Identical DYW and AdS5 Formulae: Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda_{\rm QCD}^{-1}} \zeta \, d\zeta \, J_0 \left(\frac{\zeta Q x}{\sqrt{x(1-x)}} \right) \left| \widetilde{\psi}(x,\zeta) \right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0 \left(\frac{\zeta Q x}{\sqrt{x(1-x)}} \right) = \zeta Q K_1(\zeta Q),$$

Same result for LF and AdS5

$\zeta \leftrightarrow \mathbf{z}$

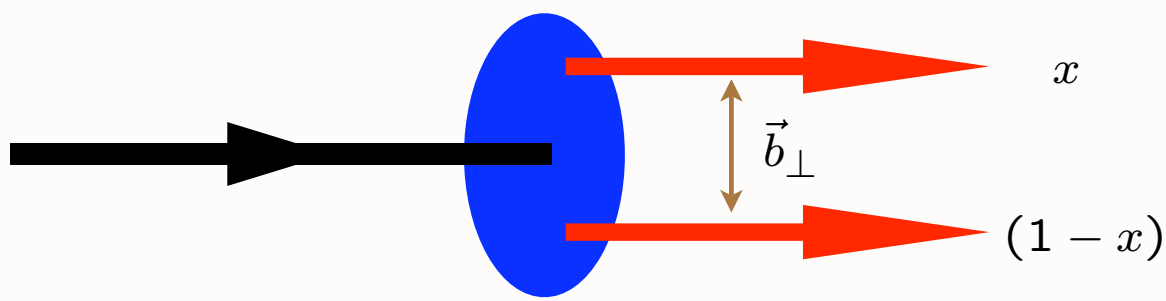
the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0 \left(\sqrt{x(1-x)} |\vec{b}_\perp| \beta_{0,1} \Lambda_{QCD} \right) \theta \left(\vec{b}_\perp^{\,2} \leq \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)} \right)}$$

the holographic LFWF for the valence Fock state of the pion $\psi_{\bar{q}q/\pi}$.

- The variable ζ, $0 \leq \zeta \leq \Lambda_{QCD}^{-1}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

LF(3+1)		AdS_5
$\psi(x, \vec{b}_\perp)$	$\longleftrightarrow$	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	$\longleftrightarrow$	z



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

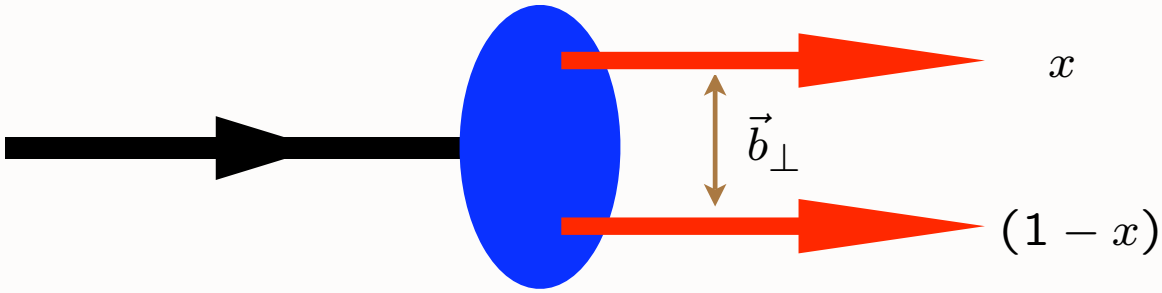
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

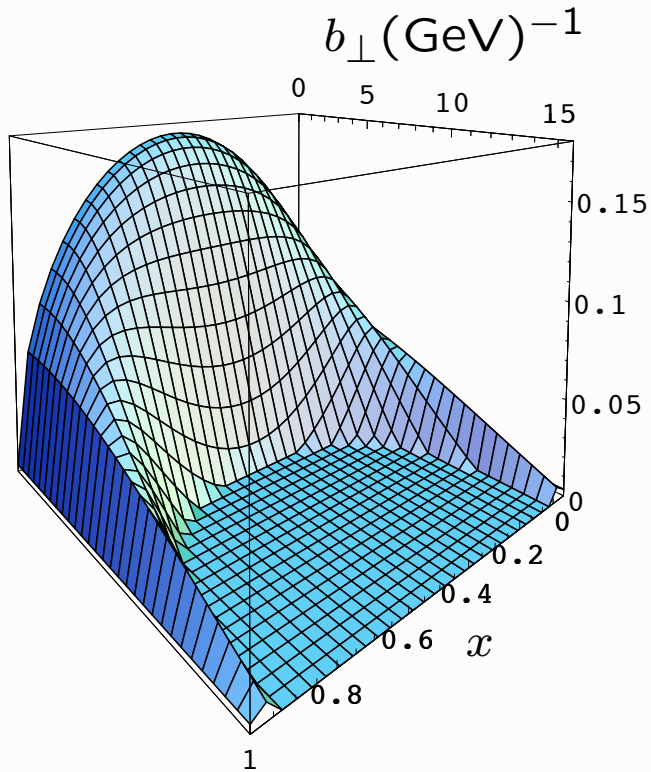
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

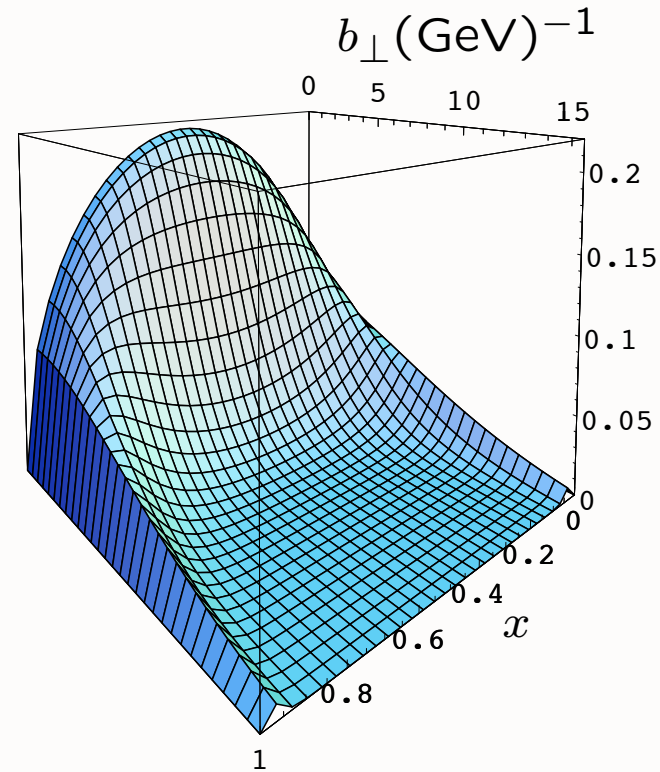
General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \leq \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$





