

## IV. Signal Processing

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# 1. The Problem

Radiation impinges on a sensor and creates an electrical signal.

The signal level is low and must be amplified to allow digitization and storage.

Both the sensor and amplifiers introduce signal fluctuations – noise.

1. Fluctuations in signal introduced by sensor
2. Noise from electronics superimposed on signal

The detection limit and measurement accuracy are determined by the signal-to-noise ratio.

Electronic noise affects all measurements:

1. Detect presence of hit:      Noise level determines minimum threshold.  
If threshold too low, output dominated by noise hits.
2. Energy measurement:      Noise “smears” signal amplitude.
3. Time measurement:      Noise alters time dependence of signal pulse.

How to optimize the signal-to-noise ratio?

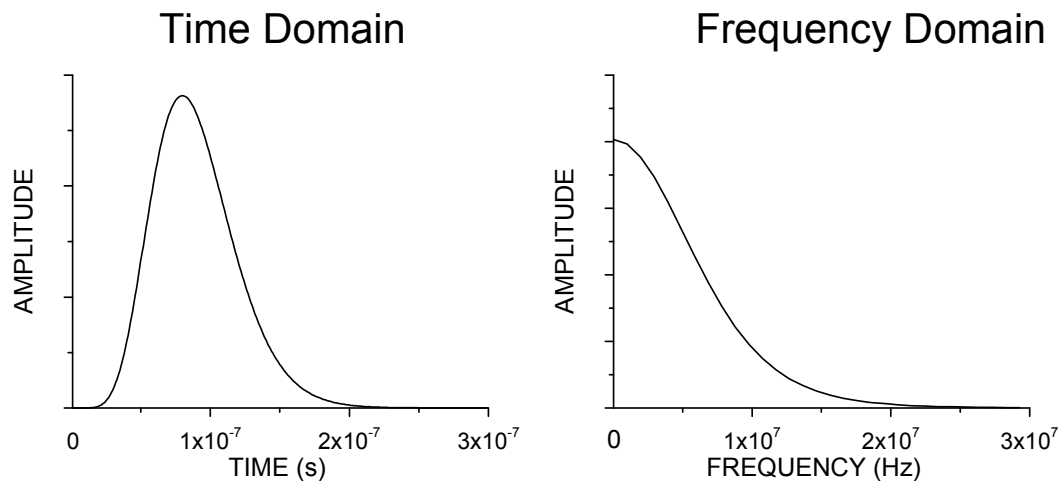
1. Increase signal and reduce noise
2. For a given sensor and signal: reduce electronic noise

Assume that the signal is a pulse.

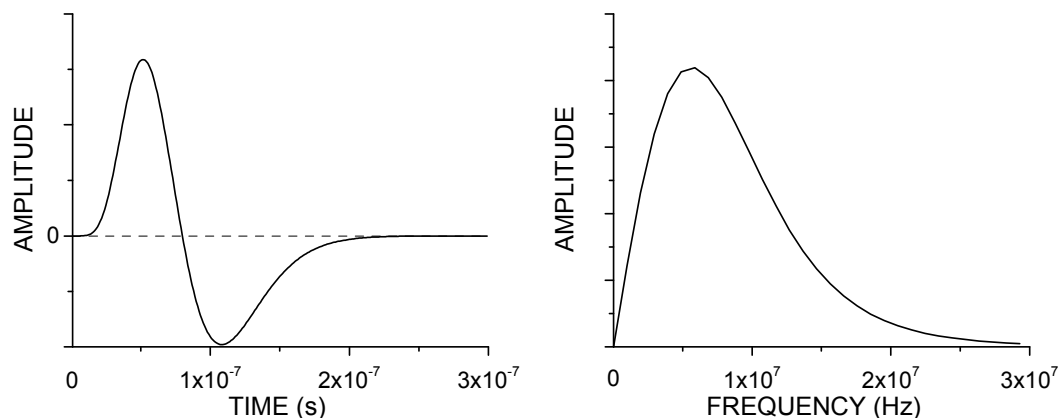
The time distribution of the signal corresponds to a frequency spectrum (Fourier transform).

