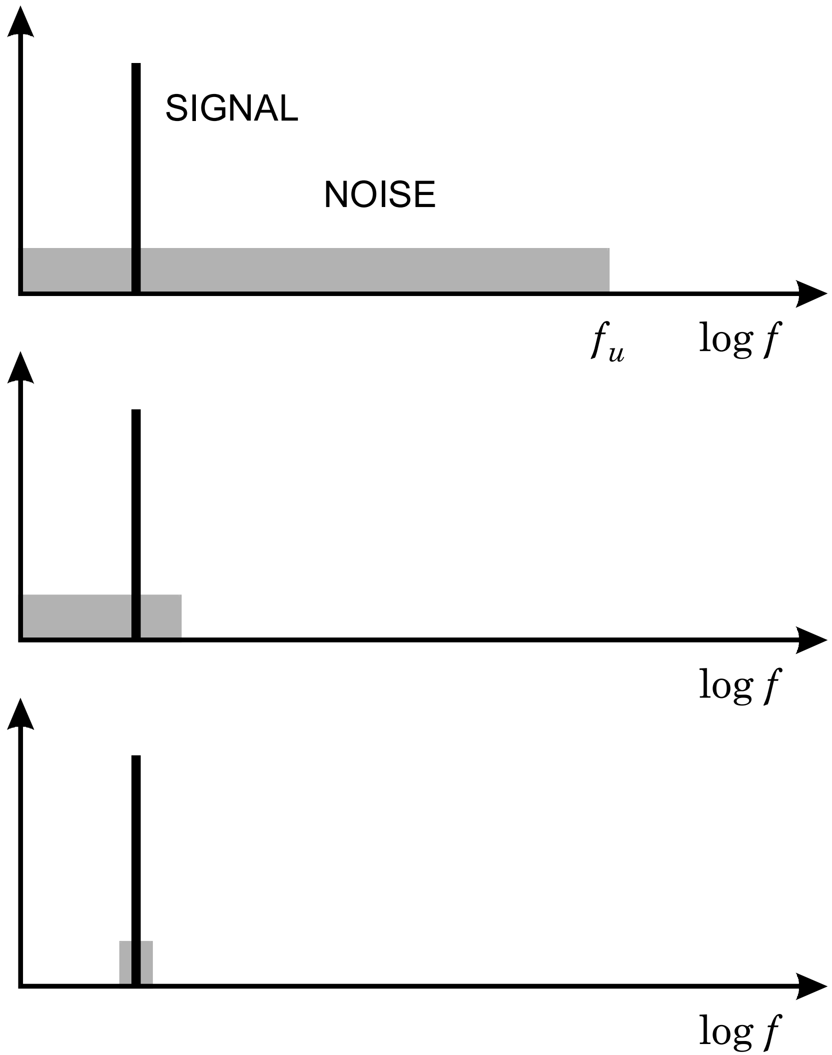
$$P_n = \frac{V_n^2}{R} = 4kTB$$

and the spectral density of the noise power in the resistor

$$\frac{dP_n}{d\nu} = 4kT \; .$$

Total noise increases with bandwidth.

Total noise is the integral over the shaded region.



S/N increases as noise bandwidth is reduced until signal components are attenuated significantly.

Shot noise

A common example of noise due to number fluctuations is “shot noise”, which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode (emission over a barrier)

Spectral noise current density: $i_n^2 = 2eI$

e = electronic charge

I = DC current

A more intuitive interpretation of this expression will be given in Part IV.

Note: Shot noise does not occur in “ohmic” conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

Low Frequency (“$1/f$ ”) Noise

Charge can be trapped and then released after a characteristic lifetime τ.

