VI. Timing Measurements

Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

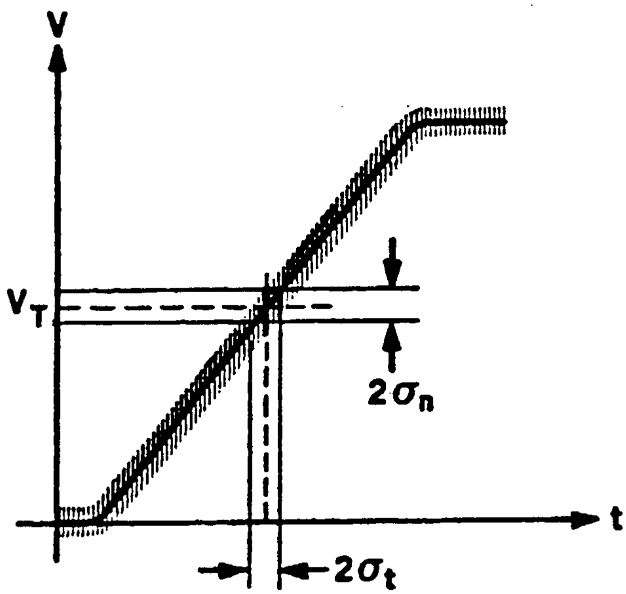
- Timing measurements optimize determination of time of occurrence.

- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

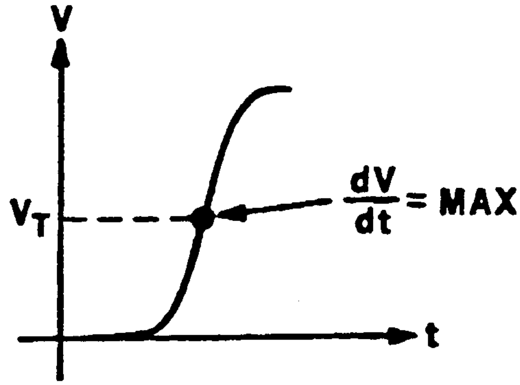
The instantaneous signal level is modulated by noise.

$\Rightarrow$ time of threshold crossing fluctuates



$$\sigma_t = \frac{\sigma_n}{\left.\dfrac{dV}{dt}\right|_{V_T}}$$

Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.



Pulse Shaping

Consider a system whose bandwidth is determined by a single *RC* integrator.

The time constant of the *RC* low-pass filter determines the

- rise time(and hence dV/dt)
- amplifier bandwidth (and hence the noise)

Time dependence

$$V_o(t) = V_0(1 - e^{-t/\tau})$$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2\tau$$

In terms of bandwidth

$$t_r = 2.2\tau = \frac{2.2}{2\pi\ f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

The frequency response of this simple system is

$$A(i\omega) \equiv \frac{V_o}{V_i} = \frac{A_0}{1 + i\omega\tau}$$

The magnitude of the gain

$$A(\omega) = \frac{A_0}{\sqrt{1 + (\omega\tau)^2}}$$

At the upper bandwidth limit

$$\omega\tau = 1 \qquad \rightarrow \qquad f_u = \frac{1}{2\pi\tau}$$

the signal response has dropped to

$$A(f_u) = \frac{A_0}{\sqrt{2}}$$

Expressed in terms of the upper bandwidth limit f_u , the frequency response

$$A(f) = \frac{A_0}{\sqrt{1 + (f / f_u)^2}}$$

A Parenthetical Practical Comment

Since it is often convenient to view the frequency response logarithmically, gain is often expressed in logarithmic scale, whose unit is the Bel.

Defined as a power ratio

$$A[\mathrm{B}] = \log \frac{P_2}{P_1}$$

Since dB tends to be a more common order of magnitude, this is usually written

$$A[\mathrm{dB}] = 10 \log \frac{P_2}{P_1}$$

which can be expressed as a voltage ratio

$$A[\mathrm{dB}] = 10 \log \frac{V_2^2 / R_2}{V_1^2 / R_1} = 20 \log \frac{V_2}{V_1} + 10 \log \frac{R_1}{R_2}$$

or if $R_1 = R_2$

$$A[\mathrm{dB}] = 20 \log \frac{V_2}{V_1}$$

V_2/V_1 = 10 corresponds to 20 dB.

Caution: In practice, voltage gains are often expressed in dB without regard to the resistance ratio. Clearly, converting such a gain figure into a power ratio can be very misleading.

At the upper cutoff frequency of the amplifier, the gain has dropped to 1/√2 of its maximum, corresponding to –3 dB.

⇒ Bandwidth limits are often referred to colloquially as “3 dB frequencies”.

Bandwidth of a Cascade of Amplifiers

Invariably, the required gain is provided by multiple amplifying stages.

If we define the bandwidth of a cascade of *n* amplifiers $f_u^{(n)}$ as the frequency where the gain has dropped by –3 dB, i.e. 1/√2

$$\left[\frac{1}{\sqrt{1+(f_u^{(n)}/f_u)^2}}\right]^n=\frac{1}{\sqrt{2}}$$

then

$$\sqrt{1+(f_u^{(n)}/f_u)^2}=\sqrt[2n]{2}$$

$$\frac{f_u^{(n)}}{f_u}=\sqrt{2^{1/n}-1}$$

Correspondingly, for the lower cutoff frequency

$$\frac{f_l^{(n)}}{f}=\frac{1}{\sqrt{2^{1/n}-1}}$$

Calculating the rise time of a cascade of *n* stages is more difficult, but to a good approximation (~ 10%)

$$t_r\approx\sqrt{t_{r1}^2+t_{r2}^2+...+t_{rn}^2}$$

Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude V_0 and a rise time t_c passing through an amplifier chain with a rise time t_{ra}.

