Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 10: Due on Tuesday, 13-Apr-99 at begin of lecture.
Discussion on Wednesday, 14-Apr-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. Traps in semiconductor materials are often characterized by a lifetime τ. A packet of charge subject to trapping will decay with time as

$$Q(t) = Q_0 e^{-t/\tau}$$

a) In an electric field E the charge will drift. What is the charge remaining after drifting a distance x?

Given a lifetime τ, a packet of charge Q_0 will decay with time as

$$Q(t) = Q_0 e^{-t/\tau}$$

In an electric field the charge drifts with a velocity $v = \mu E$. The time required to traverse a distance x is

$$t = \frac{x}{v} = \frac{x}{\mu E}$$

after which the remaining charge is

$$Q(x) = Q_0 e^{-x/\mu E\tau} \equiv Q_0 e^{-x/L}$$

Since the trapping length L is proportional to the mobility-lifetime product, $\mu\tau$ is often used as a figure of merit for the quality of the material.

b) The parameter $\mu E\tau$ is the trapping (or recombination) length L. In a detector with depletion width d, what is the induced signal charge as a function of d and L? Consider only the carrier type subject to trapping/recombination.

For a charge traversing the increment dx in a parallel-plate detector of thickness d, the induced signal charge is

$$dQ_s = Q(x)\frac{dx}{d}$$

If the charge originates at x_0 and drifts towards $x = d$, the total induced charge

$$Q_s = \frac{1}{d}\int_{x_0}^{d} Q(x)dx = \frac{1}{d}\int_{x_0}^{d} Q_0 e^{-x/L}dx$$

$$Q_s = \frac{Q_0}{d}\left[-Le^{-x/L}\right]_{x_0}^{d} = Q_0\,\frac{L}{d}\left(e^{-x_0/L} - e^{-d/L}\right)$$

If the charge originates at the origin $x_0 = 0$

$$Q_s = Q_0\,\frac{L}{d}\,\left(1 - e^{-d/L}\right)$$

c) If $d >> L$, what is the induced signal charge? How thick must the detector be for the induced charge to be 95% of the deposited charge?

Using the result obtained above for $x_0 = 0$,

$$d >> L: \qquad \frac{Q_s}{Q_0} \approx \frac{L}{d}$$

If $d >> L$, this result holds for all values of x_0 .

To determine the thickness required for $Q_s\,/\,Q_0 = 0.95$, consider the case where the charge drifts the greatest distance and suffers the maximum loss, i.e. $x_0 = 0$.

$$\frac{Q_s}{Q_0} = \frac{L}{d}\,\left(1 - e^{-d/L}\right)$$

If $d << L$, $Q_s \approx Q_0$. Since $d/L < 1$, we expand the exponential

$$\frac{Q_s}{Q_0} = \frac{L}{d}\,\left(1 - e^{-d/L}\right) \approx \frac{L}{d}\left(1 - \left(1 - \frac{d}{L} + \frac{1}{2}\left(\frac{d}{L}\right)^2 + \ldots\right)\right) = 1 - \frac{1}{2}\frac{d}{L} + \frac{1}{6}\left(\frac{d}{L}\right)^2 + \ldots$$

Solving the second order equation yields two solutions, $d/L = 0.105$ and $d/L = 2.896$, of which the first is correct. For $x_0 > 0$ the induced charge will be smaller, but if the oppositely charged carrier has a much longer trapping length, its contribution will increase linearly with x_0 , so the total induced charge will be $> 0.95Q_0$.

Although a thin detector will mitigate charge loss due to trapping, it does this at the expense of capacitance, i.e. an increase in electronic noise.

2. A Si diode with 1 cm^2 area has a reverse bias current of 1 nA at a temperature of 20 °C.

   a) In forward bias, how much voltage is required to obtain a current of 1 mA.

$$I = I_R(e^{qV/k_BT} - 1)$$

$$e^{qV/k_BT} = \frac{I}{I_R} + 1$$

$$V = \frac{k_BT}{q_e}\ln\left(\frac{I}{I_R} + 1\right)$$

   At T = 293 K, k_BT/q_e= 0.0253 V, so I = 1 mA and I_R = 1 nA yield V = 350 mV.

   b) On the same wafer there is a second identical diode, except that its diameter is 100 µm. How large is its reverse bias current?

$$A = 7.85 \text{ x } 10^{-5} \text{ cm}^2\text{, so } I_R = A \times 1 \text{ nA} = 78.5 \text{ fA .}$$

   c) How much forward bias voltage on the small diode is required for a current of 1 mA? How much voltage is required after cooling the diode to –20 °C?

   At room temperature V = 589 ≈ 590 mV.

   Reducing the temperature from 20 °C to –20 °C reduces the current by a factor

$$\frac{I_R(T_2)}{I_R(T_1)} = \left(\frac{T_2}{T_1}\right)^2 \exp\left[-\frac{E}{2k_B}\left(\frac{T_1 - T_2}{T_1T_2}\right)\right] = 2.25 \cdot 10^{-2}$$

   where E is equal to the band-gap E_g= 1.12 eV. The current reverse current drops from 76 fA at 20 °C to 1.7 pA at –20 °C. At T = 253 K, k_BT/q_e= 0.0218 V and the forward bias voltage required for a current of 1 mA is 590 mV.