Examples:

1. The pulse is unipolar, so it has a DC component and the frequency spectrum extends down to zero.

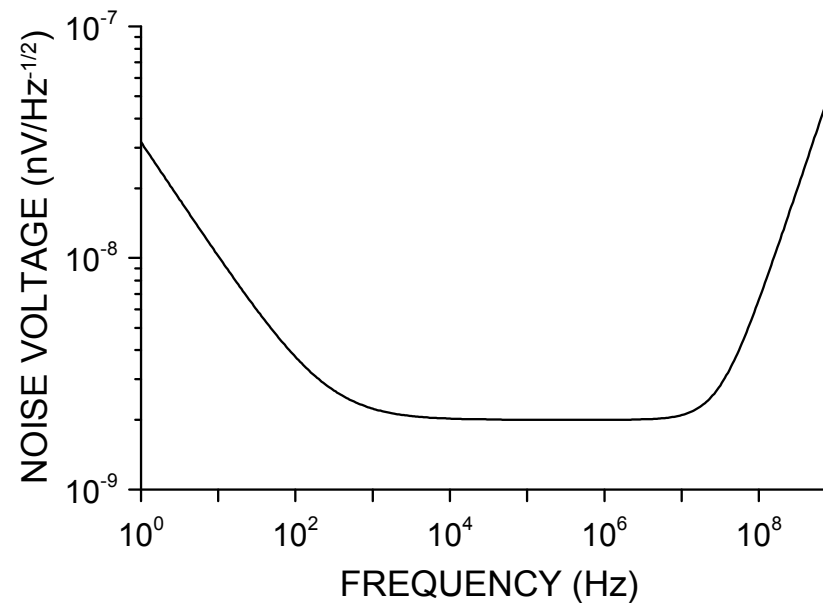


2. This bipolar pulse carries no net charge, so the frequency spectrum falls to zero at low frequencies, but extends to higher frequencies because of the faster slope.



The noise spectrum is generally not the same as the signal spectrum.

Typical Noise Spectrum:



⇒ Tailor frequency response of measurement system to optimize signal-to-noise ratio.

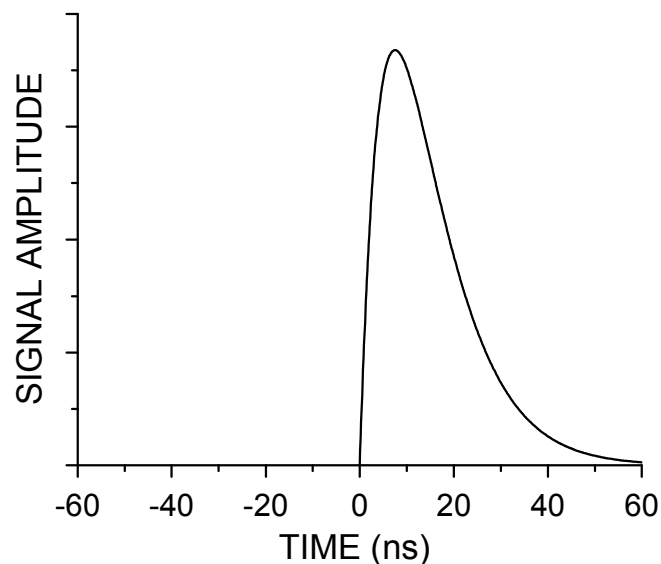
Frequency response of the measurement system affects both

- signal amplitude and
- noise.

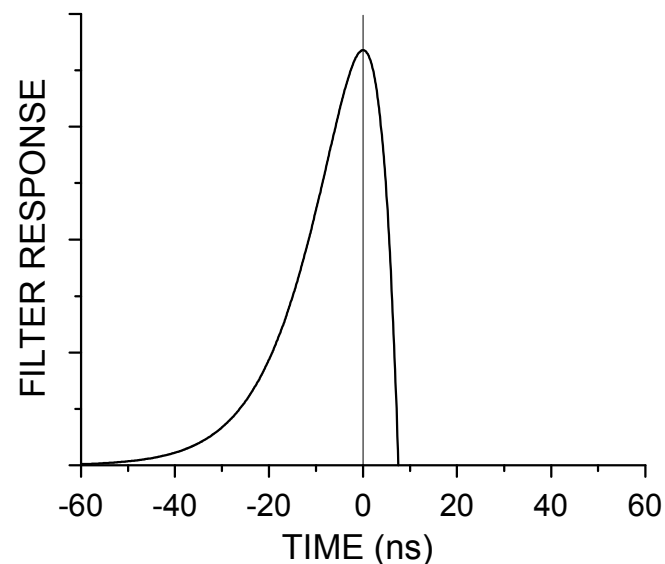
There is a general solution to this problem:

Apply a filter to make the noise spectrum white (constant over frequency). Then the optimum filter has an impulse response that is the signal pulse *mirrored in time* and shifted by the measurement time.

