MAX-PLANCK-GESELLSCHAFT

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

Testing strong-field classical and quantum electrodynamics with intense laser fields

Antonino Di Piazza*

Research Progress Meeting
Lawrence Berkeley National Laboratory,
Berkeley, June 23th 2011

*dipiazza@mpi-hd.mpg.de

OUTLINE

- Introduction
  - Electromagnetic interaction
  - Typical scales of QED
- Electron-positron pair creation
- Photon-photon scattering
  - A matterless double-slit
- Strong-field QED and its experimental feasibility
  - Furry picture and theoretical methods
  - Is strong-field QED a “weak” theory?
- The problem of radiation reaction
- Conclusion

Electromagnetic interaction

- The electromagnetic interaction is one of the four “fundamental” interactions in Nature. It is the interaction between electric charged particles (e.g. electrons and positrons) and it is mediated by the electromagnetic field (photons, in the “quantum” language)
- It is theoretically described by quantum electrodynamics (QED), whose Lagrangian depends on two parameters:
  - Electron rest mass $m=9.1\times10^{-28}$ g
  - Electron charge $-e$, with $e=4.8\times10^{-10}$ esu
- The parameters m and e, together with the fundamental constants $\hbar$ and c, determine all the typical scales and regimes of QED

Typical scales of QED

- Strength of the electromagnetic interaction: $\alpha = e^2/\hbar c = 1/137$
- Energy scale: $mc^2 = 511$ keV (time scale: $\hbar/mc^2 = 1.3\times10^{-21}$ s)
- Momentum scale: $mc = 511$ keV$/c$ (length scale: $\lambda_c = \hbar/mc = 3.9\times10^{-11}$ cm, Compton wavelength)
- Critical field of QED:

$$F_{cr} = \frac{\text{energy scale}}{(\text{positron charge}) \times (\text{length scale})}$$

$$E_{cr} = \frac{m^2c^3}{\hbar e} = 1.3 \times 10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar e} = 4.4 \times 10^{13}\ \mathrm{G}$$

$$\rightarrow \quad I_{cr} = \frac{cE_{cr}^2}{8\pi} = 2.3 \times 10^{29}\ \mathrm{W/cm^2}$$

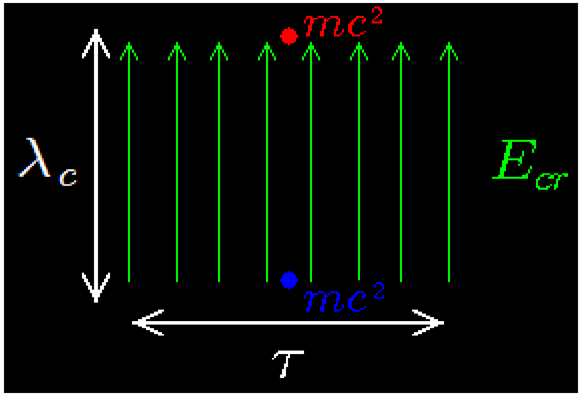
Classical vs. quantum vacuum

- In classical physics the vacuum is empty space-time
- In QED the time-energy uncertainty principle allows for the existence of "fluctuations" in the vacuum: pairs of particle-antiparticle, that spontaneously pop up and then annihilate after "living" for a very short time and covering a very short distance ($\tau = \hbar/mc^2$ and $\lambda_c = \hbar/mc$, respectively).

Physical meaning of the critical fields

$$\frac{\hbar}{mc} \times eE_{cr} \sim mc^2$$

$$\frac{e\hbar}{mc} \times B_{cr} \sim mc^2$$



- Vacuum instability under electron-positron pair production at $E \sim E_{cr}$ (Sauter 1931, Heisenberg and Euler 1936, Weisskopf 1936)

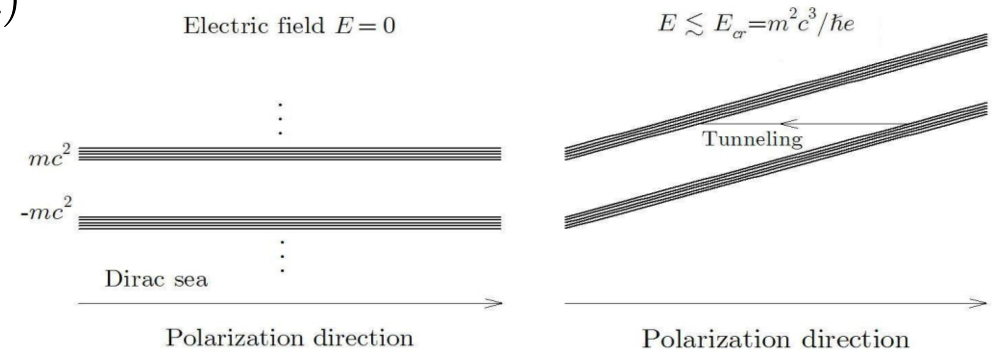
Electron-positron pair production

- Production probability per unit time and unit volume ($E \ll E_{cr}$)

