Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 12: Due on Tuesday, 27-Apr-99 at begin of lecture.
Discussion on Wednesday, 28-Apr-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. A detector with a capacitance of 10 pF is used with a JFET preamplifier. The detector and FET are cooled, so the detector and gate current noise are negligible. Assume that the electronic noise is due only to the input JFET. The pulse shaper has a peaking time of 1 µs (assume $F_i = F_v = 1$).

   a) The input JFET has a transconductance of 5 mS (mA/V) and an input capacitance of 5 pF. What is the equivalent noise charge (expressed in electrons)?

   Since the noise current sources are negligible, the equivalent noise charge

   $$Q_n^2 = i_n^2 \, T_s \, F_i + C_i^2 \, v_n^2 \, \frac{F_v}{T_s} \approx C_i^2 \, v_n^2 \, \frac{F_v}{T_s}$$

   C_i, F_v and T_s are given. The spectral density of the FET's equivalent input noise voltage is

   $$v_n^2 = 4 k_B T_0 \frac{\gamma_n}{g_m}$$

   Assume $\gamma_n = 1$, so for $T_0 = 300$ K and $g_m = 5$ mS, $v_n^2 = 3.3 \times 10^{-18}$ V^2/Hz.

   The total input capacitance is the sum of the detector capacitance and the input capacitance of the FET $C_{det} + C_{FET} = 15$ pF, so the equivalent noise charge

   $$Q_n = C_i v_n \sqrt{\frac{F_v}{T_s}} = 15 \cdot 10^{-12} \times 1.8 \cdot 10^{-9} \sqrt{\frac{1}{10^{-6}}} = 2.7 \cdot 10^{-17} \text{ As} = 171 \text{ el}$$

   b) The input transistor is replaced by another device, also with 5 mS transconductance, but with an input capacitance of 10 pF. What is the equivalent noise charge?

   The calculation is the same as in a), except that the total capacitance at the input is 20 pF, so the equivalent noise charge is 228 el.

   c) The JFET used in b) provides capacitive matching, yet the noise charge is higher than in a). Why?

   The two transistors have the same transconductance, but different input capacitances. Since the g_m / C ratio is inferior for the second FET, simple capacitive matching is not valid, as it merely increases the capacitance without reducing the noise voltage.

2. Two identical amplifiers with a voltage gain of 10 are cascaded (connected in series) to provide high gain. Both amplifiers have an input referred noise of 10 μV, integrated over their full bandwidth. What is the input referred noise of the amplifier cascade?

   At the output of the first amplifier the input noise is amplified 10-fold, so 100 μV are passed to the second amplifier, which in turn amplifies it 10-fold to 1000 μV. Correspondingly, the input noise of the second amplifier appears at its output as 100 μV, which adds in quadrature to the noise of the first amplifier, yielding

$$v_{no} = \sqrt{(v_{ni1} \cdot A_{v1} \cdot A_{v2})^2 + (v_{ni2} \cdot A_{v2})^2} = 1005\,\mu\text{V}$$

   To refer the output noise back to the input we divide by the cumulative gain of the cascaded amplifiers, so v_{ni} = 1005/100 = 10.05 μV. If the gain of the first amplifier is sufficiently large, the noise of the second amplifier is insignificant.

   A simpler way to calculate the effect is to refer the input noise of the second amplifier to the input of the first amplifier, so

$$v_{ni} = \sqrt{v_{ni1}^2 + \left(\frac{v_{ni2}}{A_{v1}}\right)^2}$$

   Additionally, the bandwidth of the amplifier cascade is reduced

$$\frac{1}{\Delta f_n} = \sqrt{\frac{1}{\Delta f_{n1}} + \frac{1}{\Delta f_{n2}}}$$

   Since the two amplifiers have the same bandwidth, the overall bandwidth is $1/\sqrt{2}$ smaller than for the single amplifier. However, since the noise voltage scales with the square root of bandwidth, the noise is only reduced by $2^{-1/4}$ from 10.05 to 8.5 mV.

