Physics 198, Spring Semester 1999 Helmuth Spieler
Introduction to Radiation Detectors and Electronics

Problem Set 2: Due on Tuesday, 2-Feb-99 at begin of lecture.
Discussion on Wednesday, 12 – 1 PM in 347 LeConte.

Solutions

1. Photoelectric absorption is often considered to be a localized interaction. After absorption of the photon energy E_γ, an electron is ejected from the shell with the energy

$$E_{el} = E_\gamma - E_b$$

where E_b is the binding energy of the electron. Using the dE/dx curve in the lecture notes, estimate the range of the ejected electron in Si for photon energies of 20 keV and 100 keV. What are the respective ranges in Ge and NaI? (The range times density is about the same for all materials.)

Answer:

The cross section for 20 keV and 100 keV is maximum for the K or L shells, for which the binding energies are 1.8 keV and 0.1 keV in Si, 11 keV and 1.4 keV in Ge and 33 keV and 5 keV in I (for which the cross section is much large than Na, as Z_I= 53 and Z_{Na}= 11 . Obviously, 20 keV photons cannot eject a K electron from I.

Thus, the respective electron energies for 20 and 100 keV photon energy are

Si:	18 keV	98 keV
Ge:	9 keV	98 keV
NaI:	15 keV	67 keV

dE/dx for 18 keV electrons in Si is 2.3 keV/μm. Since the energy loss increases with decreasing energy, the range is overestimated if one simply uses $R=E/(dE/dx)$, so let's take dE/dx at ½ the electron energy, i.e. dE/dx= 4 keV/μm. This yields a range of 4.5 μm, which is in good agreement with the actual range (see graph on the next page, which has also been added to the lecture notes). For 98 keV the range in Si is 75 μm.

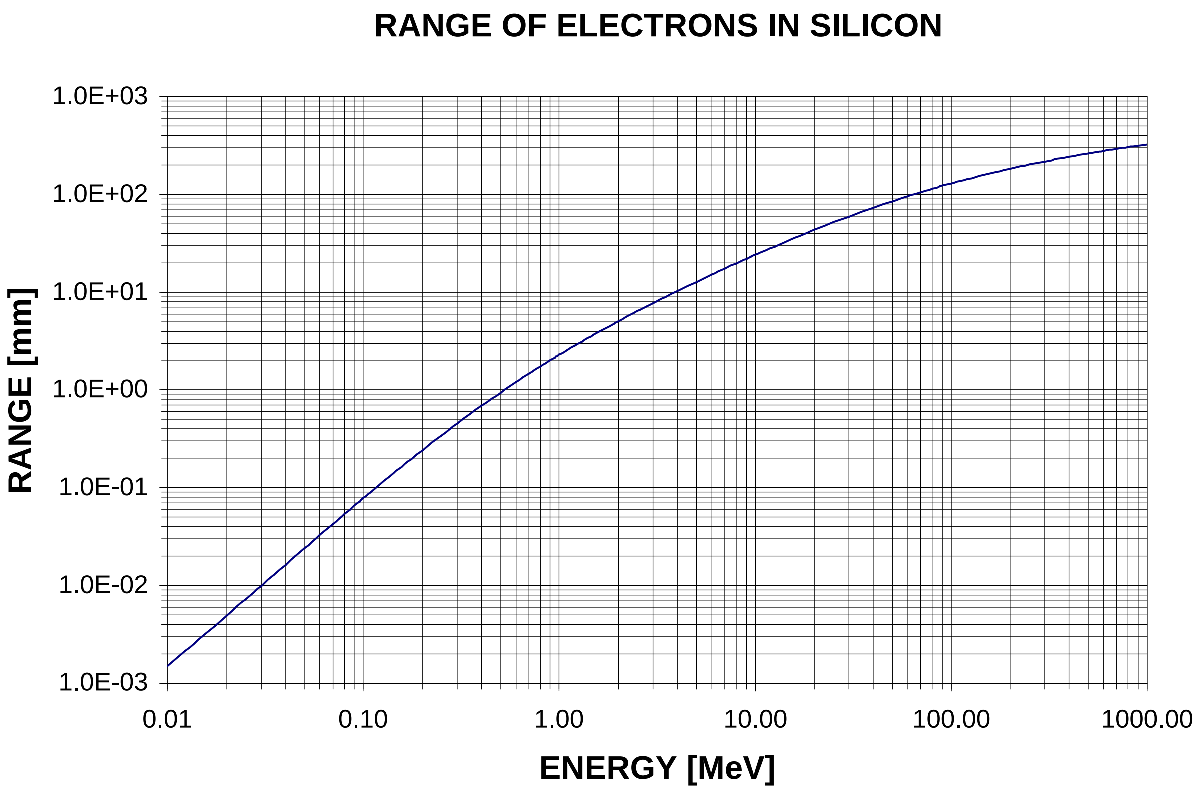
The densities of Si and Ge, 2.33 and 5.33 g/cm^3, can be found in numerous reference tomes, for example an encyclopedia, a good dictionary or the *CRC Handbook of Chemistry and Physics*. The density of NaI, 3.67 g/cm^3 is given in the table of inorganic scintillators included in the lecture notes.

