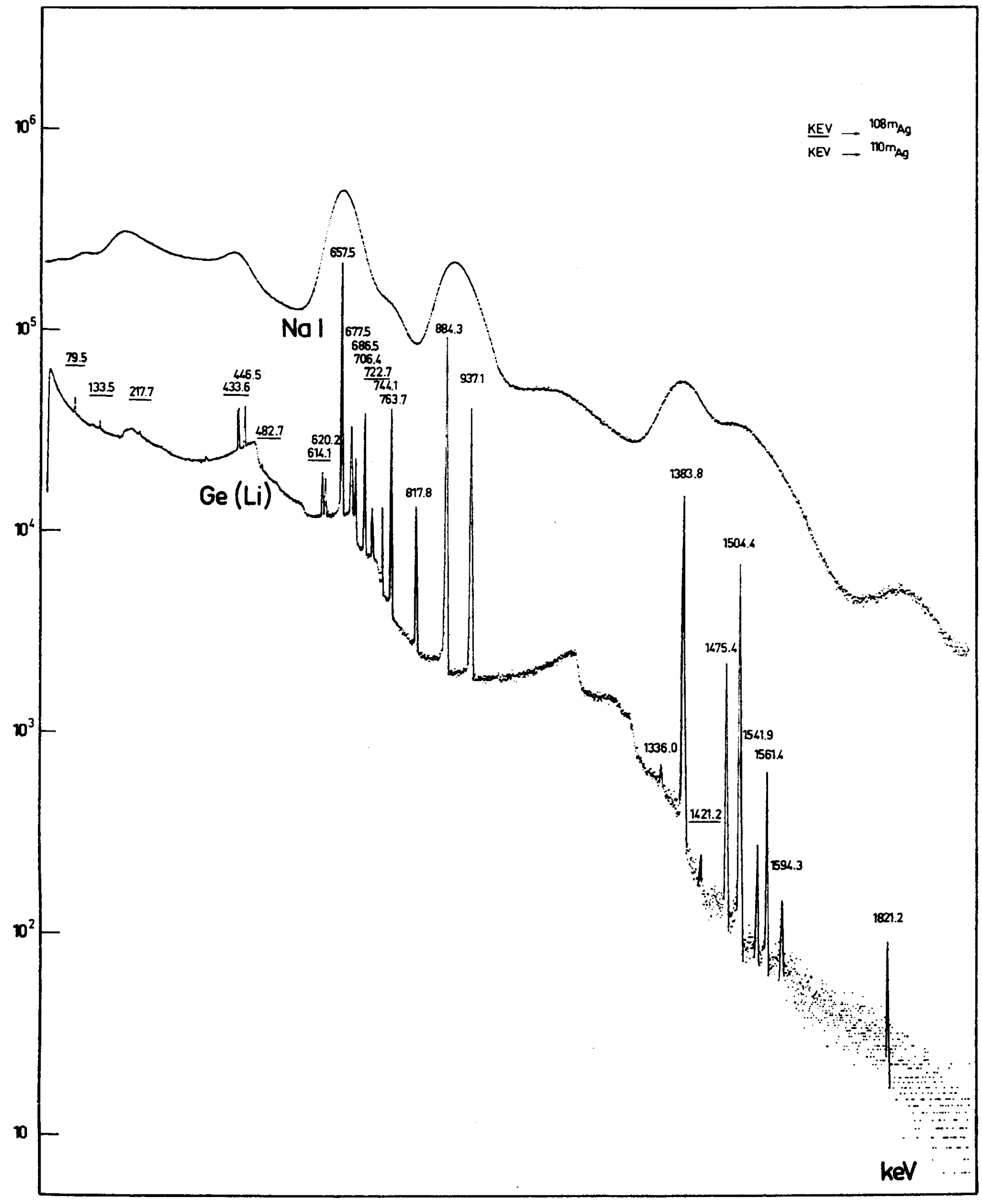
V.3. Resolution and Signal-to-Noise Ratio

Why?

a) Recognize structure in spectra

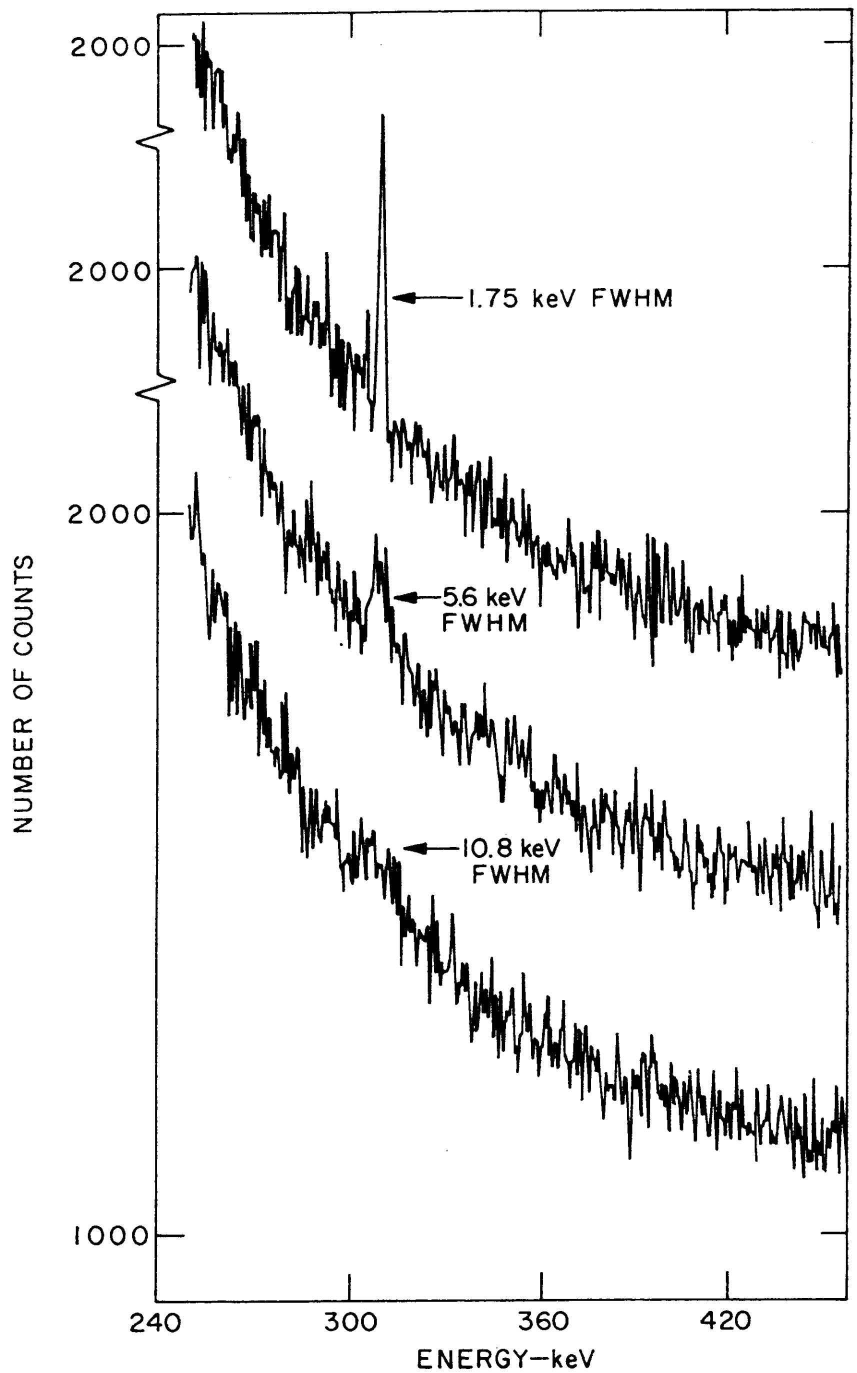
Comparison between NaI(Tl) and Ge detectors



(J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446)

b) Improve sensitivity

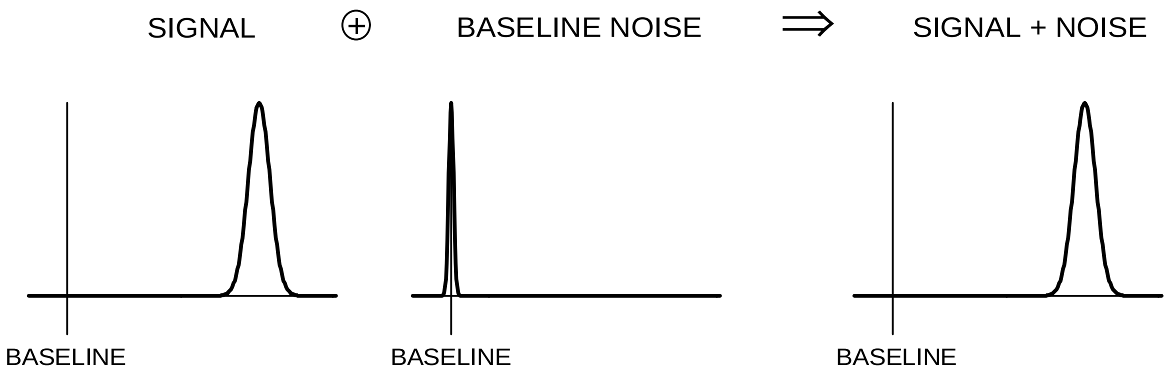
Signal to background ratio improves with better resolution (signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout, *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

What determines Resolution?