  (see also Schwinger 1951)

$$\frac{dP}{dV\,dt} = \frac{1}{8\pi^3}\left(\frac{E}{E_{cr}}\right)^2 \frac{c}{\lambda_c^4} e^{-\pi\frac{E_{cr}}{E}}$$

- Interpretation: tunneling



- Note the non-perturbative dependence on the electric field
- In time-oscillating electric fields the main role is played by the adiabaticity parameter $\gamma = m\omega_L c/eE_L$ (E_L=field amplitude, ω_L=field angular frequency) (Brezin and Itzykson 1970, Popov 1971)

$\gamma \ll 1$: tunneling regime

$$\frac{dP}{dV\,dt} = \frac{1}{8\pi^3}\left(\frac{E_L}{E_{cr}}\right)^2 \frac{c}{\lambda_c^4} e^{-\pi\frac{E_{cr}}{E_L}}$$

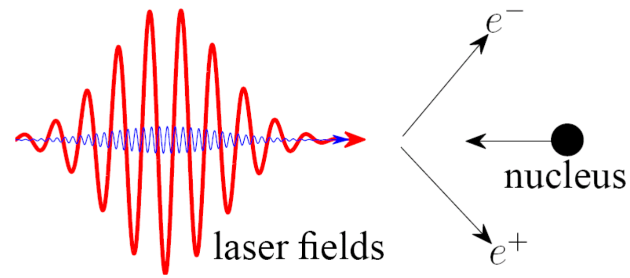
$\gamma \gg 1$: multiphoton regime

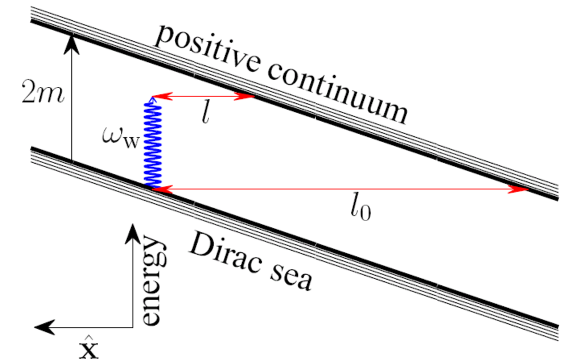
$$\frac{dP}{dV\,dt} = \frac{1}{32\pi}\left(\frac{E_L}{E_{cr}}\right)^2 \frac{c}{\lambda_c^4} (2\gamma)^{-\frac{4mc^2}{\hbar\omega_L}}$$

- About one million optical laser photons ($\hbar\omega_L \sim 1$ eV) have to be absorbed to create an electron-positron pair ($2mc^2 \sim 1$ MeV)

Units with $\hbar = c = 1$

- Suggested "mixed" setup: a weak, high-frequency field and a strong, low-frequency field collide head-on with a high-energy nucleus
- In the rest frame of the nucleus the photon energy of the weak field is below and close to the pair production threshold





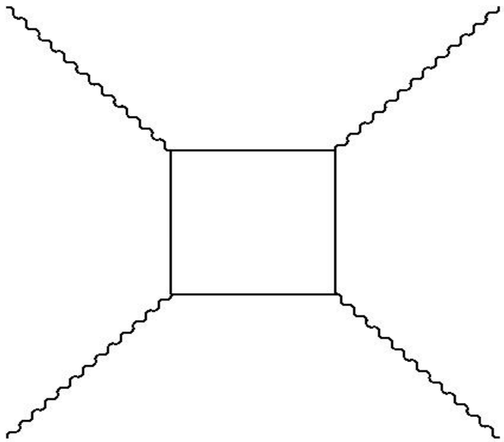
- By changing the frequency of the weak field we can control the tunneling length and enhance the production rate

