**Physics 198, Spring Semester 1999** Helmuth Spieler
**Introduction to Radiation Detectors and Electronics**

Problem Set 8: Due on Tuesday, 30-Mar-99 at begin of lecture.
Discussion on Wednesday, 31-Mar-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

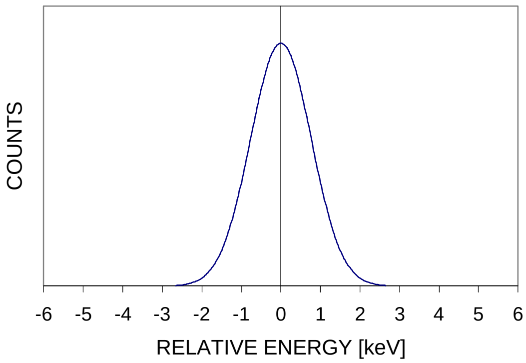
1. A Ge diode gamma-ray detector system has a resolution of 1.8 keV FWHM at a peaking time of 2 μs. The desired energy range is 2 MeV.

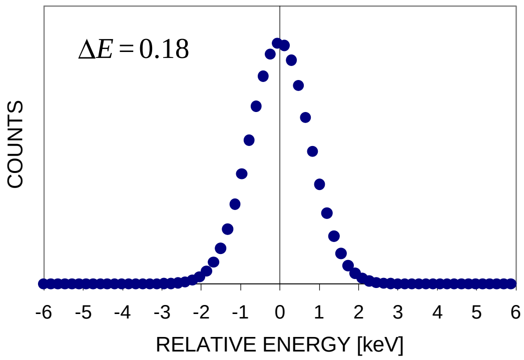
   a) What ADC resolution is required?

   The overall resolution should be limited by the detector resolution and the electronic noise level, as both are limited by rather fundamental constraints. The digitization resolution should be chosen to be so fine that it does not affect the measurement accuracy.

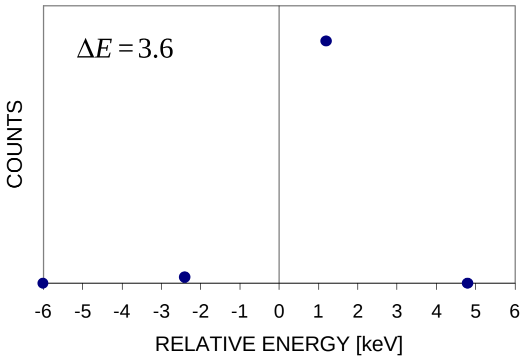
   The following plots show the continuous Gaussian signal distribution at the output of the analog signal processing chain and digitized spectra for various bin (channel) widths Δ*E* in keV.

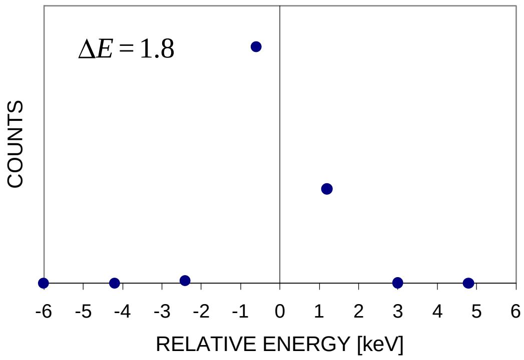
   Choosing a channel width of 0.1 x FWHM (ΔE= 0.18) allows very good reconstruction of the line shape, even without mathematical fitting.

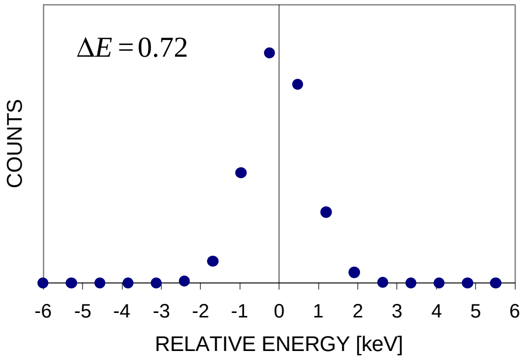


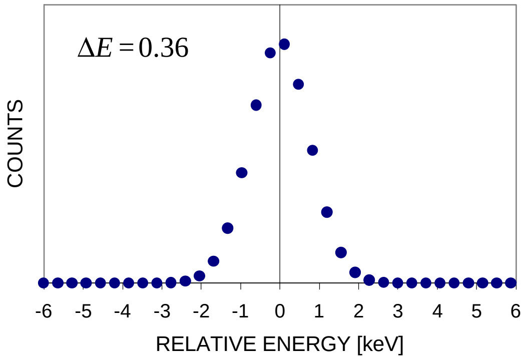


   At the other extreme, channel widths of 2 x FWHM (ΔE= 3.6) or 1 x FWHM (ΔE= 1.8) require careful fitting and good statistics for accurate centroid finding.