1. Signal Variance >> Baseline Variance



$\Rightarrow$ Electronic (baseline) noise not important

Examples:

- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

Number of photoelectrons

$$N_{pe} \approx 8 \cdot 10^4 \, [\mathrm{MeV}^{-1}] \times E_\gamma \times QE \approx 2.4 \cdot 10^4$$

Variance typically

$$\sigma_{pe} = N_{pe}^{1/2} \approx 160 \text{ and } \sigma_{pe} / N_{pe} \approx 5 - 8\%$$

Signal at PMT anode (assume Gain= 10^4)

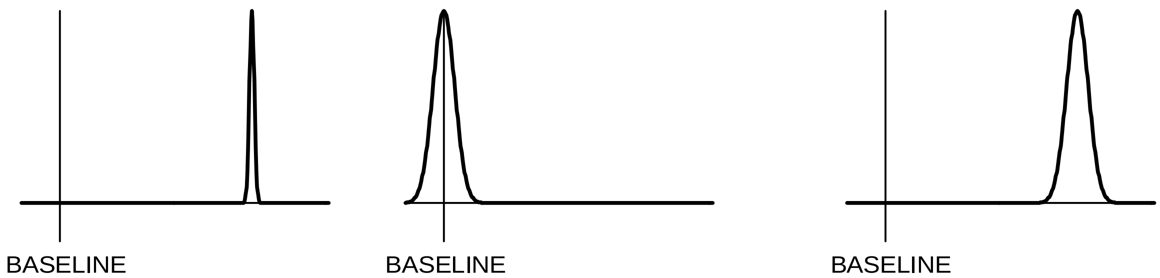
$$Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8 \text{ el and}$$

$$\sigma_{sig} = G_{PMT} \, \sigma_{pe} \approx 1.2 \cdot 10^7 \text{ el}$$

whereas electronic noise easily < 10^4 el

2. Signal Variance << Baseline Variance

SIGNAL $\oplus$ BASELINE NOISE $\Rightarrow$ SIGNAL + NOISE



$\Rightarrow$ Electronic (baseline) noise critical for resolution

Examples

- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si

Number of electron-hole pairs

$$N_{ep} = E_{dep} / (3.6 \text{ eV})$$

Variance

$$\sigma_{ep} = (F \cdot N_{ep})^{1/2}$$

(where F = Fano factor = 0.1)

For 50 keV photons

$$\sigma_{ep} \approx 40 \text{ el} \Rightarrow \quad \sigma_{ep} / N_{ep} = 7.5 \cdot 10^{-4}$$

obtainable noise levels are 100 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

Basic Noise Mechanisms

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\,e\,v}{l} \ .$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or '1/ f ' noise

Thermal noise and shot noise are both "white" noise sources, i.e.

power per unit bandwidth is constant:

(≡ spectral density)

$$\frac{dV_{noise}^2}{df} = const. \equiv v_n^2$$

or

$$\frac{dP_{noise}}{df} = const.$$

whereas for "1/ f " noise

$$\frac{dP_{noise}}{df} = \frac{1}{f^{\alpha}}$$

(typically $\alpha = 0.5 - 2$)

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f

$$\frac{dP_{noise}}{df} = 4kT$$

k = Boltzmann constant
T = absolute temperature

since

$$P = \frac{V^2}{R} = I^2 R$$

R = DC resistance

the spectral noise voltage density

$$\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$$

and the spectral noise current density

$$\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$$

The total noise depends on the bandwidth of the system.
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \, A_v^2(f)\, df$$

Note: Since spectral noise components are non-correlated, one must integrate over the noise power.

2. Shot noise

A common example of noise due to number fluctuations is "shot noise", which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode
(emission over a barrier)

Spectral noise current density:

$$i_n^2 = 2q_e I$$

q_e = electron charge
I = DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in "ohmic" conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

Noise Bandwidth vs. Signal Bandwidth

Consider an amplifier with the frequency response $A(f)$. This can be rewritten

$$A(f) \equiv A_0 G(f)$$

where A_0 is the maximum gain and $G(f)$ describes the frequency response.
For example, in the simple amplifier described above

$$A_V = g_m \left(\frac{1}{R_L} + i\omega C_o \right)^{-1} = g_m R_L \frac{1}{1 + i\omega R_L C_o}$$

and using the above convention

$$A_0 \equiv g_m R_L \quad \text{and} \quad G(f) \equiv \frac{1}{1 + i\,(2\pi f R_L C_o)}$$

If a "white" noise source with spectral density v_{ni} is present at the input, the total noise voltage at the output is

$$V_{no} = \sqrt{\int_0^\infty v_{ni}^2 \left| A_0 G(f) \right|^2 df} = v_{ni} A_0 \sqrt{\int_0^\infty G^2(f)\, df} \equiv v_{ni} A_0 \sqrt{\Delta f_n}$$

Δf_n is the "noise bandwidth".

Note that, in general, the noise bandwidth and the signal bandwidth are not the same. If the upper cutoff frequency is determined by a single RC time constant, as in the "simple amplifier", the signal bandwidth

$$\Delta f_s = f_u = \frac{1}{2\pi RC}$$

and the noise bandwidth

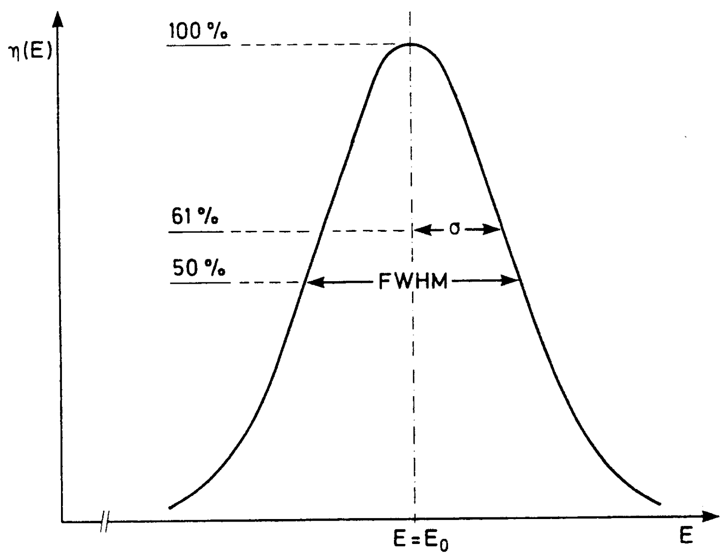
$$\Delta f_n = \frac{1}{4RC} = \frac{\pi}{2} f_u$$

Individual noise contributions add in quadrature
(additive in noise power)

$$V_{n,tot} = \sqrt{\sum_i V_{ni}^2}$$

Both thermal and shot noise are purely random.

⇒ amplitude distribution is gaussian



⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has gaussian distribution

Signal-to-Noise Ratio vs. Detector Capacitance

Voltage sensitive amplifier

Signal (at amplifier input)

$$v_s = \frac{Q_s}{C}$$

C= total capacitance at input

Noise (referred to amplifier input)

$$v_{ni}$$

for a given preamplifier and shaper

$\Rightarrow$

$$\frac{S}{N} = \frac{v_s}{v_n} = \underset{\substack{\uparrow \\ !}}{\frac{1}{C}} \; \frac{Q_s}{v_{ni}}$$

In general

- S/N cannot be *improved* by feedback.

  This result is generally valid,

  i.e. it also holds for active integrators (charge-sensitive amplifiers).

Charge-Sensitive Preamplifier
Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier the noise voltage at the output is essentially independent of detector capacitance,

i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.

The signal-to-noise ratio depends on detector capacitance, since the input signal decreases with increasing input capacitance.