For example, if the signal pulse shape is:



⇒ The response of the optimum filter:



This is an “acausal” filter, i.e. it must act before the signal appears.

⇒ Only useful if the time of arrival is known in advance.

Not good for random events

Need time delay buffer memory

⇒ complexity!

Does that mean our problem is solved (and the lecture can end)?

1. The “optimum filter” preserves all information in signal, i.e. magnitude, timing, structure.

Usually, we need only subset of the information content,  
i.e. area (charge) or time-of-arrival.

Then the raw detector signal is not of the optimum form for the information that is required.

For example, a short detector pulse would imply a fast filter function. This retains both amplitude and timing information.

If only charge information is required, a slower filter is better, as will be shown later.

2. The optimum filter is often difficult or impractical to implement

Digital signal processing would seem to remove this restriction, but this approach is not practical for very fast signals or systems that require low power.

4. Simpler filters often will do nearly as well

5. Even a digital system requires continuous (“analog”) pre-processing.

6. It’s often useful to understand what you’re doing, so we’ll spend some more time to bring out the physical background of signal formation and processing.

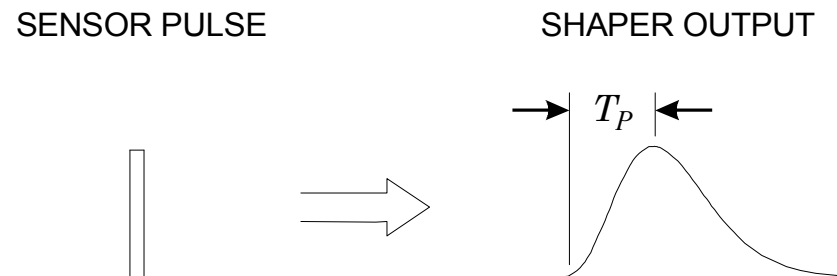
## Signal Processing Objectives

Two conflicting objectives:

### 1. Improve Signal-to-Noise Ratio $S/N$

Restrict bandwidth to match measurement time  $\Rightarrow$  Increase pulse width

Typically, the pulse shaper transforms a narrow detector current pulse to a broader pulse (to reduce electronic noise), with a gradually rounded maximum at the peaking time  $T_P$  (to facilitate measurement of the peak amplitude)



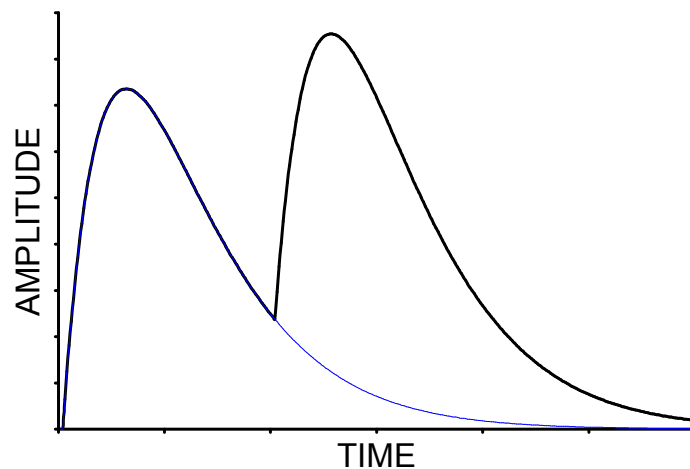
If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or pulse height analysis. The pulse height spectrum is the energy spectrum.

## 2. Improve Pulse Pair Resolution

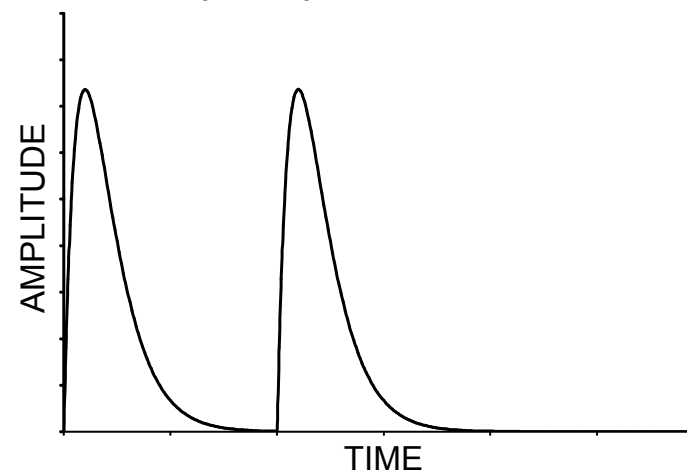


Decrease pulse width

Pulse pile-up distorts amplitude measurement.