The ranges scale as

$$R_1 \rho_1 = R_2 \rho_2 ,$$

so for Ge, R(9 keV) ≈ 0.6 μm and R(98 keV) ≈ 30 μm and for NaI, R(15 keV) ≈ 2.5 μm and R(67 keV) ≈ 25 μm.

Since position sensing detectors are capable of 1 – 10 μm resolution, the spatial extent of the ionization sets a limit for x-ray imaging.



2. a) The fraction of incident photons that have interacted within a distance *x* is

$$\frac{N_{att}}{N_0} = 1 - e^{-\mu x}$$

Derive this expression.

Answer:

In an interval *dx* the number of transmitted photons, i.e. photons that have not interacted, decreases as

$$dN = -N\mu dx$$

$$\frac{dN}{N} = -\mu dx$$

Integration yields

$$\ln N\Big|_{N_0}^{N_{tr}} = -\mu x\Big|_0^x$$

$$\ln \frac{N_{tr}}{N_0} = -\mu x$$

$$\frac{N_{tr}}{N_0} = e^{-\mu x}$$

The fraction of photons that have interacted, i.e. the attenuation factor of original photons, is then

$$\frac{N_{att}}{N} = 1 - \frac{N}{N_0} = 1 - e^{-\mu x}$$

b) Estimate the required dimensions of a NaI crystal to yield 95% absorption efficiency for 1 MeV photons.

Answer:

For N/N_0= 0.95, μx= 3. First consider the required distance for 1 MeV photons to *interact* in NaI. From the curves in the lecture notes, the attenuation coefficient is 0.053 cm^2/g. Multiplying by the density of 3.67 g/cm^3 yields μ= 0.2 cm^{-1}. For μx= 3 we get x= 15 cm.

This only tells us that 95% of the photons will interact within 15 cm, but for how many will the full energy be absorbed? Since at 1 MeV Compton scattering dominates, we need to consider the absorption of the scattered electrons and photons.

The maximum electron energy

$$E_{el} = \frac{E_\gamma}{1 + (m_0 c^2 / 2E_\gamma)} ,$$

so for E_γ= 1 MeV, the maximum electron energy is 800 keV. The range of 800 keV electrons in NaI is about 1 mm, which is small compared to the interaction length of the primary photons.

The maximum energy of the scattered photons is

$$E_C = E_\gamma - E_{el}\,|_{\Theta=\pi} = \frac{E_\gamma}{1 + 2E_\gamma / m_0 c^2} = 200\ \text{keV} ,$$

for which the mass attenuation coefficient is 0.3 cm^2/g, corresponding to μ= 1.1 cm^{-1}. If 95% of these photons are to be absorbed, μx= 3 yields x= 2.7 cm. Since this distance is within the attenuation length for the first two interactions, we need at least 18 cm length of NaI for an overall absorption of 95%.