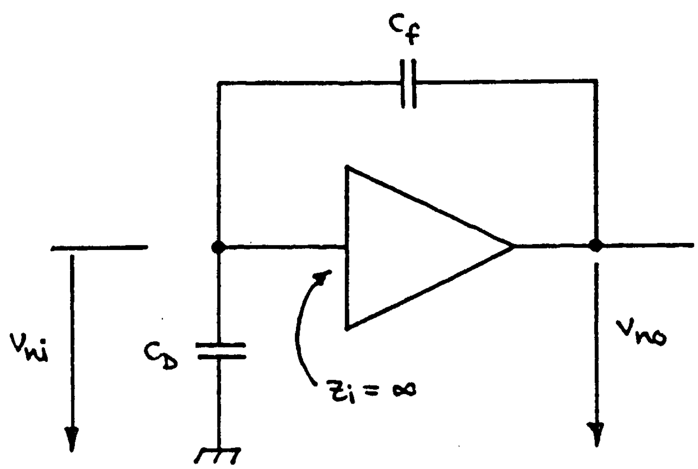
In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i >> C_{det}$).

What is the noise behavior?

> Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_{v0}$. The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.
>
> Note, that although specified specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.
>
> $\Rightarrow$ ***S/N* at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.



$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_D}}{X_{C_D}} = v_{ni} \frac{\frac{1}{\omega C_f} + \frac{1}{\omega C_D}}{\frac{1}{\omega C_D}}$$

$$v_{no} = v_{ni} \left(1 + \frac{C_D}{C_f} \right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni} \left(C_D + C_f \right)$$

Signal-to-noise ratio

$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_D + C_f)} = \frac{1}{C} \; \frac{Q_s}{v_{ni}}$$

Same result as for voltage-sensitive amplifier,

but here noise grows with increasing C.

As shown previously, pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the noise increases with C just as for the charge-sensitive amplifier.

Pulse Shaping

Two conflicting objectives:

1. Improve Signal-to-Noise Ratio S/N

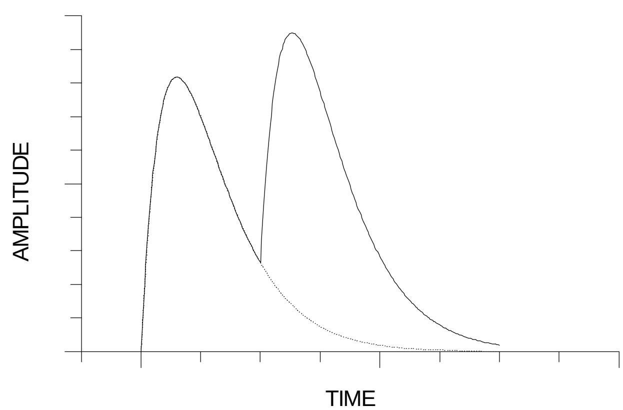
   Restrict bandwidth to match measurement time

   $\Rightarrow$ Increase Pulse Width

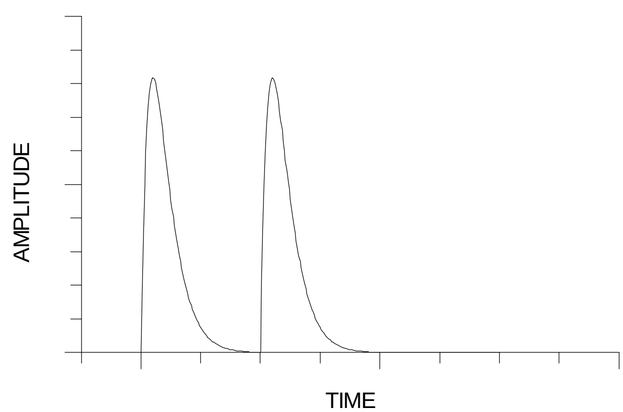
2. Improve Pulse Pair Resolution

   $\Rightarrow$ Decrease Pulse Width

Pulse pile-up distorts amplitude measurement



Reducing pulse shaping time to 1/3 eliminates pile-up.



Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

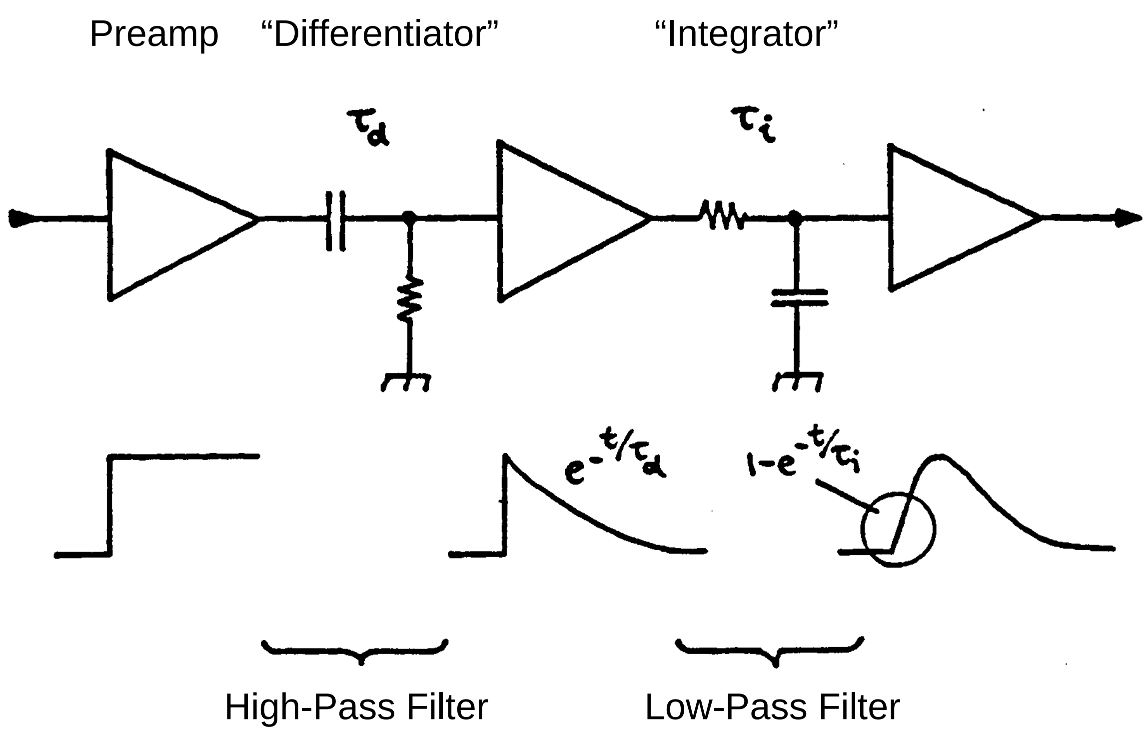
Usually, many considerations combined lead to a "non-textbook" compromise.

- *"Optimum shaping" depends on the application!*

- Shapers need not be complicated –

  Every amplifier is a pulse shaper!

Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

$\Rightarrow$ Useful for estimates, since simple to evaluate

CR Differentiator

Input voltage = voltage across C + voltage across R

$$\frac{dV_{in}(t)}{dt} = \frac{1}{C}\frac{dQ}{dt} + \frac{dV_R(t)}{dt}$$

Using $V_R(t) = R \cdot i(t)$ and $V_{out}(t) = V_R(t)$

$$\frac{dV_{in}(t)}{dt} = \frac{1}{C} i(t) + \frac{dV_{out}(t)}{dt} = \frac{1}{C}\frac{V_{out}(t)}{R} + \frac{dV_{out}(t)}{dt}$$

Setting $\tau = RC$

$$V_{out}(t) + \tau\ \frac{dV_{out}(t)}{dt} = \tau\ \frac{dV_{in}(t)}{dt}$$

If

$$\tau\ \frac{dV_{out}(t)}{dt} << V_{out}(t)$$

then

$$V_{out}(t) = \tau\ \frac{dV_{in}(t)}{dt}$$

i.e. the output is the time derivative of the input.
In practice, this condition is seldom met, but the circuit is still called a “differentiator”.