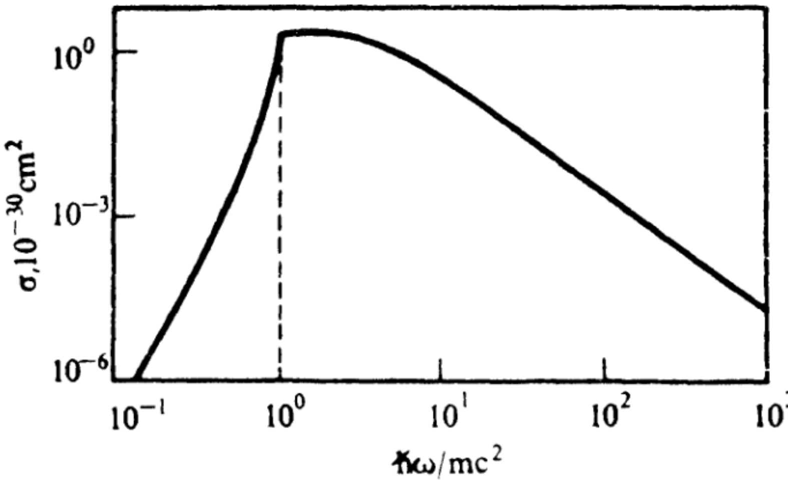
$$\dot{W} = \frac{(Z\alpha)^2 m \eta^2 \chi^2}{16\sqrt{\pi}} \sqrt{\zeta} \exp\left(-\frac{2}{3\zeta}\right)$$

Important physical parameters: $\eta = eE_w/m\omega_w$ and $\zeta = \chi/\delta^{3/2}$ where $\chi = E_s/E_{cr}$ (E_s in the rest frame of the nucleus) and $\delta = (2m - \omega_w)/m \ll 1$

A. Di Piazza, E. Loetstedt, A. I. Milstein and C. H. Keitel, Phys. Rev. Lett. **103**, 170403 (2009)

Real photon-photon scattering





- The total cross section of this process is given by (in the center-of-momentum of the two colliding photons)

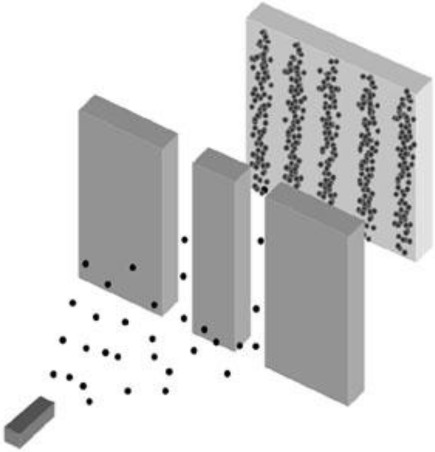
$$\sigma = \begin{cases} 3\times 10^{-2}\alpha^4\lambda_c^2\left(\frac{\hbar\omega}{mc^2}\right)^6 & \text{if } \hbar\omega \ll mc^2 \\ 4.7\alpha^4\left(\frac{c}{\omega}\right)^2 & \text{if } \hbar\omega \gg mc^2 \end{cases}$$

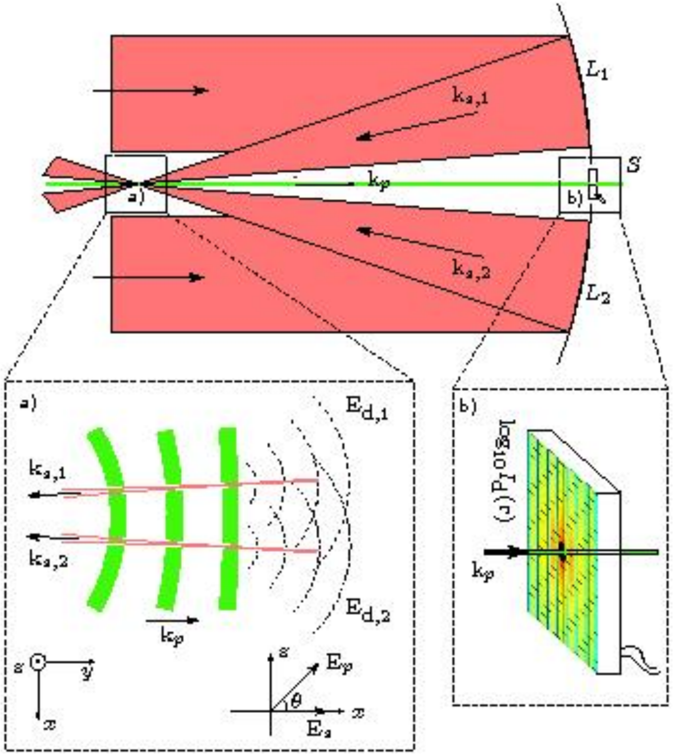
- The maximum is "only" 10^{-5} times the total cross section of Thomson scattering but it is in the MeV range
- Steep dependence on $(\hbar\omega/mc^2)^6$ at small energies
- Background

Can the large number of photons in strong optical laser beams compensate for the $(\hbar\omega/mc^2)^6$-suppression (Lundstroem et al, 2006)?