The cumulative rise time at the amplifier output (discriminator output) is

$$t_r = \sqrt{t_c^2 + t_{ra}^2}$$

The electronic noise at the amplifier output is

$$V_{no}^2 = \int v_{ni}^2 \, df = v_{ni}^2 \, \Delta f_n$$

For a single RC time constant the noise bandwidth

$$\Delta f_n = \frac{\pi}{2} f_u = \frac{1}{4\tau} = \frac{0.55}{t_{ra}}$$

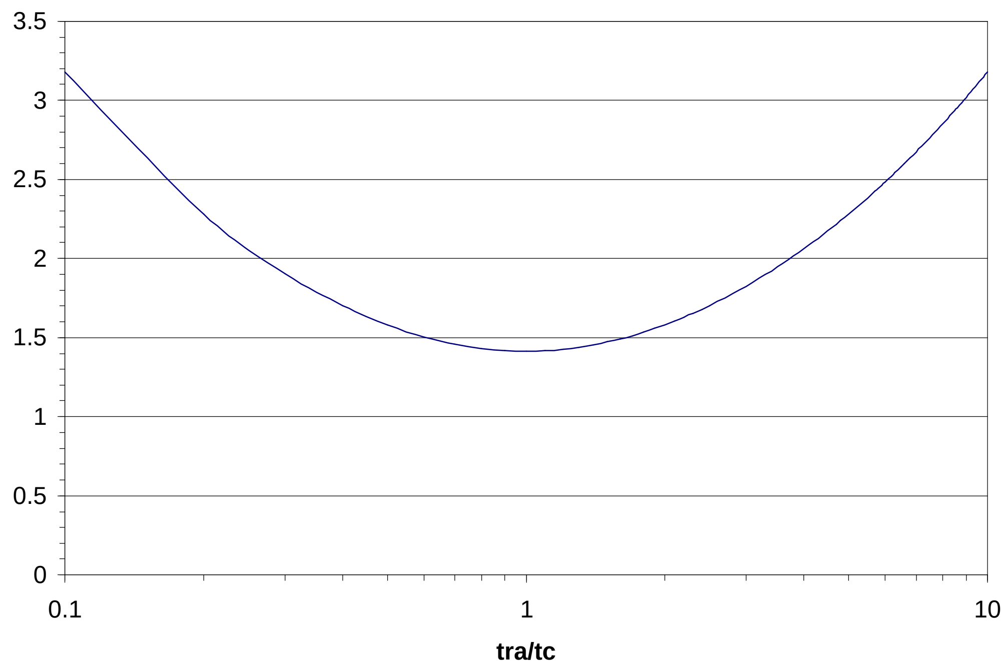
As the number of cascaded stages increases, the noise bandwidth approaches the signal bandwidth. In any case

$$\Delta f_n \propto \frac{1}{t_{ra}}$$

The timing jitter

$$\sigma_t = \frac{V_{no}}{dV/dt} \approx \frac{V_{no}}{V_0/t_r} = \frac{1}{V_0} V_{no} t_r \propto \frac{1}{V_0} \frac{1}{\sqrt{t_{ra}}} \sqrt{t_c^2 + t_{ra}^2} = \frac{\sqrt{t_c}}{V_0} \sqrt{\frac{t_c}{t_{ra}} + \frac{t_{ra}}{t_c}}$$

The second factor assumes a minimum when the rise time of the amplifier equals the collection time of the detector $t_{ra} = t_c$.



At amplifier rise times greater than the collection time, the time resolution suffers because of rise time degradation. For smaller amplifier rise times the electronic noise dominates.

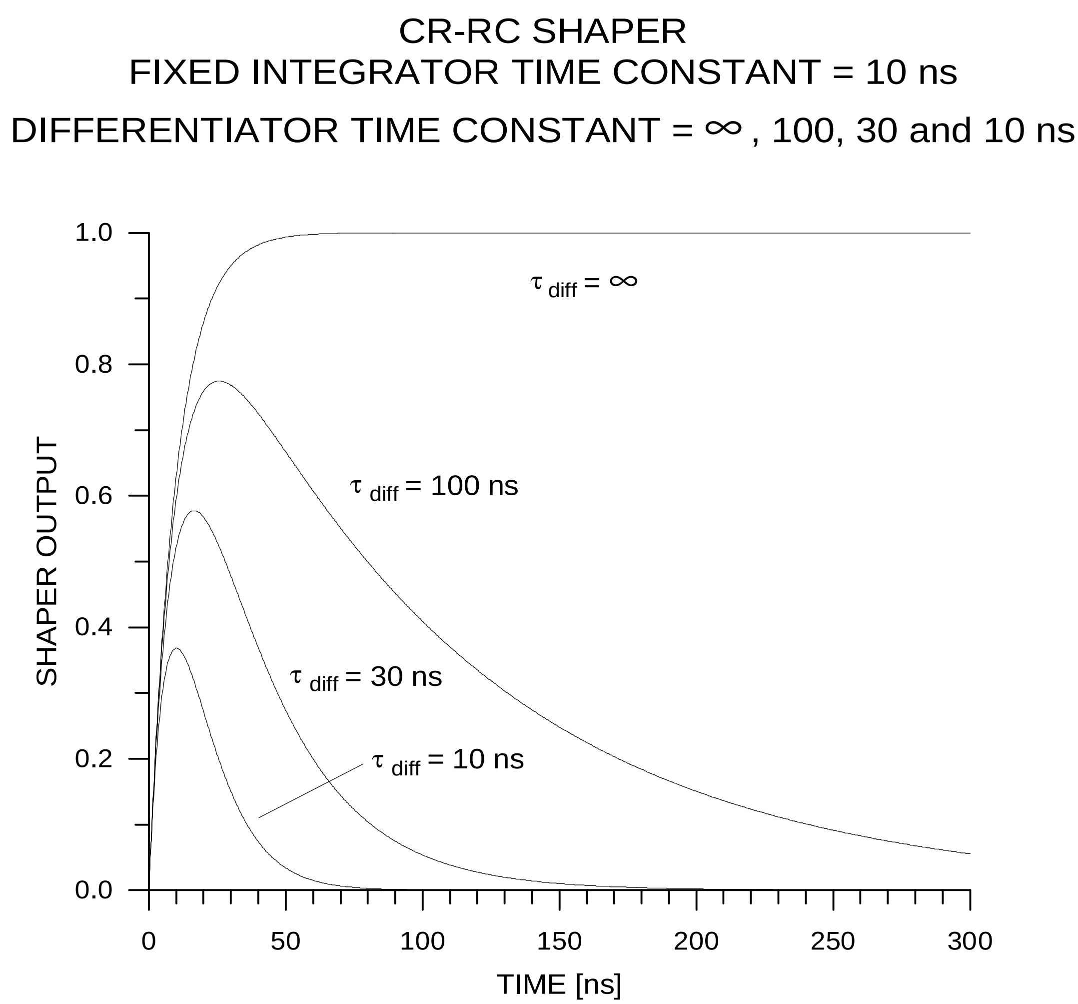
The timing resolution improves with decreasing collection time $\sqrt{t_c}$ and increasing signal amplitude V_0.

The integration time should be chosen to match the rise time.

How should the differentiation time be chosen?

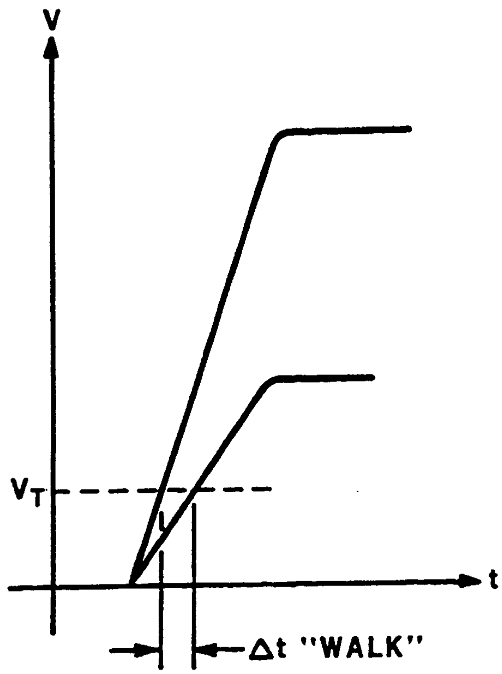
As shown in the figure below, the loss in signal can be appreciable even for rather large ratios τ_{diff}/τ_{int} , e.g. >20% for τ_{diff}/τ_{int} = 10.