For a step input $V_{in}(t) = 0$ for $t < 0$
$V_{in}(t) = V_i$ for $t \geq 0$

$$V_{out} = V_i\ e^{-t/\tau}$$

i.e. the differentiator *shortens* the pulse (decreases the fall time)

In the frequency domain

$$V_{out} = \frac{R}{R + X_C}\; V_{in} = \frac{R}{R - \dfrac{i}{\omega C}}\; V_{in}$$

$$V_{out} = \frac{i\omega RC}{1 + i\omega RC}\; V_{in} = \frac{1}{1 + 1/\,i\omega\tau}\; V_{in}$$

At frequencies $\omega << 1/\tau$ ($f << 1/2\pi\tau$)

$$V_{out} \approx i\omega\tau \cdot V_{in}$$

At high frequencies, $\omega >> 1/\tau$

$$V_{out} \approx V_{in}$$

$\Rightarrow$ the CR differentiator is a "high-pass" filter, i.e. it transmits frequencies above the cutoff frequency $1/2\pi RC$.

A similar treatment applies to the RC "integrator", which in the time domain increases the rise time and in the frequency domain acts as a low-pass filter..

Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Equivalent Noise Charge

Inject known signal charge into preamp input
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge $\equiv$ Input charge for which $S/N = 1$

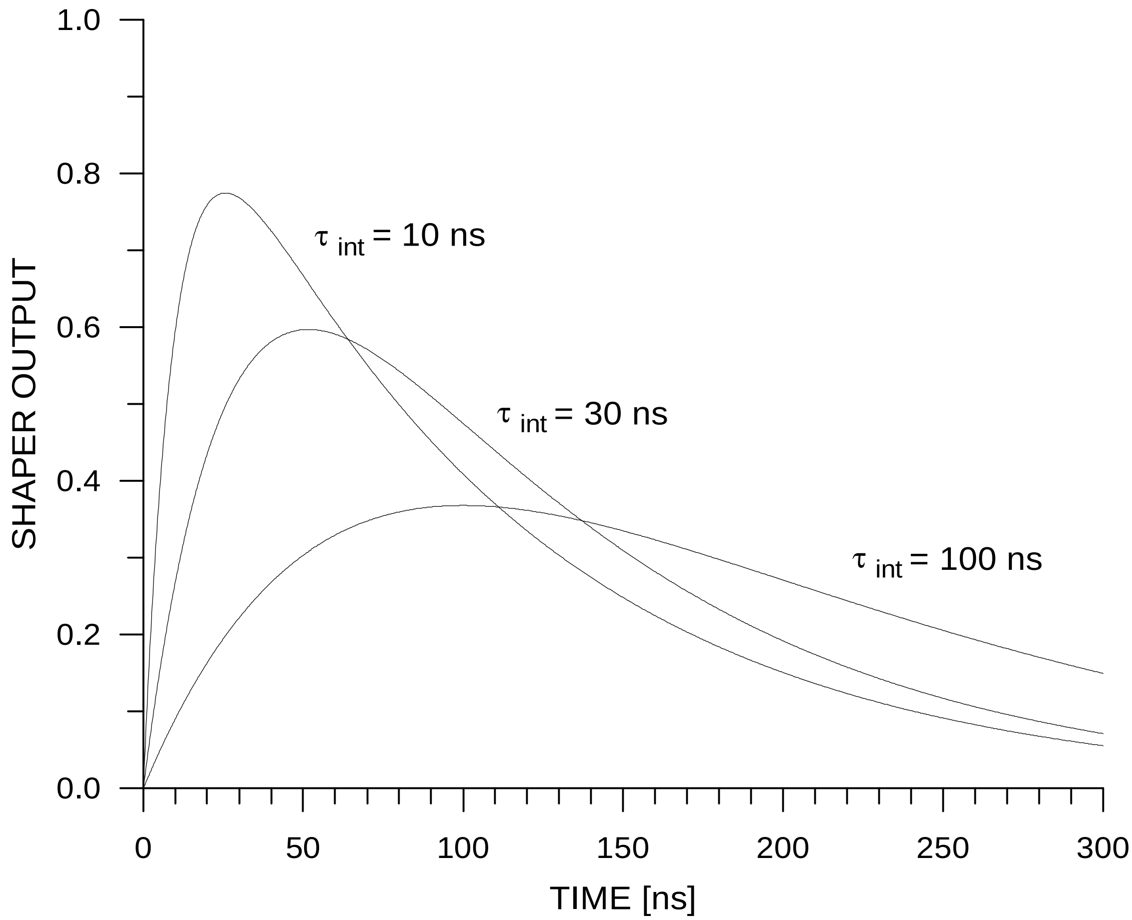
Effect of relative constants

Consider a *CR-RC* shaper with a fixed differentiator time constant of 100 ns.

Increasing the integrator time constant lowers the upper cut-off frequency, which decreases the total noise at the shaper output.

However, as shown in the following figure, the peak signal also decreases.

CR-RC SHAPER
FIXED DIFFERENTIATOR TIME CONSTANT = 100 ns
INTEGRATOR TIME CONSTANT = 10, 30 and 100 ns



Still keeping the differentiator time constant fixed at 100 ns, the next set of graphs shows the variation of

output noise

output signal amplitude

equivalent input noise charge

as the integrator time constant is increased from 10 to 100 ns:

Output Noise:

$$\frac{v_{no}(100\text{ ns})}{v_{no}(10\text{ ns})} = \frac{1}{4.2}$$

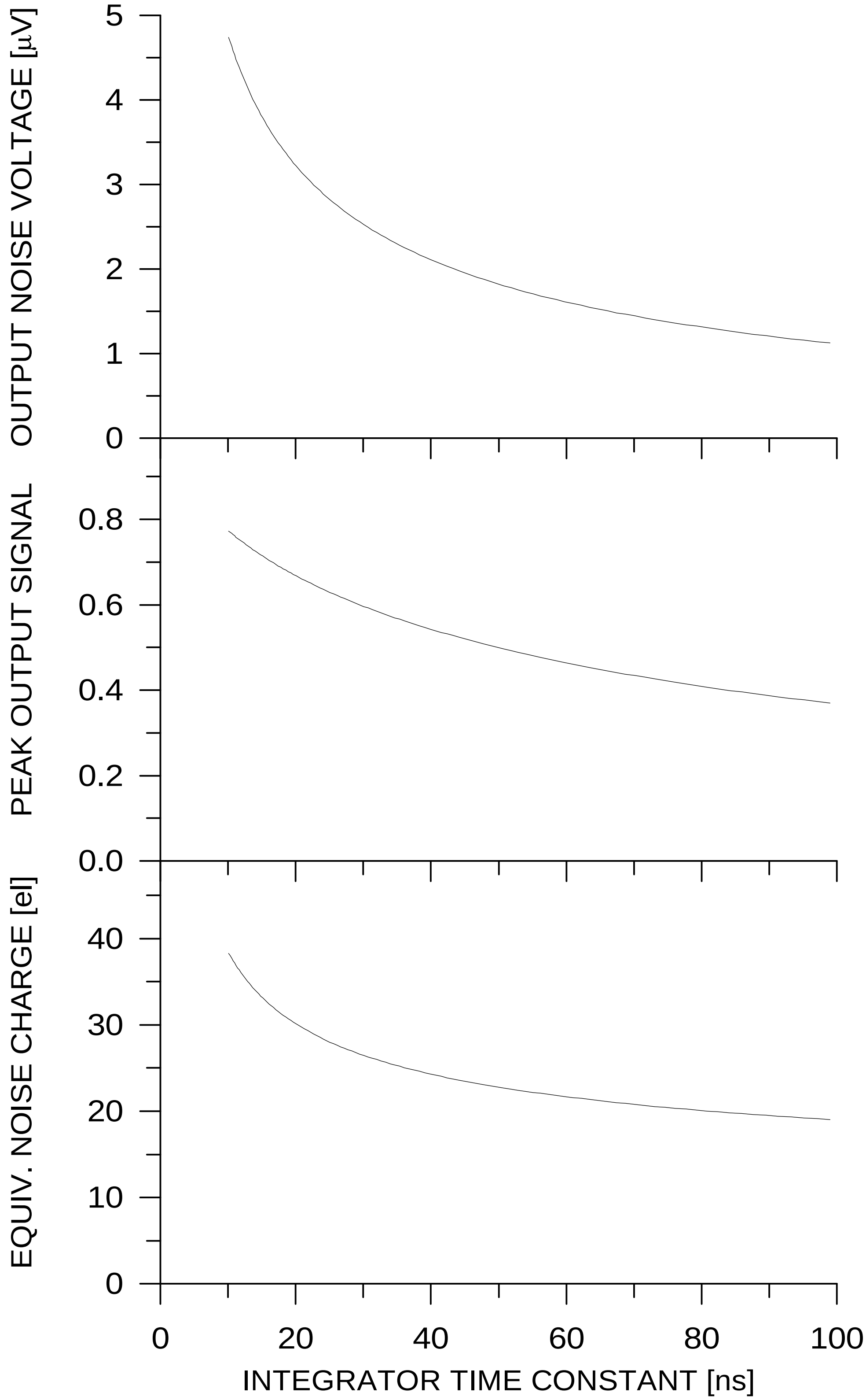
Peak Output Signal:

$$\frac{V_{so}(100\text{ ns})}{V_{so}(10\text{ ns})} = \frac{1}{2.1}$$

The roughly 4-fold decrease in noisy is partially compensated by the 2-fold reduction in signal, so that

$$\frac{Q_n(100\text{ ns})}{Q_n(10\text{ ns})} = \frac{1}{2}$$

OUTPUT NOISE, OUTPUT SIGNAL AND EQUIVALENT NOISE CHARGE
CR-RC SHAPER - FIXED DIFFERENTIATOR TIME CONSTANT = 100 ns
(e_n = 1 nV/ √ Hz, i_n = 0, C_{TOT} = 1 pF)