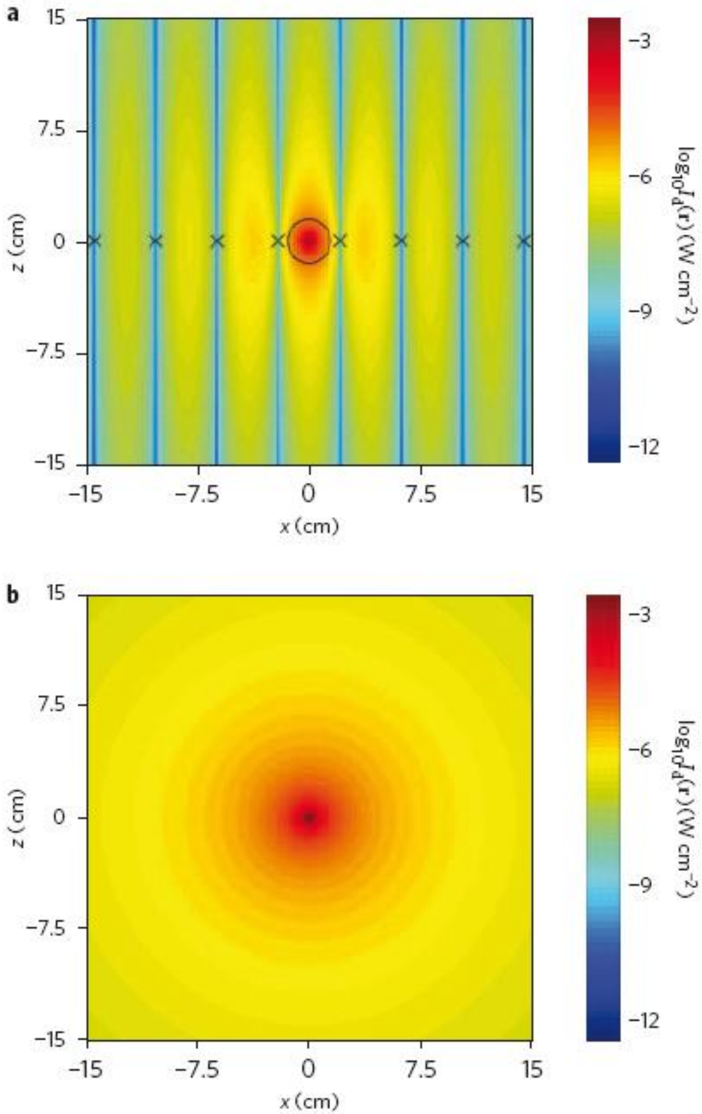
A matterless double-slit

- The double slit experiment has played a fundamental role in our understanding of quantum mechanics, in particular the so-called wave-particle duality of particles
- All double-slit schemes proposed so far have always involved matter (either the particles employed like electrons, neutrons and so on or the wall where the double slit is). By exploiting the quantum interaction among laser beams in the vacuum mediated by virtual electron-positron pairs, we have put forward a matterless double slit setup

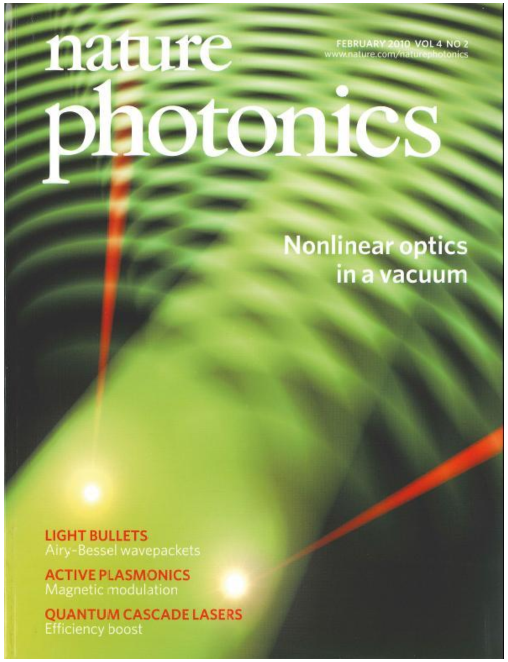




Results:



- Strong field's parameters: 150 PW (ELI, HiPER), 800 nm, 30 fs, focused to one wavelength
- Weak field's parameters: 200 TW, 527 nm, 100 fs focused to 290 μm
- The $\times$ are at: $(n+1/2)\lambda_p = D \sin\vartheta$
- With the above parameters one obtains about 4 diffracted photons per shot



B. King, A. Di Piazza and C. H. Keitel, Nature Photon. **4**, 92 (2010)

QED in a strong background field

Lagrangian density of QED in the presence of a background field $A_{B,\mu}(x)$ (Furry 1951)

