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The Basis Problem in Many-Worlds Theories *

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Abstract

It is argued that a many-worlds interpretation of quantum theory exists only to the extent that the associated basis problem is solved. The core basis problem is that the robust enduring states specified by environmental decoherence effects are essentially Gaussian wave packets that form continua of non-orthogonal states. Hence they are not a discrete set of orthogonal basis states to which finite probabilities can be assigned by the usual rules. The natural way to get an orthogonal basis without going outside the Schroedinger dynamics is to use the eigenstates of the reduced density matrix, and this idea is the basis of some recent attempts by many-worlds proponents to solve the basis problem. But these eigenstates do not enjoy the locality and quasi-classicality properties of the states defined by environmental decoherence effects, and hence are not satisfactory preferred basis states. This core problem is obscured by approaches that treat the universe as a quantum computer, but it needs to be addressed and resolved before a many-worlds-type interpretation can be said to exist.

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1. Introduction.

The basis problem of the many-worlds interpretation can be efficiently introduced by quoting a few exerpts from the entry "Everett's Relative-State Formulation of Quantum Mechanics" in the Stanford Encyclopedia of Philosophy. That entry is available online[1], and elaborates on the points summarized in the following passages:

> Everett's relative-state formulation of quantum mechanics is an attempt to solve the measurement problem by dropping the collapse dynamics from the standard von Neumann-Dirac theory of quantum mechanics. The main problem with Everett's theory is that it is not at all clear how it is supposed to work. In particular, while it is clear that he wanted to explain why we get determinate measurement results in the context of his theory, it is unclear how he intended to do this. There have been many attempts to reconstruct Everett's no-collapse theory in order to account for the apparent determinateness of measurement outcomes. These attempts have led to such formulations of quantum mechanics as the many-worlds, many-minds, many-histories, and relative-fact theories. Each of these captures part of what Everett claimed for his theory, but each also encounters problems.
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> ...
>
> Everett's no-collapse formulation of quantum mechanics was a reaction to problems that arise in the standard von Neumann-Dirac collapse theory. Everett's proposal was to drop the collapse postulate from the standard theory, then deduce the empirical predictions of the standard theory as the subjective experiences of observers who are themselves treated as physical systems described by his theory. It is, however, unclear precisely how Everett intended for this to work. Consequently, there have been many, mutually incompatible, attempts at trying to explain what

he in fact had in mind.

. . .

This is what Everett says: "We shall be able to introduce into [the relative-state theory] systems which represent observers. Such systems can be conceived as automatically functioning machines (servomechanisms) possessing recording devices (memory) and which are capable of responding to their environment. The behavior of these observers shall always be treated within the framework of wave mechanics. Furthermore, we shall deduce the probabilistic assertions of Process 1 [rule 4b] as *subjective* appearances to such observers, thus placing the theory in correspondence with experience. We are then led to the novel situation in which the formal theory is objectively continuous and causal, while subjectively discontinuous and probabilistic. While this point of view thus shall ultimately justify our use of the statistical assertions of the orthodox view, it enables us to do so in a logically consistent manner, allowing for the existence of other observers."

. . .

Everett's goal then was to show that the memory records of an observer as described by his no-collapse theory would match those predicted by the standard theory with the collapse dynamics. The main problem in understanding what Everett had in mind is in figuring out how this correspondence between the predictions of the two theories was supposed to work.

. . .

Many Worlds

DeWitt's many-worlds interpretation is easily the most popular reading of Everett. On this theory there is one world corresponding to each term in the expansion of **E** when written in the preferred basis (there are always many ways one might write the quantum-mechanical state of a system as the sum of vectors in

the Hilbert space; in choosing a preferred basis, one chooses a single set of vectors that can be used to represent a state and thus one chooses a single *preferred* way of representing a state as the sum of vectors in the Hilbert space). The theory's preferred basis is chosen so that each term in the expansion of **E** describes a world where there is a determinate measurement record. Given the preferred basis (surreptitiously) chosen above, **E** describes two worlds: one where J (or perhaps better $J1$) determinately records the measurement result "spin up" and another where J (or $J2$) determinately records "spin down".

DeWitt and Graham describe their reading of Everett as follows: "[Everett's interpretation of quantum mechanics] denies the existence of a separate classical realm and asserts that it makes sense to talk about a state vector for the whole universe. This state vector never collapses and hence reality as a whole is rigorously deterministic. This reality, which is described *jointly* by the dynamical variables and the state vector, is not the reality we customarily think of, but is a reality composed of many worlds. By virtue of the temporal development of the dynamical variables the state vector decomposes naturally into orthogonal vectors, reflecting a continual splitting of the universe into a multitude of mutually unobservable but equally real worlds, in each of which every good measurement has yielded a definite result and in most of which the familiar statistical quantum laws hold."