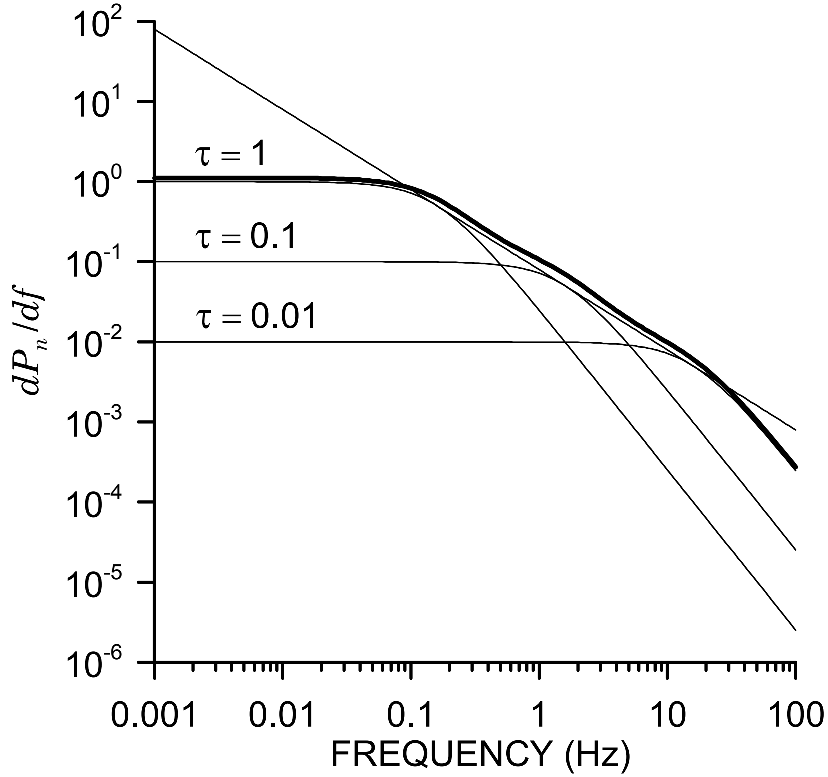
The spectral density for a single lifetime

$$S(f) \propto \frac{\tau}{1+(2\pi f \tau)^2} \; .$$

For $2\pi f \tau \gg 1$: $S(f) \propto \frac{1}{f^2}$.

However,
several traps with different time constants can yield a “$1/f$“ distribution:

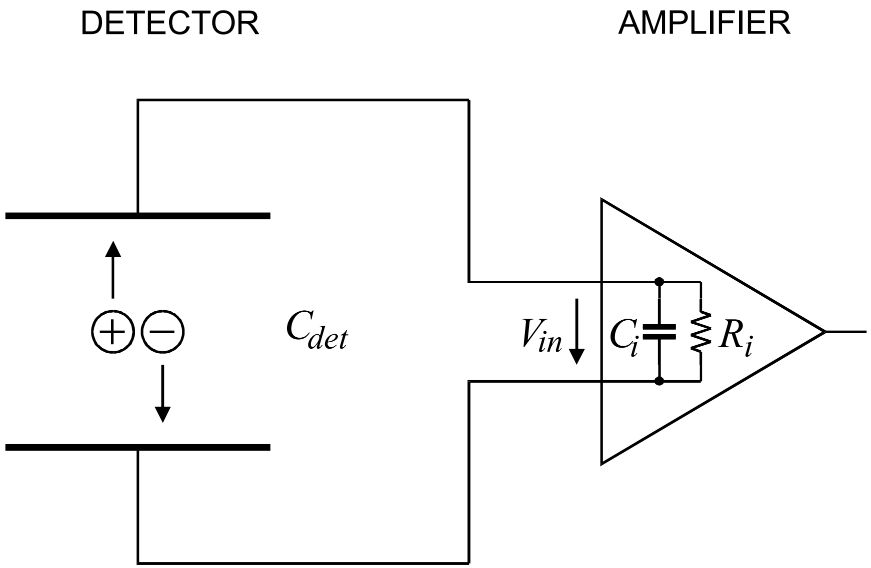
Traps with three time constants of 0.01, 0.1 and 1 s yield a $1/f$ distribution over two decades in frequency.

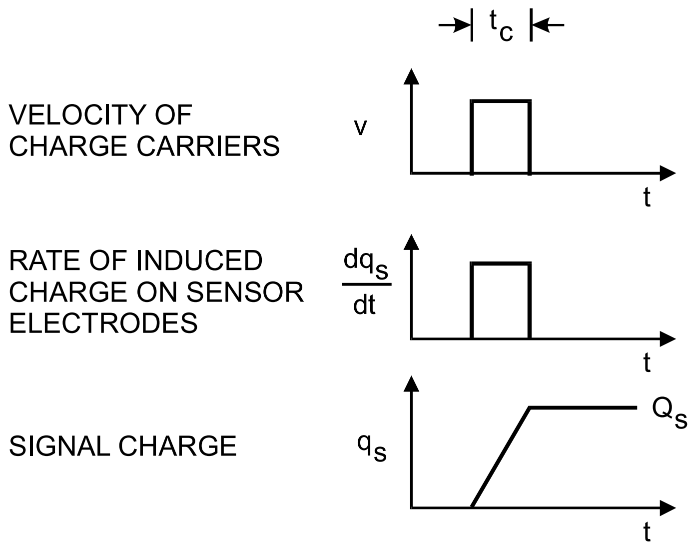


Low frequency noise is ubiquitous – must not have $1/f$ dependence, but commonly called $1/f$ noise.