Accommodating 2.5 channels in the linewidth (ΔE= 0.72) will allow good curve fitting, if the line shape is known and no additional low intensity peaks are present. Five digitizing channels within a linewidth (FWHM) allow robust peak reconstruction and centroid determination, although more may be desirable if adjacent low-intensity lines are to be resolved (depends on counting statistics).

Let's adopt 5 channels for 1.8 keV FWHM, or 360 eV per bin. For a full-scale range of 2 MeV, this requires 5555 channels. The next highest binary number is $2^{13}$ = 8192 bins, which corresponds to 244 eV/bin. A 12 bit digitizer (4096 channels) would yield 3.7 channels within the half-width and may also be a reasonable choice.

b) If a straight-line fit of the form

$$E = k \cdot N_{ch} + E_0$$

is to provide adequate energy calibration, what is the allowable integral non-linearity?

It is quite straightforward to determine the centroid position to better than 1/10 of the linewidth, so 1.8 keV FWHM allows centroid determination to better than 180 eV. The integral non-linearity should not exceed this value, which corresponds to 180/244 = 0.7 bins. Achieving this level of accuracy over the full range is very difficult, so a more sophisticated fit function may be necessary.

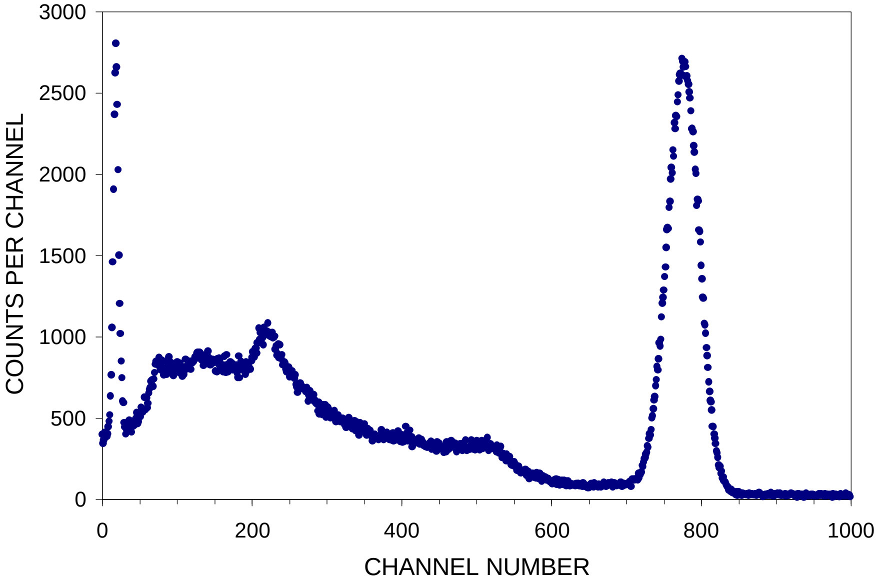
c) A Wilkinson ADC with a 400 MHz clock is used for the measurement. What is the conversion time for the 1.33 MeV line of $^{60}$Co?

A clock frequency of 400 MHz corresponds to 2.5 ns per channel, so the conversion time is $T = N_{ch}$ x 2.5 [ns]. In general, if the energy increment per channel is $\Delta E$, then the time required to convert an energy $E$ is

$$T_{conv} = \frac{E}{\Delta E} T_{clock}$$

For $\Delta E$= 244 eV per channel as calculated in a), $E$= 1.33 MeV and $T_{clock}$= 2.5 ns, the conversion time is 13.6 μs. The maximum conversion time for the full 8192 channel range (2 MeV) is 20.5 μs.

2. A NaI(Tl) scintillator shows the following spectrum when exposed to a $^{137}$Cs source.



a) A Wilkinson ADC with a 100 MHz clock was used for the measurement. The shaper was set to a peaking time of 0.5 µs. The total input counting rate was $5 \times 10^4$ s$^{-1}$. What was the dead time?

The dead time per event is the sum of the peaking time $t_p$ and the conversion time $T_{conv}$ . For events occurring at a rate $R$, the fraction of time when the system cannot process a new event is $R \times (t_p + T_{conv})$. Since a spectral measurement always involves multiple measurements within some time interval, the fraction of time when the system cannot accept a new signal is referred to as dead time. Only for small dead times does the digitized rate increases proportionally to the signal rate, so as the dead time approaches 50% , increasing the event rate hardly reduces the measurement time required to achieve a given statistical accuracy.

For a sequence of events of varying pulse height (energy), the average dead time is determined by the spectrum distribution, as the digitizing time in a Wilkinson ADC is proportional to pulse height. To determine the average conversion time one can approximate the spectrum by splitting it into two parts.

The most striking feature is the peak at channel 770. It can be approximated by a triangle with a height of 2700 counts and a base width of 100 channels, so the total number of counts in the peak is (100 ch x 2700 counts)/2= 135000 counts. To estimate the conversion time we can represent the peak by 135000 counts in channel 770.

The continuum below the dominant peak together with the low energy x-ray peak can be approximated by a rectangle with a height of 600 counts and a base of 700 channels, so this is equivalent to 420000 counts in channel 350.

