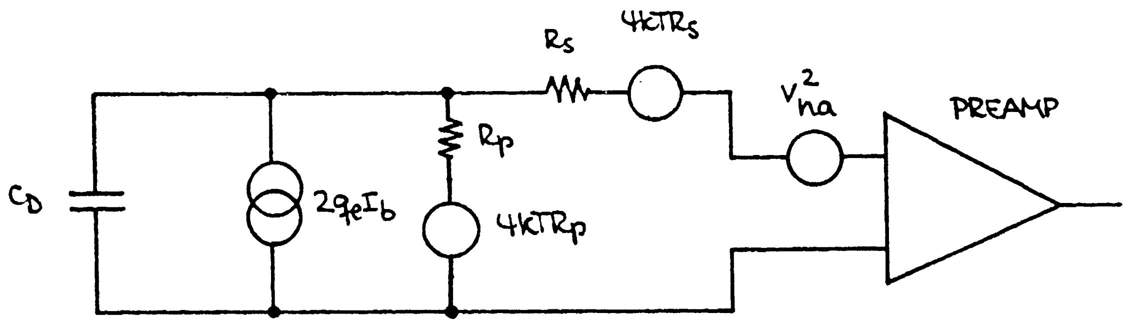
Equivalent circuit for noise analysis



detector	bias current shot noise	shunt resistance thermal noise	series resistance thermal noise	equivalent input noise voltage of amplifier

In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage appearing at the amplifier input.

Steps in the analysis:

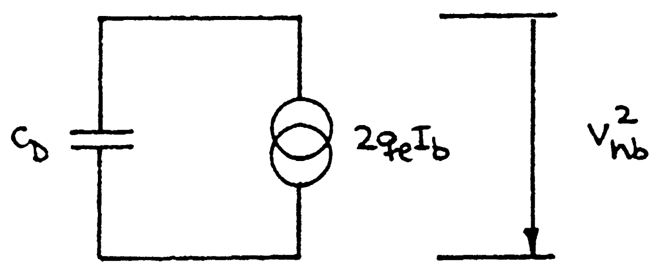
1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources

2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.

3. Determine the equivalent noise charge

Noise Contributions

1. Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite

2. The shunt resistance R_P is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

    Does this assumption make sense?

    If R_P is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_P C_D >> t_P \approx \frac{1}{\omega_P}$$

where ω_P is the midband frequency of the shaper. Therefore,

$$R_P >> \frac{1}{\omega_P C_D}$$

as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$v_{nb}^2 = i_{nb}^2 \frac{1}{(\omega C_D)^2} = 2q_e I_b \frac{1}{(\omega C_D)^2}$$

$\Rightarrow$ the noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time T. The number of electron charges measured is

$$N_e = \frac{I_b T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

2. Parallel Resistance

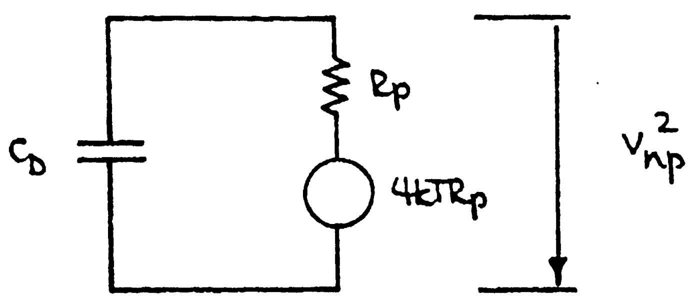
Shunt components:

1. bias noise current source
   (infinite resistance by definition)

2. detector capacitance

Current due to the noise voltage from R_P can only flow through the detector capacitance C_D.

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$v_{np}^2 = 4kTR_P \left(\frac{-i/\omega C_D}{R_P - i/\omega C_D} \right)^2$$

$$v_{np}^2 = 4kTR_P \frac{1}{1+(\omega R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^\infty v_{np}^2(\omega)d\omega = \int_0^\infty \frac{4kTR_P}{1+(\omega R_P C_D)^2}d\omega = \frac{kT}{C_D}$$

which is independent of R_P. Commonly referred to as "kTC" noise, this contribution is often erroneously interpreted as the "noise of the detector capacitance". An ideal capacitor has no thermal noise; all noise originates in the resistor.
So, why is the result independent of R_P?