3. A bipolar transistor amplifier is connected to a detector with 10 pF capacitance. Again, the detector is cooled, so its current noise is negligible. The transistor has a current gain of 150, constant over the current range of interest, and its input capacitance is 1 pF. The shaper uses simple CR-RC filtering with $\tau_i = \tau_d$.

   a) What is the minimum obtainable noise charge (expressed in electrons)?

      From the lecture notes

$$Q_{n,\min} = 772\left[\frac{el}{\sqrt{pF}}\right] \cdot \frac{\sqrt{C_{tot}}}{\sqrt[4]{\beta_{DC}}}$$

      For C_{tot} = (10 + 1) pF and β_{DC} = 150, Q_n = 732 el.

b) At what current must the transistor be operated to obtain the minimum noise at a peaking time of 1 μs?

$$I_c = 26\left[\frac{\mu A \cdot ns}{pF}\right] \cdot \frac{C_{tot}}{\tau}\sqrt{\beta_{DC}}$$

so for τ = 1 μs (in a CR-RC shaper the peaking time is equal to the time constant τ), the required collector current I_C = 3.5 μA.

c) At a peaking time of 10 ns, what is the optimum noise and the required operating current?

The optimum noise is the same as calculated in a) – as it is independent of time constant. The required current, however, scales inversely with shaping time, so I_C = 350 μA.

d) If you accept a 10% higher noise level, what is the minimum operating current at 10 ns shaping time?

The equivalent noise charge vs. current is

$$Q_n^2 = 2q_e \frac{I_C}{\beta_{DC}} F_i T_S + \frac{2(k_B T)^2}{q_e I_C} C_{tot}^2 \frac{F_v}{T_S}$$

Increasing the minimum noise of 732 el by 10% yields 805 el. In a brute-force solution one could solve the quadratic equation for the current, but it is simpler to consider that in the noise optimum the two terms of the above equation are equal. If we change the current, the first term changes by a factor k and the second by $1/k$, so if the noise is to change by a factor f

$$(fQ_n)^2 = k\frac{Q_n^2}{2} + \frac{1}{k}\frac{Q_n^2}{2}$$

which yields the quadratic equation

$$k^2 - 2f^2 k + 1 = 0$$

with the solutions

$$k = f^2 \pm \sqrt{f^4 - 1}$$

For f = 1.1 the solution corresponding to the minimum current is k = 0.53, so the current can be reduced from 350 μA to 185 μA.

e) The system is exposed to a hadron beam. Radiation damage in the transistor reduces the DC current gain to 50. At 10 ns peaking time and the current determined in d), what is the noise?

$$Q_n^2 = 2q_e \frac{I_C}{\beta_{DC}} F_i T_S + \frac{2(k_B T)^2}{q_e I_C} C_{tot}^2 \frac{F_v}{T_S}$$

At I_C = 185 μA the noise increases from 805 to 969 el, i.e. by 20%.

f) What are the optimum operating current after irradiation and the associated noise?

$$Q_{n,\min} = 772 \left[\frac{el}{\sqrt{pF}}\right] \cdot \frac{\sqrt{C_{tot}}}{\sqrt[4]{\beta_{DC}}}$$

For C_{tot} = 11 pF and β_{DC} = 50, Q_n = 963 el, which is practically the same as the noise level obtained in e).

The corresponding optimum current

$$I_c = 26 \left[\frac{\mu A \cdot ns}{pF}\right] \cdot \frac{C_{tot}}{\tau} \sqrt{\beta_{DC}} = 202\ \mu\mathrm{A}$$

As shown in d), accepting a slight increase in noise allows a substantial decrease in operating current. With radiation damage, the optimum current decreases, so the degradation in noise is still only 20%, even when running below the optimum current after irradiation.

The degradation in DC current gain used in this example corresponds to about 10 years of LHC operation at $r_\perp \approx 30$ cm from the interaction region, illustrating that good noise levels are achievable at low power, even after radiation damage.