Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 9: Due on Tuesday, 6-Apr-99 at begin of lecture.
Discussion on Wednesday, 7-Apr-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. A planar detector is made of a semiconductor material that is semi-insulating, i.e. it behaves like very high resistivity material, so thick detectors can be made with a uniform field throughout the detector. Assume a detector thickness of 1 cm. A drawback of the material is that holes are trapped after moving an average distance of 1 mm, whereas electrons can traverse the full thickness without significant losses.

   a) A photon is absorbed in the middle of the sensitive volume, i.e. 5 mm from the positive electrode, forming 1000 electron-hole pairs. After all carrier motion has ceased, how much charge (expressed in electrons) is induced on the electrodes?

   For a parallel-plate geometry the induced charge is proportional to the fraction of the detector thickness that the charges traverse

$$Q_s = Q_0 \frac{\Delta x}{d}$$

   The electrons drift 5 mm to the positive electrode, yielding the signal

$$Q_{sn} = Nq_e \frac{\Delta x}{d} = 10^3 q_e \frac{5}{10} = 500\, q_e$$

   The holes, on the other hand, only travel 1 mm on the average before being lost to trapping, so

$$Q_{sp} = Nq_e \frac{\Delta x}{d} = 10^3 q_e \frac{1}{10} = 100\, q_e$$

   The total charge $Q_s = Q_{sn} + Q_{sp} = 600\, q_e$

   b) How much charge is induced when a photon is absorbed 1 mm from the positive electrode?

   The electron moves 1 mm to the positive electrode and the hole is trapped after 1 mm, so

$$Q_s = Q_{sn} + Q_{sp} = Nq_e \frac{\Delta x_n + \Delta x_p}{d} = 1000\, q_e \frac{1+1}{10} = 200\, q_e$$

c) How much charge is induced when a photon is absorbed 1 mm from the negative electrode?

The electrons traverse 9 mm and the holes travel the 1 mm to the negative electrode, so

$$Q_s = Q_{sn} + Q_{sp} = Nq_e \frac{\Delta x_n + \Delta x_p}{d} = 1000\, q_e \frac{9+1}{10} = 1000\, q_e$$

The full charge is induced.

In materials where the trapping length of one carrier type is less than the total drift distance the measured charge depends on the interaction point, so the obtainable energy resolution is determined by the induced charge profile.

Problem set 10 will include a more exact treatment of signal loss due to trapping.

2. A detector is made of high-purity germanium using a coaxial geometry as shown in V.5., p. 10 (lecture on 04-Mar-99). The inner electrode is *n*-type with 10 mm diameter and the outer diameter is 50 mm. The detector is operated at 77 K with 5 kV bias voltage, so one can assume substantial overbias.

a) What is the field distribution within the detector?

Since the detector is operating at substantial overbias, we can neglect the field profile associated with the *pn*-junction. The field distribution is the same as in a coaxial capacitor. The inner and outer electrodes at radii r_1 and r_2 are equipotential surfaces with a potential difference equal to the applied bias voltage, so the field

$$E(r) = \frac{K}{r}$$

where K is determined by the boundary condition $\Phi(r_2) - \Phi(r_1) = V_b$. The potential

$$\Phi = \int E(r)dr = \int \frac{K}{r} dr = K \cdot \ln r$$

and the boundary conditions set the constant K with the result

$$E(r) = \frac{V_b}{r \ln \frac{r_2}{r_1}}$$

This result can also be found in many textbooks.

b) What is the maximum collection time for holes and electrons? At 77 K electrons and holes in Ge have the same mobility μ= 40,000 cm^2/Vs.