Spectral power density: $\frac{dP_{noise}}{df} = \frac{1}{f^{\alpha}}$ (typically α = 0.5 – 2)

3. Signal-to-Noise Ratio vs. Detector Capacitance





if $R_i \times (C_{det} + C_i) \gg$ collection time,

peak voltage at amplifier input $V_{in} = \dfrac{Q_s}{C} = \dfrac{\int i_s dt}{C} = \dfrac{Q_s}{C_{det} + C_i}$

↑

Magnitude of voltage depends on total capacitance at input!

The peak amplifier signal V_S is inversely proportional to the **total capacitance at the input**, i.e. the sum of

1. detector capacitance,
2. input capacitance of the amplifier, and
3. stray capacitances.

Assume an amplifier with a noise voltage v_n at the input.

Then the signal-to-noise ratio

$$\frac{S}{N} = \frac{V_S}{v_n} \propto \frac{1}{C}$$

- However, S/N does not become infinite as $C \to 0$
  (then front-end operates in current mode)

- The result that $S/N \propto 1/C$ generally applies to systems that measure signal charge.

Noise vs. Detector Capacitance – Charge-Sensitive Amplifier

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,
- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i \gg C_d$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_v$.
- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

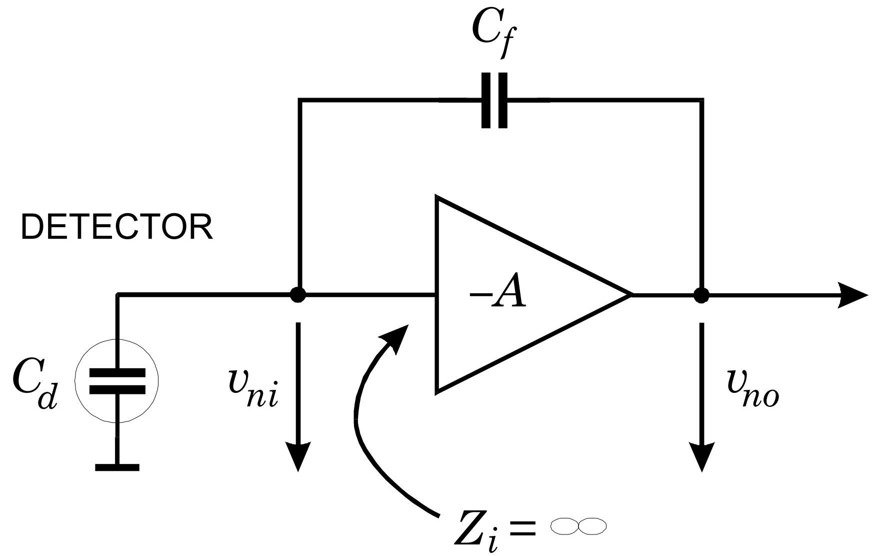
Although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier.

Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input to which the loop responds in the same manner as to a detector signal.

$\Rightarrow$ **S/N at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.



$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_d}}{X_{C_d}} = v_{ni} \frac{\dfrac{1}{\omega C_f} + \dfrac{1}{\omega C_d}}{\dfrac{1}{\omega C_d}}$$

$$v_{no} = v_{ni} \left(1 + \frac{C_d}{C_f} \right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni} \left(C_d + C_f \right)$$

Signal-to-noise ratio
$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_d + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