Units with $\hbar$=c=1

$$\mathscr{L}_{QED} = \mathscr{L}_e + \mathscr{L}_\gamma + \mathscr{L}_{\text{int}}$$

$$\begin{aligned} \mathscr{L}_e &= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\ \mathscr{L}_\gamma &= -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \\ \mathscr{L}_{\text{int}} &= e\bar{\psi}\gamma^\mu\psi(A_\mu + A_{B,\mu}) \end{aligned} \quad \longrightarrow \quad \begin{aligned} \mathscr{L}_e &= \bar{\psi}[\gamma^\mu(i\partial_\mu + eA_{B,\mu}) - m]\psi \\ \mathscr{L}_\gamma &= -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \\ \mathscr{L}_{\text{int}} &= e\bar{\psi}\gamma^\mu\psi A_\mu \end{aligned}$$

- Only the interaction between the spinor and the radiation field is treated perturbatively

1. Solve the Dirac equation $[\gamma^\mu(i\partial_\mu+eA_{B,\mu}) - m]\psi=0$
   - find the "dressed" one-particle in- and out-electron states and the "dressed" electron propagator
2. Write the Feynman diagrams of the process at hand
3. Calculate the amplitude and then the cross sections (or the rates) using "dressed" states and propagators

QED in a strong laser field

- The laser field is approximated by a plane-wave field: $A_{B,\mu}(x)=A_{L,\mu}(\phi)$, $\phi=(k_L x)=\omega_L t-\boldsymbol{k}_L\cdot\boldsymbol{r}$ and $\omega_L=|\boldsymbol{k}_L|$
  – Approximation valid for laser beams not too tightly focused

- One-particle states: Volkov states (Volkov 1936)

$$\psi_{p,\sigma} = \left[1 - \frac{e}{2(k_L p)}\hat{k}_L \hat{A}_L\right] \frac{u_{p,\sigma}}{\sqrt{2p_0}} \exp\underbrace{\left\{-i(px) + i\int_{-\infty}^{\phi} d\phi' \left[\frac{e(pA_L)}{(k_L p)} + \frac{e^2 A_L^2}{2(k_L p)}\right]\right\}}_{i(\text{Classical action})}$$

Spin term

Electron momentum and spin at $t\rightarrow-\infty$

Free constant bi-spinor

- Technical notes:
  – Volkov states are semiclassical apart from the spin term
  – In- and out-states differ only by a constant phase
  – No tadpole diagrams
  – The vacuum in the presence of a plane wave is stable