R_P determines the primary noise, but also the noise bandwidth of this subcircuit. As R_P increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of R_P.

However,
If one integrates v_{np} over a bandwidth-limited system

$$V_n^2 = \int_0^\infty 4kTR_P \left| \frac{G(i\omega)}{1-i\omega R_P C_D} \right|^2 d\omega$$

the total noise decreases with increasing R_P.

3. Series Resistance

The noise voltage generator associated with the series resistance R_S is in series with the other noise sources, so it simply contributes

$$v_{nr}^2 = 4kTR_S$$

4. Amplifier input noise voltage density

The amplifier noise sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain. Here, the output noise is referred to a voltage-sensitive input, so it appears as a noise voltage generator.

$$v_{na}^2 = \underbrace{v_{nw}^2}_{\text{“white noise”}} + \underbrace{\frac{A_f}{f}}_{1/f \text{ noise (can also originate in external components)}}$$

This noise voltage generator also adds in series with the other sources.

- Amplifiers generally also exhibit input current noise, which is physically present at the input. However, its effect is the same as for the detector bias current, so the analysis given in 1. can be applied.

Determination of equivalent noise charge

1. Calculate total noise voltage at shaper output
2. Determine peak pulse height at shaper output for a known input charge
3. Input signal for which S/N=1 yields equivalent noise charge

1. Noise voltage at shaper output

$$V_{no}^2 = \int_0^\infty \sum_k v_{ni,k}^2 \cdot G^2(\omega)\, d\omega$$

$$V_{no}^2 = \int_0^\infty \left(v_{nb}^2 + v_{np}^2 + v_{nr}^2 + v_{na}^2\right) \cdot G^2(\omega)\, d\omega$$

$$V_{no}^2 = \int_0^\infty \left(\frac{2q_e I_b}{(\omega\, C_i)^2} + \frac{4kTR_P}{1+(\omega\, R_P C_i)^2} + 4kTR_S + v_{na}^2 \right) \cdot \left(\frac{\omega^2 \tau_d^2}{2\pi\,(1+\omega^2\tau_d^2)(1+\omega^2\tau_i^2)} \right) d\omega$$

$$V_{no}^2 = \frac{1}{4C_i^2}\left(\frac{4kT}{R_P} + 2q_e I_b\right)\frac{\tau_d^2}{\tau_d + \tau_i} + (4kTR_S + v_{na}^2)\,\frac{\tau_d}{\tau_i(\tau_d+\tau_i)} + A_f\,\frac{\tau_d^2}{\tau_d^2+\tau_i^2}\ln\left(\frac{\tau_d}{\tau_i}\right)$$

Note that the last term, describing the $1/f$ noise, depends only on the ratio of the integration and differentiation time constants, i.e. the ratio of upper to lower cutoff frequencies of the shaper.

2. Peak Signal at Shaper Output

Assume the detector signal is a pulse of constant current with duration t_c (i.e. the signal charge ramps up linearly until $t = t_c$).

Assume that the shaper has no additional voltage gain. Then the input signal voltage

$$V_{si} = V_i = \frac{Q_s}{C_i}$$

At times $t < t_c$

$$V_{so}(t) = V_{si}\left[\frac{\tau_d}{t_c}\left(1 - e^{-t/\tau_i}\right) - \frac{\tau_d^2}{t_c(\tau_d - \tau_i)}\left(e^{-t/\tau_d} - e^{-t/\tau_i}\right)\right]$$

At times $t > t_c$

$$V_{so}(t) = V_{si}\left[\frac{\tau_d^2}{t_c(\tau_d - \tau_i)}\left(e^{t_c/\tau_d} - 1\right)e^{-t/\tau_d} - \frac{\tau_d \tau_i}{t_c(\tau_d - \tau_i)}\left(e^{t_c/\tau_i} - 1\right)e^{-t/\tau_i}\right]$$

The peak amplitude of the signal is

$$V_{so} = V_{si}\,\frac{\tau_d}{t_c}\,\frac{\left(e^{t_c/\tau_d} - 1\right)^{\tau_d/(\tau_d - \tau_i)}}{\left(e^{t_c/\tau_i} - 1\right)^{\tau_i/(\tau_d - \tau_i)}}$$

Note that the signal depends only on the ratio of times t_c/τ or τ_i/τ_d.