Reducing pulse shaping time to 1/3 eliminates pile-up.



Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

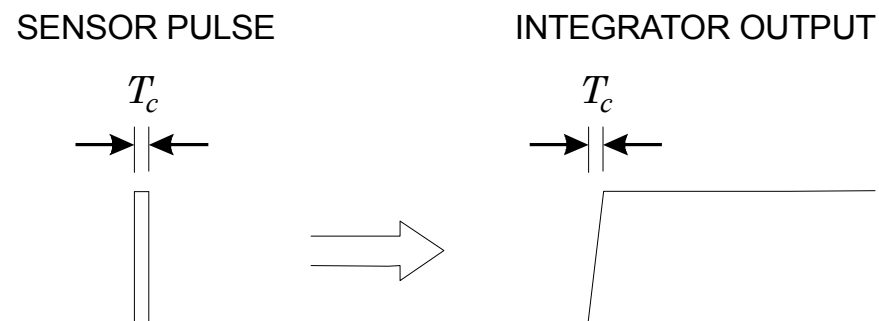
Usually, many considerations combined lead to a “non-textbook” compromise.

- “*Optimum shaping*” depends on the application!
- Shapers need not be complicated – *Every amplifier is a pulse shaper!*



Goal: Improve energy resolution

Procedure: Integrate detector signal current  $\Rightarrow$  Step impulse



Commonly approximated as  
“step” response (zero rise time).

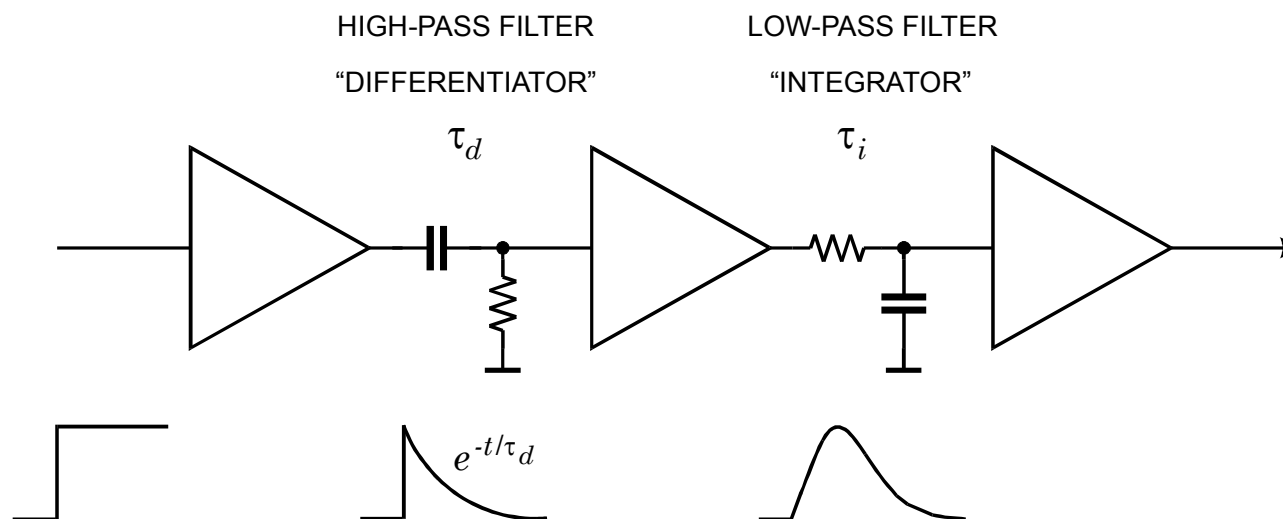
Long “flat top” allows measurements at times well beyond the collection time  $T_C$ .

$\Rightarrow$  Allows reduced bandwidth and great flexibility in selecting shaper response.

Optimum for energy measurements, but not for fast timing!

“Fast-slow” systems utilize parallel processing chains to optimize both timing and energy resolution (see Timing Measurements).

## Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

⇒ Useful for estimates, since simple to evaluate

Key elements:

- lower frequency bound ( $\hat{=}$  pulse duration)
- upper frequency bound ( $\hat{=}$  rise time)

are common to all shapers.

