Solve for convective film coefficient using: Principles of Heat Transfer by Kreith

$T_c := (273.15 - 35)K$ $T_c = 238.15\,K$ $kJ := 1000J$ $mbar := 10^{-3}bar$ $\mu Pa := 10^{-6}Pa$

$Q := 240W$ $L1 := 2m$ $\sigma := 0.012\frac{N}{m}$ $c_{liq} := 2039\frac{J}{kg\cdot K}$

$\rho_{liq} := 1096\cdot\frac{kg}{m^3}$ $\rho_v := 31\frac{kg}{m^3}$ $\mu_{liq} := 178\mu Pa\cdot s$ $\mu_v := 12.\mu Pa\cdot s$

$k_{liq} := 0.153\frac{W}{m\cdot K}$ $k_v := 0.013\frac{W}{m\cdot K}$

$h_{liq} := 123.05\frac{kJ}{kg}$ $h_v := 436.23\frac{kJ}{kg}$ $\Delta h := h_v - h_{liq}$ $\Delta h = 313.18\cdot\frac{kJ}{kg}$ $\lambda := \Delta h$

$x_i := 0.05$ $x_o := 0.85$ $mdot := \frac{Q}{(x_o - x_i)\cdot\lambda}$ $mdot = 9.579\times 10^{-4}\frac{kg}{s}$

Tube dimensions $t := .012in$ wall thickness

$d_o := 2.8mm$ $d_i := d_o - 2\cdot t$ $A_c := \frac{\pi}{4}\cdot d_i^2$ round tube

$D_h := d_i$ $D_h = 2.19\cdot mm$ $A_t := A_c$ $P_c := \pi\cdot D_h$ $L1 = 2\,m$

$G1 := \frac{mdot}{A_t}$ $G1 = 254.209\frac{kg}{m^2\cdot s}$

$h_{flux} := \frac{240W}{P_c\cdot L1}$ $h_{flux} = 1.744\times 10^4\cdot\frac{W}{m^2}$ $T_{sat} := T_c$ $T_{sat} = 238.15\,K$ fluid saturation temperature

$k_w := 200\frac{W}{m\cdot K}$ aluminum tube wall for T_{sat} = 238K the saturation pressure is 12.024bar

$P := 12.024bar$ $P = 1.202\times 10^6 Pa$ $Pr_{liq} := \frac{\mu_{liq}\cdot c_{liq}}{k_{liq}}$ $Pr_{liq} = 2.372$

$\alpha_{liq} := \frac{k_{liq}}{\rho_{liq}\cdot c_{liq}}$ $\alpha_{liq} = 6.846\times 10^{-8}\frac{m^2}{s}$ thermal diffusivity

Condition for nucleate boiling to occur

$$\Delta T_n := \left(\frac{8\cdot\sigma\cdot h_{flux}\cdot T_{sat}}{\lambda\cdot\rho_v\cdot k_{liq}}\right)^{0.5}$$ $\Delta T_n = 0.518\,K$ any differential film temp above this

Reference: Kreith

really method by Chen

$$i := 0..6 \qquad x := \begin{pmatrix} .05 \\ .1 \\ .3 \\ .5 \\ .6 \\ .7 \\ .85 \end{pmatrix}$$

this a linear change flow quality along the tube length

$$X_{ttinverse_i} := \left(\frac{x_i}{1 - x_i}\right)^{0.9} \cdot \left(\frac{\rho_{liq}}{\rho_v}\right)^{0.5} \cdot \left(\frac{\mu_v}{\mu_{liq}}\right)^{0.1} \qquad X_{ttinverse} = \begin{pmatrix} 0.321 \\ 0.628 \\ 2.118 \\ 4.54 \\ 6.54 \\ 9.734 \\ 21.632 \end{pmatrix}$$

$$Ftt_i := 2.35 \cdot \left(X_{ttinverse_i} + 0.213\right)^{0.736} \qquad Ftt = \begin{pmatrix} 1.481 \\ 2.07 \\ 4.381 \\ 7.402 \\ 9.585 \\ 12.746 \\ 22.742 \end{pmatrix}$$

convective cooling

$$h_{c_i} := 0.023 \cdot \left[\frac{G1 \cdot (1 - x_i) \cdot D_h}{\mu_{liq}}\right]^{0.8} \cdot Pr_{liq}{}^{0.4} \cdot \frac{k_{liq}}{D_h} \cdot Ftt_i \qquad h_c = \begin{pmatrix} 2017.35 \\ 2700.713 \\ 4675.668 \\ 6035.495 \\ 6537.599 \\ 6906.327 \\ 7077.704 \end{pmatrix} \cdot \frac{W}{m^2 \cdot K}$$

h_c is the cooling from forced convection. The next step is to calculate the cooling from evaporation.