Conclusion

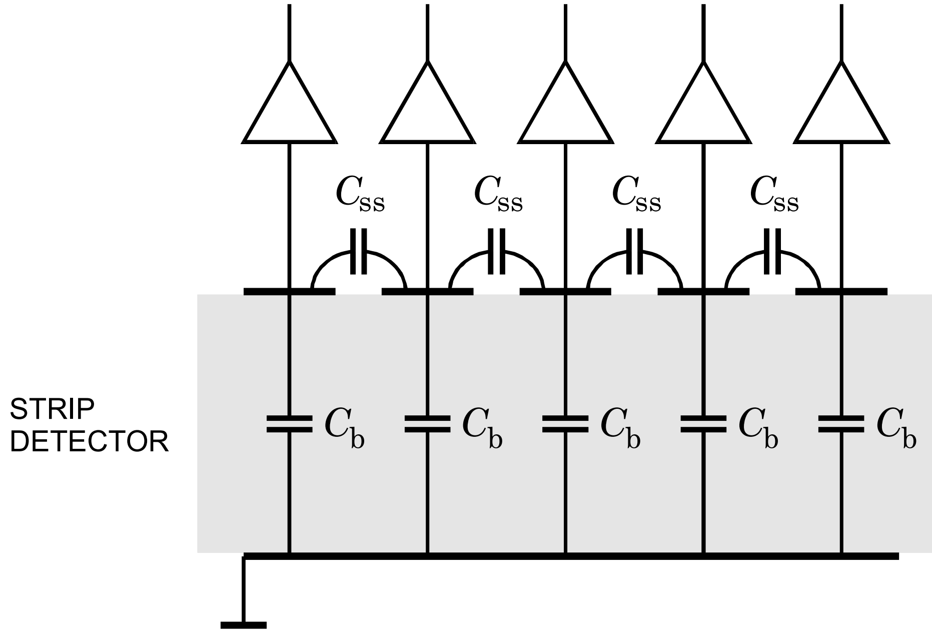
In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.
- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

4. Complex Sensors

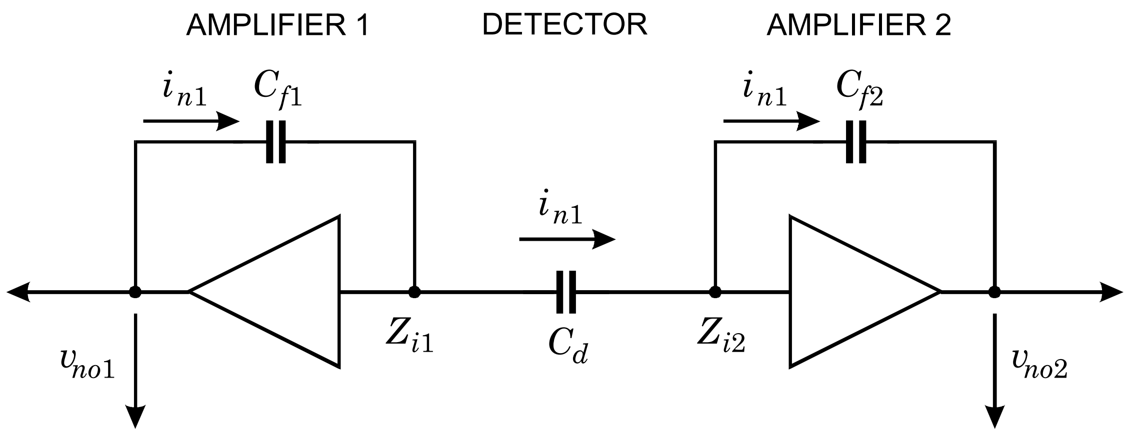
Cross-coupled noise



Noise at the input of an amplifier is cross-coupled to its neighbors.

Principle of Noise Cross-Coupling

Consider a capacitance connecting two amplifier inputs, e.g. amplifiers at opposite electrodes of a detector with capacitance C_d:



First, assume that amplifier 2 is noiseless.

The noise voltage v_{no1} causes a current flow i_{n1} to flow through the feedback capacitance C_{f1} and the detector capacitance C_d into the input of amplifier 2.

Note that for a signal originating at the output of amplifier 1, its input impedance Z_{i1} is high (∞ for an idealized amplifier), so all of current i_{n1} flows into amplifier 2.

Amplifier 2 presents a low impedance to the noise current i_{n1}, so its magnitude

$$i_{n1} = \frac{v_{no1}}{X_{C_{f1}} + X_{C_d}} = \frac{v_{no1}}{\dfrac{1}{\omega C_{f1}} + \dfrac{1}{\omega C_d}}.$$

The voltage at the output of amplifier is the product of the input current times the feedback impedance,

$$v_{no12} = \frac{i_{n1}}{\omega C_{f2}} = \frac{v_{no1}}{\dfrac{1}{\omega C_{f1}} + \dfrac{1}{\omega C_d}} \cdot \frac{1}{\omega C_{f2}} = \frac{v_{no1}}{\dfrac{C_{f2}}{C_{f1}} + \dfrac{C_{f2}}{C_d}}.$$