...

Perhaps more serious, in order to explain our determinate measurement records, the theory requires one to choose a preferred basis so that observers have determinate records (or better, determinate *experiences*) in each term of the quantum-mechanical state as expressed in this basis.

These quotes from Barrett's 1998 article in the Stanford Encyclopedia of Philosophy emphasize the central importance of specifying a basis, and it gives many references to fill in and back up the description it gives of the contemporary situation. However, that review does not mention David Deutsch's ambitious and detailed 1985 attempt[2] to solve the basis problem. That omission is justified by the fact that Deutsch's approach was found by Sara Foster and Harvey Brown[3] to be marred by many logical flaws. But in spite of these flaws, it will be useful to describe Deutsch's 1985 attempt in order to get an idea of the problems involved in the construction of a many-worlds interpretation.

2. Deutsch's 1985 Proposed Solution of the Many-Worlds Basis Problem.

Deutsch clearly recognized that the central problem in creating a many-worlds interpretation is the problem of defining the preferred basis. He calls it the 'interpretation basis', and his forty-page paper is devoted largely to task of defining it. Indeed, the other main problem, which is to understand how a relative probabilility comes to be associated with each of the alternative experiences, *all of which actually occur*, is more of a philosophical problem than a mathematical one, as it can be "talked away" by adopting a 'propensity' approach to probabilities, or by positing the existence of a large or infinite collection of parallel (or almost parallel) quasi-classical universes that are distributed according to the usual probability formula among the separate worlds, or *branches*, of the full quantum reality represented by the evolving state of the universe. The relative probabilities associated with apparently-alternative possible outcomes are then explained in terms of the ratios of the numbers of parallel universes in which the alternative possible outcomes occur.

Solving the basis problem, on the other hand, demands mathematically detailed rules that specify (1), when the full reality splits into these distinct worlds, or branches, (2), what these separate parts are, and (3), which variables associated with these different worlds, or branches, acquire, or possess,

definite values in each such world, or branch. Deutch recognized the three aspects of the problem, and tried to provide well defined answers to each.

Of course, every interpretation of quantum theory that goes beyond the pragmatic Copenhagen approach of merely providing a set of rules the connect human experiences must provide analogous rules. But the mathematical requirements on the structure of these rules is far more stringent for a many-world theory because such a theory allows no collapse, and everything must be determined exclusively in terms of the one basic reality, which is the one quantum state of the universe that evolves in accordance with the Schroedinger equation: no other process is allowed. But as Foster and Brown emphasize, in *collapse* theories the specification of these rules "need only be within the context of *some* theory, not necessarily quantum theory." The encompassing theory could allow, for example, as von Neumann's theory does, experiential realities to enter dynamically into the determination of the basis, instead of being mere consequences of some choice of basis that is itself wholly determinded by the unitary evolution of the quantum state in accordance with the Schroedinger equation.

Deutsch recognized and stressed the critical importance of this basis problem, and made a determined effort to solved it:

"The remedy which I propose in section 7 is that an extra rule be appended to the quantum formalism, specifying how the interpretational basis depends at each instant of time on the physical state and dynamical evolution of a quantum system."

The first problem that he must resolve is what he calls "the notorious difficulty of specifying the preferred instants t ... when a measurement is completed." These instants must be defined solely in terms of (1), the single evolving state of the universe, which evolves always via the Schroedinger equation, and (2), the law that governs this continuous unitary evolution.

The second problem is that at each of these distinct "instants of splitting" the unified actual universe must be decomposed in a specified way into two systems, the 'observed system' and 'observing system'. Hence the rule that

defines how the unified universe is to be split at that instant into these two special parts must be specified in terms solely of the state vector itself and its law of evolution.

The third problem is to define for each instant a set of evolving basis vectors so that the state of the universe can at every instant be decomposed in a unique way into a sum of component vectors that can be associated with different worlds, or branches of reality, that possess definite values for the quantities "measured" at each instant of splitting.

Deutsch solves the problem of dividing the unified physical world into the well defined physical parts that will ultimately acquire 'possessed values' by restricting his considerations to a model world that has only a *finite* set of states, and dividing this set of states in some way into subsystems each having a 'prime number' of states in each subsystem. One evident problem with that approach is the essential role played by this finiteness assumption and the 'prime factor' rule: Deutsch does not indicate how this 'prime number rule', which is purely kinematic (it does not involve the equations of motion) will carry over to real systems, in which every particle carries with it an infinite number of states, and in particular to the systems that carry the stored memories associated with our experiences. These systems are presumably made of many particles, and are strongly dependent on the dynamics.