   Because the ratio I/I_R enters logarithmically, the decrease in k_BT/q_e compensates for the change in reverse current. The compensation doesn't always work this well; for the large diode the forward bias voltage must be increased from 350 mV to 384 mV after cooling.

The "turn-on" voltage depends on the area of the diode. A large area Si diode has a "turn-on" voltage comparable to a small Ge diode.

3. A radiation damaged detector has a reverse bias current of 1 μA at 100 μm depletion width. The operating temperature is 20 °C.

   a) The detector is still partially depleted after quadrupling the bias voltage. How large is the reverse bias current?

   In a radiation-damaged detector the generation current sites are uniformly distributed throughout the material, so the generation current increases with the volume of the depletion region. Here, the area remains constant, so the volume increases with depletion width. As the depletion width $d \propto V^{1/2}$, quadrupling the bias voltage doubles the depletion width, so the current will double to 2 μA.

   b) What is the detector current when the temperature is decreased from 20 °C to –10 °C?

   The ratio of currents at two temperatures T_1 and T_2 is

$$\frac{I_R(T_2)}{I_R(T_1)} = \left(\frac{T_2}{T_1}\right)^2 \exp\left[-\frac{E}{2k_B}\left(\frac{T_1 - T_2}{T_1 T_2}\right)\right]$$

   In a radiation-damaged detector E= 1.2 eV, so for T_1= 293 K and T_2= 263 K

$$\frac{I_R(T_2)}{I_R(T_1)} = 5.4 \cdot 10^{-2}$$

   At 100 V the bias current is 54 nA and at 400 V it doubles to 108 nA.

4. A spectroscopy system using a Si diode at room temperature (20 °C) exhibits an equivalent noise charge of 200 eV at the optimum shaping time of 1 μs.

   a) How large are the noise contributions (in eV) due to current and voltage noise?

   The equivalent noise charge

$$Q_n^2 = i_n^2 \, T_s \, F_i + C_i^2 \, v_n^2 \, \frac{F_v}{T_s}$$

   At the optimum shaping time the current and voltage noise contributions are equal, so

$$Q_{ni} = Q_{nv} = \frac{Q_n}{\sqrt{2}} = 141\,\text{eV}$$

b) Assuming that the current noise is dominated by the reverse bias current of the detector, what is the total noise when the detector is cooled to liquid nitrogen temperature?

Cooling to liquid nitrogen temperature (77 K) reduces the reverse bias current so much that the current noise

$$i_n^2 = 2q_e I_R$$

is negligible, so practically only the voltage noise remains and Q_n = 141 eV.

c) If the detector is cooled to –20 °C, what is the optimum shaping time? Assume $F_i = F_v = 1$. What is the corresponding noise level?

As calculated in problem 2, reducing the temperature from 20 °C to –20 °C decreases the current by a factor

$$\frac{I_R(T_2)}{I_R(T_1)} = \left(\frac{T_2}{T_1}\right)^2 \exp\left[-\frac{E}{2k_B}\left(\frac{T_1 - T_2}{T_1 T_2}\right)\right] = 2.25 \cdot 10^{-2}$$

Since this is not a radiation-damaged diode E is equal to the band-gap E_g= 1.12 eV.

Thus, the spectral noise current density $i_n^2 = 2q_e I_R$ is reduced by the ratio of the currents

$$\frac{i_n^2(T = 253\,\mathrm{K})}{i_n^2(T = 293\,\mathrm{K})} = \frac{I_R(T = 253\,\mathrm{K})}{I_R(T = 293\,\mathrm{K})} = 2.25 \cdot 10^{-2}$$

and the optimum shaping time

$$T_{s.opt} = C_i \frac{v_n}{i_n}\sqrt{\frac{F_v}{F_i}}$$

increases by

$$\frac{T_{s,opt}(T = 253\,\mathrm{K})}{T_{s,opt}(T = 293\,\mathrm{K})} = \frac{i_n(T = 293\,\mathrm{K})}{i_n(T = 253\,\mathrm{K})} = \sqrt{\frac{1}{2.25 \cdot 10^{-2}}} = 6.7$$

from 1 μs to 6.7 μs. Since at the optimum shaping time both noise contributions are equal, the reduction in noise can be determined either by calculating the current or voltage noise.

The current noise contribution is reduced by

$$\frac{Q_{ni}^2(253\,\mathrm{K})}{Q_{ni}^2(293\,\mathrm{K})} = \frac{i_n^2(253\,\mathrm{K})}{i_n^2(293\,\mathrm{K})} \cdot \frac{T_{s,opt}(253\,\mathrm{K})}{T_{s,opt}(293\,\mathrm{K})} = 2.25 \cdot 10^{-2} \times 6.7 = 0.15$$

The improvement in voltage noise is simply the ratio of the shaping times, as the voltage noise density v_n and the total capacitance C remain constant

$$\frac{Q_{nv}^2(253\,\mathrm{K})}{Q_{nv}^2(293\,\mathrm{K})} = \frac{T_{s,opt}(293\,\mathrm{K})}{T_{s,opt}(253\,\mathrm{K})} = \frac{1}{6.7} = 0.15$$

The total noise improves from 200 eV to

$$Q_n = \sqrt{Q_{ni}^2 + Q_{nv}^2} = \sqrt{0.15 + 0.15} \cdot Q_n(293\,\mathrm{K}) = 110\,\mathrm{eV}\,.$$