Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 3: Due on Tuesday, 9-Feb-99 at begin of lecture.
Discussion on Wednesday, 10-Feb-99 at 12 – 1 PM in 347 LeConte.

1. A microchannel plate (MCP) consists of an array of capillaries, each of which forms a continuous electron multiplier structure. The inside of the capillary is coated to form a secondary electron emitting layer, which also exhibits the appropriate electric resistance to form a continuous voltage divider that sets up an acceleration field along the channel. In a typical device the individual channels are 10 μm diameter, with an open area ratio of ~50%, so there are 10^6 channels per cm^2. Operated at a voltage of 1000 V the gain is $5\cdot10^3$. The MCP is typically 0.6 mm thick and the resistance of each channel is about 10^{15} Ω. Metalization on the two faces of the MCP connects all channels in parallel, so only two connections are required, in addition to a collection electrode at the output.

   a) When first applying voltage to a microchannel plate, the first check is to monitor the supply current. Since MCPs must be operated in UHV to avoid contamination and random discharges, the DC current is also an indicator of the vacuum quality. For a 40 mm diameter MCP, what current draw do you expect at 1000 V?

   The area of the MCP is $2^2\pi$= 12.6 cm^2, so the total number of microchannels is $1.26\cdot10^{\mathbf{7}}$. Since the resistance of each channel is 10^{15} Ω, the resistance of all channels in parallel is 10^{15} Ω $/1.26\cdot10^{\mathbf{7}}$ = 79.6 MΩ, so the current draw at 1 kV is 12.6 μA.

   b) Assume a configuration where 1 photoelectron enters an individual channel. The gain remains constant up to an average signal current that is 10% of the standing current in a channel. What is the maximum counting rate per channel in the linear range? What is the maximum rate over the whole diameter of the MCP?

   The average signal current is

$$\bar{i}_s = Q_s \frac{dN}{dt}$$

   Since the standing current per channel I_{DC} = 1 kV/10^{15} Ω= 10^{-12} A and the charge per pulse at the output Q_s= gain x q_e= $5\cdot10^3$ x $1.6\cdot10^{-19}$ = $8\cdot10^{-16}$, the condition for the maximum rate dN/dt that does not exceed 1/10 of the standing current is

$$\frac{dN}{dt} \le \frac{I_{DC}/10}{Q_s} = \frac{10^{-13}}{5\cdot10^3 \times 1.6\cdot10^{-19}} = 125\,\mathrm{s}^{-1}$$

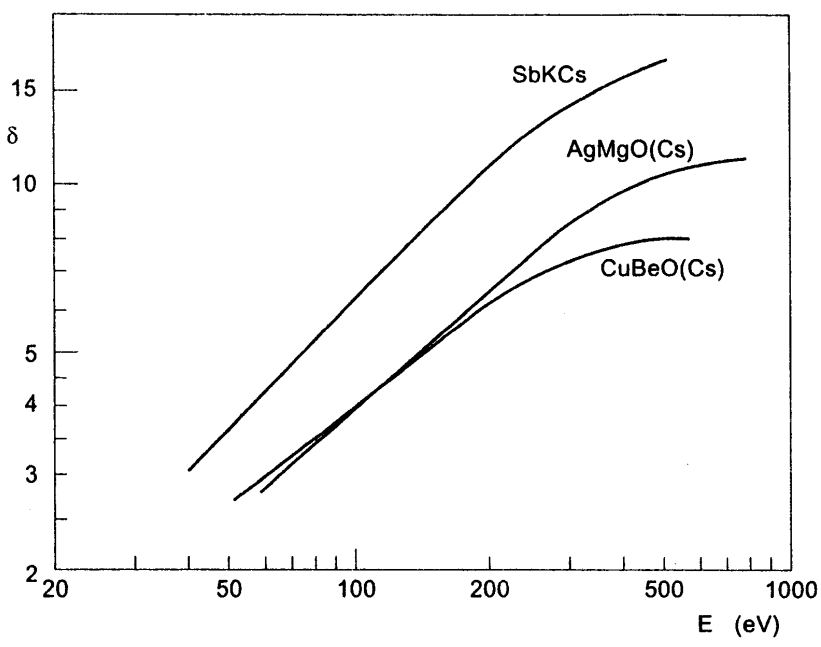
   Naively, for all channels this would allow a rate of $1.26\cdot10^{\mathbf{7}}$ x 125= $1.6\cdot10^{\mathbf{9}}$. In reality, since both the rate and the spatial distribution are random, the instantaneous rate per channel is higher and the probability of multiple hits per channel is substantial, so the allowable rate will be about 1/10 as high.

2. A 12-stage photomultiplier tube is operated at 2 kV for a gain of 10^7. This tube uses "conventional" SbKCs dynodes operated with 120 V potential difference between adjacent dynodes. Since the scintillator resolution is 7%, long term gain variations should be <1%. How stable must the power supply be to maintain this requirement?