## 2. Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

### Equivalent Noise Charge

Inject known signal charge into preamp input  
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge  $\equiv$  Input charge for which  $S/N = 1$

## Ballistic Deficit

When the rise time of the input pulse to the shaper extends beyond the nominal peaking time, the shaper output is both stretched in time and the amplitude decreases

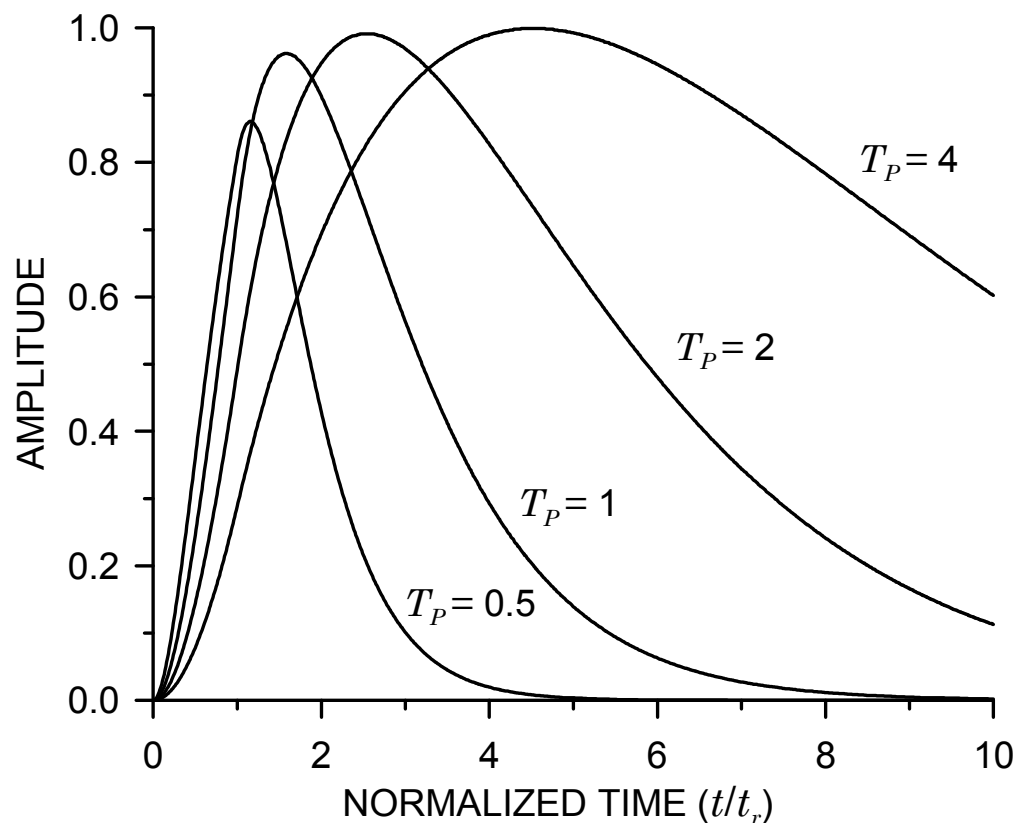
Shaper output for an input rise time

$$t_r = 1$$

for various values of nominal peaking time.

Note that the shaper with  $T_P = 0.5$

- peaks at  $t = 1.15t_r$
- and
- attains only 86% of the pulse height achieved at longer shaping times.