Once the separation of the universe into the observed and observing systems is made, Deutsch specifies the instant at which the splitting occurs to be essentially the instant at which the interaction between observed and observing system vanishes. But the ubiquitous tails of real systems would make that condition difficult to satisfy in the physical universe.

Once the instants of splittings are fixed, and the separation of the universe at each of these instants into the observed and observing system is specified, one needs to define, for *every* instant of time, the preferred basis. This basis defines the decomposition of the quantum state of the universe into a unique sum of vectors each of which is parallel to one of the vectors in the preferred set of basis vectors. Each of these particular component vec-

tors is supposed to correspond to a separate world, or branch of the universe. The decomposition must be such that if a probability is assigned to each such world in accordance with usual rule, which equates it to the square of the length of the corresponding vector, then the expectation values of certain special quantities, namely those represented by *operators whose eigenvectors are these basis vectors*, are required to agree with the general quantum mechanical rule for expectation values. This is how Deutsch proposed to endow the evolving non-collapsing quantum universe with some definite outcomes in a way concordant with the quantum probability rules: each individual branch would have 'possessed values' appropriate to that branch, and the branch would be given the appropriate statistical weight. But the entire construction depends crucially on the idea that a particular well-defined set of preferred basis states is specified by the evolving non-collapsing quantum state of the universe.

Deutsch tried to deduce the structure of this preferred basis set by imposing the requirement of no-faster-than-light-influence, but his argument is, as was correctly noted by Foster and Brown, logically flawed. The primary condition on a many-world interpretation is that there be no actual collapse: the *values* associated with the preferred basis vectors are supposed to be "possessed" by the systems, or to exist within nature, without there being any collapse that actually eliminates the cross terms. However, the crucial equation (Eq. 50) that gives the structure of the preferred basis does not follow from Deutsch's argument, insofar as the no collapse condition is maintained. The result *would* follow if the appropriate collapse were to occur. That is, Deutsch's proof would be valid if he were trying to define the preferred basis in the context of a collapse-type theory, but it is not valid in the many-worlds context. This error is not just a small slip-up that can by fixed by a minor correction. Foster and Brown emphasize this problem, and also problems with the existence and uniqueness of solutions to the conditions that Deutsch proposes. Michael Lockwood reports in his 1989 book *Mind, Brain, and the Quantum,* (p. 236) that Deutsch "has failed so far to find any

plausible way to meet Foster and Brown's objections." Lockwood's own solution brings in "Mind" in an important way, and is thus not a "Many-Worlds Interpretation" in the DeWitt-Deutsch sense of this term.

3. The Core Basis Problem

This section identifies the core basis problem.

The first point is that if the universe has been evolving since the big bang in accordance with the Schroedinger equation, then it must by now be an amorphous mass in which every device is a smeared-out cloud of a continuum of different possibilities. Indeed, the planet earth would not have a well defined location, nor would the rivers and oceans, nor the cities built on their banks. Each particle would, due to the uncertainty principle, be by now smeared out continuously over immumerable positions.

However, there is a countervailing process. Each subsystem is interacting with all of the particles of its environment, and these interactions produce an "environmental decoherence" effect that has been extensively examined [4]. One of the effects of this interaction is to convert what could otherwise be a wave packet that can extend coherently over a large distance, in various dimensions, into what is essentially a *mixture* of narrow gaussian wave packets. The mixture is, essentially, a sum over a *continuum* of displaced narrow Gaussians. The basis problem arises basically from the fact that these Gaussians *overlap*: they are not orthogonal.

More specifically, the effect of environmental decoherence is to reduce a typical state to a collection of non-interfering states of the form

$$\Psi_x(x') = N \exp -a^2(x' - x)^2, \tag{2.1}$$

where N is a normalization constant, and the subscript x, which labels the center-point, or peak, of the Gaussian wave packet, ranges over a *continuum* of values, and none of these states is orthogonal to any other. I am, to make the idea clear, exhibiting only one single continuum variable, x. But all position-type variables, and also all angular-type variables, would be smeared

out in a similar way. Thus the preferred axis of a Stern-Gerlach apparatus would not be well defined. Nor would the location of a Geiger counter.