   For simplicity, assume that the gain G is distributed evenly among the 12 dynodes. If the gain per dynode varies by δg, then the gain variation for n dynodes is $(g \pm \delta g)^n = G(1 \pm \delta g/g)^n$. If overall gain variation is $\Delta G/G$, then $(1 \pm \Delta G/G) = (1 \pm \delta g/g)^n$ or $\Delta G/G = n(\delta g/g)$. If $\Delta G/G \leq 1\%$, then $\delta g/g \leq 8.3 \cdot 10^{-4}$.

   Data on gain vs. supply voltage can be determined from the PMT's data sheet, but even without these data, a simple estimate is possible. To first order, the secondary electron yield is proportional to the kinetic energy of the incident electron, so the gain per stage will increase roughly proportionally to the voltage between dynodes. If anything, this will overestimate the sensitivity, as the penetration depth into the dynode will increase with incident energy, but losses will also increase as the low-energy secondaries need to travel farther to reach the surface.

   The gain curves in the lecture notes allow a test of this reasoning. One can determine the electron yield at 120 eV, and the derivative by evaluating the change in gain at 100 and 120 eV, for example.



   The 17% change in energy (voltage) corresponds to a 14% change in gain, so this increase is slightly less than linear as surmised. If the relative gain per stage is to change no more than $8.3 \cdot 10^{-4}$, then the relative voltage change may not exceed 10^{-3}.

   Since the dynode voltages are derived from a resistive divider fed by the 2 kV supply, the relative voltage change per dynode equals the relative voltage change of the total supply voltage, so the 2 kV supply must be stable to 2 V. This is quite practical, but requires some care.

3. Typically, 511 keV photons interacting in a NaI(Tl) scintillator provide about 3000 photoelectrons at the first dynode, yielding a theoretical resolution of 2% rms. Assume that non-uniformities in light collection and gain increase this resolution to 3%.

a) What resolution do you expect for 30 keV x-rays? Express the resolution both as a percentage and as an energy spread, and compare with the result for 511 keV. In general, how does the resolution scale with energy?

The energy resolution has a statistical and a systematic component. The statistical contribution depends on the number of photoelectrons reaching the first dynode. The systematic component due to non-uniformities in light collection and gain are contribute a constant factor to the resolution. In this case

$$\frac{\Delta E}{E} = 1.5 \frac{1}{\sqrt{N}}$$

where N is the number of photoelectrons. For 511 keV, 3000 photoelectrons yield $\Delta E/E$= 3% or 15 keV rms. 30 keV x-rays will produce 176 photoelectrons, yielding $\Delta E/E$= 11% or 3.4 keV rms.

b) Replace the NaI(Tl) scintillator with a $PbWO_4$ crystal and compare the results for 30 and 511 keV with NaI(Tl).

$PbWO_4$ yields only 500 photons/MeV whereas NaI(Tl) yields 38000. The number of photoelectrons for 511 keV thus reduces to 40, and the resolution $\Delta E/E$= 24% or 121 keV rms. For 30 keV, only 2.3 photoelectrons are produced, so the distribution is not Gaussian. For a mean number of photoelectrons $\overline{n}$ the number of instances where n are observed is

$$N_n = \frac{(\overline{n})^n}{n!} e^{-\overline{n}}$$

so the distribution is not symmetrical for small $\overline{n}$.

4. A CsI(Tl) scintillator exposed to a ^{137}Cs source yields $7 \cdot 10^9$ electrons at the output of a photomultiplier tube.

a) What is the peak anode current? Assume that the rise time is negligible.

Since the rise time is negligible, the output current is

$$i(t) = i_0 e^{-t/\tau}$$

where τ is the decay time of the phosphor. Integrating over the current pulse yields the charge as a function of time

$$Q_s(t) = \int i(t) = Q_0 (1 - e^{-t/\tau})$$

For $t = \infty$, the integral must equal the total signal charge. Differentiating this expression yields the signal current as a function of total charge and decay time

$$\frac{d}{dt} Q_s(t) = \frac{Q_0}{\tau} e^{-t/\tau} \ ,$$

so the peak signal current is Q_0/τ.

For CsI(Tl) the decay time τ = 800 ns, so the peak current is $7 \cdot 10^9 \, q_e/\tau$ = 1.4 mA (or 70 mV into 50 Ω).

b) Replace the CsI(Tl) crystal by a BaF_2 scintillator. How does the peak anode current change?

BaF_2 has both a slow and a fast component. The slow component yields 10000 photons/MeV (compared to 60000 for CsI(Tl)) at a decay time of 630 ns, so for the 662 keV Cs gammas $Q_0 = 1.2 \cdot 10^9$ el and the peak current $1.2 \cdot 10^9 \, q_e/\tau$ = 0.3 mA. For the fast component, only 1800 photons are produced for 1 MeV, but the decay time is 0.8 ns, so the peak current is $2.1 \cdot 10^8 \, q_e/\tau$ = 42 mA. Despite the much lower light output, the peak current is more than 10 times larger than for the slow component.