- In analyzing pulse shapers it is convenient to assume that the collection time of the detector is much smaller than the shaping time and to account for the ballistic deficit separately.

As the pulse shape is independent of the absolute time scale, the peak output signal can be written in the form

$$V_{so} = \frac{Q_s}{C_i} F_s(\tau_d / \tau_i) \equiv \frac{Q_s}{C_i} F_s$$

where F_s is a general property of the pulse shape, independent of the time scale.

3. Signal-to-Noise Ratio

The peak signal at the shaper output depends only on relative times, i.e. it is independent of the absolute time scale.

The noise at the output, however, does depend on the time scale (as it determines the noise bandwidth).

$$V_{no}^2 = \frac{1}{4C_i^2} \left(\frac{4kT}{R_P} + 2q_e I_b \right) \frac{\tau_d^2}{\tau_d + \tau_i} +$$

$$+ (4kTR_S + v_{na}^2) \frac{\tau_d}{\tau_i(\tau_d + \tau_i)} + A_f \frac{\tau_d^2}{\tau_d^2 + \tau_i^2} \ln\left(\frac{\tau_d}{\tau_i} \right)$$

First, to simplify the expression for the output noise, combine the terms for

current noise $$i_n^2 = 2q_e I_b + \frac{4kT}{R_P}$$

and voltage noise $$v_n^2 = 4kTR_S + v_{na}^2$$

and introduce a characteristic time T, which we'll arbitrarily set equal to the peaking time t_p , the time at which the pulse assumes its maximum value.

With these conventions

$$V_{no}^2 = \frac{1}{4C_i^2} \; i_{\mathrm{n}}^2 \; T \; \frac{\tau_d}{t_p} \frac{\tau_d}{\tau_d + \tau_i} + \; v_n^2 \; \frac{1}{T} \; \frac{t_p}{\tau_i} \frac{\tau_d}{\tau_d + \tau_i} + A_f \; \frac{\tau_d^2}{\tau_d^2 + \tau_i^2} \ln\left(\frac{\tau_d}{\tau_i}\right)$$

We now have an expression for the output noise that depends on a characteristic time T and otherwise depends only on the ratio of time constants.

This allows us to characterize the shaper in terms of noise indices that depend only on the ratios of times – i.e the pulse shape – and whose effect is scalable by the characteristic time T.

Recall that the last term, describing the $1/f$ noise, depends only on the ratio of the integration and differentiation time constants, so it is unaffected by the introduction of the characteristic time T.

Since the equivalent noise charge is equal to the signal level for $S/N = 1$

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

and we obtain a general expression for the equivalent noise charge that applies to any shaper

$$Q_n^2 = i_n^2\, T\, F_i \;+\; C_i^2\, v_n^2\, \frac{1}{T} F_v \;+\; C_i^2\, A_f\, F_{vf}$$

The first term describes the current noise contribution, whereas the second and third terms describe the voltage noise contributions due to white and $1/f$ noise sources.

In the specific case of the CR-RC shaper, the indices F_i, F_v and F_{vf} are given by

$$F_i \equiv \frac{1}{F_s} \cdot \frac{\tau_d}{4t_p} \frac{1}{1+\tau_i/\tau_d}$$

$$F_v \equiv \frac{1}{F_s} \cdot \frac{t_p}{\tau_i} \frac{1}{1+\tau_i/\tau_d}$$

$$F_{vf} \equiv \frac{1}{F_s} \cdot \frac{1}{1+(\tau_i/\tau_d)^2} \ln\left(\frac{\tau_d}{\tau_i}\right)$$

- Generally, the noise indices or “form factors” F_i , F_v and F_{vf} characterize the type of shaper, for example $CR\text{-}RC$ or $CR\text{-}(RC)^4$.
- They depend only on the ratio of time constants τ_d/τ_i, rather than their absolute magnitude.
- The noise contribution then scales with the characteristic time T. The choice of characteristic time is somewhat arbitrary. so any convenient measure for a given shaper can be adopted in deriving the noise coefficients F.