$\Lambda_{\rm QCD} = 0.32$ GeV

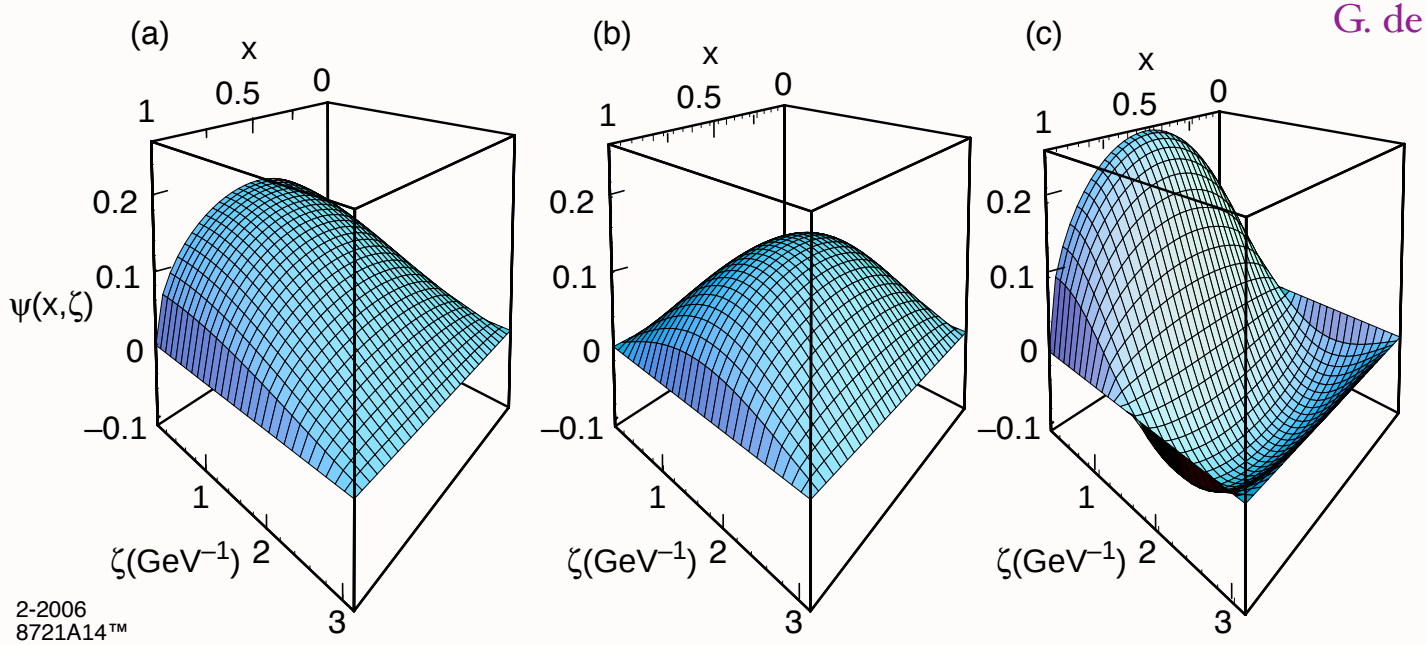
$\kappa = 0.76$ GeV

Truncated Space

Harmonic Oscillator

AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \leq \Lambda_{\rm QCD}^{-1}\right)$$

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

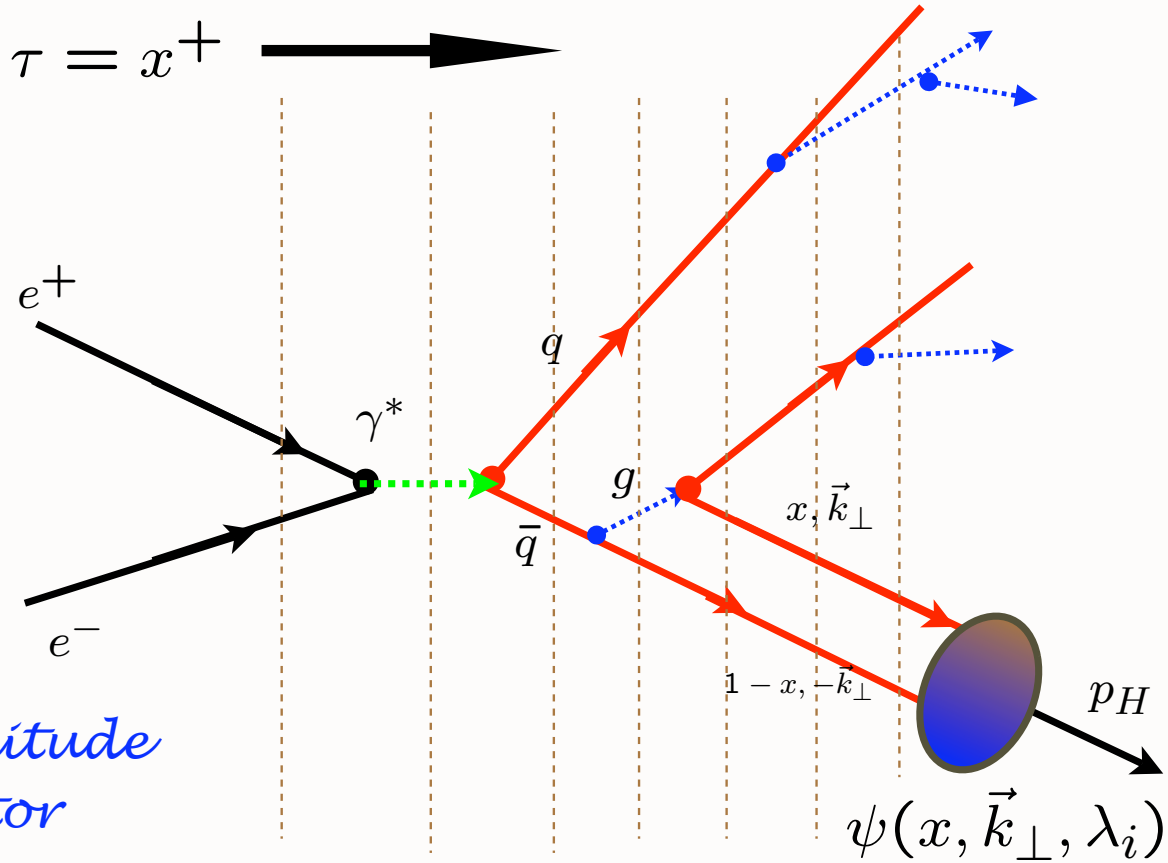
- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$\boxed{z = \sqrt{\frac{x}{1-x}} \, |\sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}|}$$

Hadronization at the Amplitude Level

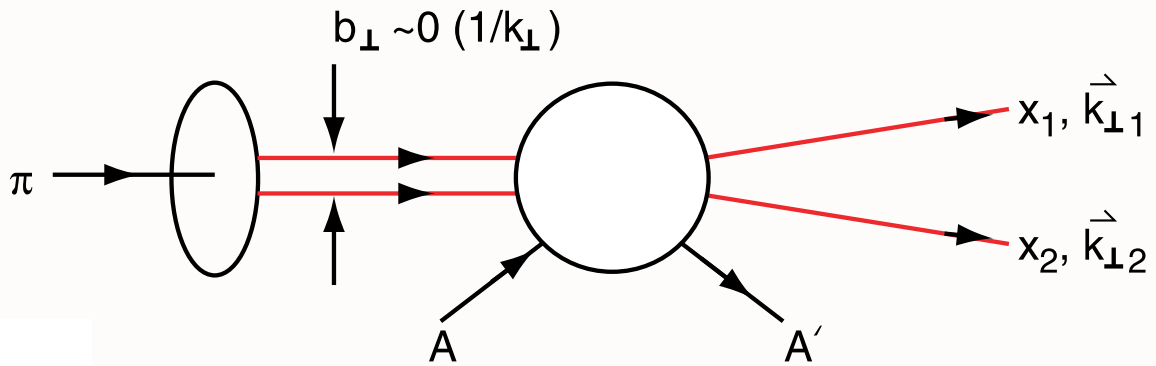


Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



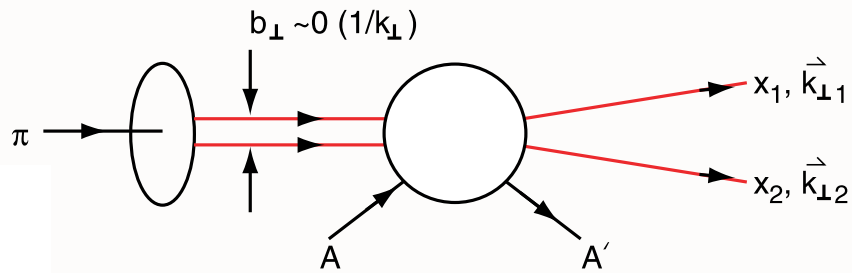
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