Since the time resolution improves directly with increasing peak signal amplitude, the differentiation time should be set to be as large as allowed by the required event rate.



Time Walk

For a fixed trigger level the time of threshold crossing depends on pulse amplitude.



⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)

If the rise time is known, "time walk" can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

> This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

Lowest Practical Threshold

Single RC integrator has maximum slope at $t = 0$.

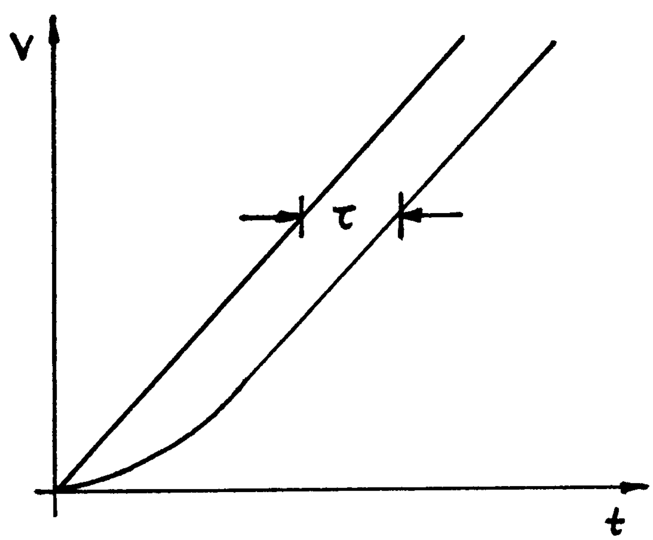
$$\frac{d}{dt}(1-e^{-t/\tau}) = \frac{1}{\tau}e^{-t/\tau}$$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small t the slope at the output of a single RC integrator is linear, so initially the pulse can be approximated by a ramp $\alpha\, t$.

Response of the following integrator

$$V_i = \alpha\, t \quad \rightarrow \quad V_o = \alpha(t-\tau) - \alpha\tau\, e^{-t/\tau}$$



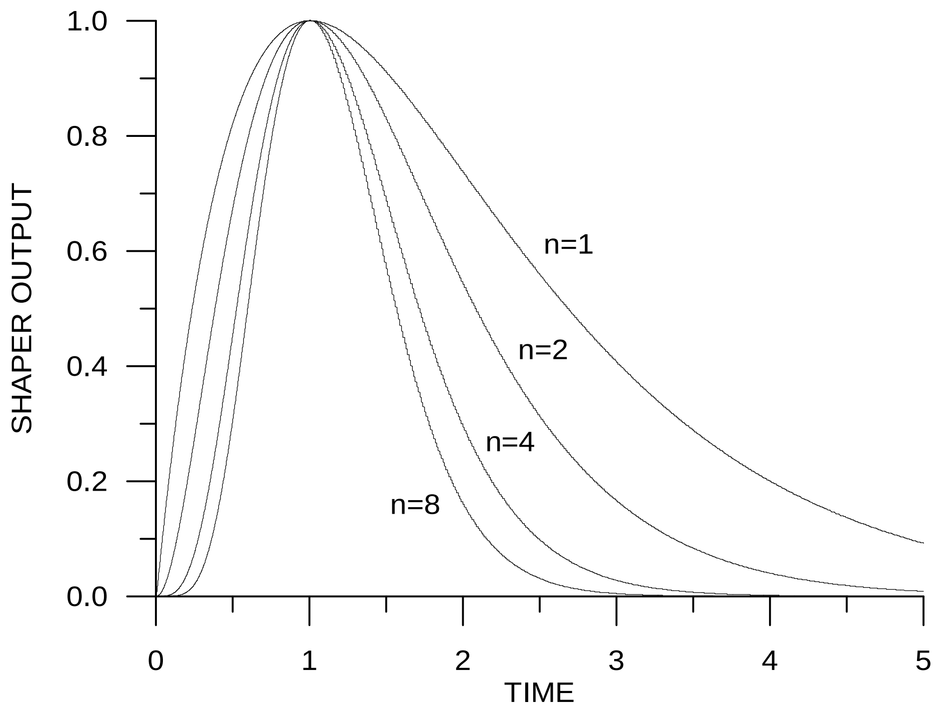
⇒ The output is delayed by τ and curvature is introduced at small t.

Output attains 90% of input slope after $t = 2.3\tau$.

Delay for n integrators= $n\tau$

Output pulse shape for multiple *RC* integrators

Time constants changed to preserve the peaking time
($\tau_n = \tau_{n=1}/n$)



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

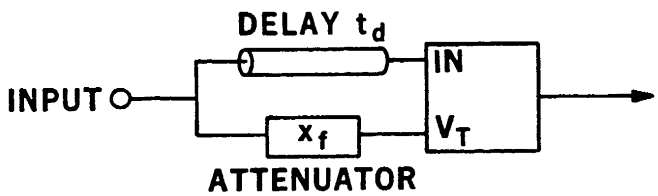
$\Rightarrow$ improved rate capability at the same peaking time

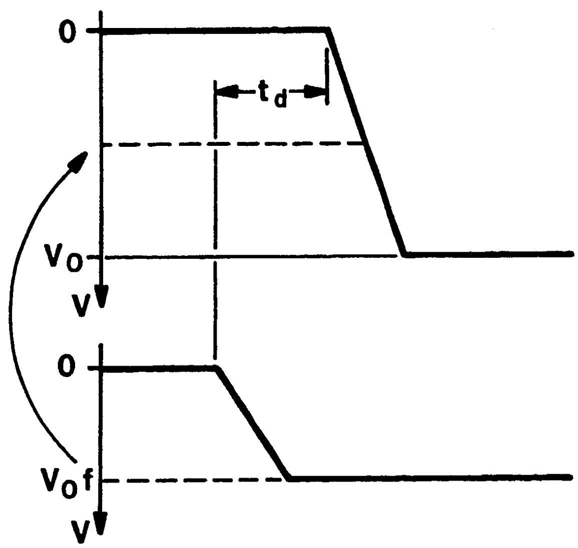
but … increased curvature at beginning of pulse limits minimum threshold in timing measurements

Constant Fraction Timing

Basic Principle:

make the threshold track the signal





The threshold is derived from the signal by passing it through an attenuator $V_T = f\,V_s$.

The signal applied to the comparator input is delayed so that the transition occurs after the threshold signal has reached its maximum value $V_T = f\,V_0$.