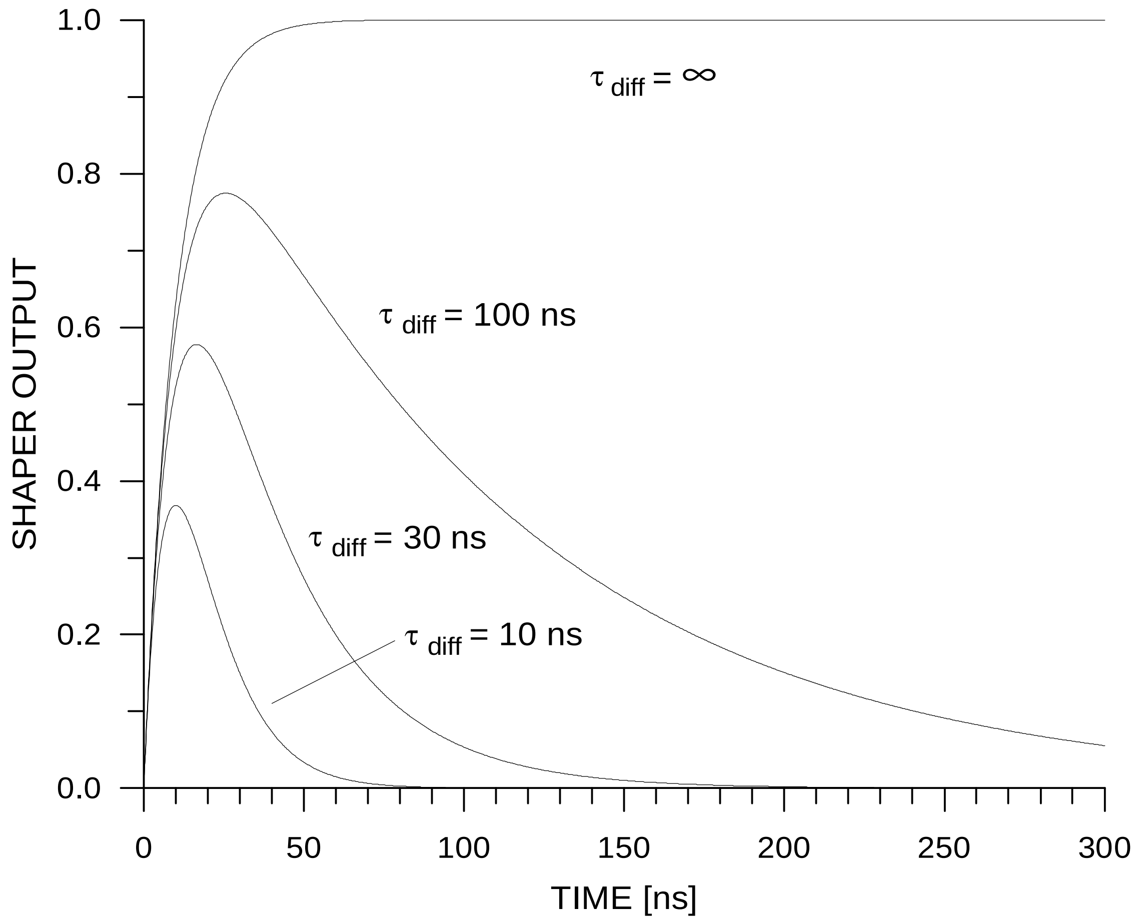
For comparison, consider the same *CR-RC* shaper with the integrator time constant fixed at 10 ns and the differentiator time constant variable.

As the differentiator time constant is changed, the peak signal amplitude at the shaper output varies as shown in the following graph.

Note that the need to limit the pulse width incurs a significant reduction in the output signal.

Even at a differentiator time constant τ_{diff} = 100 ns = 10 τ_{int} the output signal is only 80% of the value for $\tau_{diff} = \infty$, i.e. a system with no low-frequency roll-off.

CR-RC SHAPER
FIXED INTEGRATOR TIME CONSTANT = 10 ns
DIFFERENTIATOR TIME CONSTANT = ∞, 100, 30 and 10 ns



Keeping the integrator time constant fixed at 10 ns,
the next graph shows

output noise

output signal amplitude

equivalent input noise charge

as the differentiator time constant is changed from 10 to 100 ns.

Since changing the low-frequency cut-off does not affect the total noise bandwidth appreciably, the change in output noise is modest

$$\frac{v_{no}(100\ \mathrm{ns})}{v_{no}(10\ \mathrm{ns})} = 1.3\ ,$$

whereas the signal amplitude changes appreciably.

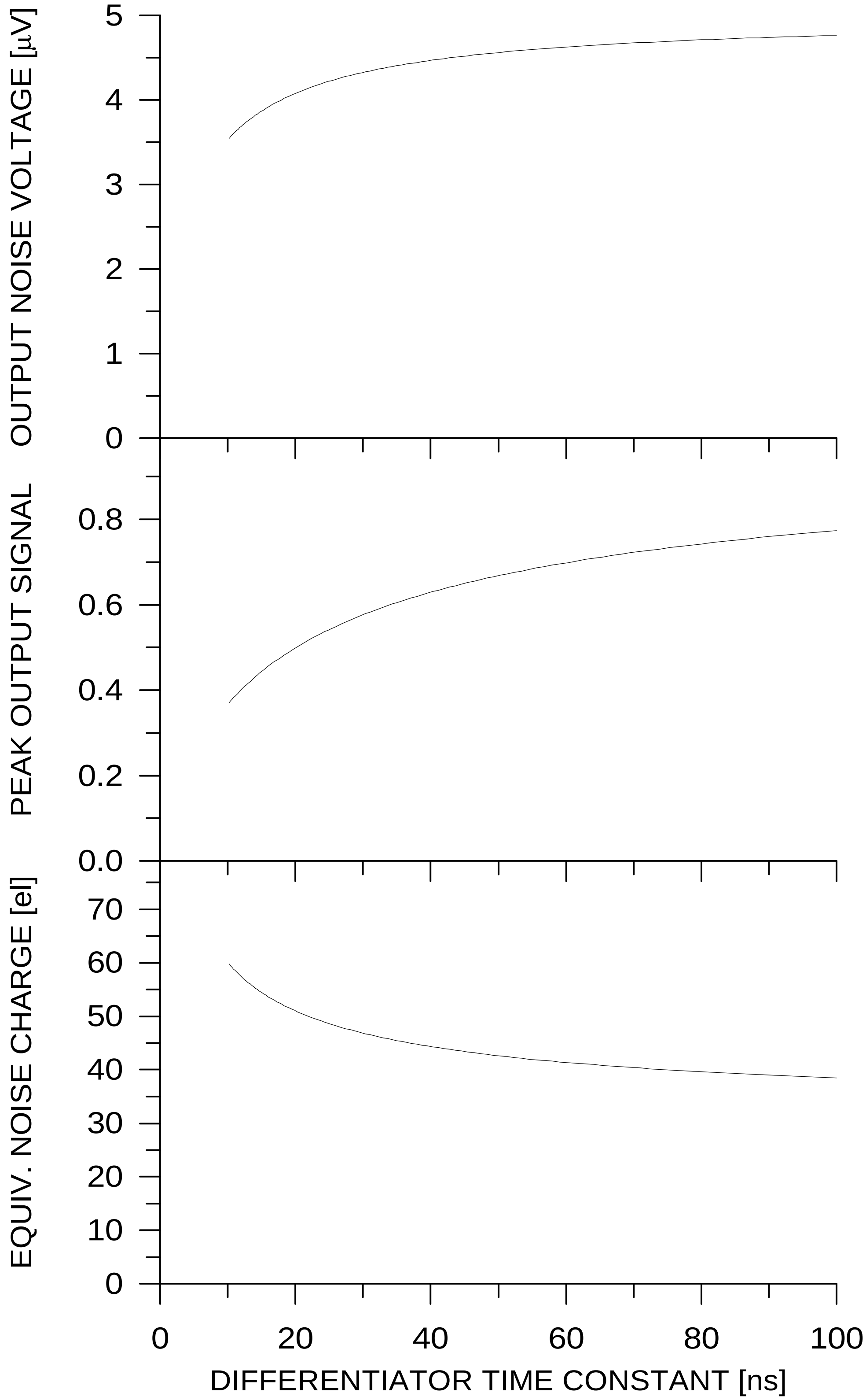
$$\frac{V_{so}(100\ \mathrm{ns})}{V_{so}(10\ \mathrm{ns})} = 2.1$$

Although the noise grows as the differentiator time constant is changed from 10 to 100 ns, it is outweighed by the increase in signal level so that the net signal-to-noise ratio improves.

