Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 11: Due on Tuesday, 20-Apr-99 at begin of lecture.
Discussion on Wednesday, 21-Apr-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. Consider a spectroscopy system whose resolution is determined by electronic noise.

   a) Find a quiet place and clearly recite 100 times "Noise contributions add in quadrature."

   b) The current noise contribution is 120 eV and the voltage noise contribution is 160 eV. What is the total noise?

$$Q_n = \sqrt{Q_{ni}^2 + Q_{nv}^2} = \sqrt{120^2 + 160^2} = 200\,\mathrm{eV}$$

   c) After cooling the detector the current noise is 10 eV and the voltage noise remains unchanged at 160 eV. What is the total noise?

$$Q_n = \sqrt{Q_{ni}^2 + Q_{nv}^2} = \sqrt{10^2 + 160^2} = 160.3\,\mathrm{eV}$$

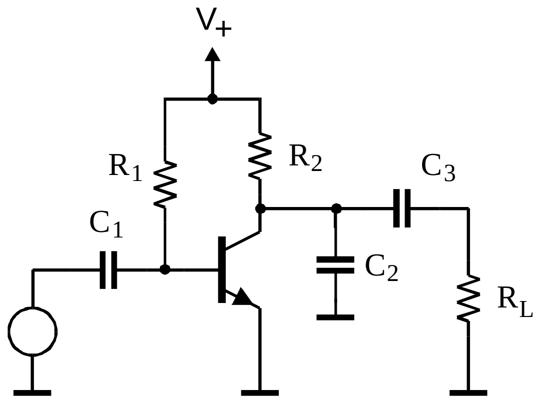
   i.e. in practice the current noise contribution is not discernible.

2. If the overall resolution of a system is determined by the convolution of multiple Gaussian distributions, the individual resolutions add in quadrature.

   In a time-of-flight system the start detector has a time resolution of 100 ps and the stop detector has 50 ps resolution. What is the overall time resolution?

$$\Delta t = \sqrt{\Delta t_1^2 + \Delta t_1^2} = \sqrt{100^2 + 50^2} = 112\,\mathrm{ps}$$

3. Calculate the properties of a simple transistor amplifier. A small-signal silicon transistor is used with a current gain of 100 with an Early voltage >100 V. The supply voltage $V_+ = 12$ V.



a) The transistor is to operate at a collector current of 5 mA. What is the required value of the base bias resistor R_1? (Assume $V_{BE} = 0.6$ V)

To establish a collector current with a current gain of 50, the required base current is

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{5 \cdot 10^{-3}}{100} = 50\ \mu\text{A}$$

The only path for the base current is R_1. The voltage difference across R_1 is $V_+ - V_{BE}$, so

$$R_1 = \frac{V_+ - V_{BE}}{I_B} = \frac{12 - 0.6}{50 \cdot 10^{-6}} = 228\ \text{k}\Omega$$

b) What value of the collector resistor R_2 is required to obtain 6 V at the collector?

The voltage drop across R_2 is V_+ - V_C with a current flow if I_C, so

$$R_2 = \frac{V_+ - V_C}{I_C} = \frac{12 - 6}{5 \cdot 10^{-3}} = 1.2\ \text{k}\Omega$$

c) The output of the amplifier is measured using a high-impedance probe, so the load resistor R_L= 1 MΩ. If the coupling capacitances C_1 and C_3 are sufficiently large to be irrelevant at the measurement frequency (and neglecting C_2), what is the voltage gain for a sinusoidal input?

The output resistance of the transistor and the resistances R_L and R_2 are in parallel for AC, so the effective load resistance

$$\frac{1}{R_{L,eff}} = \frac{1}{R_0} + \frac{1}{R_2} + \frac{1}{R_L}$$

Since the Early voltage > 100 V, $R_0 = V_A / I_C > 100/0.005 = 20$ kΩ. Since both R_0 and $R_L >> R_2$, the effective load resistance is about equal to R_2 and the voltage gain

$$A_V = g_m R_2$$

The transconductance of a bipolar transistor

$$g_m = \frac{q_e}{k_B T} I_C$$

so, using $k_B T/q_e$ = 26 mV (T = 300 K)

$$A_V = g_m R_2 = \frac{q_e}{k_B T} I_C R_2 = \frac{1}{0.026} \cdot (5 \cdot 10^{-3}) \cdot 1200 \approx 230$$

d) What is the small signal input resistance measured at the base of the transistor?

As derived in the lecture, the input resistance of the transistor

$$R_i = \frac{k_B T}{q_e} \cdot \frac{\beta}{I_C} = 0.026 \cdot \frac{100}{5 \cdot 10^{-3}} = 520$$

The base bias resistor R_1 is effectively in parallel with R_i , but since $R_1 >> R_i$, the effective input resistance is R_i .

e) The lower cut-off frequency of the amplifier is to be 160 kHz. What is the required value of C_1? Assume that the signal source has zero source resistance.

C_1 together with the input resistance R_i form a low-pass filter with the time constant

$$\tau = R_i C_1 = \frac{1}{2\pi f_L}$$

so

$$C_1 = \frac{1}{2\pi f_L R_i} = \frac{1}{2\pi \cdot (160 \cdot 10^3) \cdot 520} = 1.9 \text{ nF}$$

e) The capacitance to ground at the output of the amplifier C_2 is 133 pF. What is the upper cutoff frequency?

The effective load resistance R_2 in parallel with the shunt capacitance C_2 when driven by a current source (the transistor) forms an integrator (low-pass filter) with the time constant

$$\tau = R_2 C_2 = \frac{1}{2\pi f_U}$$

and

$$f_U = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi \cdot 1200 \cdot (133 \cdot 10^{-12})} = 997\ \mathrm{kHz}$$

f) If the signal source provides a step impulse, what is the pulse shape at the output? (Assume that C_3 is sufficiently large to be negligible.)

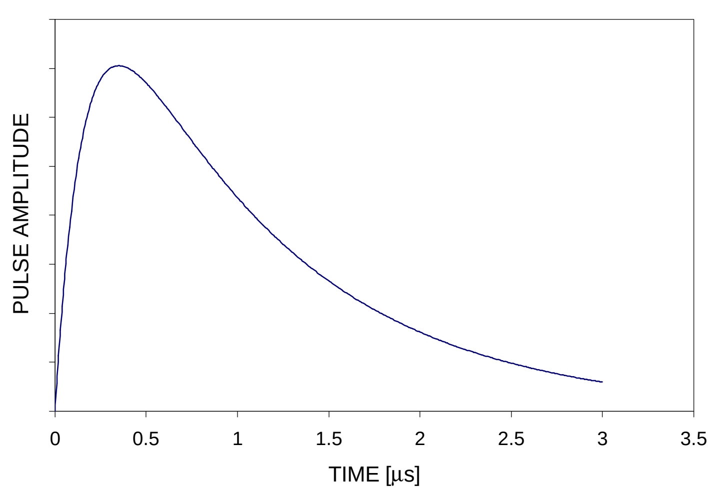
The capacitive coupling at the input forms a differentiator with the time constant

$$\tau_d = R_i C_1 = \frac{1}{2\pi f_L} = 1\ \mu\mathrm{s}$$

At the output the parallel combination of R_2 and C_2 forms an integrator with the time constant

$$\tau_i = R_2 C_2 = 160\ \mathrm{ns}$$

so the output pulse shape (for a negative input step, since the amplifier inverts)



i.e. the amplifier is a simple CR-RC shaper. The peak signal will be inverted and 160 times larger than the input (voltage gain from c) times 0.7 shaper attenuation).