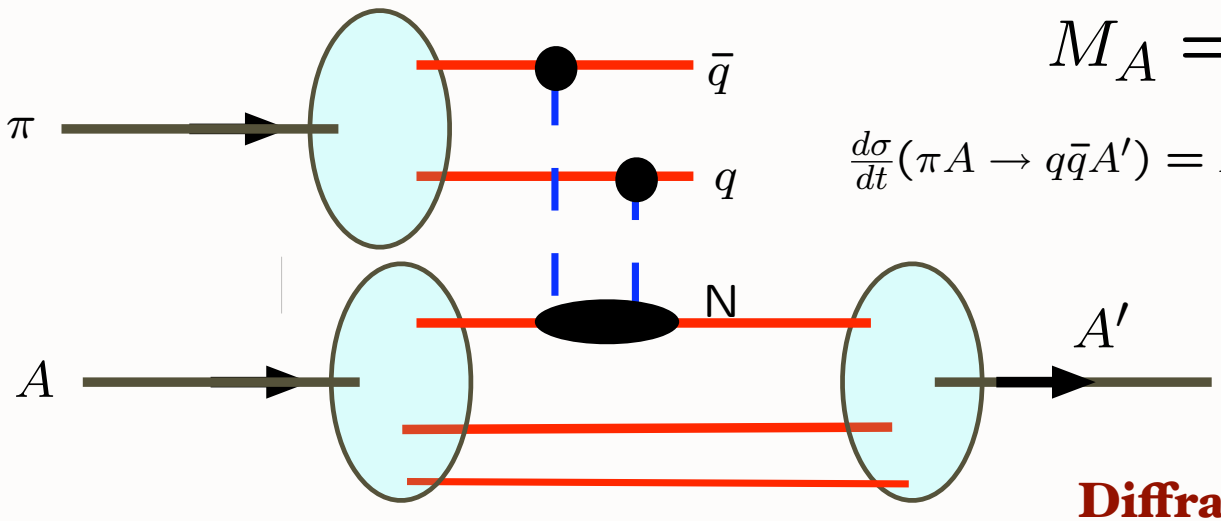
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed;
interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A \; M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2 \; \frac{d\sigma}{dt}(\pi N \to q\bar{q}N') \; F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

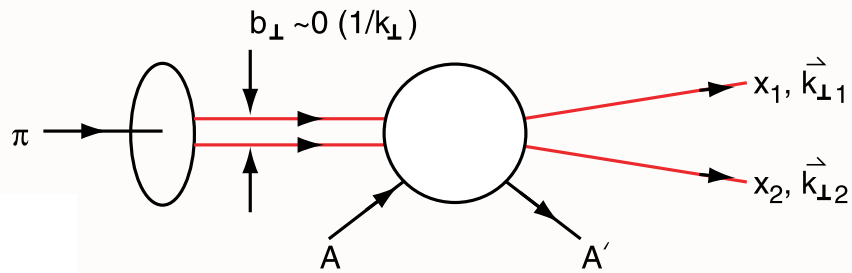
Ashery E791

α (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out !

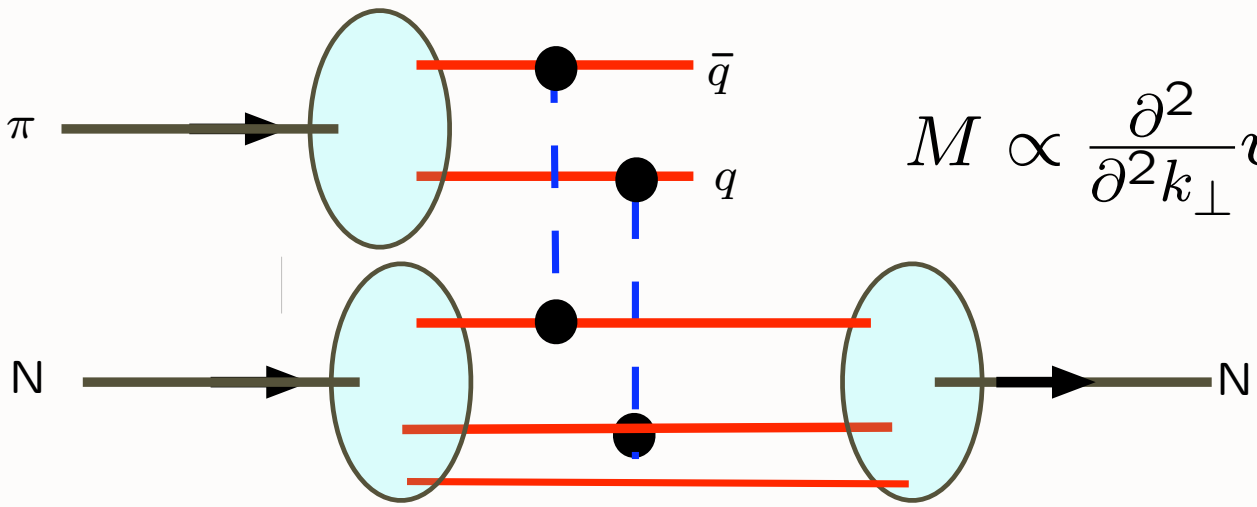
Factor of 7

Key Ingredients in Ashery Experiment



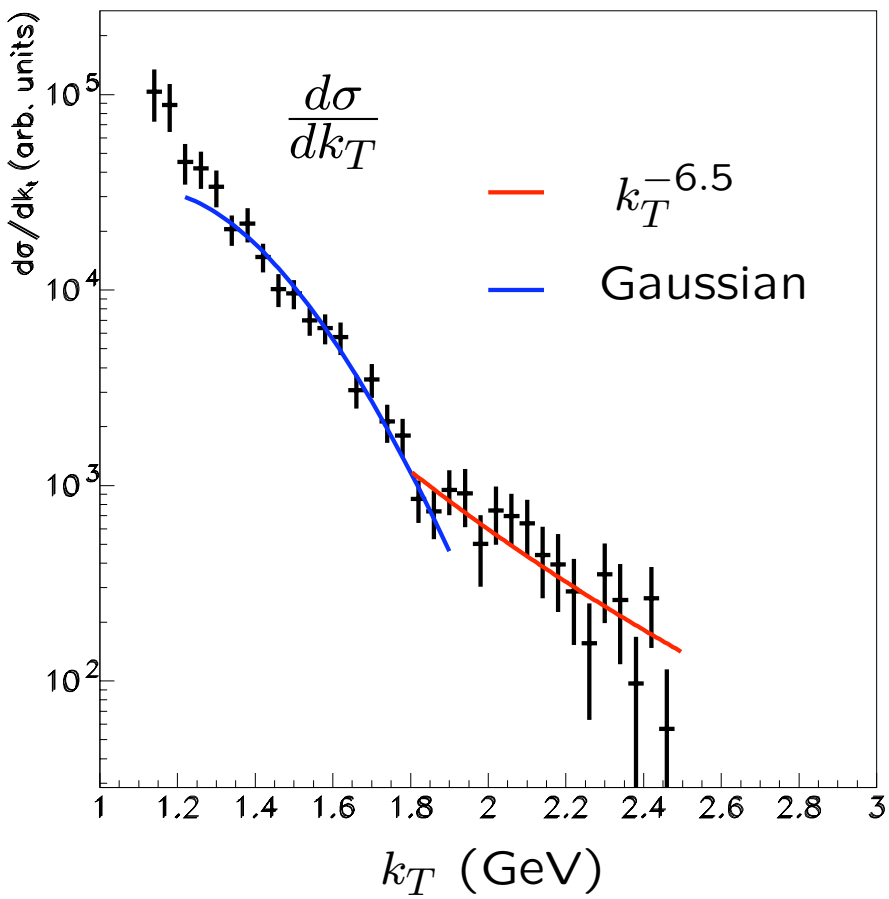
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution

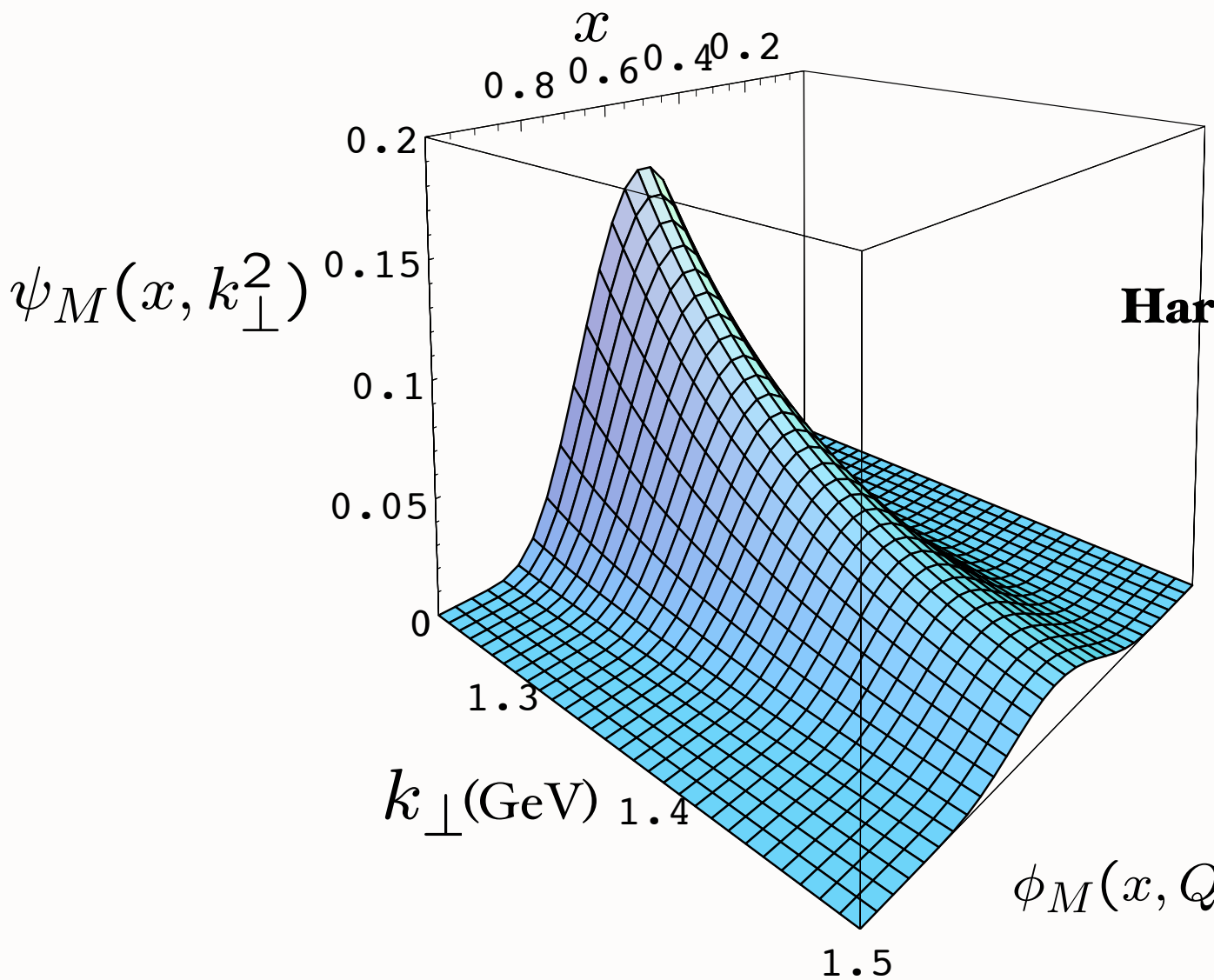


Two Components

High Transverse momentum dependence consistent with PQCD, ERBL Evolution $k_T^{-6.5}$

Gaussian component similar to AdS/CFT HO LFWF

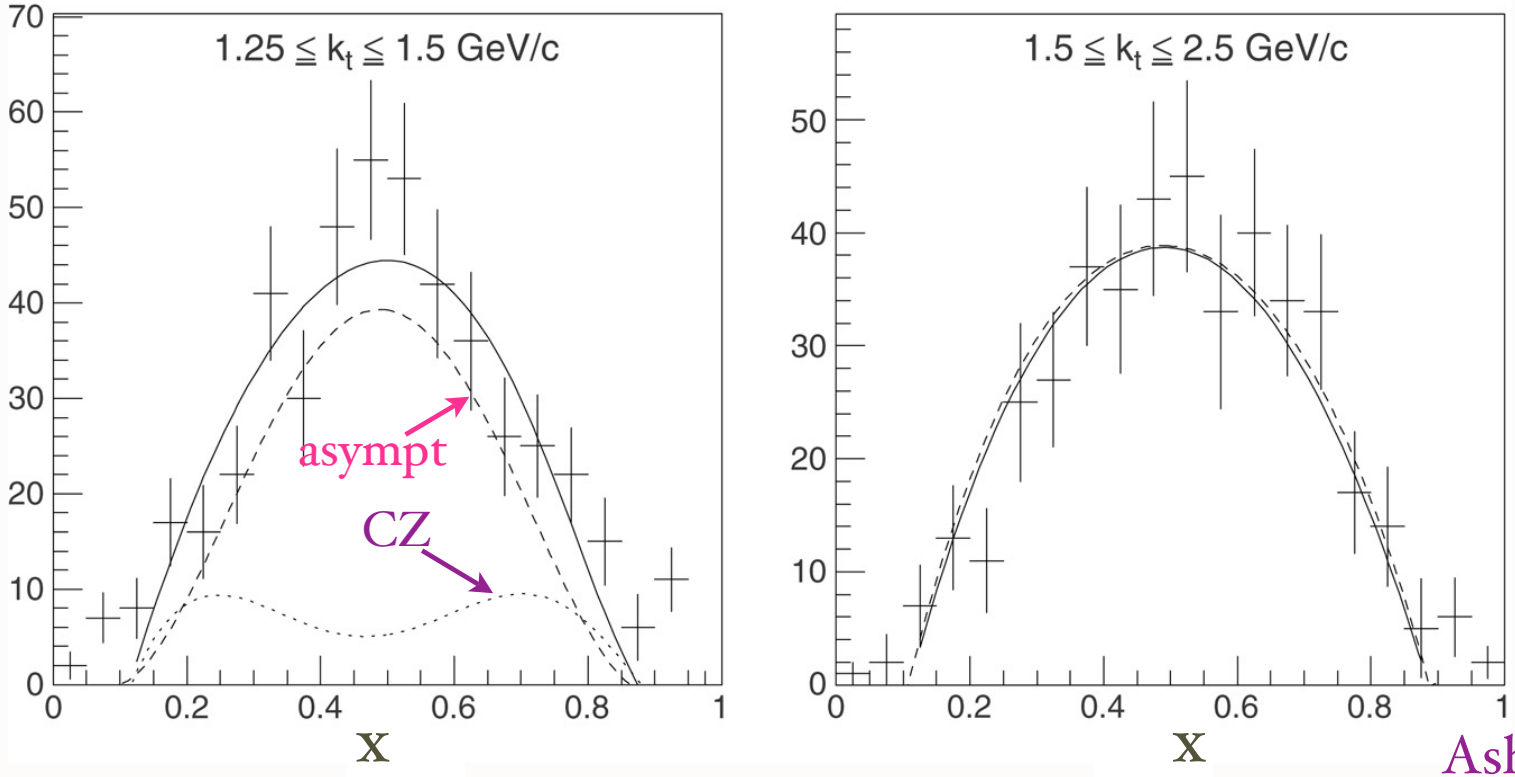
Prediction from AdS/CFT: Meson LFWF



Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$



Ashery E791

Narrowing of x distribution at higher jet transverse momentum

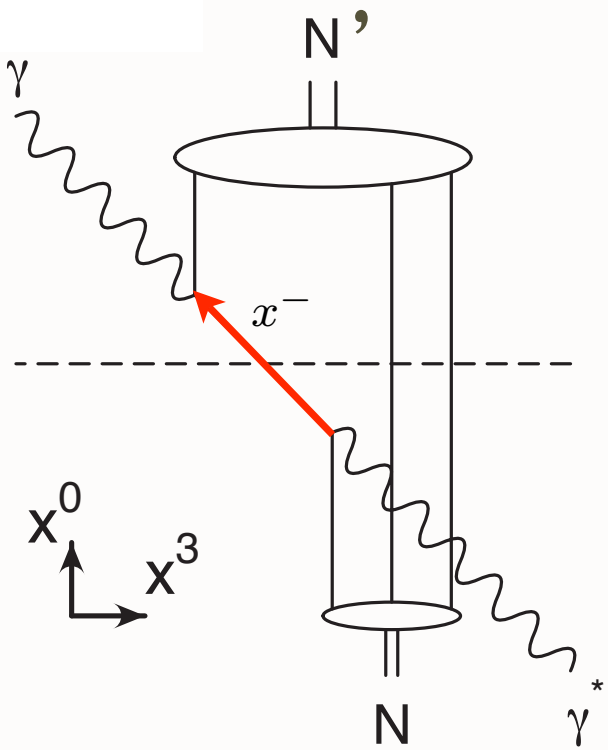
x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

