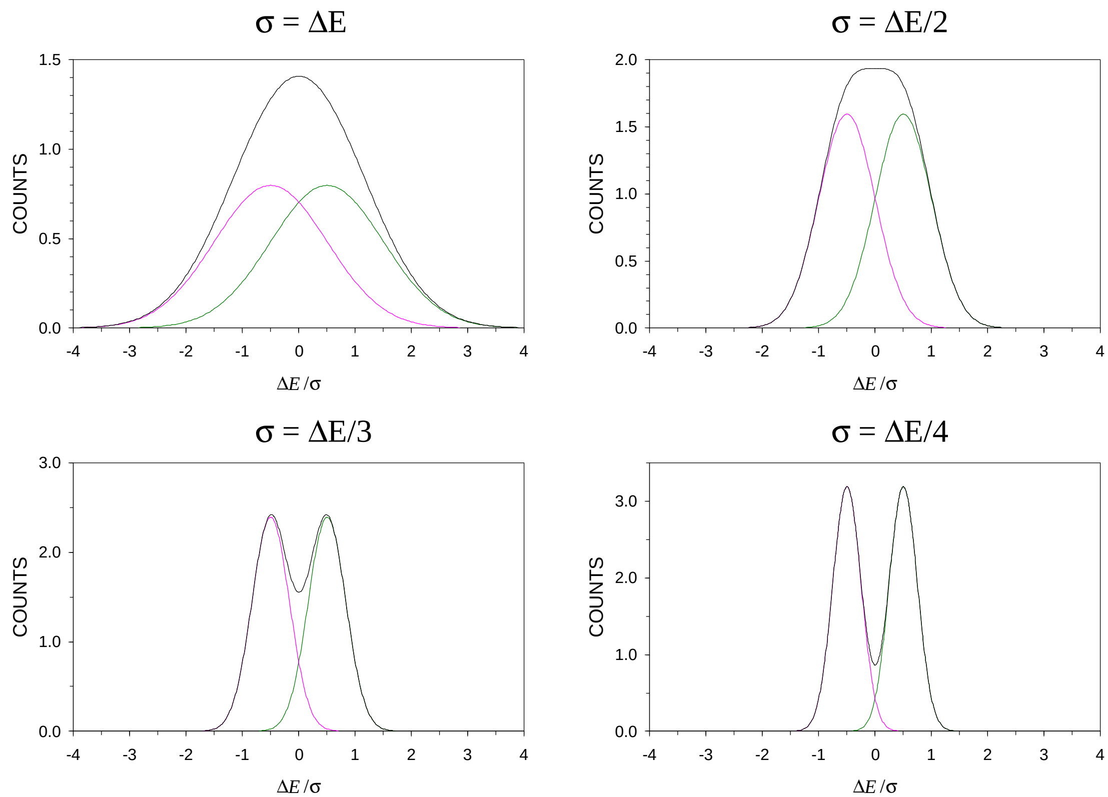
Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 5: Due on Tuesday, 2-Mar-99 at begin of lecture.
Discussion on Wednesday, 3-Mar-99 at 12 – 1 PM in 347 LeConte.
Office hours: Mondays, 3 – 4 PM in 420 LeConte

1. An x-ray spectroscopy system is to resolve the Tl $K_{\alpha 1}$ and $K_{\alpha 2}$ emissions from a ^{203}Hg source. The $K_{\alpha 1}$ and $K_{\alpha 2}$ energies are 72.87 and 70.83 keV, at about equal intensities.

   a) Determine the energy resolution required to separate the two x-ray peaks.

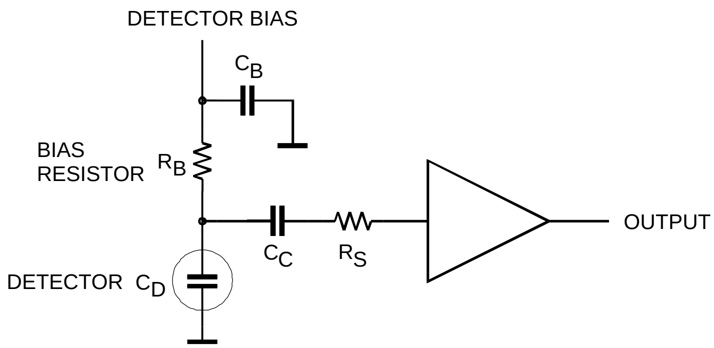


The two Gaussian peaks are adequately resolved at $\sigma = \Delta E/3$, so since ΔE= 72.87 - 70.83 = 2.04 keV, the required resolution σ_E = 0.68 keV or 1.6 keV FWHM. Note that in systems dominated by electronic noise specifying absolute resolution is more useful than relative resolution, as the linewidth is essentially independent of energy.

   b) The intrinsic energy resolution of the detector is 160 eV. What is the allowable electronic noise contribution?

Since the individual resolutions add in quadrature $\sigma_E^2 = \sigma_{\text{det}}^2 + \sigma_n^2$, the allowable electronic noise is σ_n= 660 eV.

2. A spectroscopy system has the front-end components shown below.



The Si detector draws a reverse bias current of 100 nA and has a capacitance of 100 pF. The bias resistor $R_B = 10$ MΩ and the connections between the detector and the preamplifier input have a total resistance of 10 Ω. The preamplifier has an equivalent input noise voltage of 1 nV/Hz$^{1/2}$ with negligible 1/*f* noise.

a) The system utilizes a simple CR-RC pulse shaper with integration and differentiation time constants of 1 μs. What is the electronic noise expressed in electrons and in eV? How large are the contributions of the individual noise sources?

