Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 7: Due on Tuesday, 16-Mar-99 at begin of lecture.
Discussion on Wednesday, 17-Mar-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

Compare the time resolution obtainable in a scintillation detector using either a photodiode or a photomultiplier. As an example, consider a scintillator array, as was shown in the lecture on March 4, V.5. Semiconductor Detectors – Examples. An 8 x 8 array of crystals is coupled either to a PMT with a 1 x 1 in^2 faceplate or to an array of photodiodes on the other side. The light output of the scintillator as a function of time is given by

$$\frac{dN(t)}{dt} = N_0 \frac{e^{-t/\tau_2} - e^{-t/\tau_1}}{\tau_2 - \tau_1}$$

where N_0 is the total number of photons, τ_1 is 5 ns and τ_2 is 200 ns. The number of scintillation photons from a 511 keV gamma incident on the detector is 15000.

1. First consider the photomultiplier system. Its quantum efficiency is 20%.

   a) Integrated over the light pulse, how many electrons will reach the first dynode?

   (number of scintillation photons) x (quantum efficiency) = 15000 x 0.2 = 3000

   b) What is the photoelectron current vs. time?

$$i(t) = q_e \frac{dN(t)}{dt} = q_e N_0 \frac{e^{-t/\tau_2} - e^{-t/\tau_1}}{\tau_2 - \tau_1}$$

   where N_0 = 3000, τ_1= 5 ns and τ_2= 200 ns.

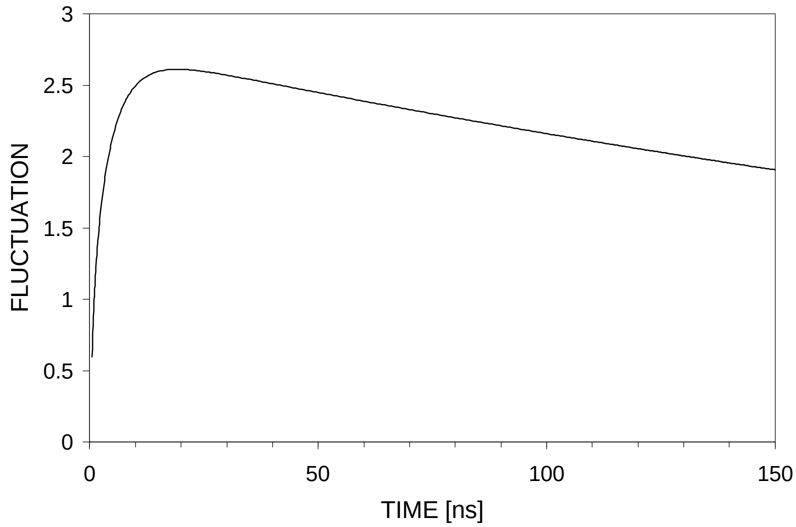
   c) In a photomultiplier system the instantaneous fluctuation of the signal is not due to electronic noise, but to photoelectron statistics. For the purposes of this analysis, assume that the electron-multiplication structure provides a constant gain and mean transit time, but that the transit time jitter is such that the structure effectively integrates over 500 ps. What is the fluctuation of the anode current pulse as a function of time? (Subdivide the pulse into 500 ps intervals and determine the number of photoelectrons and their statistical fluctuation in each interval).

   The number of photelectrons in a time interval ΔT is

$$n = \frac{dN}{dt} \Delta T$$

so (provided n is sufficiently large) the statistical fluctuation

$$\Delta n = \sqrt{n} = \sqrt{\frac{dN}{dt}\Delta T}$$



Note that for small times the numbers are too small for the simple approximation used above to be accurate, but the calculation is sufficient to illustrate the behavior, as will be shown in d).

d) What is the timing jitter for various trigger levels I_T/I_0, where I_0 is the peak current?

Expressed in terms of current the timing jitter

$$\sigma_t = \frac{\sigma_n}{\left(\frac{di}{dt}\right)_{i_T}}$$

The fluctuation σ_n was calculated in the c). The time derivative

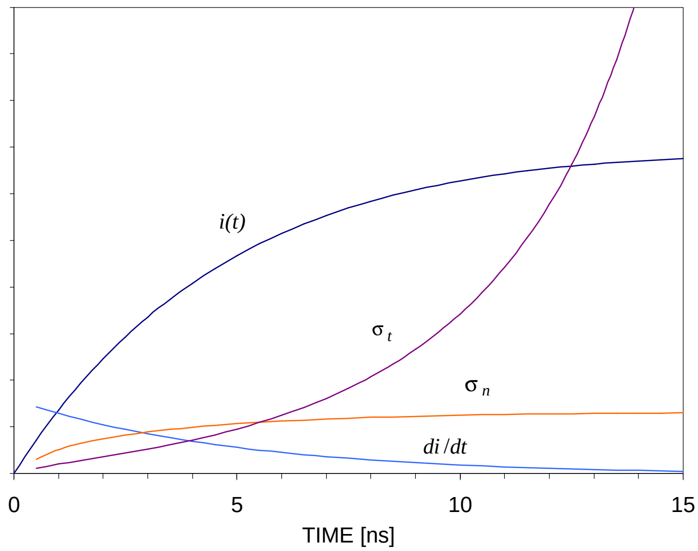
$$\frac{di}{dt} = \frac{N_0 q_e}{\tau_2 - \tau_1}\left(\frac{1}{\tau_1}e^{-t/\tau_1} - \frac{1}{\tau_2}e^{-t/\tau_2}\right)$$