The drift velocity

$$v(r) = \mu E(r) = \mu \frac{V_b}{r \ln \frac{r_2}{r_1}}$$

so the time dt required to traverse a radial increment dr is

$$dt = \frac{\ln(r_2 / r_1)}{\mu V_b} \cdot r dr$$

so the transit time of the electrons moving from the interaction point r to the electrode at r_1

$$t_{ce} = \frac{\ln(r_2 / r_1)}{2\mu V_b} \cdot (r^2 - r_1^2)$$

and of the holes

$$t_{cp} = \frac{\ln(r_2 / r_1)}{2\mu V_b} \cdot (r_2^2 - r^2)$$

Traversing the distance r_2 - r_1 takes

$$t_c = \frac{\ln(r_2 / r_1)}{2\mu V_b} \cdot (r_2^2 - r_1^2)$$

This is the maximum collection time, which for r_1= 5 mm and r_2= 25 mm is 24 ns.

c) Assume 100 keV deposited at a radius of 10 mm. The ionization energy in Ge at 77 K is 3.0 eV. What is the induced signal current vs. time for the electrons, the holes, and for the total signal?

The induced current is given by the carrier velocity and the weighting field from Ramo's theorem. The distribution of the weighting field is the same as of the electric field, except that unit potential is applied.

$$F(r) = \frac{1}{r \ln \frac{r_2}{r_1}}$$

so for N electron-hole pairs the induced current at a given radius

$$i_s = Nq_e v(r) F(r) = Nq_e \mu E(r) F(r) = Nq_e \mu \frac{V_b}{r \ln \frac{r_2}{r_1}} \cdot \frac{1}{r \ln \frac{r_2}{r_1}} = \frac{Nq_e \mu V_b}{(\ln(r_2 / r_1))^2} \cdot \frac{1}{r^2}$$

The position of the carrier vs. time was calculated in a). For electrons to move from the interaction point r_x to radius r

$$t = \frac{\ln(r_2 / r_1)}{2\mu V_b} \cdot (r_x^2 - r^2)$$

$$r^2(t) = r_x^2 - \frac{2\mu V_b}{\ln(r_2 / r_1)}\, t$$

and for holes

$$t = \frac{\ln(r_2 / r_1)}{2\mu V_b} \cdot (r^2 - r_x^2)$$

$$r^2(t) = r_x^2 + \frac{2\mu V_b}{\ln(r_2 / r_1)}\, t$$

Inserting these results into the induced current calculated above yields the current vs. time for electrons

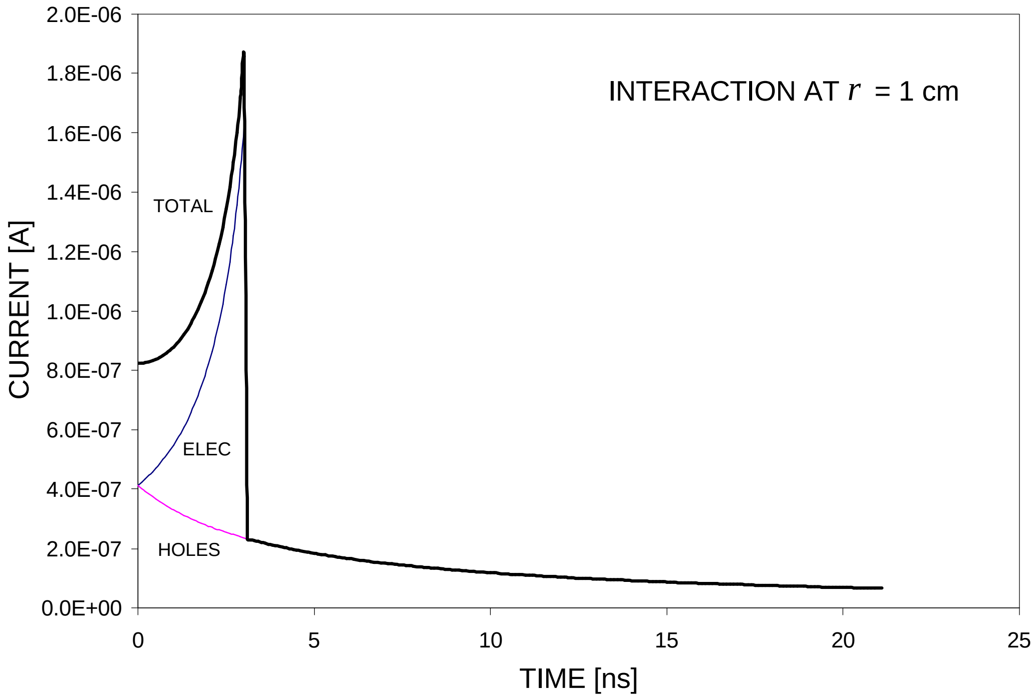
$$i_{sn}(t) = \frac{N q_e \mu V_b}{\left(\ln(r_2 / r_1)\right)^2} \cdot \frac{1}{r_x^2 - \dfrac{2\mu V_b}{\ln(r_2 / r_1)}\, t}$$