Note that the curves in the notes also show an absorption mass coefficient of 0.053 cm^2/g for 1 MeV photons. This corresponds to μ= 0.12 cm^{-1}, so μx= 3 requires x= 25 cm.

3. An arrangement commonly used to determine the charge number Z of low energy nuclei (1 – 10 MeV/amu) consists of a thin transmission detector and a thick detector to stop the particles. The thin detector measures an incremental energy loss ΔE and the stop detector measures the residual energy. Derive a simple algorithm to determine Z from these measurements.

Answer:

For small velocities

$$-\frac{dE}{dx} \propto \left(\frac{z}{v}\right)^2 \cdot NZ = z^2 \frac{2E}{m} \cdot NZ$$

where z and v are the charge and velocity of the projectile. Assuming a thin absorber, $\Delta E \propto dE/dx$, so

$$\frac{z^2}{m} \propto \frac{\Delta E}{E}$$

or plot ΔE vs. E as shown in the Lecture Notes of February 2 for the Plastic Ball.

4. Consider a beam of 5 MeV protons and α particles, i.e. both particle types have the same energy. Describe a simple arrangement that allows discrimination between the two types of particles without requiring an energy measurement.

Answer:

From the Lecture Notes …

For E = 5 MeV in Si:	p	R = 220 μm
	α	R = 25 μm

A similar relationship between the ranges holds for other materials.

A combination of two detectors, the first just thick enough to stop the α's (say 30 to 50 μm) and the other thick enough to absorb a sizable fraction of the proton energy, allows a "binary" analysis. If a signal appears only in the first detector, it's from an α, if both detectors show signals at the same time, it's a proton.

5. a) What is the required time resolution in a time-of-flight PET system to achieve a position resolution of 1 mm?

Answer:

The time difference between the two detectors indicates the displacement from the center of the chord connecting the two detectors, so a displacement reduces the one flight time and increases the other $(L+\Delta x) - (L-\Delta x) = 2\Delta x$.

$$\Delta t = \frac{2\Delta x}{c}$$

For $\Delta x = 1$ mm, $\Delta t = 6.7 \cdot 10^{-12}$ s = 6.7 ps.

b) In a recent press conference Prof. Seren Dipity from the F. Gump Institute of Advanced Studies announced that using scintillation detectors and "state-of-the-art space-age technology" this time resolution has been achieved. The scintillator crystals use a rather standard geometry, i.e. a face of 10 x 10 mm^2 and 30 mm length. Do you believe his claim? Why?

Answer:

You shouldn't believe his claim for various reasons, but here's one. In principle, there is no fundamental obstacle to obtaining ps time resolutions. However, given the size of the scintillation crystal, transit time differences from the interaction point to the photodetector will be much greater.

$$\Delta t = \frac{L}{(c/n)}$$

For $n = 1.5$, the speed of light in the crystal corresponds to about 50 ps/cm, so for a 3 cm long crystal the transit time in the crystal is 150 ps. If the trajectories of the scintillation photons were just a linear continuation of the incident photon's, the interaction position wouldn't matter, as all photons propagate at the speed of light, so the sum of transit times within the crystal for the incident and scintillation photons would be constant. However, scintillation photons are emitted isotropically, so internal reflections can easily introduce time differences of 150 ps or more.

In principle, one could think of adding a detection mechanism that determines the position of the interaction to correct for the transit time, but this would require a position resolution comparable to the range of the Compton electrons, which introduce a fundamental ambiguity. Note that detecting scattered photons incident on the sides of the crystal reduces the signal (and may destroy the required position information), as detection of optical photons is destructive.

Two schemes could avoid the problem of transit time differences.

1. Absorb all scintillation photons that strike either the sides or the entrance face. This would approximate the "straight through" trajectory mentioned above, but requires a scintillator with sufficient light output to accommodate these losses.

2. Devise a truly novel scintillator that provides high efficiency with a thickness of about 1 mm or less.

The bottom line is that 7 ps time resolution with scintillators is quite impractical, but interesting to think about.