For the single CR-RC shaper with equal differentiation and integration time constants $\tau_d = \tau_i = \tau$ the output pulse is

$$V_o(t) = \frac{Q_s}{C_i}\frac{t}{\tau}e^{-t/\tau} \qquad \text{and} \qquad \frac{V_{so}}{V_{si}} = \frac{1}{e}$$

where e is the base of the natural logarithm. In this special case the current and voltage noise indices are equal $F_i = F_v = 0.924$ and the equivalent noise charge

$$Q_n^2 = \left(\frac{e^2}{8}\right)\left[\underbrace{\left(2q_eI_b + \frac{4kT}{R_P}\right)\cdot\tau}_{\uparrow} + \underbrace{\left(4kTR_S + v_{na}^2\right)\cdot\frac{C_i^2}{\tau}}_{\uparrow} + \underbrace{4A_fC_i^2}_{\uparrow}\right]$$

current noise	voltage noise	$1/f$ noise
$\propto \tau$	$\propto 1/\tau$	independent
independent of C_D	$\propto C_D^2$	of τ
		$\propto C_D^2$

- Current noise is independent of detector capacitance, consistent with the notion of “counting electrons”.

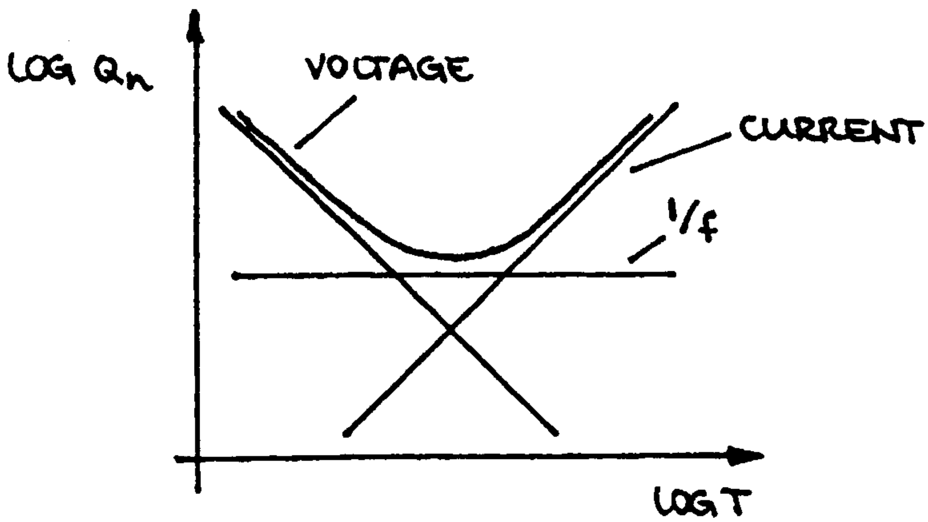
- Voltage noise increases with detector capacitance (reduced signal voltage)

- $1/f$ noise is independent of shaping time.
  In general, the total noise of a $1/f$ depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If τ_d and τ_i are scaled by the same factor, this ratio remains constant.

The equivalent noise charge Q_n assumes a minimum when the current and voltage noise contributions are equal.

dominated by voltage current noise

↓ ↓



For a CR-RC shaper the noise minimum obtains for $\tau_d = \tau_i = \tau$.

This criterion does not hold for more sophisticated shapers.

Caution: Even for a CR-RC shaper this criterion only applies when the differentiation time constant is the primary parameter, i.e. when the pulse width must be constrained.