Thus, the total number of counts in the spectrum is 555000, of which 24% can be assigned to channel 770 and 76% to channel 350. Correspondingly, the total rate of 5 x $10^4$ $s^{-1}$ splits into about 12000 $s^{-1}$ in channel 770 and 38000 $s^{-1}$ in channel 350.

The corresponding conversion times are (0.5 μs + 10 ns x 770) = 8.2 μs and (0.5 μs + 10 ns x 350) = 4 μs. At a rate of 38000 $s^{-1}$ in channel 350, the converter is dead for ( 4 μs x 38000 $s^{-1}$ ) = 0.15 of the time and for the high-energy peak the relative dead time is ( 8.2 μs x 12000 $s^{-1}$ ) = 0.10, so for the total rate the system is dead 25% of the time.

b) The spectrum shown above was taken with more ADC resolution than needed. What is the minimum conversion range (number of channels full-scale) required for this spectrum? How many bits are required for the data word?

The required resolution depends on the peaks that are to be measured. The low-energy x-ray peak has about 9 ch FWHM, so half the ADC resolution would be adequate. The 662 keV peak has about 40 ch FWHM, so 1/8 the digitizing resolution would do. Since this is a 10 bit ADC (1024 conversion range), either a 512 ch range (for the low-energy peak) or 128 ch range (for the high-energy peak) would be sufficient. These ranges correspond to 9 and 7 bits, respectively.

c) What is the dead time for the conversion range determined in b)? What is the maximum counting rate that will not increase the dead time beyond the value determined in a)?

Using the 128 channel range reduces the basic ADC conversion time by 1/8, so the corresponding conversion times are (0.5 μs + 10 ns x 350/8) = 0.9 μs and (0.5 μs + 10 ns x 770/8) = 1.5 μs. For the low-energy portion the converter is dead for ( 0.9 μs x 38000 $s^{-1}$ ) = 0.034 of the time and for the high-energy peak the relative dead time is ( 1.5 μs x 12000 $s^{-1}$ ) = 0.018, so for the total rate the system is dead 5.2% of the time. The rate can be increased roughly five-fold to 2.4 x $10^5$ $s^{-1}$, while maintaining a 25% dead time.

3. A precision time-of-flight system has a time resolution of 20 ps rms. The maximum flight time to be measured is 100 ns.

a) What is the required resolution of the time digitizer (TDC)? Keep in mind that the number of measurement bins corresponding to an *n* bit data word is $2^n$.

Again assuming 5 bins within full width half-maximum, 20 ps rms corresponds to about 50 ps FWHM, so 10 ps per bin is appropriate. For a full scale range of 100 ns this requires $10^4$ bins. The next highest binary number is 16384, or 14 bits (13 bits may also suffice).

b) The time digitizer uses a capacitor charging scheme coupled with a Wilkinson ADC. The start pulse switches on a current source that charges up a memory capacitor of 100 pF. The stop pulse switches off the current source, establishing a voltage on the capacitor that is proportional to $t_{stop} - t_{start}$. If a stop pulse does not occur within the TDC range, internal reset circuitry discharges the capacitor and allows acceptance of a new start pulse after 0.5 µs. Both start and stop inputs are disabled until the reset or conversion sequences are complete.

Full scale corresponds to a voltage of 5 V on the capacitor. What is the charging current?

To charge a capacitor $C$ to a voltage $V$ in a time $\Delta T$ requires a charge $I\,\Delta T = CV$, so for $C$ = 100 pF, $V$ = 5 V and $\Delta T$ = 100 ns, $I$ = 5 mA.

c) The Wilkinson ADC that measures the capacitor voltage uses a 50 MHz clock. What is the maximum conversion time and the required discharge current?

For 16384 channels the maximum conversion time is 20 ns x 16384= 328 µs.

The current required to discharge the 100 pF capacitor form 5 V to 0 in 328 µs is 1.5 µA.

d) The system is used at an accelerator that provides very sharp (~ ps) beam pulses at a bunch frequency of 5 MHz. Beam particles impinge on a target. A detector placed at a known distance from the target determines the time-of-arrival of the reaction products. The accelerator control system provides a timing pulse synchronized with the particle bunches for use as a time reference. How do you connect the detector and beam pulses to the start and stop inputs of the time digitizer?

Since not all beam pulses will necessarily lead to a stop signal from the detector (depending on beam current, reaction cross section, detector solid angle, …), using the beam signal to start the converter will lead to excessive dead time. Each start pulse incurs at least 0.5 µs dead time for reset, so starting the TDC at a 5 MHz rate, i.e. at a period of 200 ns, means that only every 3rd start pulse could accept a stop pulse, i.e. the dead time would be 50% from resets alone.

Instead, the converter should be started with the lowest rate in the system, which comes from the detector. Specifically, the "stop" detector starts the TDC and the subsequent beam pulse stops it. This leads to an inverted time spectrum $T_{TOF}$ = 100 ns – $T_{TDC}$, but since the maximum time of flight is less than the beam period, the measurement is unambiguous.