In order to become relevant to human experience the location of the detection device, and also the outcome, must become recorded in some memory device. But this memory device, including eventually the brain of the human observer, is also in a smeared-out continuum of non-orthogonal states. So the core problem is to assign probabilities to the states of such a smeared-out continuum of overlapping non-orthogonal states. The normal procedure at this point is to invoke some outside process to specify a *denumerable set* of discrete possibilities associated with a discrete set of orthogonal projection operators P_i that satisfy $P_iP_j = \delta_{ij}P_i$, and sum to unity. Then a finite probability is assigned to each of these projection operators P_i by the normal rules of quantum theory, and these probabilities sum to unity.

The natural candidate for the discrete set of operators associated with the continuum of states (2.1) are the eigenstates of the reduced density matrix associated with that set. That is, in fact, the choice made by Deutsch in his 1985 paper. However, that choice is not felicitous. One wants states that are highly localized, like a classical state. But the eigenstates of the reduced density matrix associated with the states (2.1) are spread out over large distances, rather than being compressed into small regions, like the states (2.1) themselves are. Taking the eigenstates of the reduced density matrix goes in the wrong direction. Because this natural choice is unsatisfactory one is left with the problem of how the quantities, or operators, that have well defined values and probabilities are to be determined by the amorphous evolving state of the universe itself, without any appeal to an outside process. That is the core basis problem.

4. Deutsch's Multiverse Proposal

Recently Deutsch has put forward a new many-worlds concept labelled the "multiverse". In the popular account given in his book *The Fabric of Reality* the multiverse would appear to be a collection of classical-type universes each one similar to the single quasi-classical world introduced by David

Bohm in his famous pilot-wave model. But that cannot be Deutsch's real meaning because Bohm's classical-type universe is governed, in part, by non-local forces, and a key virtue of the many-worlds approach, often emphasized by Deutsch, is that it does not involve or entail any kind of faster-than-light transfer of information.

In his recent paper *The Structure of the Multiverse* [5] Deutsch gives a more detailed glimpse into his thinking, and he emphasizes at the outset the existence of the basis problem, and the demand for no faster-than-light transfer of information. Deutsch's new proposal involves parallel quasi-classical aspects of the quantum universe that persist for finite temporal intervals but can later *recombine* to produce quantum interference effects. This idea is much closer to standard ideas than Bohm's model, in which the individual quasi-classical world persists for all time, and quantum interference effects are explained by special nonlocal forces, not by a merging of previously distinct quasi-classical components of the evolving state of the universe.

In his new work Deutsch again emphasizes the basis problem, and I think it is fair to say that a many-worlds interpretation is not defined until it solves in principle the basis problem. Indeed, the basis problem is *the* problem that any interpretation of quantum theory must resolve in some way. Thus the central idea of the Copenhagen interpretation was to imbed the quantum system in a larger system that specifies the preferred basis by bringing in "measuring devices" that are set in place by a classically conceived process. In von Neumann's formulation there is the infamous "Process I", which likewise lies outside the process governed by the Schroedinger equation. The challenge that must be met by a satisfactory many-worlds interpretation is to solve this basis problem within quantum theory itself, without bringing in any outside dynamical process.

Deutsch's program is to create a satisfactory many-worlds-type theory by considering nature to be a quantum computer. Deutsch cites Feynman's paper[6] on quantum mechanical computers. In this paper Feynman says:

> Of course, there are external things involved in making the measurements and determining all this [setting up the computer], which are not parts of our computer. Surely a computer has eventually to be in interaction with the external world, both for putting in data and for taking it out.

Here Feynman is following the orthodox idea that a computer is a quantum system, and as such must be considered imbedded in a larger world in which we prepare initial conditions, including the structures of all devices, and then extract outcomes. Deutsch's problem is, accordingly, to show how by considering the whole universe to be a quantum computer one can solve this heretofore unresolved interpretational problem that has beset those trying to create a satisfactory many-worlds theory.

To emphasize the problems that others have encountered I quote some passsages from Zurek's 1998 review[7] of the status of attempts to use environmental decoherence (abetted, if helpful, by coherent histories) to solve the interpretational problem.

> It will be emphasized that while significant progress has been made ... much remains to be done on several fronts which all have implications on the overarching question of interpretation. We can mention two such open questions right away: both the formulation of the measurement problem and its resolution through appeal to decoherence require a universe split into two systems. Yet it is far from clear how one can define systems given [only] an overall Hilbert space 'of everything' and the total Hamiltonian. Moreover, while the paramount importance of information has been recognized, I do not believe it is as yet sufficiently thoroughly understood. Thus, while what follows is perhaps the most complete discussion of the interpretation implied by decoherence, it is still only a report of partial progress.
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> ...