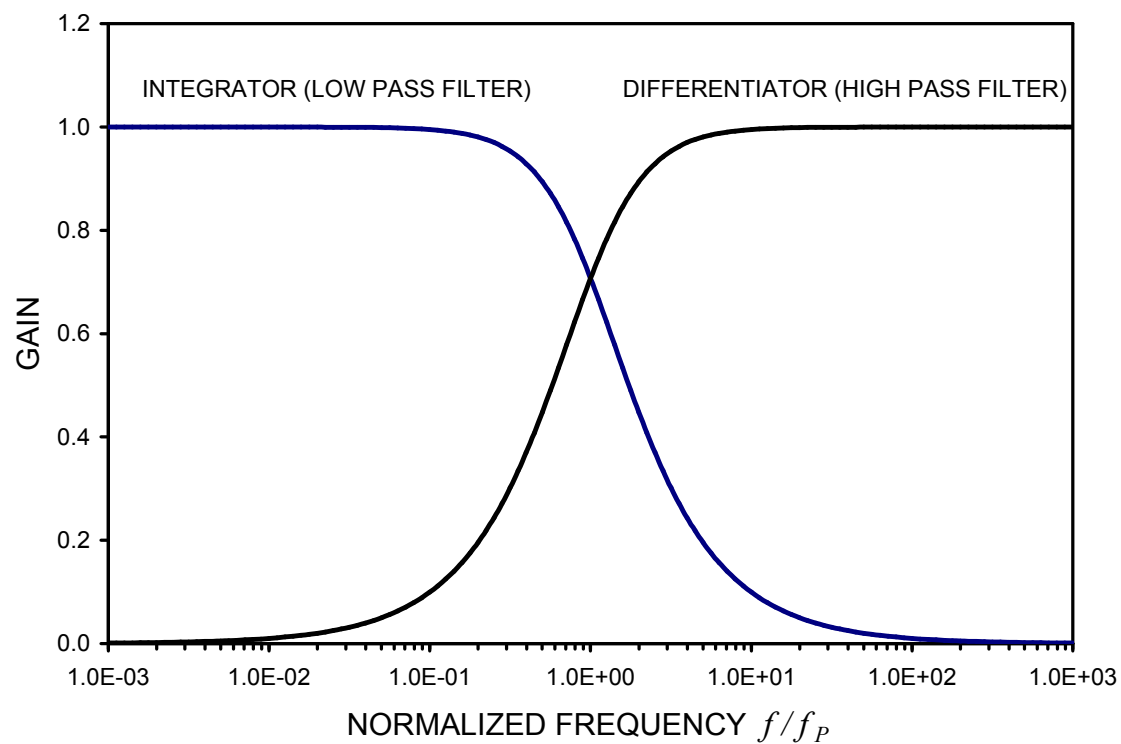
⇒ Increased equivalent noise charge

## Noise Charge vs. Shaping Time

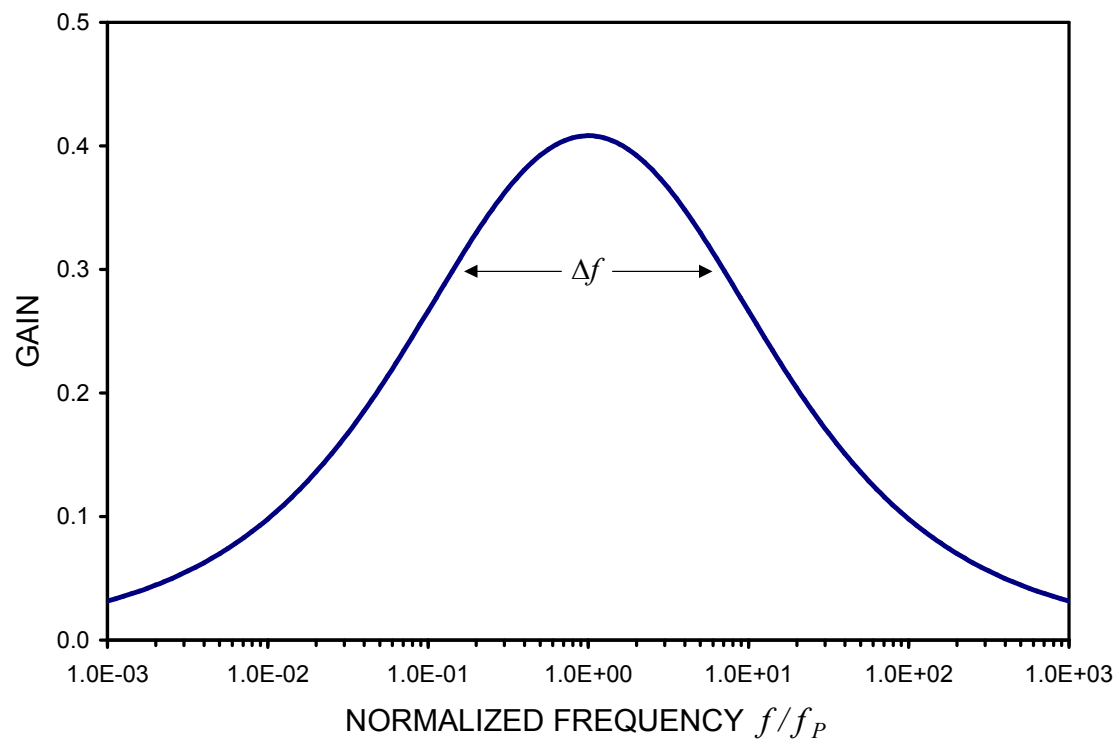
Assume that differentiator and integrator time constants are equal  $\tau_i = \tau_d \equiv \tau$ .

$\Rightarrow$  Both cutoff frequencies equal:  $f_U = f_L \equiv f_P = 1 / 2\pi\tau$ .