5. In rummaging around the lab you find a silicon detector. Of course, the data sheet is nowhere to be found, but apparently the detector was designed for charged particle spectroscopy, so you have good reason to expect that it is less than a mm thick and has an "asymmetrical" junction.

a) How can you determine the correct polarity of the bias voltage?

Apply voltage, gradually increasing the level from 0 to ~1 V, first with one polarity and then the opposite polarity. Compare the magnitude of the current, which should be quite different for voltages > 0.5 V or so, and determine forward and reverse bias.

b) You have a source of 1 MeV electrons. How can you determine the depletion voltage and the thickness of the detector?

The range of 1 MeV electrons is about 2 mm, so they will traverse the detector. As the depletion width increases with reverse bias, the magnitude of the measured charge signal will increase. When the detector is fully depleted, the signal plateaus. By measuring the deposited energy ΔE beyond full depletion, one can determine the thickness of the detector $d = R(1 \text{ MeV}) - R(1 \text{ MeV} - \Delta E)$.

c) Assume that the detector is 300 μm thick with a depletion voltage of 120 V. What is the resistivity of the material?

From the lecture notes:

n-type silicon (*V* in volts and ρ in Ω·cm): $W = 0.5\,\mu m \text{ x } \sqrt{\rho_n (V + V_{bi})}$

and in *p*-type material: $W = 0.3\,\mu m \text{ x } \sqrt{\rho_p (V + V_{bi})}$

If the detector is *n*-type, the resistivity is 3 kΩcm. For *p*-type material it is 8.3 kΩcm.

d) You've measured the diameter of the detector to be 20 mm. What is the capacitance at 0, 30, 60 and 120V?

The detector capacitance is

$$C = \varepsilon \frac{A}{W} = A \sqrt{\frac{\varepsilon\, q_e N}{2(V_b + V_{bi})}}$$

The ratio of capacitances at two bias voltages is

$$\frac{C_1}{C_2} = \sqrt{\frac{V_{b2} + V_{bi}}{V_{b1} + V_{bi}}}$$

We know one of these capacitances, as at full depletion the detector forms a capacitor with a thickness W= 300 μm and area A= π cm^2.

$$C = \varepsilon \frac{A}{W} = \varepsilon_r \varepsilon_0 \frac{A}{W}$$

The dielectric constant of Si ε_r= 11.9 and ε_0= 8.85x10^{-14} F/cm, so the capacitance at 120 V is 110 pF.

To scale to the other voltages, as a first approximation we'll neglect V_{bi}. Then

$$\frac{C_1}{C_2} = \sqrt{\frac{V_{b2}}{V_{b1}}}$$

and we obtain	V_b= 120 V	C= 110 pF
	V_b= 60 V	C= 156 pF
	V_b= 30 V	C= 220 pF
	V_b= 0 V	C= ∞ pF

Obviously, we must consider the built-in voltage to obtain a realistic capacitance at 0 V. Assume V_{bi}= 0.6 V. With that we obtain

V_b= 120 V	C= 110 pF
V_b= 60 V	C= 155 pF
V_b= 30 V	C= 219 pF
V_b= 0 V	C= 1560 pF

from which we see that the effect of the built-in voltage is negligible except at low voltages.

An alternative solution obtains the doping concentration N from the total thickness of the detector d and the depletion voltage V_d .

$$d = \sqrt{\frac{2\varepsilon(V_d + V_{bi})}{q_e N}}$$

$$N = \frac{2\varepsilon(V_d + V_{bi})}{q_e d^2}$$

Again assume V_{bi}= 0.6 V. Then the depletion voltage of 120 V and the detector thickness of 300 μm yield the doping concentration

$$N = \frac{2 \cdot 11.9 \cdot 8.85 \cdot 10^{-14}(120 + 0.6)}{1.60 \cdot 10^{-19} \cdot (300 \cdot 10^{-4})^2} = 1.76 \cdot 10^{12} \ \ [\mathrm{cm}^{-3}]$$

The capacitance as a function of voltage is

$$C = A\sqrt{\frac{\varepsilon\, q_e N}{2(V_b + V_{bi})}} = \pi \cdot \sqrt{\frac{11.9 \cdot 8.85 \cdot 10^{-14} \cdot 1.60 \cdot 10^{-19} \cdot 1.76 \cdot 10^{12}}{2(V_b + 0.6)}} = \frac{1.21\ [\mathrm{nF} \cdot \mathrm{V}^{1/2}]}{\sqrt{V_b + 0.6}},$$

which yields the same results as above.