The equivalent input noise charge

$$\frac{Q_n(100\ \mathrm{ns})}{Q_n(10\ \mathrm{ns})} = \frac{1}{1.6}$$

OUTPUT NOISE, OUTPUT SIGNAL AND EQUIVALENT NOISE CHARGE
CR-RC SHAPER - FIXED INTEGRATOR TIME CONSTANT = 10 ns
(e_n = 1 nV/ √ Hz, i_n = 0, C_{TOT} = 1 pF)



Summary

To evaluate shaper noise performance

- Noise spectrum alone is inadequate

Must also

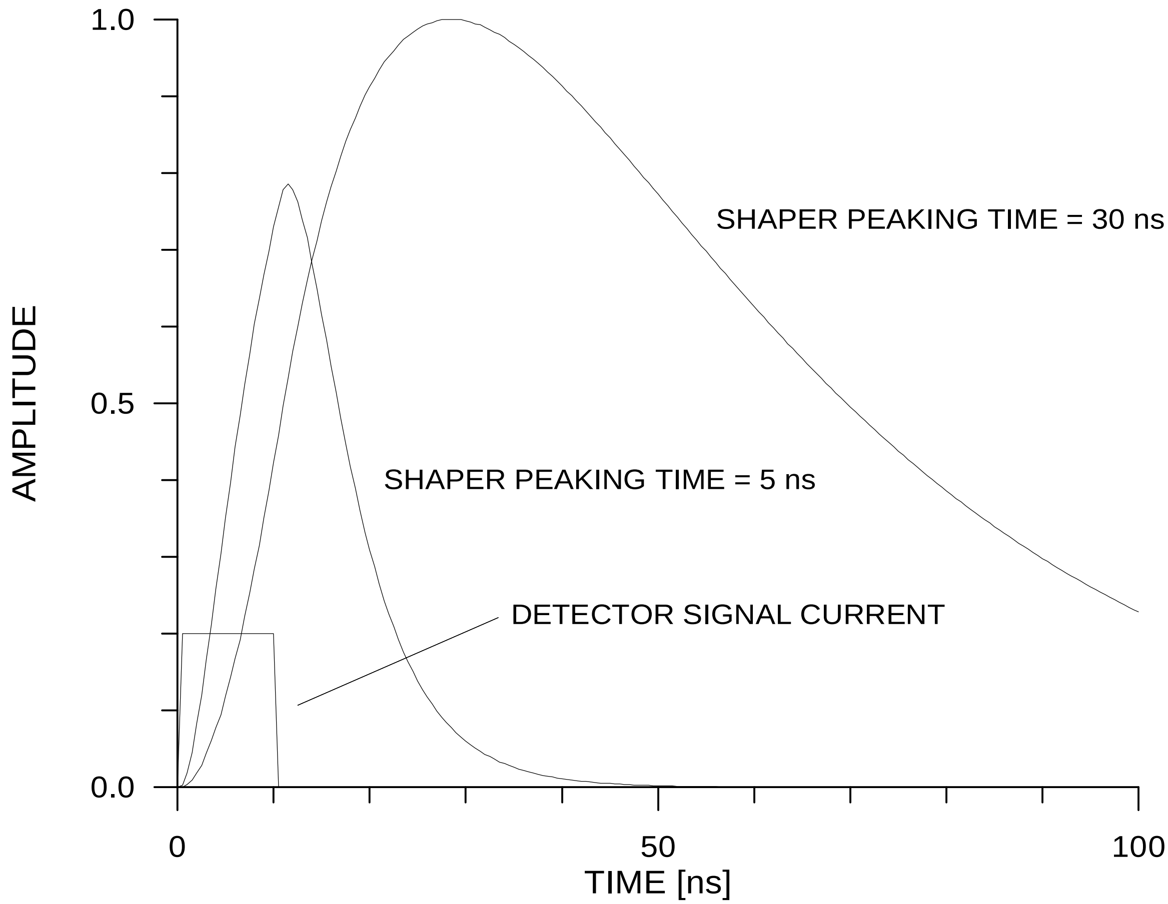
- Assess effect on signal

Signal amplitude is also affected by the relationship of the shaping time to the detector signal duration.

If peaking time of shaper < collection time

$\Rightarrow$ signal loss (“ballistic deficit”)

Loss in Pulse Height (and Signal-to-Noise Ratio) if Peaking Time of Shaper < Detector Collection Time

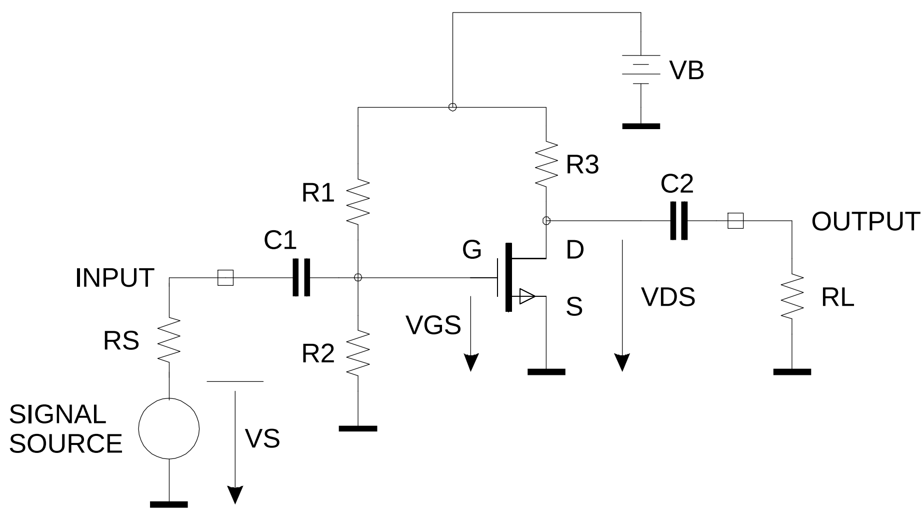


Note that although the faster shaper has a peaking time of 5 ns, the response to the detector signal peaks after full charge collection.

Equivalent Circuits

Take a simple amplifier as an example.

a) full circuit diagram



First, just consider the DC operating point of the circuitry between $C1$ and $C2$:

1. The n-type MOSFET requires a positive voltage applied from the gate G to the source S.

$$V_{GS} = \frac{R2}{R1 + R2} V_B$$

2. The gate voltage V_{GS} sets the current flowing into the drain electrode D.

3. Assume the drain current is I_D. Then the DC voltage at the drain is

$$V_{DS} = V_B - I_D R3$$

Next, consider the AC signal V_S provided by the signal source.

Assume that the signal at the gate G is dV_G/dt.

1. The current flowing through $R2$ is

$$\frac{dI}{dt}(R2) = \frac{dV_G}{dt} \cdot \frac{1}{R2}$$

2. The current flowing through $R1$ is

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{d}{dt}(V_G + \mathrm{V_B})$$

Since the battery voltage $\mathrm{V_B}$ is constant,

$$\frac{d\mathrm{V_B}}{dt} = 0$$

so that

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{dV_G}{dt}$$

3. The total time-dependent input current is

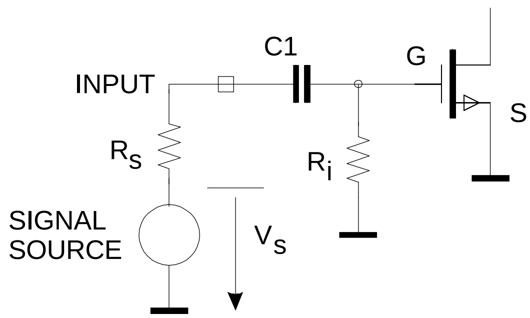
$$\frac{dI}{dt} = \frac{dI_{R1}}{dt} + \frac{dI_{R2}}{dt} = \left(\frac{1}{R1} + \frac{1}{R2}\right) \cdot \frac{dV_G}{dt} \equiv \frac{1}{R_i} \cdot \frac{dV_G}{dt}$$

where

$$R_i = \frac{R1 \cdot R2}{R1 + R2}$$

is the parallel connection of $R1$ and $R2$.