Solving for evaporation requires assuming a temperature difference between the tube wall and the saturation temperature of the fluid. After calculating this contribution we can solve for this temperature difference through the combination of both cooling mechanisms. If the end result agrees with the assumption, the process stops otherwise the process is repeated with a new assumed ΔT_{sat}

$$Re_{TP_i} := \frac{G1 \cdot (1 - x_i) \cdot D_h}{\mu_{liq}} \cdot (Ftt_i)^{1.25} \cdot 10^{-4}$$

needed to calculate S_{tt}

$$Re_{TP} = \begin{pmatrix} 0.485 \\ 0.699 \\ 1.388 \\ 1.91 \\ 2.11 \\ 2.26 \\ 2.33 \end{pmatrix} \qquad Stt_i := \left[1 + 0.12\cdot\left(Re_{TP_i}\right)^{1.14}\right]^{-0.1} \qquad Stt = \begin{pmatrix} 0.995 \\ 0.992 \\ 0.984 \\ 0.978 \\ 0.976 \\ 0.974 \\ 0.973 \end{pmatrix}$$

essentially constant over the tube length

For the assumed ΔT_{sat} (wall temp-fluid saturation temp) we calculate the value of ΔP_{sat} for CO_2. This value is the change in saturation pressure corresponding to the change in fluid temperature. At -35C the change in saturation pressure per degree C is 45080Pa, or for 2.5C ΔT_{sat}, the change is 112700Pa

$\Delta T_{sat} := 2.55K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 114954Pa \quad \Delta p_{sat} = 1.15 \times 10^3 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_0} := .00122\cdot\left(\frac{k_{liq}^{0.79}\cdot c_{liq}^{0.45}\cdot\rho_{liq}^{0.49}}{\sigma^{0.5}\cdot\mu_{liq}^{0.29}\cdot\lambda^{0.24}\cdot\rho_v^{0.24}}\right)\cdot\Delta T_{sat}^{0.24}\cdot\Delta p_{sat}^{0.75}\cdot Stt_0$$ Chen

$h_{b_0} = 4815.221\cdot\frac{W}{m^2\cdot K} \qquad h_0 := h_{b_0} + h_{c_0} \qquad h_0 = 6.833 \times 10^3\cdot\frac{W}{m^2\cdot K}$

$\Delta t_{b_0} := \frac{h_{flux}}{h_0} \qquad \Delta t_{b_0} = 2.552K$ ΔT assumed was 2.5C

For the next quality 0.1

$\Delta T_{sat} := 2.4K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 108192Pa \quad \Delta p_{sat} = 1.082 \times 10^3 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_1} := .00122\cdot\left(\frac{k_{liq}^{0.79}\cdot c_{liq}^{0.45}\cdot\rho_{liq}^{0.49}}{\sigma^{0.5}\cdot\mu_{liq}^{0.29}\cdot\lambda^{0.24}\cdot\rho_v^{0.24}}\right)\cdot\Delta T_{sat}^{0.24}\cdot\Delta p_{sat}^{0.75}\cdot Stt_1$$ Chen

$h_{b_1} = 4523.196\cdot\frac{W}{m^2\cdot K} \qquad h_1 := h_{b_1} + h_{c_1} \qquad h_1 = 7.224 \times 10^3\cdot\frac{W}{m^2\cdot K}$

$\Delta t_{b_1} := \frac{h_{flux}}{h_1} \qquad \Delta t_{b_1} = 2.414K$ ΔT assumed was 2.4C

For the next quality 0.3

$\Delta T_{sat} := 2.05K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 92414Pa$ $\Delta p_{sat} = 924.14 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_2} := .00122 \cdot \left(\frac{k_{liq}^{0.79} \cdot c_{liq}^{0.45} \cdot \rho_{liq}^{0.49}}{\sigma^{0.5} \cdot \mu_{liq}^{0.29} \cdot \lambda^{0.24} \cdot \rho_v^{0.24}} \right) \cdot \Delta T_{sat}^{0.24} \cdot \Delta p_{sat}^{0.75} \cdot Stt_2$$ Chen

$h_{b_2} = 3837.293 \cdot \frac{W}{m^2 \cdot K}$ $h_2 := h_{b_2} + h_{c_2}$ $h_2 = 8.513 \times 10^3 \cdot \frac{W}{m^2 \cdot K}$