When the rise time, i.e. the integration time constant, is the primary consideration, it is advantageous to make $\tau_d > \tau_i$, since the signal will increase more rapidly than the noise, as was shown previously

Note:

Although the parallel resistor was analyzed as a noise voltage source, it appears as a current noise source.

$\Rightarrow$ Modeling the parallel resistor as a noise current source is more appropriate.

$\Rightarrow$ judicious choice of model simplifies calculation.

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

$\Rightarrow$ In the detector community voltage and current noise are often called “series” and “parallel” noise.

The rest of the world uses equivalent noise voltage and current. Since they are physically meaningful, use of these widely understood terms is preferable.

Numerical expression for the noise of a CR-RC shaper

(note that some units are "hidden" in the numerical factors)

$$Q_n^2 = 12\, I_B\, \tau \;+\; 6\cdot 10^5\, \frac{\tau}{R_P} + \; 3.6\cdot 10^4\; v_n^2\, \frac{C^2}{\tau} \quad [\text{rms electrons}^2]$$

where

τ shaping time constant [ns]

I_B detector bias current + amplifier input current [nA]

R_P input shunt resistance [kΩ]

v_n equivalent input noise voltage spectral density [nV/√Hz]

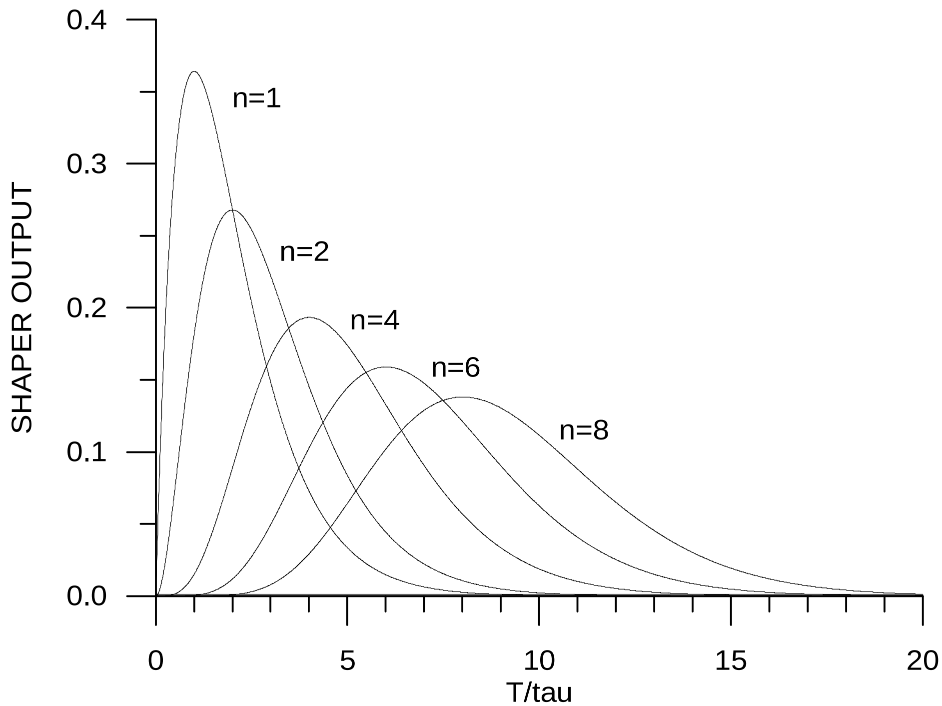
C total input capacitance [pF]

Q_n= 1 el corresponds to 3.6 eV in Si
2.9 eV in Ge

(see Spieler and Haller, IEEE Trans. Nucl. Sci. **NS-32** (1985) 419)