is maximal at $t = 0$, where

$$\frac{di}{dt} = \frac{N_0 q_e}{\tau_1 \tau_2} = 0.48\ \frac{\text{A}}{\text{s}}$$

At t = 0.5 ns, the derivative is somewhat smaller, 0.43 A/s. Since σ_n increases with time and di/dt decreases with time, the time resolution will be best at small times, i.e. small trigger fractions i_T/i_0 .

The graph shows how the signal current, its derivative, the signal fluctuation and the time resolution change with time.



Using the results obtained above the approximate time resolution vs. time and trigger fraction i_T/i_0 is

		QE= 0.2	QE= 1
t [ns]	i_T/i_0	σ_t [ns]	σ_t [ns]
0.5	4.75E-04	0.21	0.09
1.0	9.04E-04	0.40	0.18
1.5	1.29E-03	0.55	0.25
2.0	1.64E-03	0.71	0.32
2.5	1.95E-03	0.87	0.39
3.0	2.24E-03	1.04	0.47
3.5	2.49E-03	1.23	0.55
4.0	2.72E-03	1.43	0.64

For comparison the time jitter for 100% quantum efficiency is also shown. The jitter improves with the square root of the quantum efficiencies, as the slope di/dt increases linearly with the increased number of photoelectrons, whereas the fluctuation σ_n increases with the square root.

The approach used here was only chosen for illustration, as it only works well for large photon fluxes where the signal is continuous. For small photon fluxes with just a photoelectron or less per integration interval, analysis of the arrival times is better.

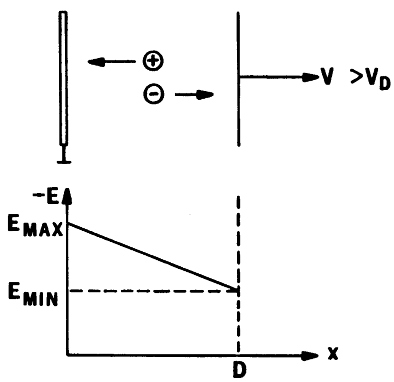
2. Now consider a photodiode readout. The photodiode array is fabricated on a 300 μm thick *n*-type substrate with a donor concentration of $1.0 \cdot 10^{12}$ cm^{-3}. It has the usual asymmetric structure with a thin, highly-doped *p*-layer. Light is incident on the n^{+}-contact, which has been formed by a special technique to make it very transparent to scintillation light. The detector is operated at 100 V and the reverse bias current is 0.4 nA/cm^2. The quantum efficiency is 90%, which means that for 90% of the incident photons a scintillation photon will form an electron-hole pair.

a) What is the depletion voltage? Sketch the field profile in the detector.

The depletion voltage

$$V_D = \frac{D^2 q_e N}{2\varepsilon} = 68.5\,\mathrm{V} \ ,$$

so the detector is fully depleted with the field profile



$$E(x) = \frac{2V_D}{D^2}(D-x) + \frac{V-V_D}{D}$$

$$E_{\min} = \frac{V-V_D}{D}, \quad E_{\max} = \frac{V+V_D}{D}$$

where $E_{\min}$= 1.05 x 10^3 V/cm and $E_{\max}$= 5.6 x 10^3 V/cm .

b) What is the capacitance of an individual photodiode?

Since the total area of 1 x 1 in^2 is covered by an 8 x 8 array of photodiodes, each photodiode is (2.54 / 8) cm on a side, so the area is $(2.54/8)^2$ cm^2 = 0.1 cm^2.

The diode is fully depleted, so the capacitance

$$C = \varepsilon_{Si}\varepsilon_0 \frac{A}{D} = 3.5\ \mathrm{pF}$$

c) The absorption coefficient of the scintillation light is about 10^4 cm^{-1}. Which carrier type dominates the detector signal? What is the collection time t_c ?

The fraction of absorbed photons is

$$\frac{N}{N_0} = 1 - e^{-\mu x}$$

so for $\mu = 10^4$ cm^{-1}, 95% of the photons are absorbed within 3 μm, i.e. 1% of the sensitive thickness. Within the detector holes drift toward the *p*-side and electrons toward the *n*-side. Since practically all of the electrons are already at the *n*-side, where the light is absorbed, the only charges that move and cause current flow are the holes, all of which traverse about 300 μm.