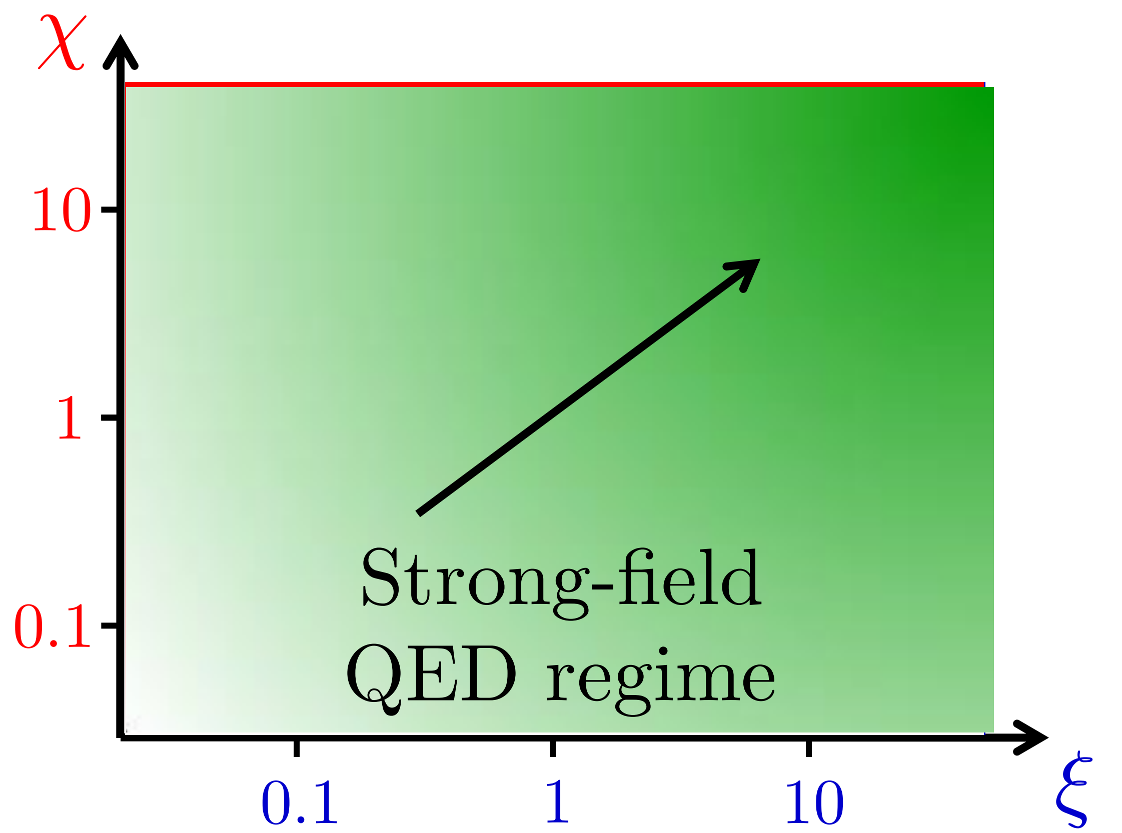
Regimes of QED in a strong laser field

A particle (e^-, e^+ or γ) with energy $\mathcal{E}$ ($\hbar\omega$ for a photon) collides head on with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

Relevant parameters
(Ritus 1985):

$$\xi = \frac{1}{2\pi}\frac{eE_L\lambda_L}{mc^2} = \frac{eE_L\lambda_c}{\hbar\omega_L}$$

$$\chi = 2\frac{\hbar\omega}{mc^2}\frac{E_L}{E_{cr}} = \left.\frac{E_L}{E_{cr}}\right|_{\mathrm{r.f.}}$$



Optical laser and electron accelerator technology

Optical laser technology ($\hbar\omega_L$=1 eV)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al. (2008))	10	30	1	2×10^{22}
Soon (2012) (POLARIS, Vulcan, Astra-Gemini, BELLA etc...)	10÷100	10÷100	1	$10^{22}\div10^{23}$
Near future (2020) (ELI, HiPER)	10^4	10	1	$10^{25}\div10^{26}$

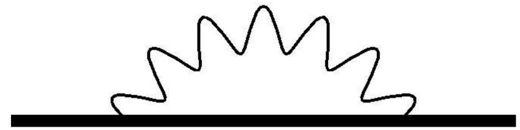
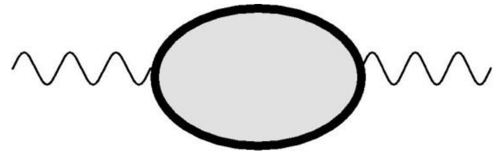
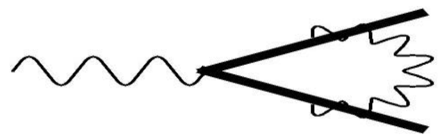
Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Conventional accelerators (PDG)	10÷50	$10^3\div10^4$	10÷100	$10^{10}\div10^{11}$
Laser-plasma accelerators (Leemans et al. 2006)	0.1÷1	50	5	$10^9\div10^{10}$

$$\xi = 110\frac{\sqrt{I_L[10^{22}\ \mathrm{W/cm^2}]}}{\hbar\omega_L[\mathrm{eV}]}$$

$$\chi = 0.82\,\mathcal{E}[\mathrm{GeV}]\sqrt{I_L[10^{22}\ \mathrm{W/cm^2}]}$$

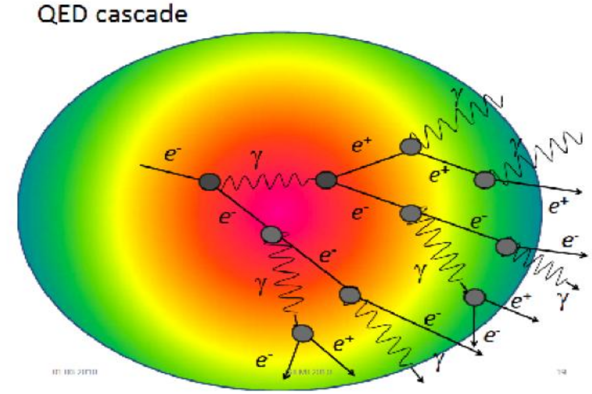
- Present technology allows in principle the experimental investigation of strong-field QED

- Radiative corrections in strong-field QED at $\chi \gg 1$ (Ritus 1985)

Radiative correction	Feynman diagram	Matrix element
Mass operator (Ritus 1972, Baier et al. 1975)		$\sim \alpha \chi^{2/3}$
Polarization operator (Ritus 1972, Baier et al. 1976)		$\sim \alpha \chi^{2/3}$
Vertex correction		Not yet calculated (work in progress…)

- The solid thick lines indicate electron (positron) Volkov states
- At $\alpha\chi^{2/3} \sim 1$, i.e. at $\chi \sim 10^3$, QED in a strong laser field is expected to become a "strong" interaction
- Doubts have been cast on the possibility of reaching the ($\chi \gg 1$)-regime due to avalanche processes and consequent disruption of the laser beam at $\chi \sim 1$ (Bell and Kirk 2008, Bulanov et al. 2010, Sokolov et al. 2010, Nerush et al. 2011)

QED cascade



Radiation reaction in classical electrodynamics

What is the equation of motion of an electron in an external, given electromagnetic field $F^{\mu\nu}(x)$?

