Reference: Boiling and Condensation and Gas-Liquid Flow, Whalley

Frictional Pressure Drop Analysis-Stave Barrel CO_2, Quality of 5% after injection using Friedel correlation

$t := .012in$

$Q := 240W$ $\quad L1 := 2m$ tube length, round trip

$d_o := 2.8mm$ $\quad d_i := d_o - 2\cdot t$ $\quad A_c := \frac{\pi}{4}\cdot d_i^2$ round tube

$D_h := d_i$ $\quad D_h = 2.19\cdot mm$ $\quad A_t := A_c$

CO_2 Fluid Properties at -35C

$T_c := (273.15 - 35)K$ $\quad T_c = 238.15K$ $\quad kJ := 1000J$ $\quad mbar := 10^{-3}bar$ $\quad \mu Pa := 10^{-6}Pa$

$\sigma := 0.012\frac{N}{m}$ $\quad c_{liq} := 2039\frac{J}{kg\cdot K}$

$\rho_{liq} := 1096\cdot\frac{kg}{m^3}$ $\quad \rho_v := 31\frac{kg}{m^3}$ $\quad \mu_{liq} := 178\mu Pa\cdot s$ $\quad \mu_v := 12.\mu Pa\cdot s$

$k_l := 0.153\frac{W}{m\cdot K}$ $\quad k_v := 0.013\frac{W}{m\cdot K}$

$h_{liq} := 123.05\frac{kJ}{kg}$ $\quad h_v := 436.23\frac{kJ}{kg}$ $\quad \Delta h := h_v - h_{liq}$ $\quad \Delta h = 313.18\cdot\frac{kJ}{kg}$ $\quad \lambda := \Delta h$

$x_i := 0.05$ $\quad x_o := 0.85$ $\quad mdot := \frac{Q}{(x_o - x_i)\cdot\lambda}$ $\quad mdot = 9.579\times 10^{-4}\frac{kg}{s}$

Tube Fluid Parameters, based on inlet and exit flow quality

$G_{liq} := mdot\cdot(1 - x_i)$ $\quad G_{liq} = 9.1\times 10^{-4}\frac{kg}{s}$ $\quad G_v := mdot - G_{liq}$ $\quad G_v = 4.79\times 10^{-5}\frac{kg}{s}$

$G_t := \frac{mdot}{A_t}$ $\quad R_{liq} := \frac{G_t\cdot D_h}{\mu_{liq}}$ $\quad R_{liq} = 3.128\times 10^3$

$R_{fgo} := \frac{G_t\cdot D_h}{\mu_v}$ $\quad R_{fgo} = 4.64\times 10^4$ $\quad C_{fgo} := 0.079\cdot R_{fgo}^{-0.25}$ $\quad C_{fgo} = 5.383\times 10^{-3}$

$R_{flo} := \frac{G_t\cdot D_h}{\mu_{liq}}$ $\quad R_{flo} = 3.128\times 10^3$ $\quad C_{flo} := 0.079\cdot R_{flo}^{-0.25}$ $\quad C_{flo} = 0.011$

$$\Phi2^2 = E + \frac{3.24 \cdot F2 \cdot Hf}{(Fr)^{0.045} \cdot (We)^{0.035}}$$

basic equation for two phase flow correction to single phase flow pressure drop

$$a1 := \frac{\rho_{liq} \cdot C_{fgo}}{\rho_v \cdot C_{flo}} \qquad a1 = 18.015 \qquad b1 := \frac{{G_t}^2 \cdot D_h}{\sigma} \qquad d1 := \frac{{G_t}^2}{g \cdot D_h} \qquad \rho_h := \left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^{-1}$$

$$Hf := \left(\frac{\rho_{liq}}{\rho_v}\right)^{0.91} \cdot \left(\frac{\mu_v}{\mu_{liq}}\right)^{0.19} \cdot \left(1 - \frac{\mu_v}{\mu_{liq}}\right)^{0.7} \qquad Hf = 14.633$$

$$We := \frac{b1}{\left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^{-1}} \qquad Fr := d1 \cdot \left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^2 \qquad F2 := x^{0.78} \cdot (1-x)^{0.224} \qquad E := (1-x)^2 + x^2 \cdot a1$$

$$dpdz_{lo} := \frac{2 \cdot C_{flo} \cdot {G_t}^2}{D_h \cdot \rho_{liq}}$$

frictional pressure drop based on fluid being solely single phase

$$\Delta P_f := dpdz_{lo} \cdot \frac{L1}{0.85 - 0.05} \cdot \int_{0.05}^{0.85} (1-x)^2 + x^2 \cdot (a1) + \frac{3.24 \cdot \left[x^{0.78} \cdot (1-x)^{0.224}\right] \cdot Hf}{\left[\frac{d1}{\left[\left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^{-1}\right]^2}\right]^{0.045} \cdot \left[\frac{b1}{\left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^{-1}}\right]^{0.035}} \, dx$$

$$\Delta P_f = 1.998 \times 10^4 \cdot Pa \qquad \Delta P_f = 199.786 \cdot mbar$$