The collection time with $\mu = 480$ cm^2/Vs

$$t_c = \frac{D^2}{2\mu V_D} \ln \frac{V + V_D}{V - V_D} = 23 \text{ ns}$$

d) A current sensitive amplifier is used to sense the instantaneous signal current. The signal current due to *n* charges traversing the detector is

$$i_s = \frac{n q_e}{t_c}$$

where t_c is the time required to traverse the whole width of the detector. Estimate the optimum trigger level.

The time derivative of the signal current, using the quantum efficiency *QE*

$$\frac{di_s}{dt} = \frac{d}{dt}\left(\frac{n q_e}{t_c}\right) = \frac{q_e}{t_c}\frac{dn}{dt} = \frac{q_e}{t_c} QE \frac{dN}{dt} = I_0 \frac{e^{-t/\tau_2} - e^{-t/\tau_1}}{\tau_2 - \tau_1}$$

where

$$I_0 = \frac{q_e}{t_c} QE \cdot N_0$$

The time derivative is determined by the incident photon flux. This is because the detector effectively integrates the photon flux during the collection time, i.e. “new” carriers are released while the “old” ones are still in motion. However, as soon as the carriers generated first have traversed the detector, they no longer contribute to the current. Because of this the above expression is only valid for $t < t_c$.

The flux attains its maximum at the time

$$t_m = \frac{\ln(\tau_2 / \tau_1)}{\dfrac{1}{\tau_1} - \dfrac{1}{\tau_2}} = 18.9 \text{ ns}$$

Since this is less than the collection time of 23 ns, the maximum slope will occur at $t = t_m$. The corresponding current level is

$$i_s(t) = \int_0^{t_m} \frac{di_s}{dt} dt = \frac{I_0}{\tau_2 - \tau_1}\left(\tau_1 (e^{-t_m/\tau_1} - 1) - \tau_2 (e^{-t_m/\tau_2} - 1)\right) = 0.067 I_0$$

Since $I_0 = 94$ nA, $I_T = 6.3$ nA.

e) What is the optimum rise time of the amplifier? What is the electronic noise current level required for a time jitter of 300 ps?

The rise time constant of the signal current is τ_1= 5 ns, so the optimum rise time constant τ_a of the amplifier is also 5 ns.

Note that the rise time of the signal convolved with the rise time of the amplifier degrades the slope at the origin

$$i_i = \frac{di_s}{dt} t \quad \rightarrow \quad i_o = \frac{di_s}{dt}(t - \tau_a) - \frac{di_s}{dt} \tau_a \, e^{-t/\tau_a}$$

The amplifier output attains 90% of the original slope after t = 2.3τ_a = 11.5 ns, well before the trigger time of 18.9 ns.

The time jitter

$$\sigma_t = \frac{\sigma_n}{di_s / dt} = \frac{I_n}{(di_s / dt)_{t=t_m}}$$

The integrated noise current of the amplifier is

$$I_n = i_n \sqrt{f_n} = i_n \sqrt{\frac{1}{4\tau_a}}$$

The signal slope at the trigger time is 0.43 A/s, so

$$\sigma_t = \frac{I_n}{(di_s / dt)_{t=t_m}} = i_n \frac{7.07 \cdot 10^3 \, [\sqrt{\text{Hz}}]}{0.43 [\text{A/s}]} = 1.64 \cdot 10^4 \cdot i_n$$

If σ_t = 300 ps, then i_n = 18 fA/Hz$^{1/2}$.

f) A realistic spectral noise current density referred to the input of the amplifier is 20 pA/Hz$^{1/2}$. Is this adequate for a time jitter of 300 ps? What is the noise contribution from the detector bias current?

An amplifier spectral noise density of 20 pA/Hz$^{1/2}$ is roughly 1000 times larger than the 18 fA/Hz$^{1/2}$ required for a time resolution of 300 ps. This is why the “noiseless” gain of photomultiplier tubes makes them superior to photodiodes for fast timing.

The spectral noise density originating from the detector bias current of 0.4 nA is

$$i_n = \sqrt{2 q_e I_b} = 11.3 \;\; \text{fA}/\sqrt{\text{Hz}}$$

If the electronic noise were sufficiently small to obtain 300 ps resolution, the noise current from the detector would be a significant contribution.

g) Compare the current fluctuations due to electronic noise with the current fluctuations due to photon statistics.

For a spectral noise density of 20 pA/Hz$^{1/2}$ and an amplifier time constant of 5 ns the total noise current

$$I_n = i_n \sqrt{f_n} = i_n \sqrt{\frac{1}{4\tau_a}} = 141\,\mathrm{nA}$$

For the 18 fA/Hz$^{1/2}$ required for a time resolution of 300 ps, I_n = 127 pA, so the signal-to-noise ratio at the trigger time is 6.3 nA/ 127 pA= 50.

In the PMT system the optimum trigger time is 0.5 to 1 ns, where the average number of photoelectrons is one. Good time resolution is obtained by virtue of the high *di*/*dt*.