Frequency response of individual pulse shaping stages



## Combined frequency response

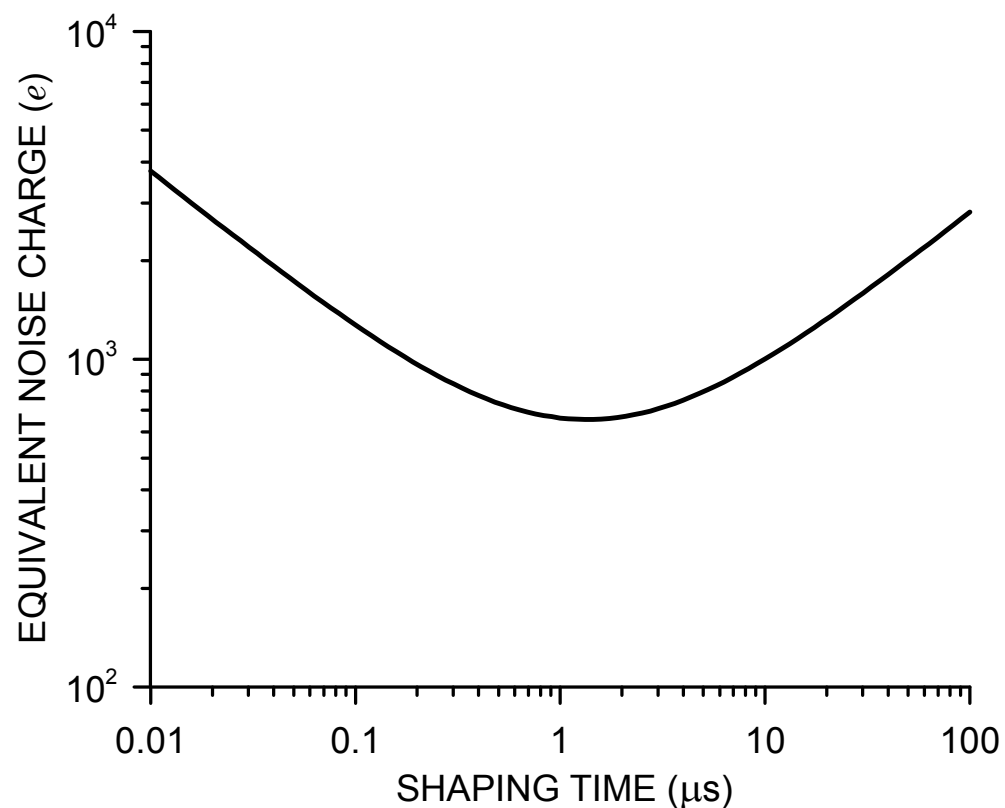


Logarithmic frequency scale  $\Rightarrow$  shape of response independent of  $\tau$ .

However, bandwidth  $\Delta f$  decreases with increasing time constant  $\tau$ .

$\Rightarrow$  for white noise sources expect noise to decrease with bandwidth,  
i.e. decrease with increasing time constant.

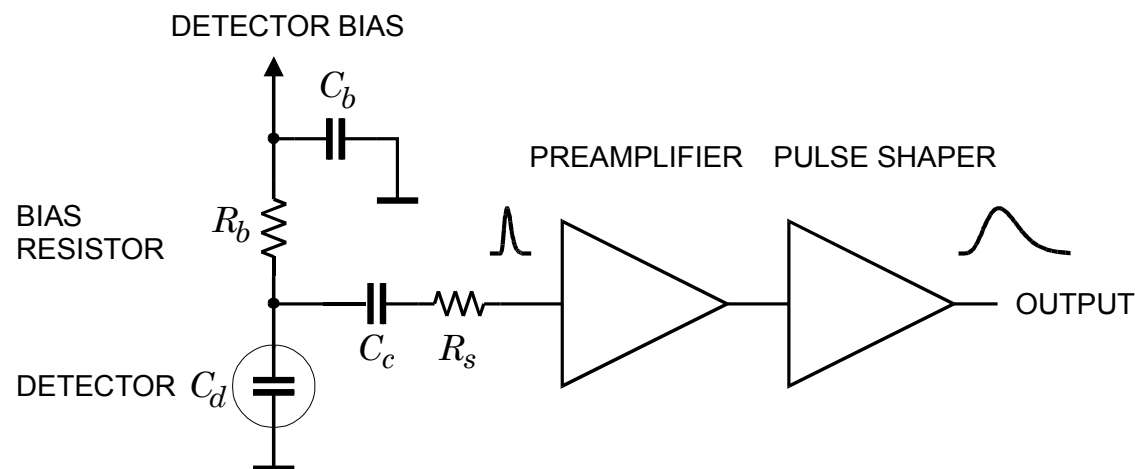
Result of typical noise measurement vs. shaping time



Noise sources (thermal and shot noise) have a flat (“white”) frequency distribution.