CR-RC Shapers with Multiple Integrators

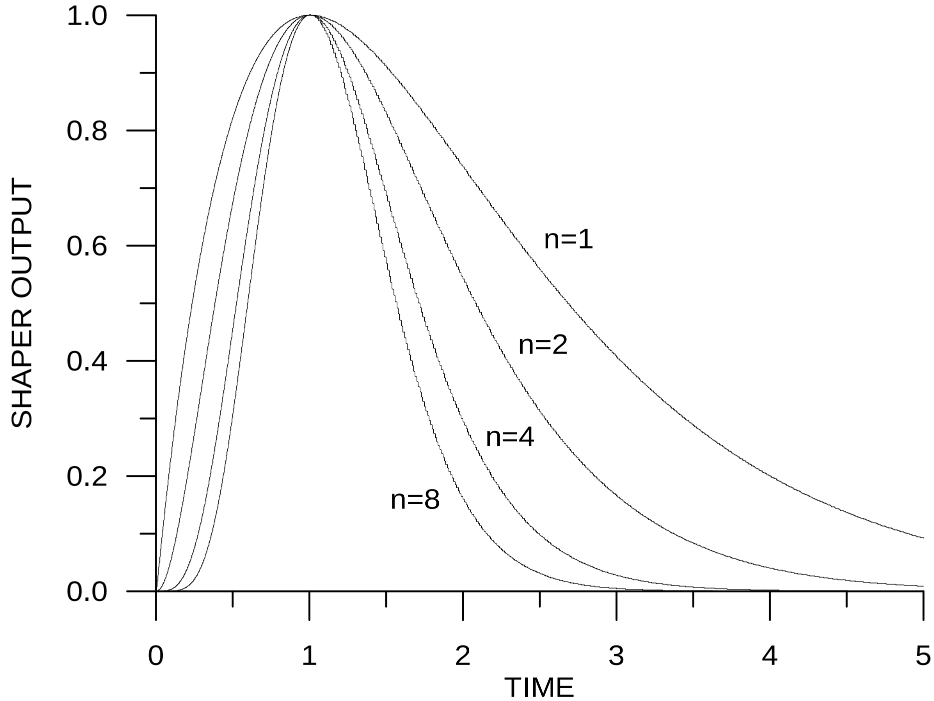
a. Start with simple *CR-RC* shaper and add additional integrators (n= 1 to n= 2, ... n= 8) with the same time constant τ .



With additional integrators the peaking time T_p increases

$$T_p = n\tau$$

b) Time constants changed to preserve the peaking time ($\tau_n = \tau_{n=1}/n$)



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

$\Rightarrow$ improved rate capability at the same peaking time

Shapers with the equivalent of 8 RC integrators are common. Usually, this is achieved with active filters (i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

Summary

Two basic noise mechanisms: input noise current i_n
input noise voltage v_n

Equivalent Noise Charge:

$$Q_n^2 = i_n^2 \, T_s \, F_i + C_i^2 \, v_n^2 \, \frac{F_v}{T_s}$$

where T_s Characteristic shaping time (*e.g.* peaking time)

F_i , F_v "Form Factors" that are determined by the shape of the pulse.

C_i Total capacitance at the input node (detector capacitance + input capacitance of preamplifier + stray capacitance + …)

Typical values of F_i , F_v

CR-RC shaper	F_i = 0.924	F_v = 0.924
CR-(RC)^4 shaper	F_i = 0.45	F_v = 1.02
CR-(RC)^7 shaper	F_i = 0.34	F_v = 1.27
CAFE chip	F_i = 0.4	F_v = 1.2

Note that $F_i < F_v$ for higher order shapers.
Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).

The noise analysis of shapers is rather straightforward if the frequency response is known.

On the other hand, since we are primarily interested in the pulse response, shapers are often designed directly in the time domain, so it seems more appropriate to analyze the noise performance in the time domain also.

Clearly, one can take the time response and Fourier transform it to the frequency domain, but this approach becomes problematic for time-variant shapers.

The CR-RC shapers discussed up to now utilize filters whose time constants remain constant during the duration of the pulse, i.e. they are time-invariant.

Many popular types of shapers utilize signal sampling or change the filter constants during the pulse to improve pulse characteristics, i.e. faster return to baseline or greater insensitivity to variations in detector pulse shape.