Noise current sources

detector bias current $\quad i_{ni}^2 = 2q_e I_B$

bias resistor $\quad i_{nR}^2 = \dfrac{4kT}{R_B}$

Noise voltage sources

series resistance $\quad v_{nR}^2 = 4kTR_S$

amplifier $\quad v_{na}^2 = 10^{-18}\ V^2 / Hz$

Noise indices: $F_i = F_v = 0.924$

Equivalent noise charge

$$Q_n^2 = i_n^2\, T_s\, F_i + C_i^2\, v_n^2\, \frac{F_v}{T_s}$$

$$Q_n^2 = \left(2q_e I_B + \frac{4kT}{R_B}\right) \cdot T_s \cdot F_i \ + \ C_\mathrm{i}^2 \cdot (4kTR_S + v_{na}^2) \cdot \frac{F_v}{T_s}$$

$$Q_n^2 = (3.2 \cdot 10^{-26} + 1.66 \cdot 10^{-27}) \cdot 10^{-6} \cdot 0.924 + 10^{-20}(1.67 \cdot 10^{-19} + 10^{-18}) \cdot \frac{0.924}{10^{-6}}$$

The contributions of the individual noise sources are

Detector bias current:	Q_n = 1075 el
Bias resistor:	Q_n = 245 el
Series resistor:	Q_n = 246 el
Amplifier:	Q_n = 601 el

The total noise is 1279 el or 4605 eV rms (10.8 keV FWHM).

b) Assume a CR-RC shaper with adjustable peaking time, where both the integration and differentiation time constants are adjusted simultaneously to be equal. What are the noise current and noise voltage contributions at 1 μs shaping time? Determine the time constant that yields minimum noise.

As calculated in a) the noise current contribution is

$$Q_{ni} = \sqrt{1075^2 + 245^2} = 1103 \text{ el}$$

and the voltage noise contribution is

$$Q_{nv} = \sqrt{246^2 + 601^2} = 649 \text{ el}$$

Since minimum noise obtains when the current and voltage noise contributions are equal, the shaping time must be decreased.

Using the calculated values from a)

$$3.11 \cdot 10^{-26} T = \frac{1.08 \cdot 10^{-38}}{T},$$

so T_{opt}= 589 ns and $Q_{n,min}$= 1196 el. Problem c) gives a general formula.

c) Replace the simple CR-RC shaper by a CR-RC^7 shaper, i.e. a shaper with a single differentiator and 7 integrators. At what peaking time does this shaper provide the minimum noise?

Equality of current and voltage noise yields the optimum shaping time

$$T_{s.opt} = C_i \frac{v_n}{i_n} \sqrt{\frac{F_v}{F_i}}$$

For the CR-RC[7] shaper $F_i = 0.34$ and $F_v = 1.27$, so given the same C_i, v_n *and* i_n, the optimum shaping time scales with the square root of the noise indices and T_{opt}= 1.14 μs. $Q_{n,min}$= 1009 el, 84% of the minimum noise of the CR-RC shaper.

d) Using the CR-RC shaper at the optimum shaping time determined in b), what is the minimum value of bias resistor that will degrade the overall noise by less than 1%?

Without the bias resistor, the noise is 1181 el. For the resistor to add 1% to the total, its noise may be 2% of 1181 el or 24 el, so $R_B > 34$ MΩ.

3. A detector system using a 100 μm thick detector exhibits minimum noise at 10 μs shaping time. The detector is replaced by another of the same material and area, but with 1 mm thickness.

a) What is the improvement in electronic noise, assuming that the shaping time is kept constant?

$$Q_n^2 = i_n^2 \, T_s \, F_i + C_i^2 \, v_n^2 \, \frac{F_v}{T_s} = Q_{ni}^2 + Q_{nv}^2$$

Minimum noise obtains when the current and voltage contributions Q_{ni} and Q_{nv} are equal.

$$Q_{ni} = Q_{nv} = \frac{Q_n}{\sqrt{2}}$$

Assume that the detector capacitance dominates. Then, increasing the thickness 10-fold reduces the capacitance to 1/10 of the original value, so $Q_{nv}= Q_{ni}/10$ and the total noise

$$Q_n(C=0.1) = \sqrt{\frac{Q_n^2}{2} + \left(\frac{1}{10}\right)^2 \frac{Q_n^2}{2}} = 0.711 Q_n(C=1)$$

b) What is the optimum shaping time for the thick detector and what is the corresponding noise level?

Equality of current and voltage noise determines the optimum shaping time

$$T_{s.opt} = C_i \frac{v_n}{i_n} \sqrt{\frac{F_v}{F_i}}$$

If the input capacitance is 1/10 of the original value, the optimum shaping time also decreases to 1/10 of the original 10 μs, or 1 μs.

The noise at the optimum shaping time is

$$Q_n^2 = 2 C_i v_n i_n \sqrt{F_i F_v}$$

so 1/10 of the capacitance yields $1/10^{1/2} = 0.32$ of the original noise. When the shaping time is optimized with capacitance changes, the noise scales with $C^{1/2}$.