Consequently, for the AC input signal the circuit is equivalent to



At the output, the voltage signal is formed by the current of the transistor flowing through the combined output load formed by R_L and $R3$.

For the moment, assume that $R_L >> R3$. Then the output load is dominated by $R3$.

The voltage at the drain D is

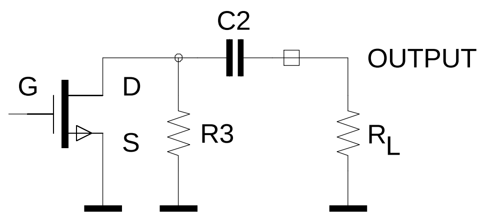
$$V_o = \mathrm{V_B} - i_D R3$$

If the gate voltage is varied, the transistor drain current changes, with a corresponding change in output voltage

$$\frac{dV_o}{di_D} = \frac{d}{dI_D}(\mathrm{V_B} - i_D R3) = R3$$

$\Rightarrow$ The DC supply voltage does not directly affect the signal formation.

If we remove the restriction $R_L >> R3$, the total load impedance for time-variant signals is the parallel connection of $R3$ and $(X_{C2} + R_L)$, yielding the equivalent circuit at the output



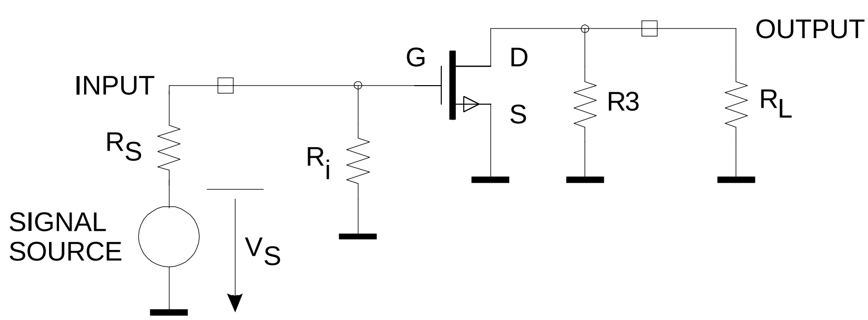
If the source resistance of the signal source $R_S << R_i$, the input coupling capacitor $C1$ and input resistance R_i form a high-pass filter. At frequencies where the capacitive reactance is $<<R_i$, i.e.

$$f >> \frac{1}{2\pi \, R_i \, C1}$$

the source signal v_s suffers negligible attenuation at the gate, so that

$$\frac{dV_G}{dt} = \frac{dV_s}{dt}$$

Correspondingly, at the output, if the impedance of the output coupling capacitor $C2<<R_L$, the signal across R_L is the same as across $R3$, yielding the simple equivalent circuit



Note that this circuit is only valid in the “high-pass” frequency regime.

Equivalent circuits are an invaluable tool in analyzing systems, as they remove extraneous components and show only the components and parameters essential for the problem at hand.

Often equivalent circuits are tailored to very specific questions and include simplifications that are not generally valid. Conversely, focussing on a specific question with a restricted model may be the only way to analyze a complicated situation.

Evaluation of Equivalent Noise Charge

A. Experiment

Inject an input signal with known charge using a pulse generator set to approximate the detector signal (possible ballistic deficit). Measure the pulse height spectrum.

peak centroid $\Rightarrow$ signal magnitude

peak width $\Rightarrow$ noise (FWHM= 2.35 rms)

If pulse-height digitization is not practical:

1. Measure total noise at output of pulse shaper

   a) measure the total noise power with an rms voltmeter of sufficient bandwidth
   or
   b) measure the spectral distribution with a spectrum analyzer and integrate (the spectrum analyzer provides discrete measurement values in N frequency bins Δf_n)

$$V_{no} = \sqrt{\sum_{n=0}^{N} \left(v_{no}^2(n) \cdot \Delta f \right)}$$

   The spectrum analyzer shows if “pathological” features are present in the noise spectrum.

2. Measure the magnitude of the output signal V_{so} for a known input signal, either from detector or from a pulse generator set up to approximate the detector signal.

3. Determine signal-to-noise ratio $S/N = V_{so} / V_{no}$

   and scale to obtain the equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

B. Numerical Simulation (e.g. SPICE)

This can be done with the full circuit including all extraneous components. Procedure analogous to measurement.

1. Calculate the spectral distribution and integrate

$$V_{no} = \sqrt{\sum_{n=0}^{N} v_{no}^2(n) \cdot \Delta f}$$

2. Determine the magnitude of output signal V_{so} for an input that approximates the detector signal.

3. Calculate the equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

C. Analytical Simulation

1. Identify individual noise sources and refer to input
2. Determine the spectral distribution at input for each

$$v_{ni,k}^2(f)$$

source k

3. Calculate the total noise at shaper output ($G(f)$ = gain)

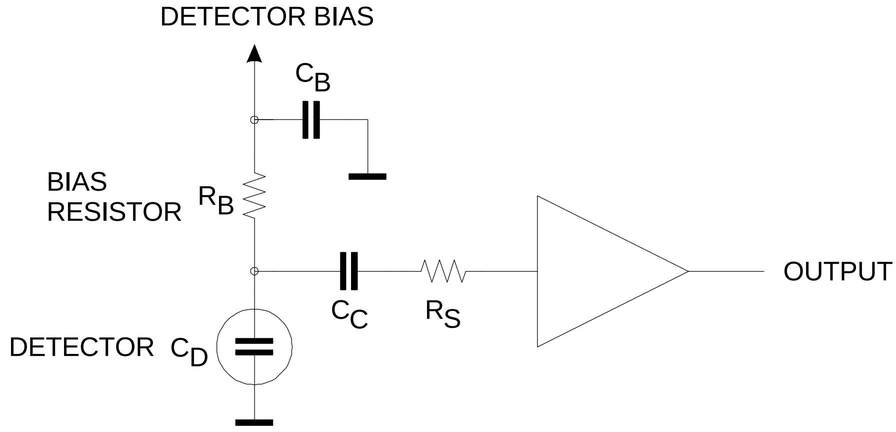
$$V_{no} = \sqrt{\int_0^\infty G^2(f)\left(\sum_k v_{ni,k}^2(f)\right) df} \equiv \sqrt{\int_0^\infty G^2(\omega)\left(\sum_k v_{ni,k}^2(\omega)\right) d\omega}$$

4. Determine the signal output V_{so} for a known input charge Q_s and realistic detector pulse shape.

5. Equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

Analytical Analysis of a Detector Front-End



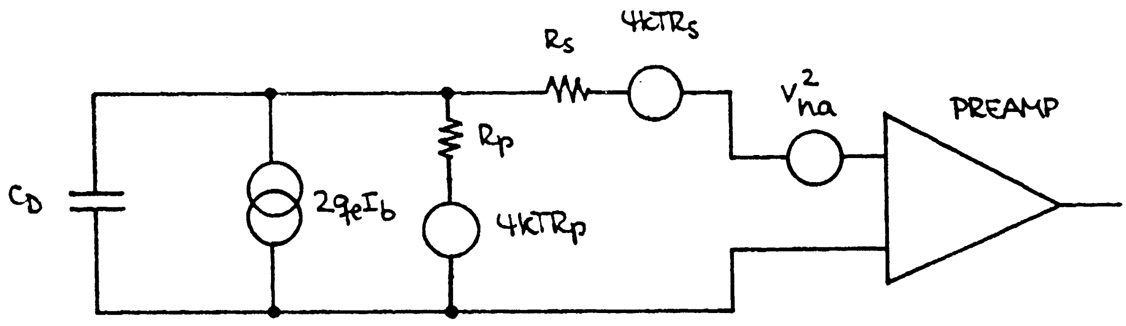
Detector bias voltage is applied through the resistor R_B. The bypass capacitor C_B serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the “far end” of the bias resistor to ground, so that R_B appears to be in parallel with the detector.