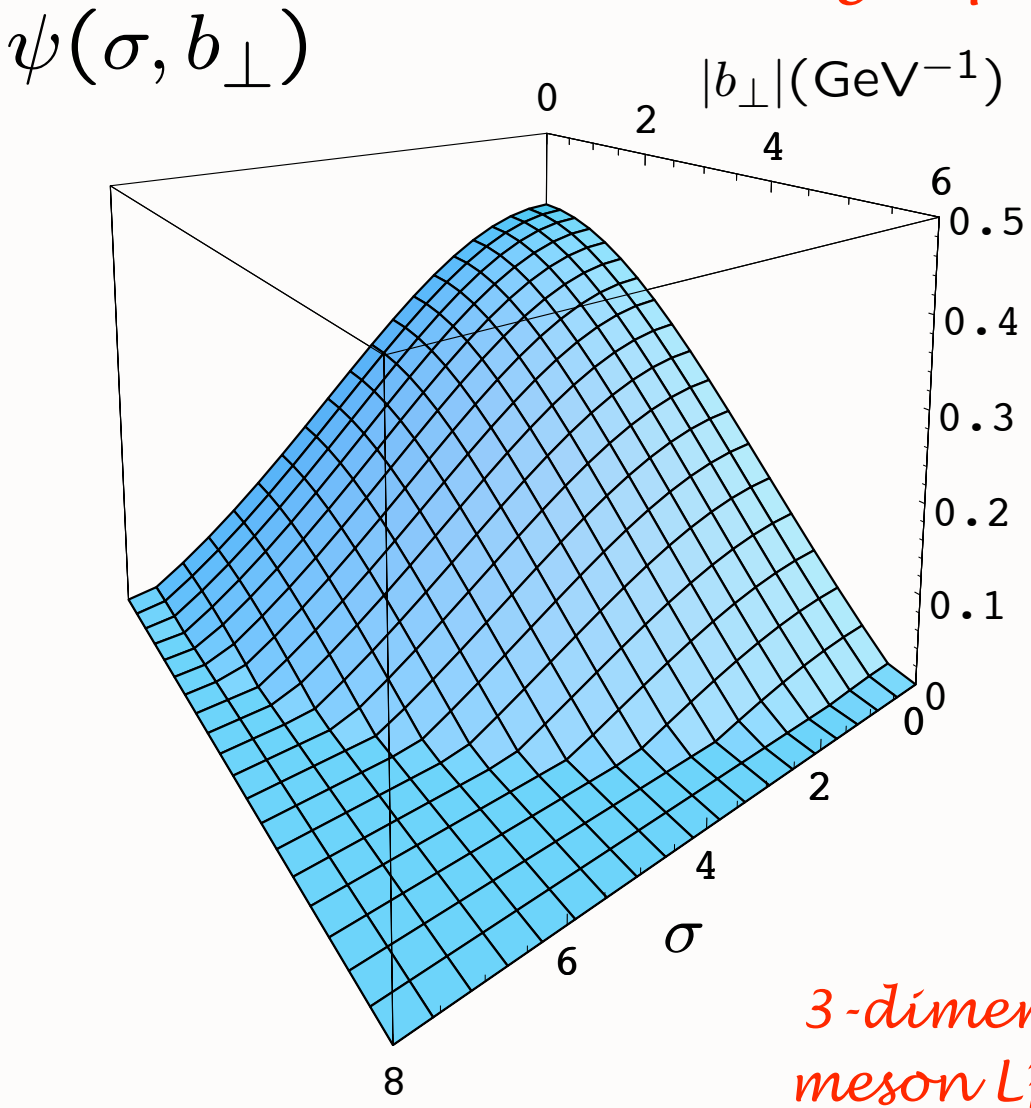
Measure x- distribution from DVCS:
Take Fourier transform of skewness,
the longitudinal momentum transfer

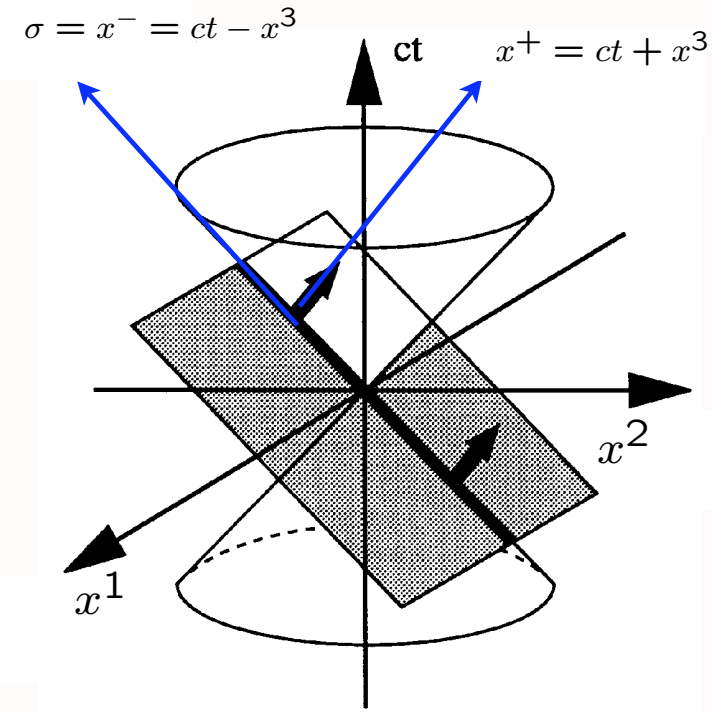
$$\zeta = \frac{Q^2}{2p \cdot q}$$

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

AdS/CFT Holographic Model

G. de Teramond
SJB





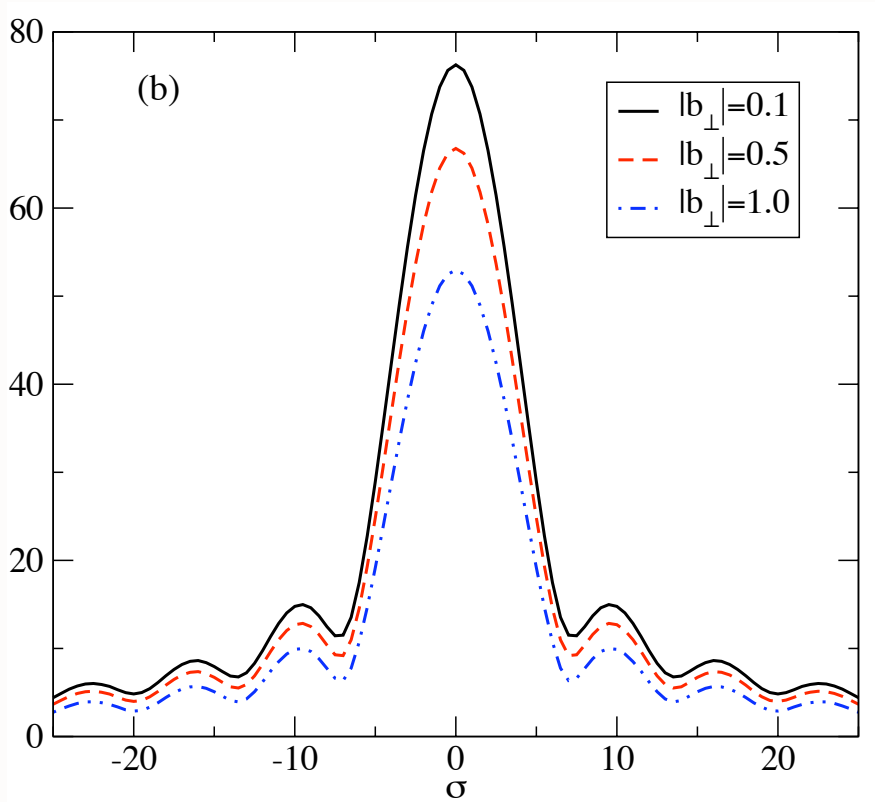
The front form

3-dimensional photograph:
meson LFWF at fixed LF Time

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi}\int d\zeta e^{i\sigma\zeta}\tilde{A}(b_\perp, \zeta) \qquad \sigma = \tfrac{1}{2}x^- P^+ \qquad \zeta = \frac{Q^2}{2p\cdot q}$$