$$\int_{0.05}^{0.85} (1-x)^2 + x^2 \cdot (a1) + \frac{3.24 \cdot \left[x^{0.78} \cdot (1-x)^{0.224}\right] \cdot Hf}{\left[d1 \cdot \left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)^2\right]^{0.045} \cdot \left[b1 \cdot \left(\frac{x}{\rho_v} + \frac{1-x}{\rho_{liq}}\right)\right]^{0.035}} \, dx = 14.052$$

this is the correction for the two phase flow, as a multiplier

Reference: Evaporative Cooling-Conceptual Design for ATLAS SCT, T.O. Niinikoski

Pressure drop due to momentum transfer , inlet to outlet, $\Delta P_m = \Phi_m mdot^2/(At^2 \rho_{liq})$

$x_{in} := 0.05 \qquad x_{out} := 0.85$

$$\rho_{hi} := \left(\frac{x_{in}}{\rho_v} + \frac{1 - x_{in}}{\rho_{liq}}\right)^{-1}$$ homogeneous flow density $\rho_{hi} = 403.276\frac{kg}{m^3}$ at inlet

$\frac{\rho_{liq}}{\rho_{hi}} = 2.718 \qquad \frac{\rho_v}{\rho_{hi}} = 0.077$ volume fraction of constituents at the inlet

$$\rho_{ho} := \left(\frac{x_{out}}{\rho_v} + \frac{1 - x_{out}}{\rho_{liq}}\right)^{-1}$$ homogeneous flow density $\rho_{ho} = 36.289\frac{kg}{m^3}$ at outlet

Relative volume fraction of liquid and vapor phases at the inlet and exit

$$j_{in} := \frac{x_{in} \cdot mdot}{\rho_v \cdot A_t} + \frac{\left(1 - x_{in}\right) \cdot mdot}{\rho_{liq} \cdot A_t}$$ $j_{in} = 0.63\frac{m}{s}$ total volume flux at inlet per unit area

$$j_o := \frac{x_o \cdot mdot}{\rho_v \cdot A_t} + \frac{\left(1 - x_o\right) \cdot mdot}{\rho_{liq} \cdot A_t}$$ $j_o = 7.005\frac{m}{s}$ total volume flux at exit

$$j_{vin} := \frac{x_{in}}{x_{in} + \left(1 - x_{in}\right) \cdot \frac{\rho_v}{\rho_{liq}}}$$ $j_{vin} = 0.65$ volume fraction of vapor at inlet

$j_{liqin} := 1 - j_{vin}$ $\qquad j_{liqin} = 0.35$ volume fraction of liquid at inlet

$$j_{vo} := \frac{x_o}{x_o + \left(1 - x_o\right) \cdot \frac{\rho_v}{\rho_{liq}}}$$ $j_{vo} = 0.995$ volume fraction of vapor at inlet

$j_{liqo} := 1 - j_{vo}$ $\qquad j_{liqo} = 4.967 \times 10^{-3}$ volume fraction of liquid at inlet

$$\Phi_m := \frac{\left(1 - x_o\right)^2}{j_{liqo}} - \frac{\left(1 - x_{in}\right)^2}{j_{liqin}} + \left(\frac{x_o^{\,2}}{j_{vo}} - \frac{x_{in}^{\,2}}{j_{vin}}\right)\frac{\rho_{liq}}{\rho_v}$$ $\Phi_m = 27.484$

Pressure difference to momentum change ΔP_m

$$\Delta P_m := \Phi_m \cdot \frac{mdot^2}{A_t^{\,2} \cdot \rho_{liq}}$$ $\Delta P_m = 16.205 \cdot mbar$

Total Pressure Drop $\Delta P := \Delta P_f + \Delta P_m$ $\Delta P = 215.991 \cdot mbar$

R. Reinhard, Y. Hwang Vapor Compression Heat Pumps with Refrigerant Mixtures.

Martinelli and Nelson Correlation

C_{fgo} agrees with their f_{fo} and sincedpdz$_{lo}$ = their same term

$$\Delta P_{fnew} := dpdz_{lo} \cdot \frac{L1}{(0.85 - 0.05)} \cdot \int_{0.05}^{0.85} \left(1 + \frac{1}{x^{0.5}}\right)^4 \cdot (1 - x)^{1.75} \, dx$$

$\Delta P_{fnew} = 6.584 \times 10^4 Pa$ $\Delta PTnew := \Delta P_{fnew} + \Delta P_m$

$\Delta PTnew = 6.746 \times 10^4 Pa$ $\Delta PTnew = 674.625 \cdot mbar$

Change in Temparture

The change in saturation temperature corresponding to the pressure drop is as follows:

use maximum of two methods and the change in ΔT/ΔP

at -35C the ΔT/ΔP= K/45080Pa

$\Delta PTnew = 674.625\, mbar$ **use 675mbar for Table 4**

$$\Delta Tsat := \Delta PTnew \frac{K}{45080Pa}$$

$\Delta Tsat = 1.497\, K$ **use 1.5C for Table 4**