For simplicity assume a linear leading edge

$$V(t) = \frac{t}{t_r} V_0 \quad \text{for } t \le t_r \qquad \text{and} \qquad V(t) = V_0 \quad \text{for } t > t_r$$

so the signal applied to the input is

$$V(t) = \frac{t - t_d}{t_r} V_0$$

When the input signal crosses the threshold level

$$fV_0 = \frac{t - t_d}{t_r} V_0$$

and the comparator fires at the time

$$t = f\, t_r + t_d \qquad (t_d > t_r)$$

at a constant fraction of the rise time independent of peak amplitude.

If the delay t_d is reduced so that the pulse transitions at the signal and threshold inputs overlap, the threshold level

$$V_T = f \frac{t}{t_r} V_0$$

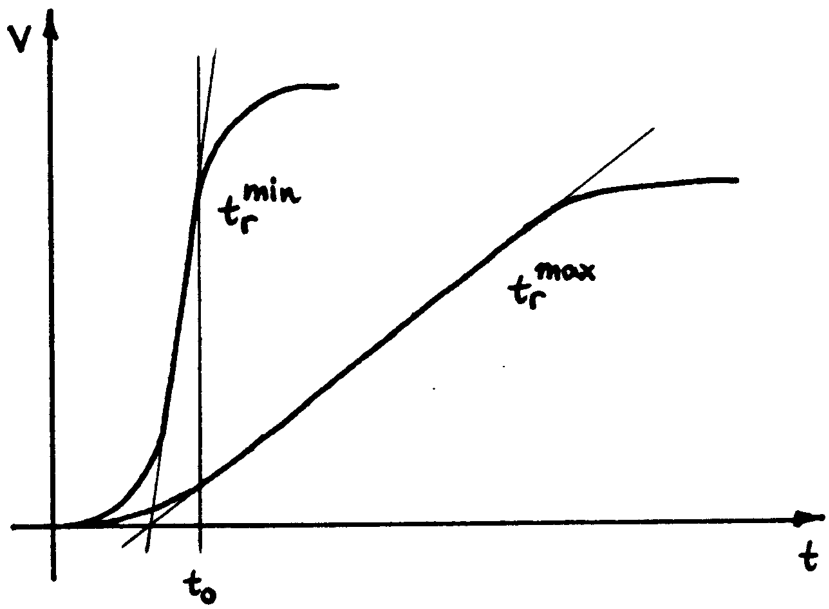
and the comparator fires at

$$f \frac{t}{t_r} V_0 = \frac{t - t_d}{t_r} V_0$$

$$t = \frac{t_d}{1 - f} \qquad (t_d < (1 - f)\, t_r)$$

independent of both amplitude and rise time (amplitude and rise-time compensation).

The circuit compensates for amplitude and rise time if pulses have a sufficiently large linear range that extrapolates to the same origin.



The condition for the delay must be met for the minimum rise time:

$$t_d \leq (1-f)\, t_{r,\min}$$

In this mode the fractional threshold V_T/V_0 varies with rise time.

For all amplitudes and rise times within the compensation range the comparator fires at the time

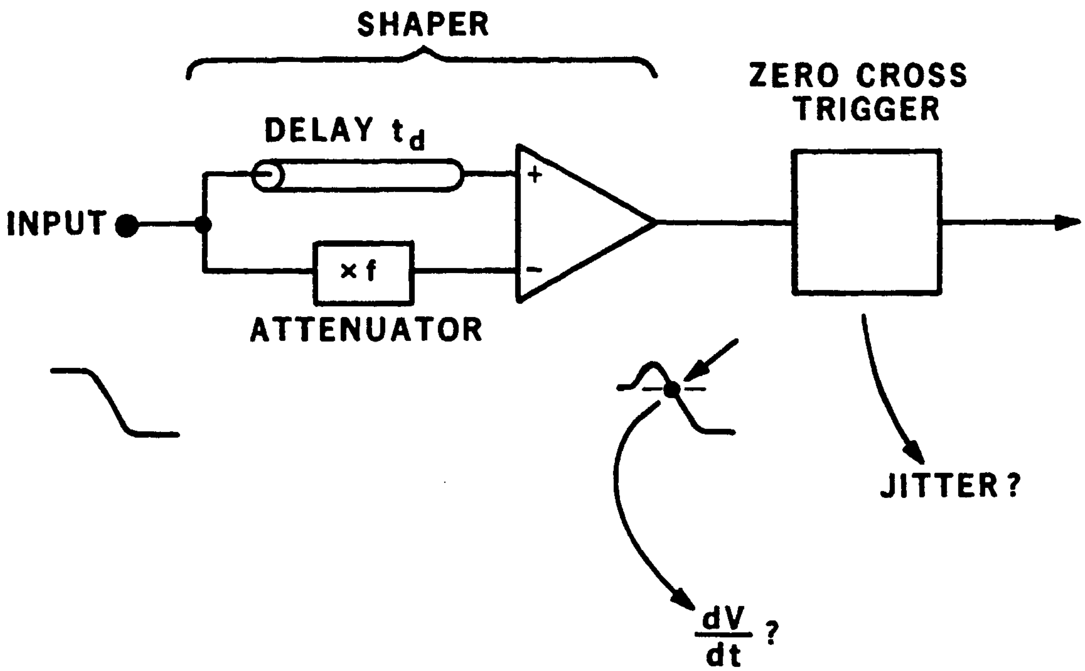
$$t_0 = \frac{t_d}{1-f}$$

Another View of Constant Fraction Discriminators

The constant fraction discriminator can be analyzed as a pulse shaper, comprising the

- delay
- attenuator
- subtraction

driving a trigger that responds to the zero crossing.