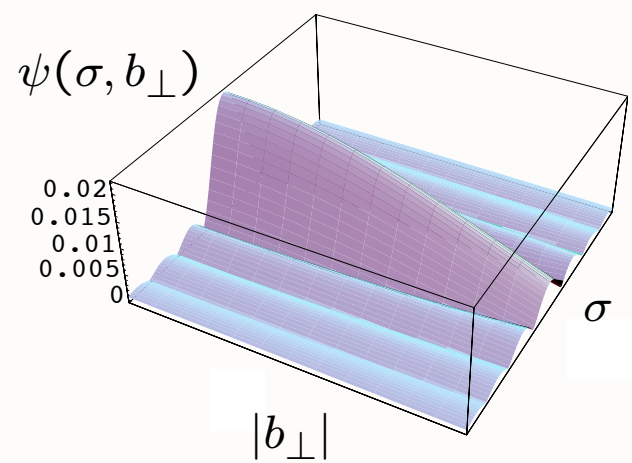


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$



Hadron Dynamics at the Amplitude Level

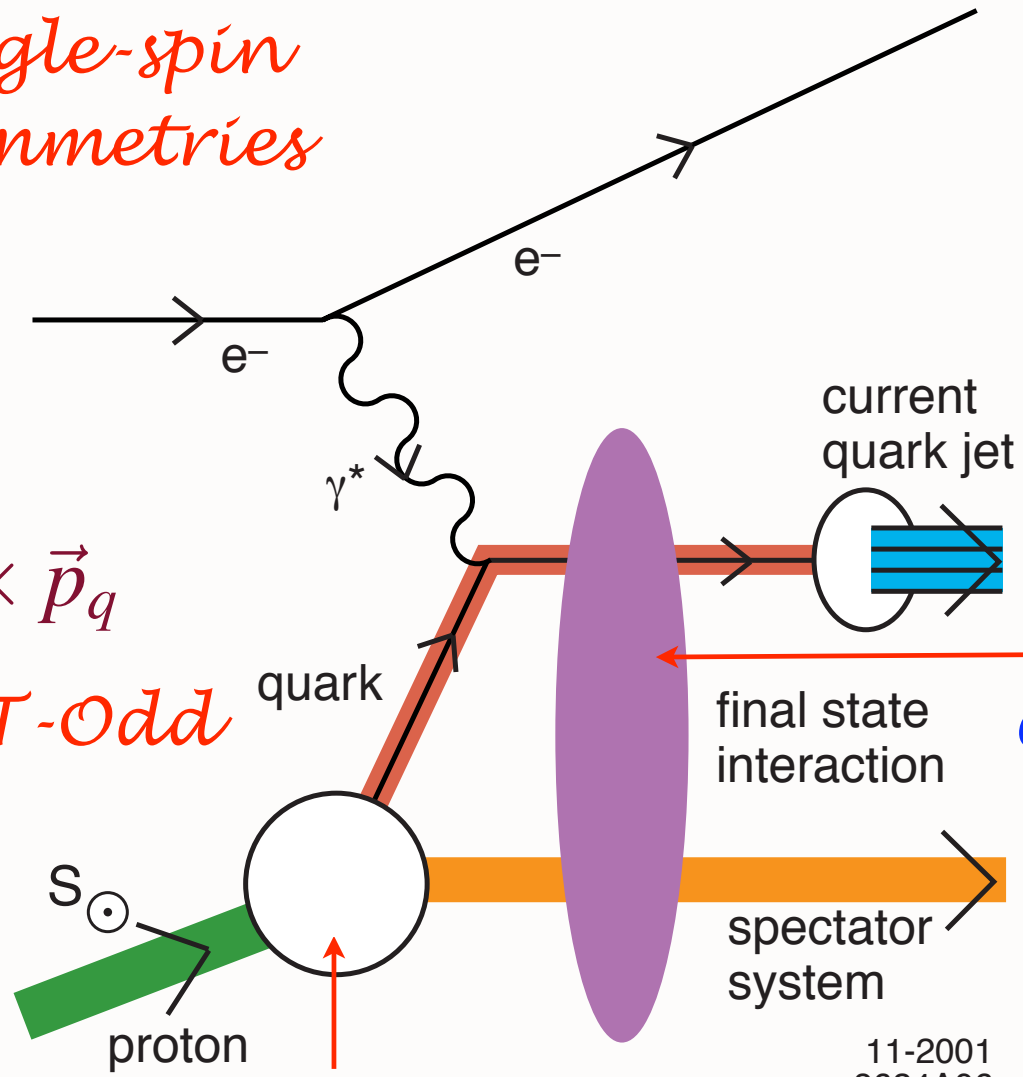
- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Leading Twist Sivers Effect

Single-spin asymmetries

$$i\,\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd



QCD S- and P- Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

Features of Light-Front Formalism

- *Hidden Color* Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- *Intrinsic glue, sea quarks, intrinsic charm*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

$$\nu = L$$

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^\nu_{LF}\phi_\nu = \mathcal{M}^2_\nu\phi_\nu \quad \textbf{Bilinear} \quad H^\nu_{LF} = \Pi^\dagger_\nu \Pi_\nu,$$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $$b_\nu^\dagger = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad b_\nu^\dagger b_\nu = b_{\nu+1} b_{\nu+1}^\dagger$$

Ladder Operator $$b_\nu^\dagger|\nu\rangle = c_\nu|\nu+1\rangle$$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

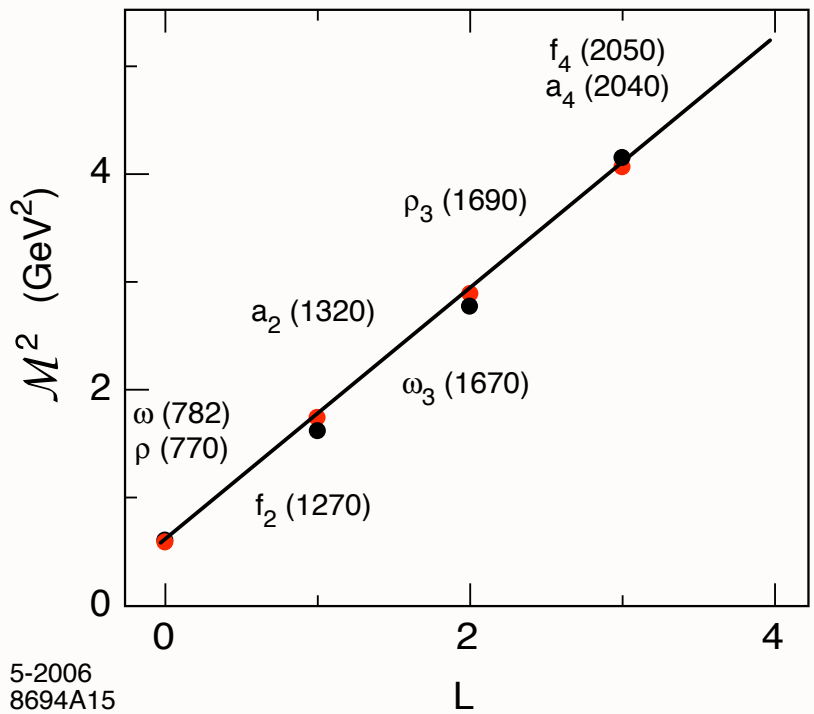
defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