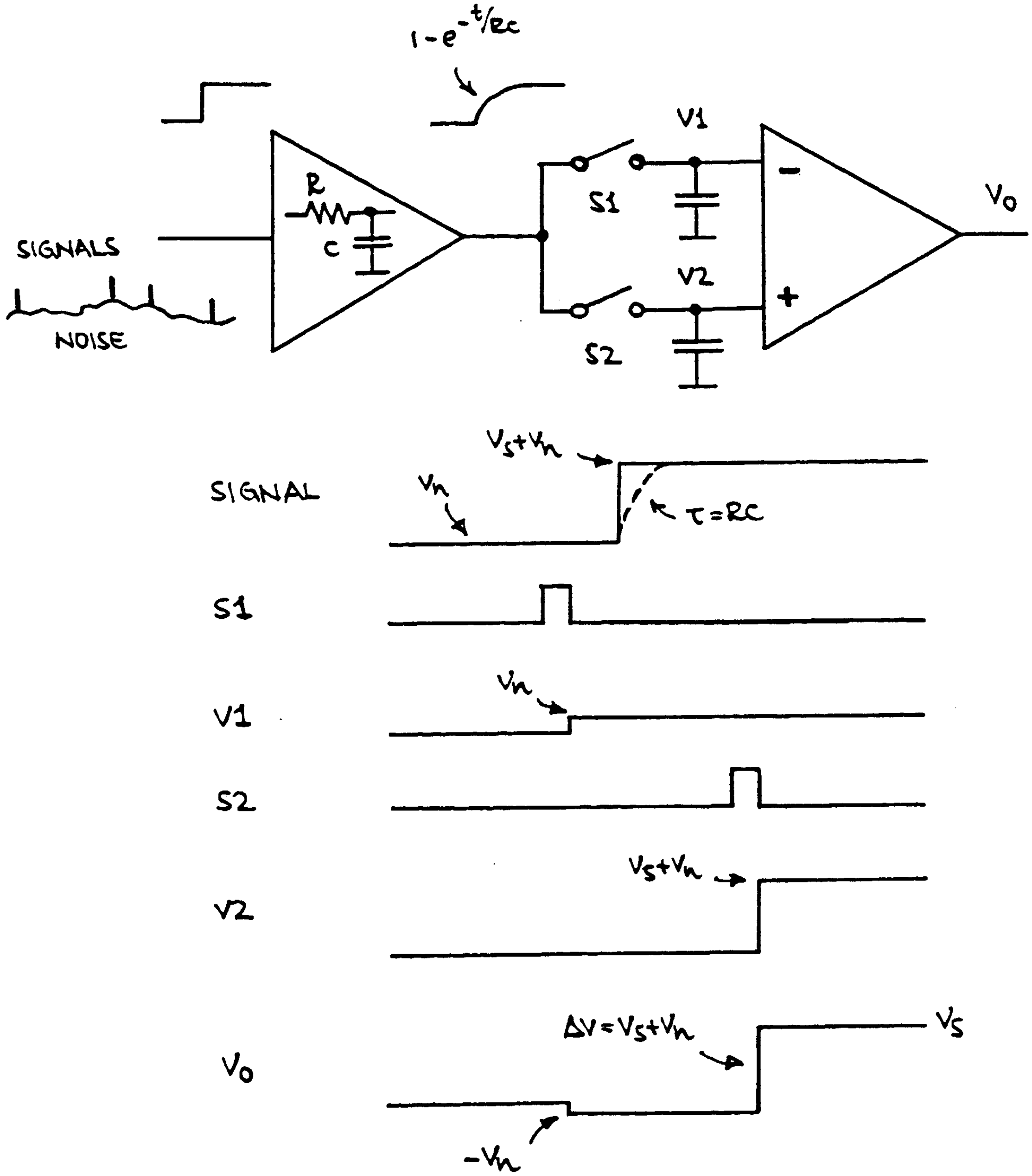
These time-variant shapers cannot be analyzed in the manner described above. Various techniques are available, but some shapers can be analyzed only in the time domain.

Example:

A commonly used time-variant filter is the correlated double-sampler.

This shaper can be analyzed exactly only in the time domain.

Correlated Double Sampling



1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal