Dynamic Range of a Series Array SQUID

We compare a series array SQUID comprising N SQUIDs with a single SQUID using the same input inductance as the complete array. We assume that both devices use the same SQUID loop, so that for a individual SQUIDs the sensitivity V_Φ and the output noise V_{on1} are the same.

1. Maximum Output Signal

Let the total input inductance of the array be L_i. Then the input inductance of each SQUID in the array is $L_{i1} = L_i / N$ and the mutual inductance $M_{i1} = \sqrt{L_{i1} L_{SQ}} = M_i / \sqrt{N}$, where M_i is the mutual inductance of the conventional single SQUID. A signal current I leads to a flux $\Phi_{i1} = M_{i1} I$ and the summed output voltage from all of the SQUIDs in the array is $V_o = N M_{i1} I V_\Phi = N\sqrt{\frac{L_i}{N} L_{SQ}}\, I V_\Phi = \sqrt{N L_i L_{SQ}}\, I V_\Phi$, which is $\sqrt{N}$ times larger than for the single SQUID. In other words, the transresistance dV_o / dI of the array is is $\sqrt{N}$ times larger than for the single SQUID.

2. Signal-to-Noise Ratio

The noise at the output of a single SQUID is V_{on1}, so the total output noise of the array is $V_{on} = \sqrt{N} V_{on1}$ and the signal to noise ratio

$$\frac{V_o}{V_{on}} = \frac{N M_{i1} I V_\Phi}{\sqrt{N} V_{on1}} = \frac{N\sqrt{L_{i1} L_{SQ}}\, I V_\Phi}{\sqrt{N} V_{on1}} = \frac{N\sqrt{\frac{L_i}{N} L_{SQ}}\, I V_\Phi}{\sqrt{N} V_{on1}}$$

$$\frac{V_o}{V_{on}} = \frac{\sqrt{L_i L_{SQ}}\, I V_\Phi}{V_{on1}}$$

This is the same as for the single SQUID.

Assume a maximum allowable flux $\Phi_{\max}$ in each SQUID. Then maximum input current

$$I_{\max} = \frac{\Phi_{\max}}{M_{i1}} = \frac{\Phi_{\max}}{\sqrt{L_{i1} L_{SQ}}} = \sqrt{N}\,\frac{\Phi_{\max}}{\sqrt{L_i L_{SQ}}} \quad ,$$

which is $\sqrt{N}$ larger than for a single SQUID with the same input inductance as the array.

The maximum output voltage of the array $V_o = N \Phi_{\max} V_\Phi$. Thus, the maximum signal-to-noise ratio

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$$\frac{V_{o\max}}{V_{on}} = \frac{N\Phi_{\max}V_{\Phi}}{\sqrt{N}V_{on1}} = \sqrt{N}\,\frac{\Phi_{\max}V_{\Phi}}{V_{on1}} \;,$$

and the dynamic range of the series array SQUID is $\sqrt{N}$ times larger than of the single SQUID.

3. Noise Matching

The equivalent input noise current of the array is the same as for an individual SQUID

$$i_n = \frac{\sqrt{S_V}}{M_{i1}V_{\Phi}} = \sqrt{N}\,\frac{\sqrt{S_V}}{M_i V_{\Phi}} \;,$$

which is *N* times larger than for a single SQUID with the same input inductance as the array.

The input noise voltage, however, is the quadrature sum of noise voltages of the individual SQUIDs. For a single SQUID

$$e_{n1} = -\mathbf{i}\omega M_{i1}\sqrt{S_I}$$

and for the array

$$e_n = -\mathbf{i}\omega\sqrt{N}M_{i1}\sqrt{S_I} \;.$$

Since $L_{i1} = L_i / N$ and the mutual inductance $M_{i1} = \sqrt{L_i L_{SQ} / N} = M_i / \sqrt{N}$, the equivalent input noise voltage of the array

$$e_n = -\mathbf{i}\omega M_i\sqrt{S_I} \;.$$

is the same as for a single SQUID with the input inductance of the array. Thus, the optimum source resistance

$$R_{opt} = \left|\frac{e_n}{i_n}\right| = \frac{\omega M_i^2 V_{\Phi}}{\sqrt{N}}\sqrt{\frac{S_I}{S_V}} \;.$$

For a single SQUID with input inductance L_i

$$R_{opt} = \omega M_i^2 V_{\Phi}\sqrt{\frac{S_I}{S_V}} \;,$$

so the optimum source resistance of the series array is $1/\sqrt{N}$ times smaller than for the single comparison SQUID.