Subtract Vacuum Energy

$$\mathcal{M}^2 \to \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$

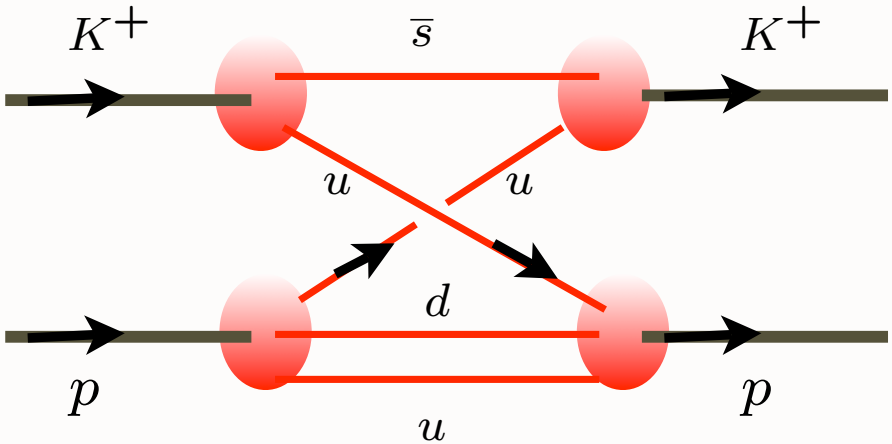


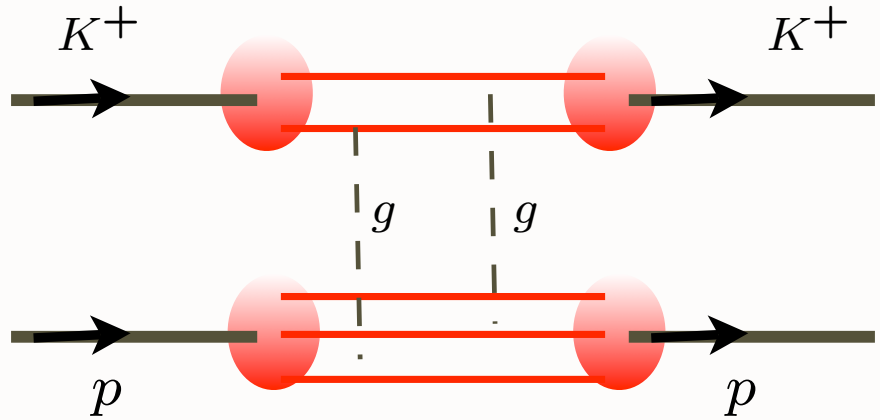
$J = L + 1$ vector meson Regge trajectory for $\kappa \simeq 0.54$ GeV

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange
(Spin exchange in atom-atom scattering)

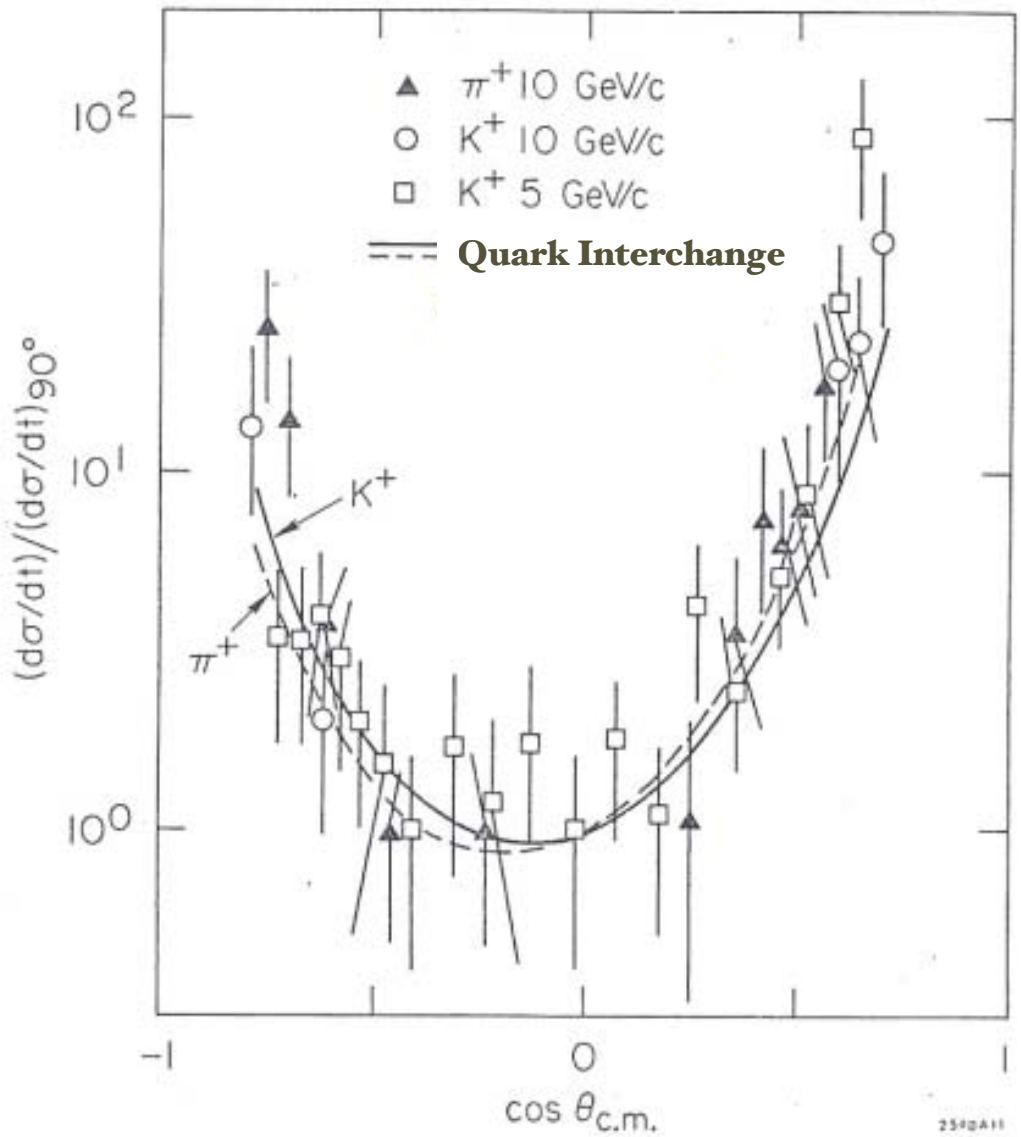
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \; \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

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and

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(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

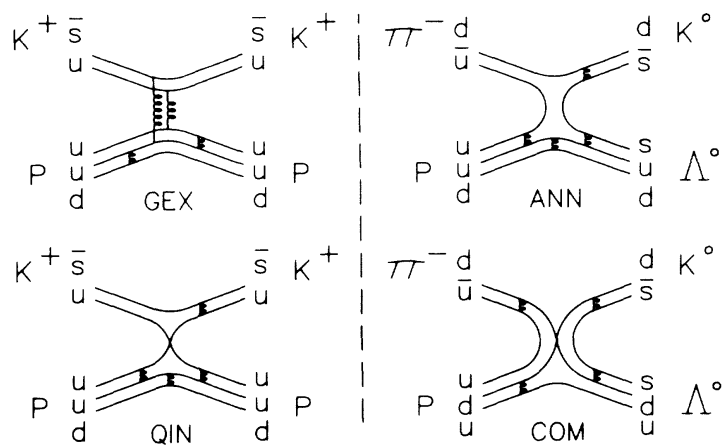
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^0 K^0, \Sigma^0 K^0$,

$p^{\pm}p \to pp^{\pm}$.