- The Lorentz equation

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu$$

  does not take into account that while being accelerated the electron generates an electromagnetic radiation field and it loses energy and momentum

- One has to solve self consistently the coupled Lorentz and Maxwell equations

$$\begin{aligned} m_0\frac{du^\mu}{ds} &= -eF_T^{\mu\nu}u_\nu \\ \partial_\mu F_T^{\mu\nu} &= -e\int ds\,\delta(x - x(s))u^\nu \end{aligned} \xrightarrow{\text{Lorentz gauge}} \begin{aligned} m_0\frac{du^\mu}{ds} &= -e(\partial^\mu A_T^\nu - \partial^\nu A_T^\mu)u_\nu \\ \Box A_T^\nu &= -e\int ds\,\delta(x - x(s))u^\nu \end{aligned}$$

where now m_0 is the electron's bare mass and $F_{T,\mu\nu}=\partial_\mu A_{T,\nu}-\partial_\nu A_{T,\mu}$ is the total electromagnetic field (external field plus the one generated by the electron, Dirac 1938)

- One first solves the inhomogeneous wave equation exactly with the Green-function method

$$\Box A_T^\nu = -e\int ds\delta(x - x(x))u^\nu = j^\nu(x) \longrightarrow A_T^\nu(x) = A^\nu(x) + \int dx'\mathcal{D}(x - x')j^\nu(x')$$

and then re-substitute the solution into the Lorentz equation:

$$(m_0 + \delta m)\frac{du^\mu}{ds} = -eF^{\mu\nu}(x)u_\nu + \frac{2}{3}\alpha\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- After "renormalization" one obtains the Lorentz-Abraham-Dirac equation

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu + \frac{2}{3}\alpha\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- The Lorentz-Abraham-Dirac equation is plagued by serious inconsistencies: runaway solutions, preacceleration
- In the realm of classical electrodynamics, i.e. if quantum effects are negligible, the Lorentz-Abraham-Dirac equation can be approximated by the so-called Landau-Lifshitz equation (Landau and Lifshitz 1947)

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu - \frac{2}{3}\alpha\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu + \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha - \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

- If $F^{\mu\nu}(x)$ is a plane wave the Landau-Lifshitz equation can be solved exactly analytically A. Di Piazza, Lett. Math. Phys. **83**, 305 (2008)
- The analytical solution indicates that in the ultrarelativistic case radiation-reaction effects
  1. are mainly due to the "Larmor" damping term

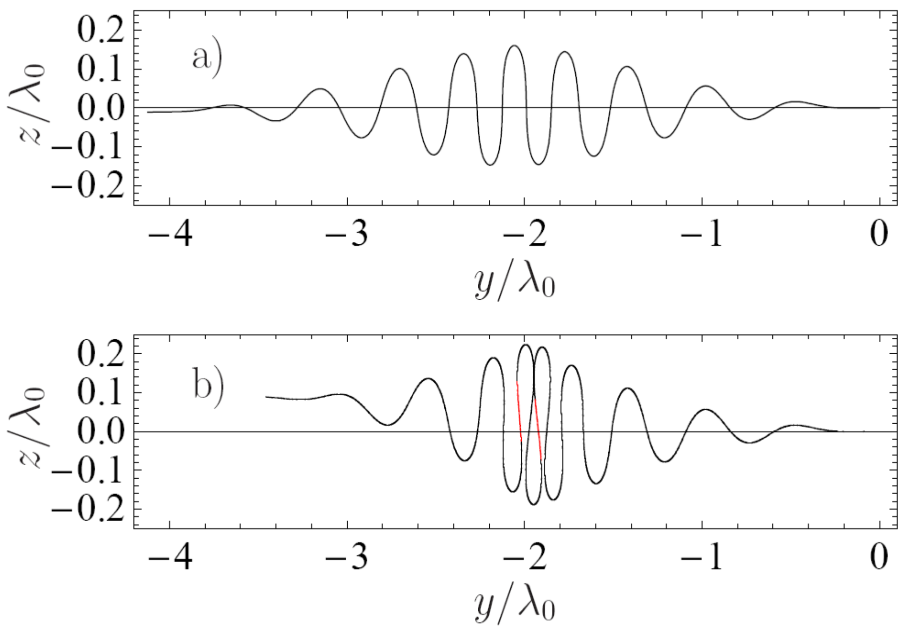
$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu - \frac{2}{3}\alpha\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu + \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha - \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