and for holes

$$i_{sp}(t) = \frac{N q_e \mu V_b}{\left(\ln(r_2 / r_1)\right)^2} \cdot \frac{1}{r_x^2 + \dfrac{2\mu V_b}{\ln(r_2 / r_1)}\, t}$$

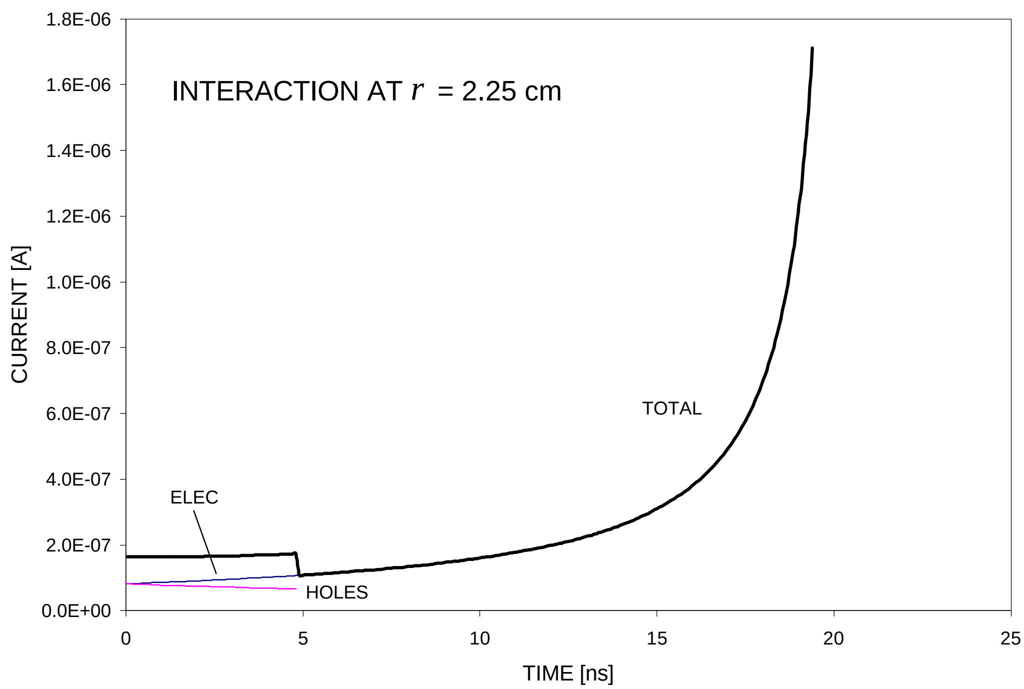
Both of these expressions are only valid for times less than the respective collection times, which for this interaction point are 3.0 ns for the electrons and 21.1 ns for the holes.

The current pulses are plotted on the next page.



d) What are the current pulse shapes for 100 keV deposited at a radius of 22.5 mm?

At this interaction point the collection times are 19.4 ns for the electrons and 4.8 ns for the holes.



Note the differences in pulse shapes for the two interaction points.