$\Delta t_{b_2} := \frac{h_{flux}}{h_2}$ $\Delta t_{b_2} = 2.048\,K$ ΔT assumed was 2.04C

For the next quality 0.5

$\Delta T_{sat} := 1.83K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 82496Pa$ $\Delta p_{sat} = 824.96 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_3} := .00122 \cdot \left(\frac{k_{liq}^{0.79} \cdot c_{liq}^{0.45} \cdot \rho_{liq}^{0.49}}{\sigma^{0.5} \cdot \mu_{liq}^{0.29} \cdot \lambda^{0.24} \cdot \rho_v^{0.24}} \right) \cdot \Delta T_{sat}^{0.24} \cdot \Delta p_{sat}^{0.75} \cdot Stt_3$$ Chen

$h_{b_3} = 3407.788 \cdot \frac{W}{m^2 \cdot K}$ $h_3 := h_{b_3} + h_{c_3}$ $h_3 = 9.443 \times 10^3 \cdot \frac{W}{m^2 \cdot K}$

$\Delta t_{b_3} := \frac{h_{flux}}{h_3}$ $\Delta t_{b_3} = 1.847\,K$ ΔT assumed was 1.83C

For the next quality 0.6

$\Delta T_{sat} := 1.75K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 78890Pa$ $\Delta p_{sat} = 788.9 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_4} := .00122 \cdot \left(\frac{k_{liq}^{0.79} \cdot c_{liq}^{0.45} \cdot \rho_{liq}^{0.49}}{\sigma^{0.5} \cdot \mu_{liq}^{0.29} \cdot \lambda^{0.24} \cdot \rho_v^{0.24}} \right) \cdot \Delta T_{sat}^{0.24} \cdot \Delta p_{sat}^{0.75} \cdot Stt_4$$

$h_{b_4} = 3252.5 \cdot \frac{W}{m^2 \cdot K}$ $h_4 := h_{b_4} + h_{c_4}$ $h_4 = 9.79 \times 10^3 \cdot \frac{W}{m^2 \cdot K}$

$\Delta t_{b_4} := \frac{h_{flux}}{h_4}$ $\Delta t_{b_4} = 1.781\,K$ ΔT assumed was 1.75C

For the next quality 0.7

$\Delta T_{sat} := 1.7K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 76636Pa$ $\Delta p_{sat} = 766.36 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_5} := .00122 \cdot \left(\frac{k_{liq}^{0.79} \cdot c_{liq}^{0.45} \cdot \rho_{liq}^{0.49}}{\sigma^{0.5} \cdot \mu_{liq}^{0.29} \cdot \lambda^{0.24} \cdot \rho_v^{0.24}} \right) \cdot \Delta T_{sat}^{0.24} \cdot \Delta p_{sat}^{0.75} \cdot Stt_5$$

$h_{b_5} = 3154.901 \cdot \frac{W}{m^2 \cdot K}$ $h_5 := h_{b_5} + h_{c_5}$ $h_5 = 1.006 \times 10^4 \cdot \frac{W}{m^2 \cdot K}$

$\Delta t_{b_5} := \frac{h_{flux}}{h_5}$ $\Delta t_{b_5} = 1.733\,K$ ΔT assumed was 1.7C

For the next quality 0.85

$\Delta T_{sat} := 1.7K$ temperature difference between wall and fluid, assumed to start the process.

$\Delta p_{sat} := 76636Pa$ $\Delta p_{sat} = 766.36 \cdot mbar$ pressure difference for temperature difference of 2C

$$h_{b_6} := .00122 \cdot \left(\frac{k_{liq}^{0.79} \cdot c_{liq}^{0.45} \cdot \rho_{liq}^{0.49}}{\sigma^{0.5} \cdot \mu_{liq}^{0.29} \cdot \lambda^{0.24} \cdot \rho_v^{0.24}} \right) \cdot \Delta T_{sat}^{0.24} \cdot \Delta p_{sat}^{0.75} \cdot Stt_6$$

$h_{b_6} = 3152.299 \cdot \frac{W}{m^2 \cdot K}$ $h_6 := h_{b_6} + h_{c_6}$ $h_6 = 1.023 \times 10^4 \cdot \frac{W}{m^2 \cdot K}$

$\Delta t_{b_6} := \frac{h_{flux}}{h_6}$ $\Delta t_{b_6} = 1.705\,K$ ΔT assumed was 1.7C