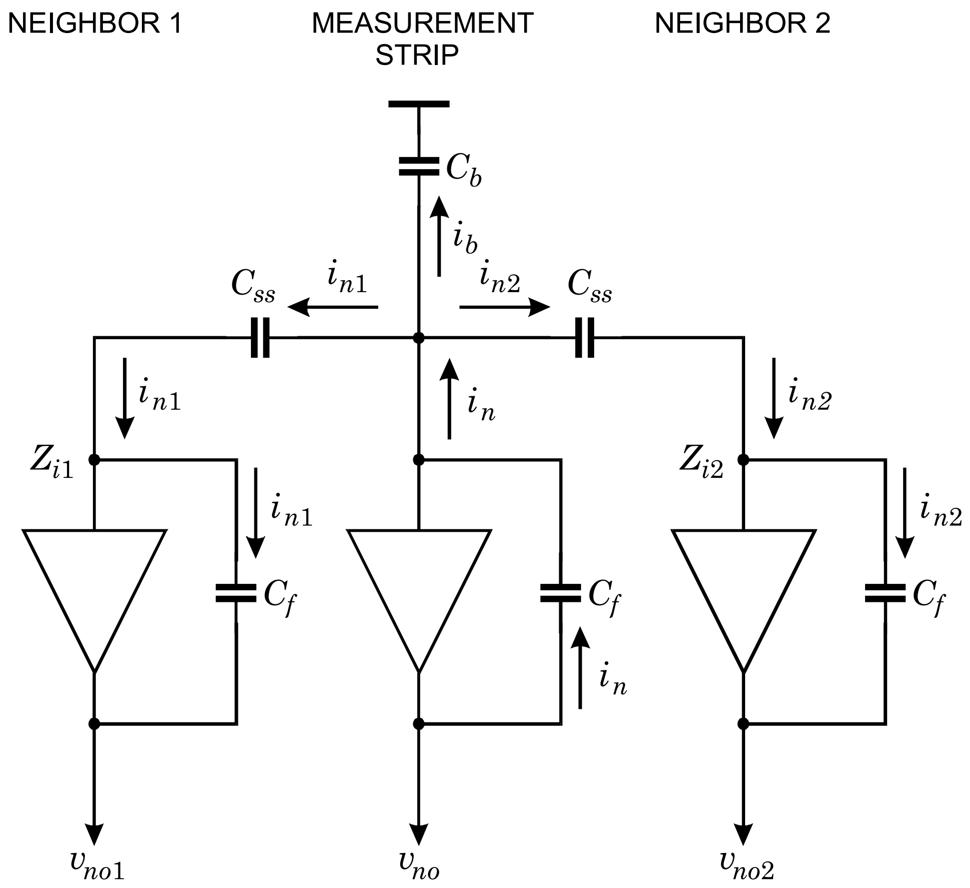
For identical amplifiers $C_{f1} = C_{f2}$. Furthermore, $C_{f2} \ll C_d$, so the additional noise from amplifier 1 at the output of amplifier 2 is

$$v_{no12} = v_{no1}.$$

This adds in quadrature to the noise of amplifier 2. Since both amplifiers are same, $v_{no1} = v_{no2}$, so cross-coupling increases the noise by a factor $\sqrt{2}$.

Cross-Coupling in Strip Detectors

The backplane capacitance C_b attenuates the signal transferred through the strip-to-strip capacitance C_{ss}.



The additional noise introduced into the neighbor channels

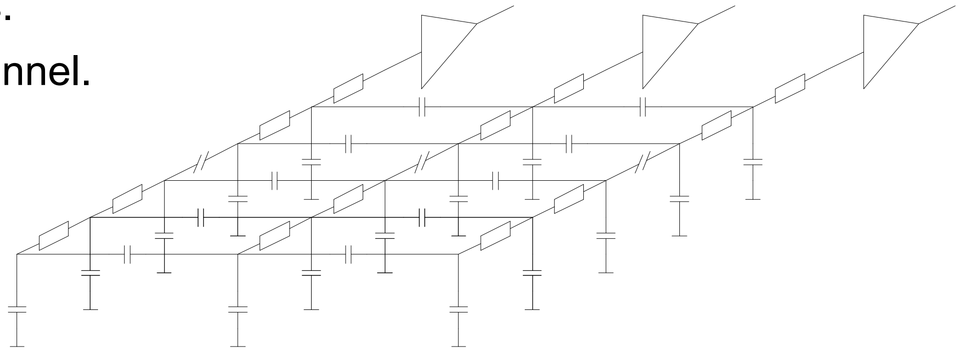
$$v_{no1} = v_{no2} \approx \frac{v_{no}}{2} \frac{1}{1 + 2C_b / C_{ss}}$$

For $C_b = 0$, $v_{no1} = v_{no2} = v_{no} / 2$ and the total noise increases by a factor

$$\sqrt{1 + 0.5^2 + 0.5^2} = 1.22$$

For a backplane capacitance $C_b = C_{ss} / 10$ the noise increases by 16%.