> The advantages of Everett's original vision was to reinstate quantum theory as a key tool in search of its own interpretation. The disadvantages (which were realized only some years later, after the original proposal became more widely known) include (i) the ambiguity in what constitutes the branches ... (ii) re-emergence of the questions about the origin of probabilities ... (iii) it was never clear how to reconcile unique experiences of observers with the multitude of alternatives present in the MWI wave function.
> ...
> The interpretation based on ideas of decoherence and [environmentally induced] einselection has not really been spelled out to date in any detail. I have made a few half-hearted attempts in this direction, but, frankly, I was hoping to postpone this task, since the ultimate questions tend to involve such 'anthropic' attributes of the 'observership' as 'perception', 'awareness' or 'consciousness',
> ...
> Einselection chooses a preferred basis ...İt will often turn out that it is overcomplete. Its states may not be orthogonal, and, hence, they could never follow from the diagonalization of the density matrix.

These quotations show that Zurek recognizes the core problem that I have been describing.

How then does Deutsch deal with this basic problem, which is that a quantum universe evolving via the Schroedinger equation does not pick out a *discrete set* of preferred directions in Hilbert space that specify quasi-classical properties that can be assigned definite "possessed values" with well defined probabilities.

His approach is first to define, in terms of his quantum-computer formulation of quantum theory, certain mathematical requirements for a portion of

the full quantum computer to be 'behaving classically'. These requirements depend upon certain directions in Hilbert space being singled out as the ones that correspond to 'possessed values'. Those special directions are not specified beforehand, but are defined by the dynamics, in the sense that *if* with some choice of these directions a certain system behaves classically then these special directions are picked out by that classicality condition itself. Thus the preferred basis is defined dynamically, by a classicality condition.

This is eminently reasonable, as far as it goes. By it does not come to grips with the central problem, which is that in the full amorphous many-worlds universe there will be neighboring environmentally robust configurations in which the Geiger counters are infinitesimally displaced, and hence multiple continua of *non-orthogonal equally-preferred states*, and hence no singled-out discrete set of preferred basis states to which finite probabilities are ascribed by the normal quantum rules.

5. Zurek's Argument

Deutsch's multiverse theory is in general accord with Zurek's approach[7], and in the end Deutsch appeals to Zurek's arguments to support the idea that the 'classically behaving' aspects of the quantum universe will explain the 'classical world' in which we live and act. However, Zurek's theory uses computer concepts, and likewise fails to come to grips with the continuum problem. Zurek's treatment is wrapped up with his treatment of probabilities. In this connection he says:

> A true 'frequency' with a classical interpretation cannot be defined at a level which does not allow 'events'—quantum evolution which leads to objectively existing states—to be associated with the individual members of that ensemble. ... The reduced density matrix ρ, which emerges following the interaction with the environment, and a partial trace will always be diagonal in the *same* basis of einselected pointer states $\{|i>\}$. These states help define elementary 'events'. Probabilities of such events can be

> inferred from their coefficients in ρ, which have the desired 'Born rule' form.

Zurek's discussions of 'existing' values and probabilities rests on this model, in which the reduced density matrix is diagonal in the *same* states that specify the values that are supposed to 'exist'. But that model ignores the core difficulty, which is emphasized elsewhere in his paper by statements such as:

> In that case [which in fact arises in the quantum field theoretic treatment of the measurement process] the states which are most immune to decoherence in the long run turn out to be ... Gaussians. ... Hence coherent states [which are Gaussians] are favored by decoherence. ... Einselection chooses a preferred basis in the Hilbert space in recognition of its predictability. That basis will be determined by the dynamics of the open system in the presence of environmental monitoring. It will often turn out that it is over complete. [This is precisely the case for the Gaussian's mentioned above.] Its states may not be orthogonal, and, hence, they would never follow from diagonalization of the density matrix.

Thus Zurek, while clearly recognizing the problem elsewhere, does not actually solve the core basis problem, which is the fact that in a quantum field theory context the 'preferred' states picked out by environmental decoherence belong to a *continuum of non-orthogonal states*. Hence they *cannot* be the eigenstates of the reduced density matrix. (*If* they were *then* they would define the required set of orthogonal projection operators P_i that sum to unity.) Consequently, Zurek's argument, like that of Deutsch, does not show how the Schroedinger dynamics, by itself, picks out a preferred set of orthogonal projection operators that sum to unity. No substitute for this orthodox way of specifying, in a realistic relativistic quantum field-theoretic context, which properties have well defined values, and what probabilities

are assigned to these values, has yet been spelled out by proponents of the many-worlds approach. Thus, as Zurek himself emphasizes, from a technical standpoint no many-worlds interpretation yet exists.

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