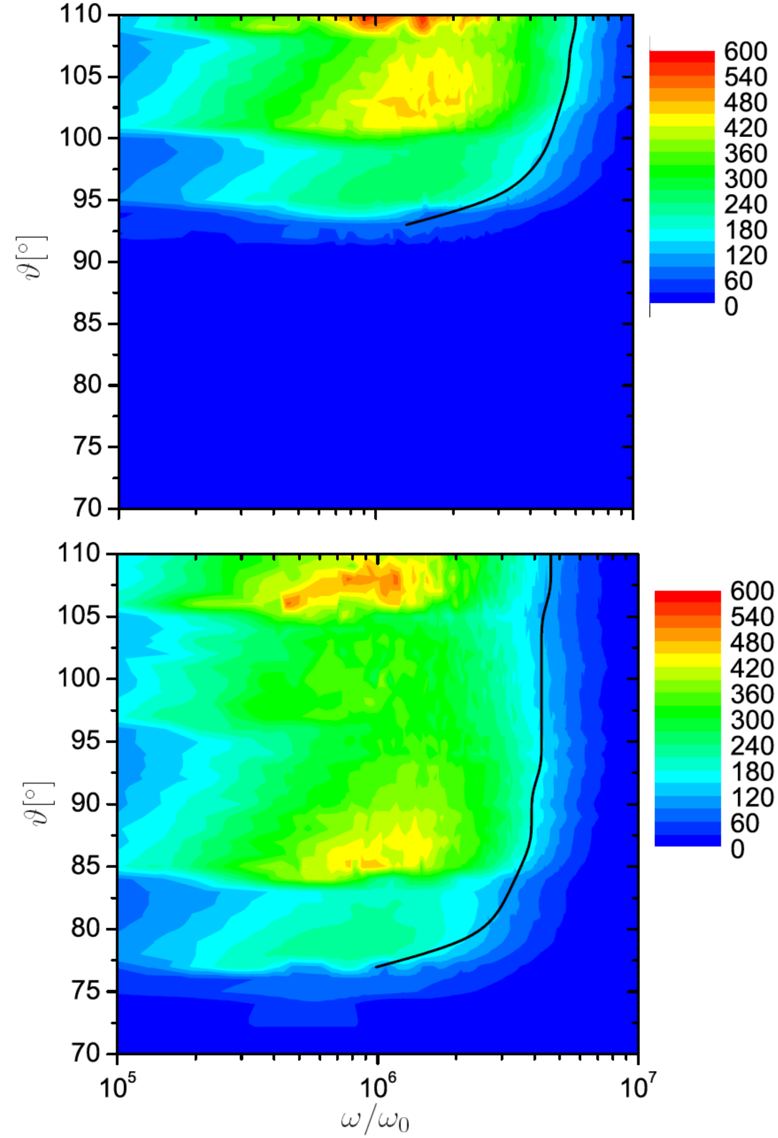
  2. scale with the parameter $R_C\Phi$, where Φ is the total phase of the laser pulse and

$$R_C = \alpha\frac{\omega_0}{m}\gamma_0(1+\beta_0)\xi^2 = \alpha\chi\xi$$

- The condition $R_C \approx 1$ means that the energy emitted by the electron in one laser period is of the order of the initial energy (classical radiation dominated regime) but it is experimentally demanding
- However, one sees that if $2\gamma_0 \approx \xi$, the initial electron's longitudinal momentum is almost compensated by the laser field and this regime turns out to be very sensitive to radiation reaction. A regime has been found, where the electron is reflected by the laser field only if radiation reaction is taken into account
- Recall that an ultrarelativistic electron mainly emits along its velocity within a cone of aperture $\sim 1/\gamma$ (Landau and Lifshitz 1947)

- Numerical parameters: electron energy 40 MeV ($2\gamma_0$=156), laser wavelength 0.8 μm, laser intensity 5×10^{22} W/cm^2 (ξ=150), pulse duration 30 fs, focused to 2.5 μm (note that R_C=0.08!)
- Electron trajectories and emission spectra without and with radiation reaction





- The red parts of the trajectory are those where the longitudinal velocity of the electron is positive
- The black lines indicate the cut-off position from the formula $\omega_c=3\omega_0\gamma^3$

A. Di Piazza, K. Z. Hatsagortsyan and C. H. Keitel, Phys. Rev. Lett. **102**, 254802 (2009)

CONCLUSION

- Classical and quantum electrodynamics are well established theories but there are still areas to be investigated both theoretically and experimentally
- Intense laser fields can be employed
  - to test the predictions of quantum electrodynamics under the extreme conditions generated by "critical" electromagnetic fields
  - to investigate the non-perturbative regime of tunneling electron-positron pair creation
  - to shed light on the classical problem of radiation reaction and to test the validity of the Landau-Lifshitz equation