Strip Detector Model for Noise Simulations



Noise coupled from neighbor channels.

Analyze signal and noise in center channel.

Includes:
a) Noise contributions from
 neighbor channels
b) Signal transfer to
 neighbor channels
c) Noise from distributed strip resistance
d) Full SPICE model of preamplifier

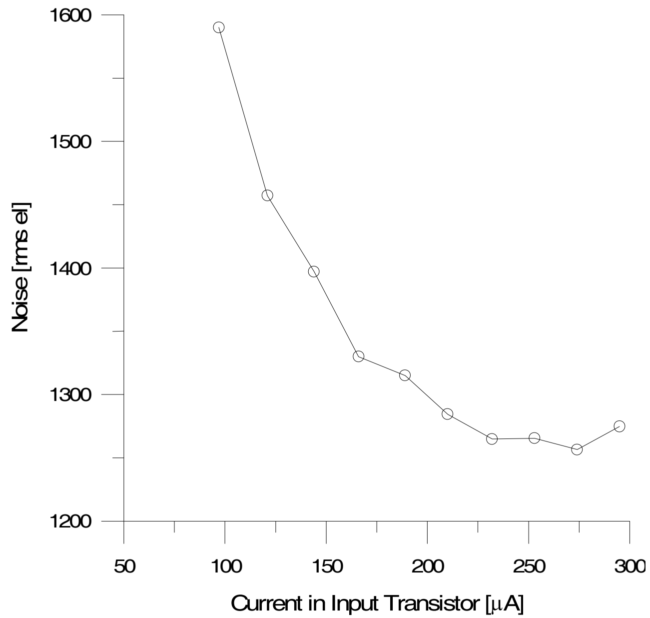
Measured Noise of Module:

p-strips on n-bulk, BJT input transistor

Simulation Results: 1460 el (150 μA)

1230 el (300 μA)

⇒ Noise can be predicted with good accuracy.



5. Quantum Noise Limits in Amplifiers

What is the lower limit to electronic noise?

Can it be eliminated altogether, for example by using superconductors and eliminating devices that carry shot noise?

Starting point is the uncertainty relationship

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Consider a narrow frequency band at frequency ω. The energy uncertainty can be given in terms of the uncertainty in the number of signal quanta

$$\Delta E = \hbar \omega \Delta n$$

and the time uncertainty in terms of phase

$$\Delta t = \frac{\Delta \varphi}{\omega},$$

so that

$$\Delta \varphi \Delta n \geq \frac{1}{2}$$

We assume that the distributions in number and phase are Gaussian, so that the equality holds.

Assume a noiseless amplifier with gain G, so that n_1 quanta at the input yield

$$n_2 = Gn_1$$

quanta at the output.

Furthermore, the phase at the output φ_2 is shifted by a constant relative to the input.

Then the output must also obey the relationship $\Delta\varphi_2 \Delta n_2 = \dfrac{1}{2}$

However, since $\Delta n_2 = G\Delta n_1$ and $\Delta\varphi_2 = \Delta\varphi_1$:

$$\Delta\varphi_1 \Delta n_1 = \frac{1}{2G} ,$$

which is smaller than allowed by the uncertainty principle.

This contradiction can only be avoided by assuming that the amplifier introduces noise per unit bandwidth of

$$\frac{dP_{no}}{d\omega} = (G-1)\hbar\omega \ ,$$

which, referred to the input, is

$$\frac{dP_{ni}}{d\omega} = \left(1 - \frac{1}{G}\right)\hbar\omega$$

If the noise from the following gain stages is to be small, the gain of the first stage must be large, and then the minimum noise of the amplifier

$$\frac{dP_{ni}}{d\omega} = \hbar\omega$$

At 2 mm wavelength the minimum noise corresponds to about 7K.

This minimum noise limit applies to phase-coherent systems. In systems where the phase information is lost, e.g. bolometers, this limit does not apply.

For a detailed discussion see C.M. Caves, Phys. Rev. D **26** (1982) 1817-1839
H.A. Haus and J.A. Mullen, Phys. Rev. 128 (1962) 2407-2413