The coupling capacitor C_C in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why a capacitor serving this role is also called a “blocking capacitor”).

The series resistor R_S represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients (“input protection”)
- ... etc.

Equivalent circuit for noise analysis



detector	bias current shot noise	shunt resistance thermal noise	series resistance thermal noise	equivalent input noise voltage of amplifier

In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage applied to the amplifier input.

Steps in the analysis:

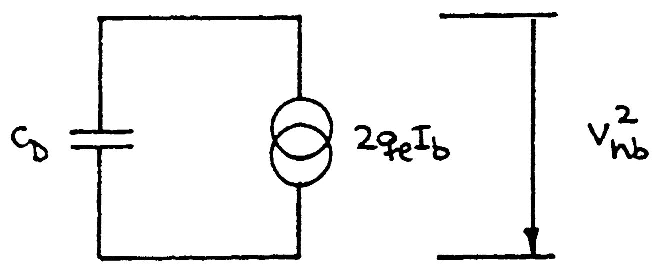
1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources

2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.

3. Determine the equivalent noise charge

Noise Contributions

1. Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite

2. The shunt resistance R_P is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

   Does this assumption make sense?

   If R_P is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_P C_D >> t_P \approx \frac{1}{\omega_P}$$

   where ω_P is the midband frequency of the shaper. Therefore,

$$R_P >> \frac{1}{\omega_P C_D}$$

   as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$v_{nb}^2 = i_{nb}^2 \frac{1}{(\omega C_D)^2} = 2q_e I_b \frac{1}{(\omega C_D)^2}$$

⇒ the noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time T. The number of electrons charges measured is

$$N_e = \frac{I_b T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?

Since shot noise is a fluctuation, the current suffers both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

2. Parallel Resistance

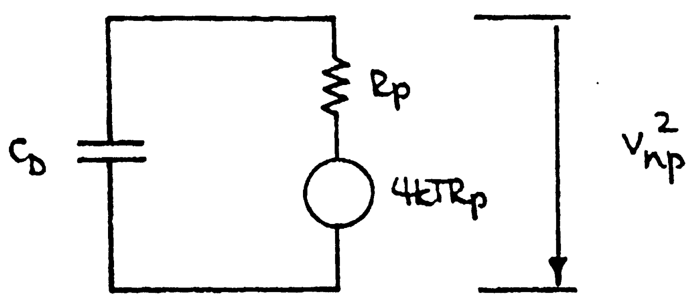
Shunt components:

1. bias noise current source
   (infinite resistance by definition)

2. detector capacitance

Current due to the noise voltage from R_P can only flow through the detector capacitance C_D.

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$v_{np}^2 = 4kTR_P \left(\frac{-i/\omega C_D}{R_P - i/\omega C_D} \right)^2$$

$$v_{np}^2 = 4kTR_P \frac{1}{1 + (\omega R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^\infty v_{np}^2(\omega)d\omega = \int_0^\infty \frac{4kTR_P}{1+(\omega R_P C_D)^2} d\omega = \frac{kT}{C_D}$$

which is independent of R_P. Commonly referred to as "kTC" noise, this contribution is often erroneously interpreted as the "noise of the detector capacitance". An ideal capacitor has no thermal noise; all noise originates in the resistor.
So, why is the result independent of R_P?

R_P determines the primary noise, but also the noise bandwidth of this subcircuit. As R_P increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of R_P.

However,
If one integrates v_{np} over a bandwidth-limited system

$$V_n^2 = \int_0^\infty 4kTR_P \left| \frac{G(i\omega)}{1 - i\omega R_P C_D} \right|^2 d\omega$$

the total noise decreases with increasing R_P.

3. Series Resistance

The noise voltage generator associated with the series resistance R_S is in series with the other noise sources, so it simply contributes

$$v_{nr}^2 = 4kTR_S$$

4. Amplifier input noise voltage density

The amplifier noise sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain. Here, the output noise is referred to a voltage-sensitive input, so it appears as a noise voltage generator.

$$v_{na}^2 = v_{nt}^2 + \frac{A_f}{f}$$

$$\uparrow \qquad \uparrow$$

"white noise"

$1/f$ noise (can also originate in external components)

This noise voltage generator also adds in series with the other sources.

Amplifiers generally also exhibit input current noise, which is physically present at the input. However, its effect is the same as for the detector bias current, so the analysis given in 1. can be applied.

Equivalent noise charge at the shaper output

1. Assume a CR-RC shaper

2. Equal differentiation and integration time constants
   $\tau_d = \tau_i = \tau$.

Integrating over frequency for all noise contributions and accounting for the signal transmission through the shaper yields the equivalent noise charge

$$Q_n^2 = \left(\frac{e^2}{8}\right)\left[\underbrace{\left(2q_e I_b + \frac{4kT}{R_P}\right)\cdot\tau}_{\uparrow} + \underbrace{\left(4kTR_S + v_{na}^2\right)\cdot\frac{C_D^2}{\tau}}_{\uparrow} + \underbrace{4A_f C_D^2}_{\uparrow}\right]$$

current noise	voltage noise	$1/f$ noise
$\propto \tau$	$\propto 1/\tau$	independent
independent of C_D	$\propto C_D^2$	of τ
		$\propto C_D^2$

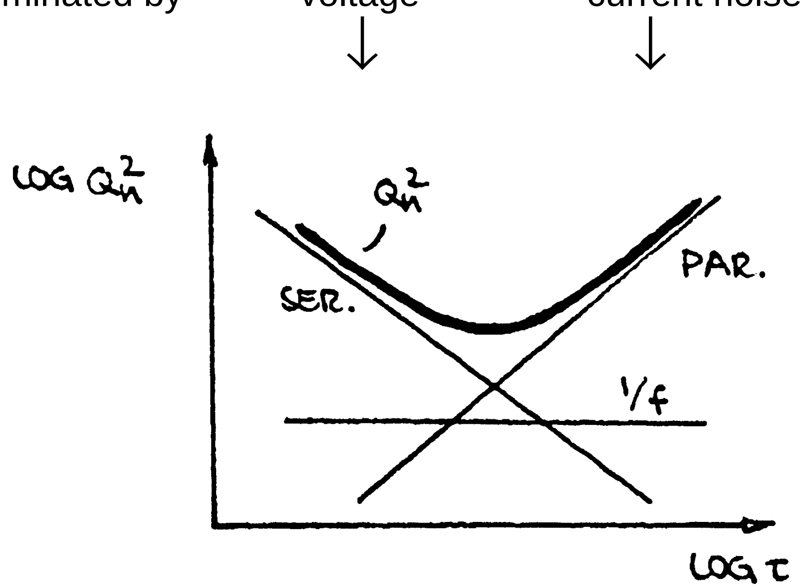
- Current noise is independent of detector capacitance, consistent with the notion of “counting electrons”.

- Voltage noise increases with detector capacitance (reduced signal voltage)

- $1/f$ noise is independent of shaping time.
  In general, the total noise of a $1/f$ depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If τ_d and τ_i are scaled by the same factor, this ratio remains constant.

Q_n assumes a minimum when the current and voltage noise contributions are equal.

dominated by voltage current noise



For a CR-RC shaper the noise minimum obtains for $\tau_d = \tau_i = \tau$.

This is not true for more sophisticated shapers.

Note:

Although the parallel resistor was analyzed as a noise voltage source, it appears as a current noise source.

⇒ Modeling the parallel resistor as a noise current source is more appropriate.

⇒ judicious choice of model simplifies calculation.

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

The rest of the world uses equivalent noise voltage and current. Since they are physically meaningful, use of these widely understood terms is preferable.

Numerical expression for the noise of a CR-RC shaper

(note that some units are "hidden" in the numerical factors)

$$Q_n^2 = 12\, I_B\, \tau \;+\; 6\cdot 10^5 \frac{\tau}{R_P} \;+\; 3.6\cdot 10^4\, v_n^2\, \frac{C^2}{\tau} \quad \text{[rms electrons]}$$

where

τ	shaping time constant [ns]
I_B	detector bias current + amplifier input current [nA]
R_P	input shunt resistance [kΩ]
v_n	equivalent input noise voltage spectral density [nV/$\sqrt{}$Hz]
C	total input capacitance [pF]

Q_n= 1 el corresponds to 3.6 eV in Si
2.9 eV in Ge

(see Spieler and Haller, IEEE Trans. Nucl. Sci. **NS-32** (1985) 419)