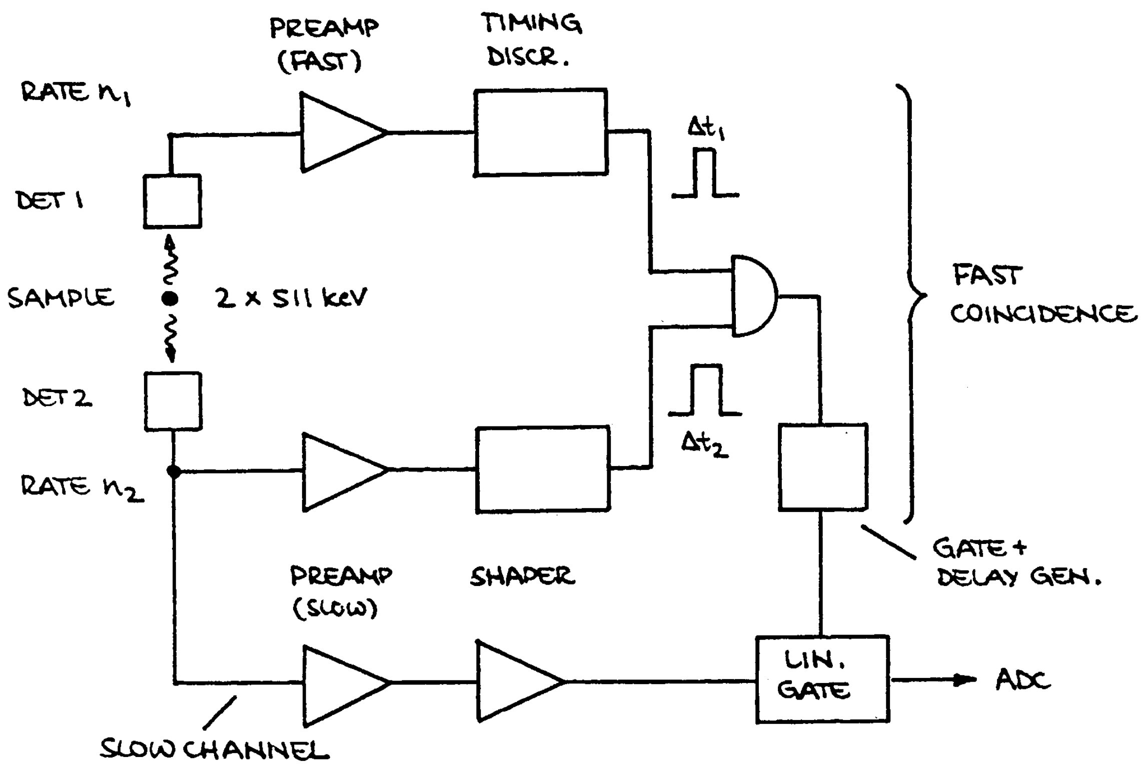


The timing jitter depends on

- the slope at the zero-crossing
  (depends on choice of f and t_d)
- the noise at the output of the shaper
  (this circuit increases the noise bandwidth)

Examples

1. γ -γ coincidence (as used in positron emission tomography)



Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.

- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output if the two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.

This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are n_1 and n_2, and the timing pulse widths are Δt_1 and Δt_2, the probability of a pulse from the first source occuring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate n_2 , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$

$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

i.e. in the arrangement shown above, the chance coincidence rate increases with the square of the source strength.

Example: $n_1 = n_2 = 10^6$ s^{-1}

$\Delta t_1 = \Delta t_1 = 5$ ns

$\Rightarrow \quad n_c = 10^4$ s^{-1}

2. Nuclear Mass Spectroscopy by Time-of-Flight

Two silicon detectors

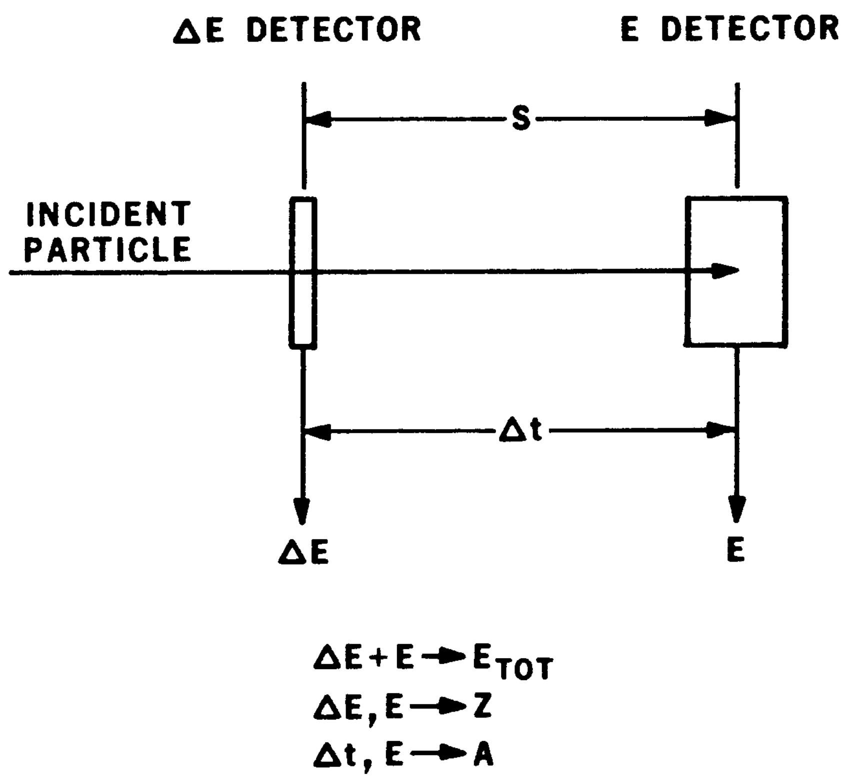
First detector thin, so that particle passes through it (transmission detector)

$\Rightarrow$ differential energy loss ΔE

Second detector thick enough to stop particle

$\Rightarrow$ Residual energy E

Measure time-of-flight Δt between the two detectors



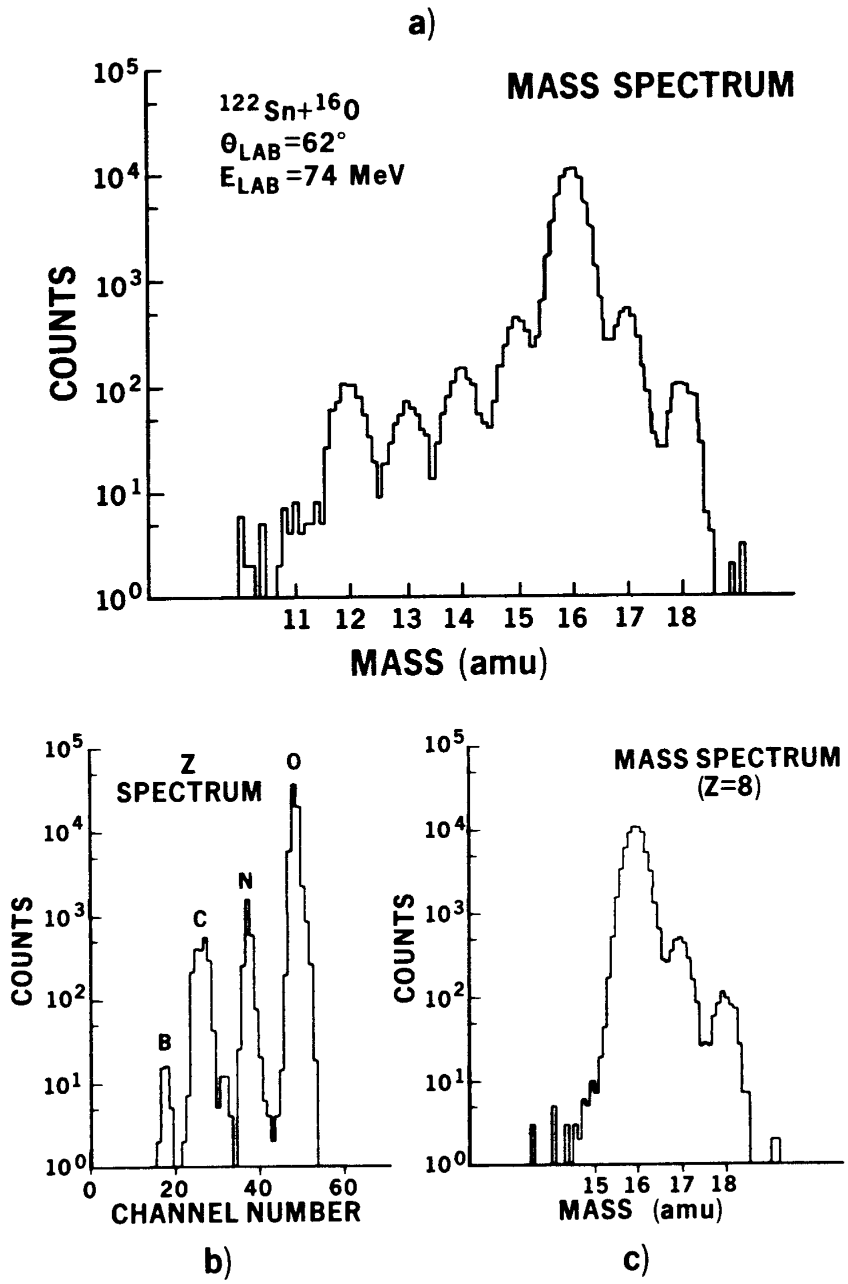
$$E_{tot} = \Delta E + E \qquad Z \propto \sqrt{\Delta E \cdot E_{tot}} \qquad A \propto E \cdot (\Delta t / s)^2$$

“Typical” Results

Example 1

Flight path 20 cm, $\Delta t \approx 50$ ps FWHM

$\sigma_t = 21$ ps



(H. Spieler et al., Z. Phys. **A278** (1976) 241)

Example 2

1. ΔE-detector: 27 μm thick, A= 100 mm^2, $\langle E \rangle$=1.1·10^4 V/cm

2. E-detector: 142 μm thick, A= 100 mm^2, $\langle E \rangle$=2·10^4 V/cm

For 230 MeV ^{28}Si: ΔE = 50 MeV $\Rightarrow$ V_s= 5.6 mV
E = 180 MeV $\Rightarrow$ V_s= 106 mV

$\Rightarrow$ Δt = 32 ps FWHM
σ_t = 14 ps

Example 3

Two transmission detectors,

each 160 μm thick, A= 320 mm^2

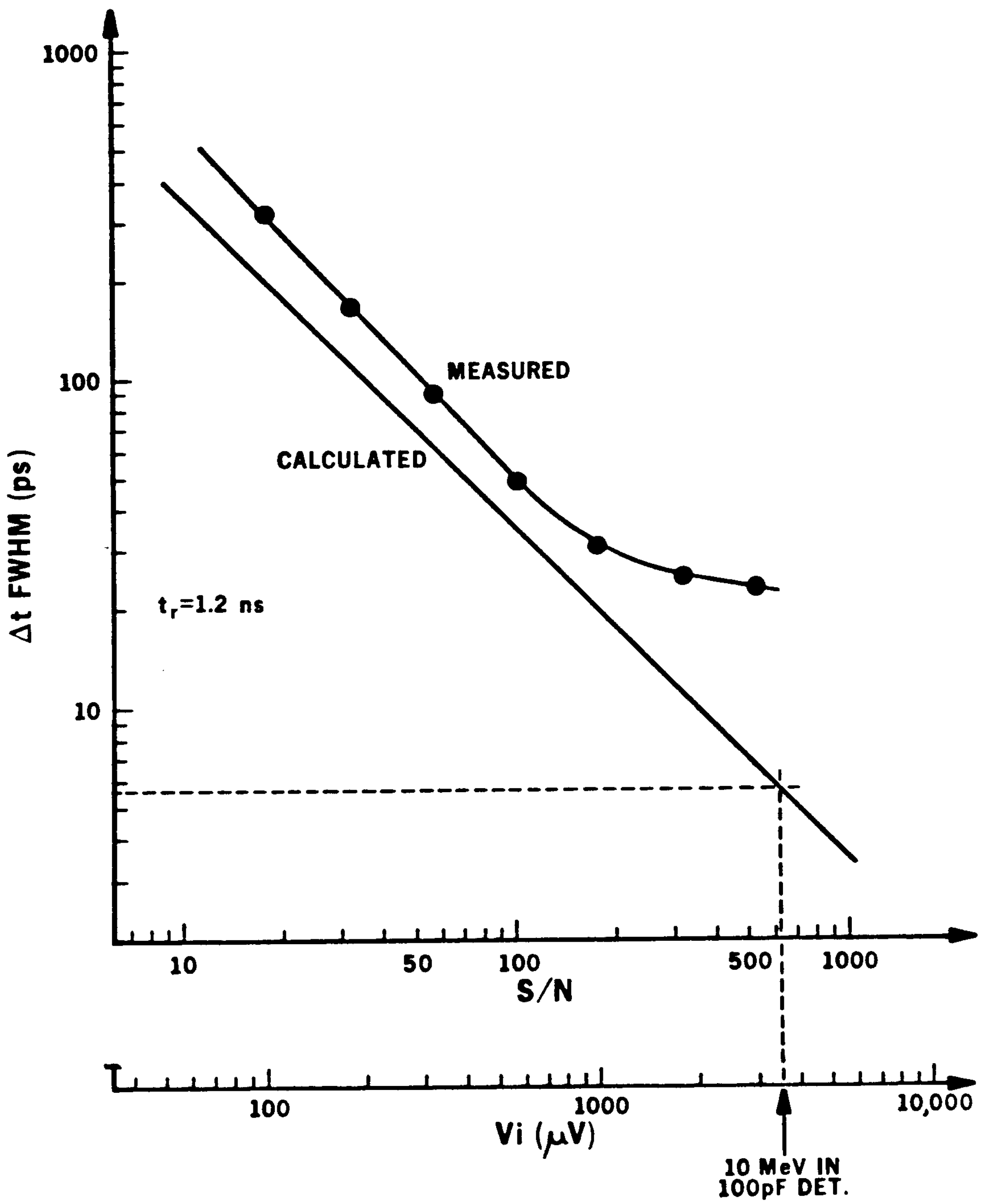
For 650 MeV/u ^{20}Ne: ΔE = 4.6 MeV $\Rightarrow$ V_s= 800 μV

$\Rightarrow$ Δt = 180 ps FWHM
σ_t = 77 ps

For 250 MeV/u ^{20}Ne: ΔE = 6.9 MeV $\Rightarrow$ V_s= 1.2 mV

$\Rightarrow$ Δt = 120 ps FWHM
σ_t = 52 ps

Fast Timing: Comparison between theory and experiment



At S/N<100 the measured curve lies above the calculation because the timing discriminator limited the rise time.
At high S/N the residual jitter of the time digitizer limits the resolution.

For more details on fast timing with semiconductor detectors, see H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.

Timing with Photomultiplier Tubes

1. Scintillator

Assume a scintillator with a single decay constant τ , which coupled to a photomultiplier tube yields a total number of photoelectrons N.

The probability of the emission of a single photoelectron in the time interval $t,\ t+dt$ is

$$P(t) = \frac{1}{\tau} e^{-t/\tau} dt$$

The mean emission rate is

$$\frac{d\overline{N}}{dt} = \frac{\overline{N}}{\tau} e^{-t/\tau}$$

The probability that the nth photon is emitted in the time interval $t,\ t+dt$ is

$$P_n(t)dt = \frac{\left[f(t)\right]^{n-1} e^{-f(t)} \left[df(t)/dt\right]\ dt}{(n-1)!}$$

where $f(t)$ is the average or expected number of photons emitted up to the time t. (R.F. Post and L.I. Schiff, Phys. Rev. **80** (1950) 1113)

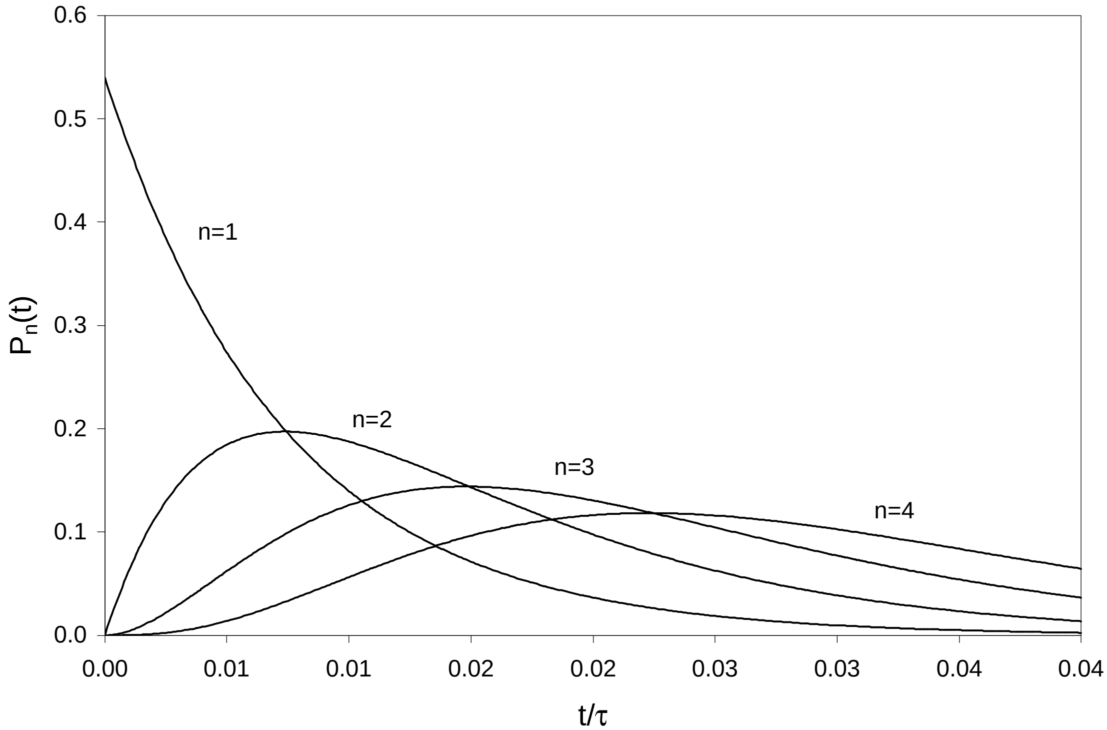
For the simple exponential decay shown above

$$f(t) = \overline{N}(1 - e^{-t/\tau})$$

and

$$P_n(t) = \frac{\overline{N}^n (1 - e^{-t/\tau})^{n-1} \exp[-\overline{N}(1 - e^{-t/\tau})]\ e^{-t/\tau}}{\tau\ (n-1)!}$$

Time distributions for triggering on the 1st, 2nd, 3rd and 4th photoelectron



The time resolution is best for $n = 1$, which at small times yields an exponential timing distribution with the time constant $\tau / \overline{N}$

$$P_1(t) = \frac{\overline{N}}{\tau} e^{-t/\tau} \exp[-\overline{N}(1 - e^{-t/\tau})] \approx \frac{\overline{N}}{\tau} e^{-\overline{N}t/\tau}$$

(F. Lynch, IEEE Trans. Nucl. Sci., NS-13/3 (1966) 140)

2. The Photomultiplier

Photoelectrons emitted from the photocathode are subject to time variations in reaching the anode.

For a typical fast 2” PMT (Philips XP2020) the transit time from the photocathode to the anode is about 30 ns at 2000V.

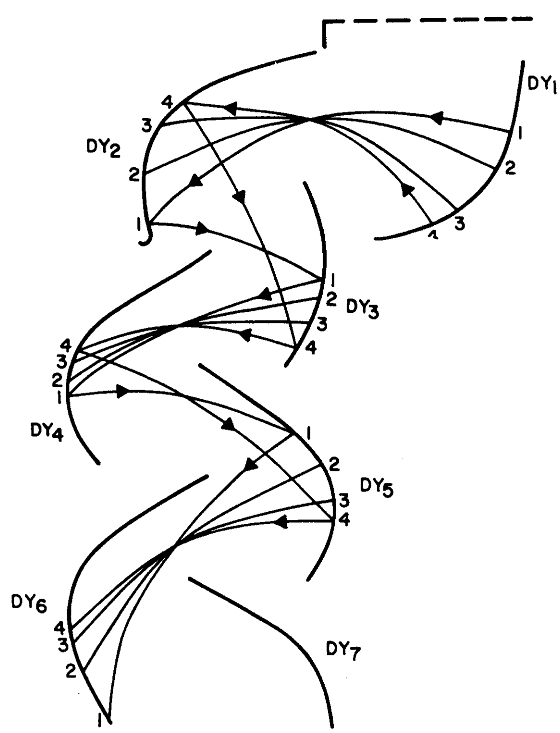
The intrinsic rise time is 1.6 ns, due to broadening of the initial electron packet in the course of the multiplication process.

The transit time varies by 0.25 ns between the center of the photocathode and a radius of 18 mm.

For two tubes operating in coincidence at a signal level of 1500 photoelectrons, a time resolution of 230 ps is possible.

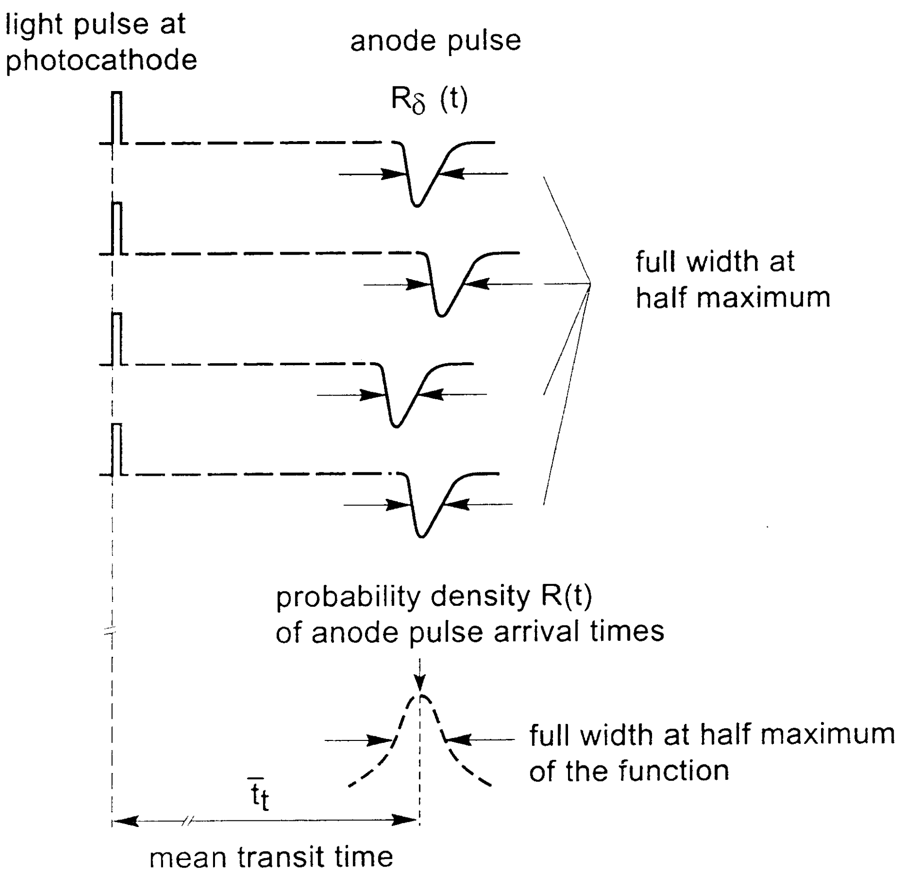
Special dynode structures are used to reduce transit time spread.

Example: time compensating structure



(from *Burle Photomultiplier Handbook*)

Even for photons impinging on a given position, the transit time through a photomultiplier varies from photon to photon, since photoelectrons are emitted from the photocathode with varying velocities and directions.



(from *Photomultiplier Tubes*, Philips Photonics)

The transit time "jitter" distribution is often gaussian, yielding an instantaneous anode current

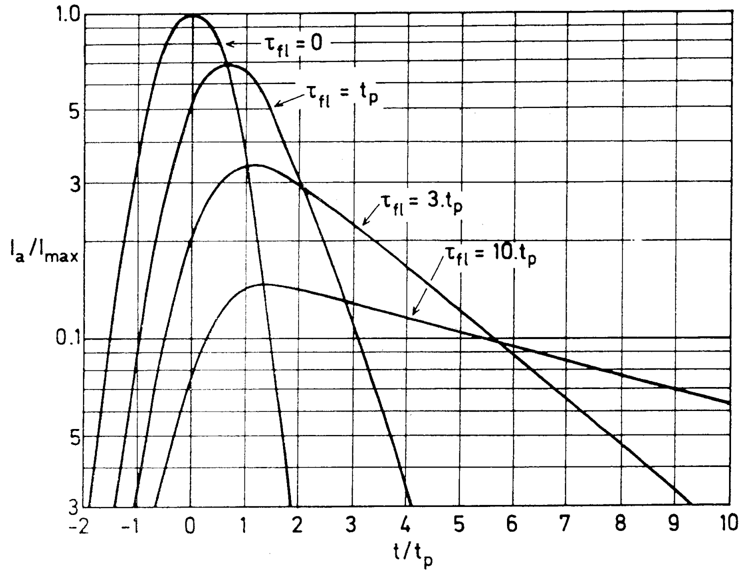
$$i_a(t) = \frac{\overline{A}e}{t_p\sqrt{\pi}} e^{-(t/t_p)^2}$$

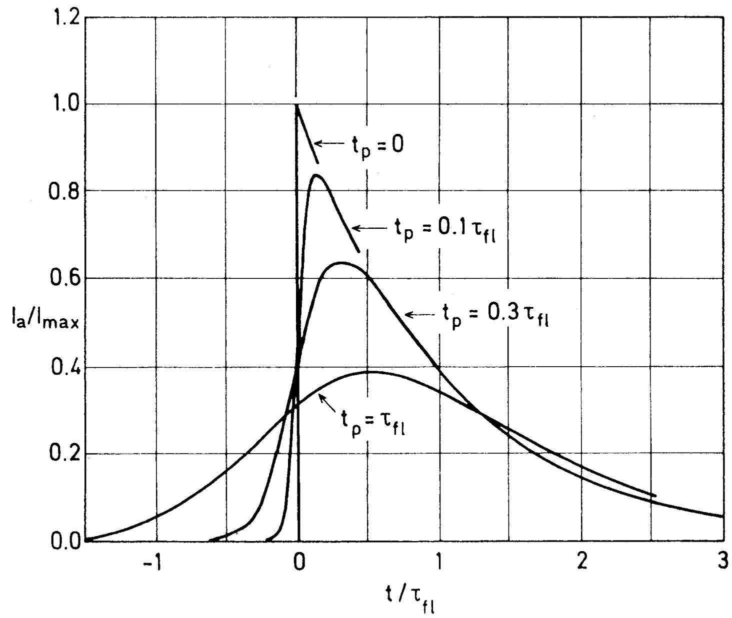
Then the response to the scintillator becomes the convolution of the exponential decay function with the Gaussian transit time spread

$$I_a(t) = \frac{\overline{A}e\overline{N}}{\tau\, t_p} \cdot \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-[(t-t')/t_p]^2} e^{-t'/\tau} dt'$$

The transit time spread imparts a finite rise time to the output pulse, due to the smearing of the arrival times of electrons at the anode.

PMT output pulses for various values of scintillator decay time τ_{fl} and transit time jitter t_p.





(from Kowalski, *Nuclear Electronics*)

If the decay time of the scintillator is short with respect to the transit time spread, signals from successive photoelectrons will be comingled and the signal from the first photon can no longer be distinguished from the response to successive photons.

Now it may become advantageous to trigger after some integration time. As a consequence, fast scintillators often provide the best timing at a fixed fraction of their peak output signal, typically 0.1 to 0.3. Only for relative slow scintillators, NaI(Tl) with a decay time of 250 ns for example, is the fraction very small, of order 1%.

Many scintillators exhibit an inherent rise time

$$N(t) = \frac{e^{-t/\tau_2} - e^{-t/\tau_1}}{\tau_2 - \tau_1}$$

Here, τ_1 represents the non-radiative transitions that feed the optically active states, which emit photons with the time constant τ_2.

This expression is accurate for binary solution scintillators and is a good approximation for ternary solution scintillators. Here triggering at 10 to 30% of the peak pulse height is nearly always advantageous, since the rise time of the scintillator masks the transit time spread of the photomultiplier.