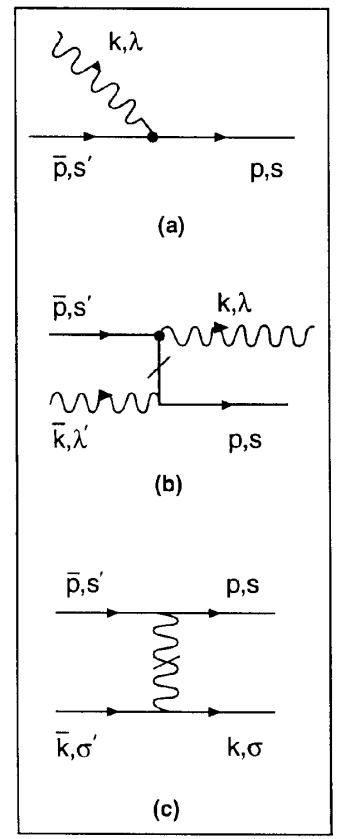
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

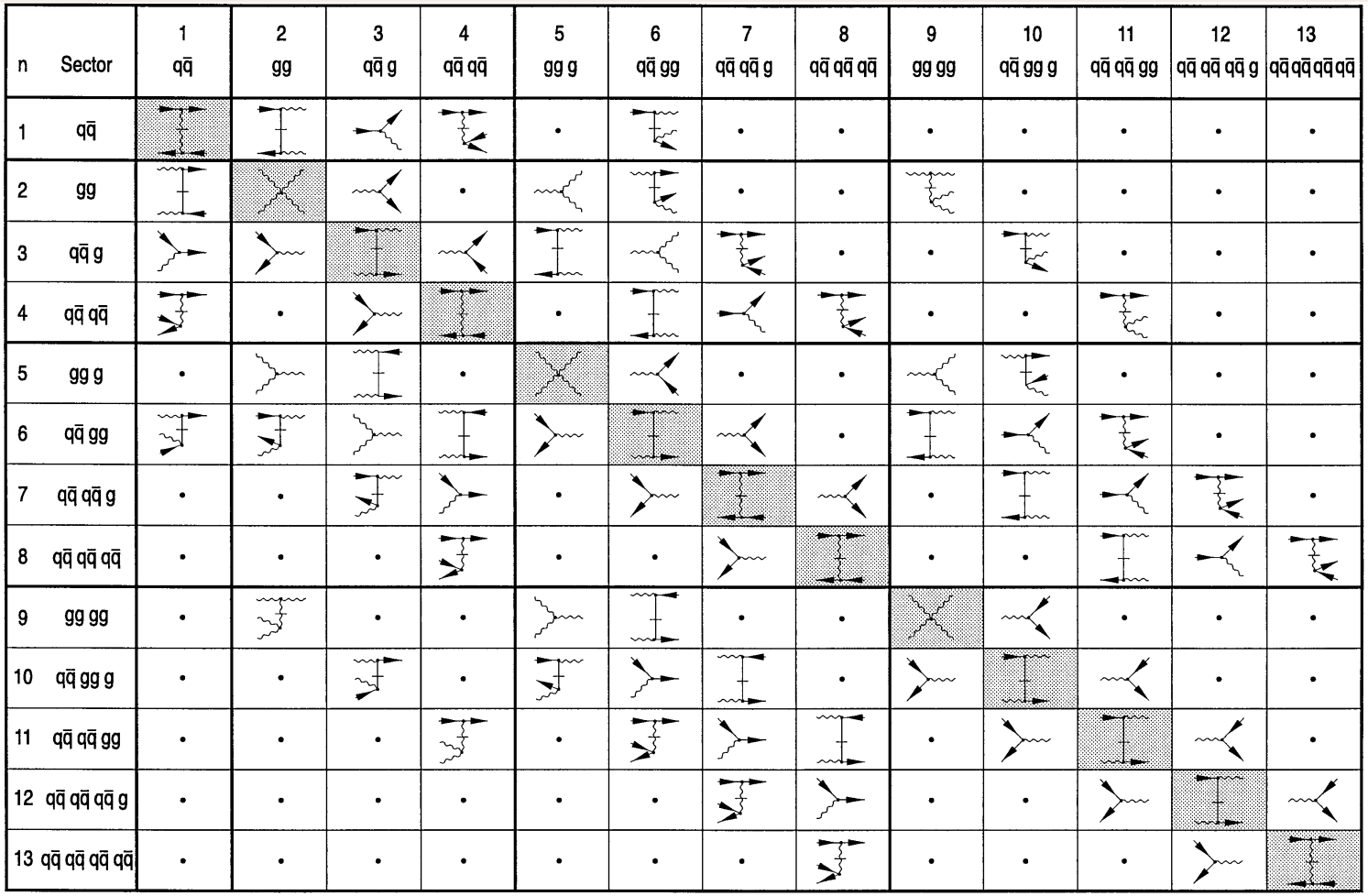
- Good initial approximant
- Better than plane wave basis — Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion — Vary, Harinandrath, sjb
- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ





Use AdS/QCD basis functions

Pauli, Pinsky, sjb

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

AdS/QCD

- New initial approximation to QCD based on conformal invariance, and confinement
- Underlying principle: Conformal Window
- AdS5: Mathematical representation of conformal gauge theory
- Systematically improve using DLCQ
- Successes: Hadron spectra, LFWFs, dynamics
- QCD at the Amplitude Level

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:
  Polchinski and Strassler, hep-th/0109174.
- Deep inelastic structure functions at small x:
  Polchinski and Strassler, hep-th/0209211.
- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:
  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.
- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:
  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

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- Kent Hornbostel

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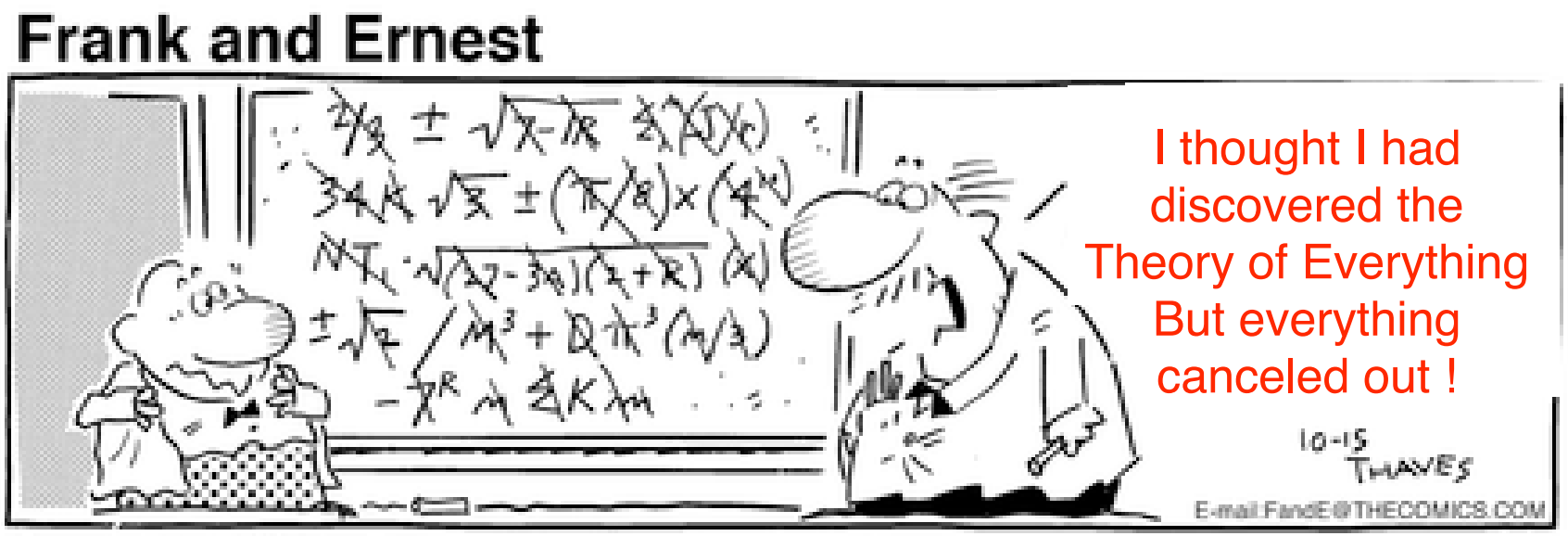
A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.



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