Why doesn't the noise decrease monotonically with increasing shaping time (decreasing bandwidth)?

## Analytical Analysis of a Detector Front-End



Detector bias voltage is applied through the resistor  $R_b$ . The bypass capacitor  $C_b$  serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the “far end” of the bias resistor to ground, so that  $R_b$  appears to be in parallel with the detector.

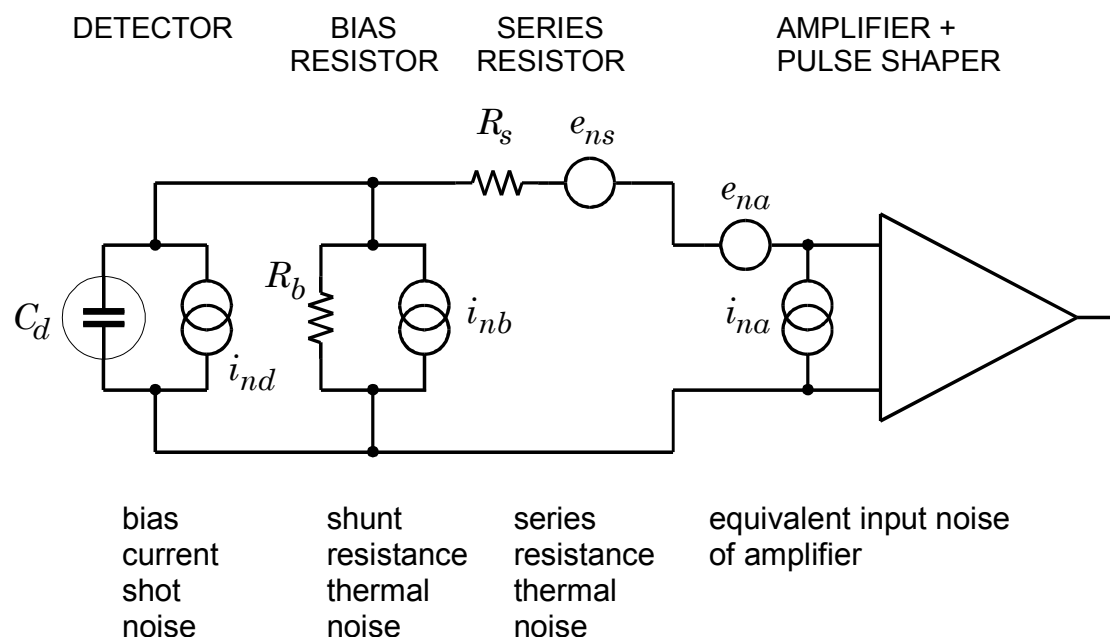
The coupling capacitor  $C_c$  in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why this capacitor is also called a “blocking capacitor”).

The series resistor  $R_s$  represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients (“input protection”)



## Equivalent circuit for noise analysis



In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage appearing at the amplifier input.

Resistors can be modeled either as voltage or current generators.

- Resistors in parallel with the input act as current sources.
- Resistors in series with the input act as voltage sources.

## Steps in the analysis:

1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources
2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.
3. Determine the output signal for a known signal charge and calculate equivalent noise charge (signal charge for  $S/N= 1$ )

First, assume a simple *CR-RC* shaper with

equal differentiation and integration time constants  $\tau_d = \tau_i = \tau$  ,

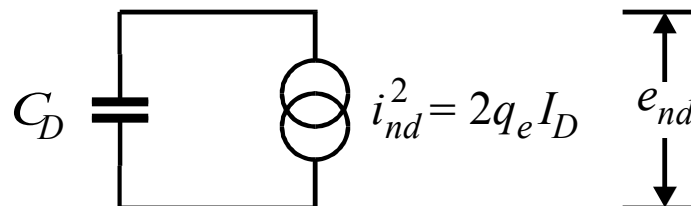
which in this special case is equal to the peaking time.

For a given differentiation time constant – chosen to set the pulse length – the *CR-RC* shaper achieves optimum noise when the integration time constant is set to the same value.

This is not a general condition for other shapers.

## Noise Contributions

### a) Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite
2. The shunt resistance  $R_p$  is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

*Does this assumption make sense?*

If  $R_p$  is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_p C_D \gg T_P \approx \frac{1}{\omega_p},$$

where  $\omega_p$  is the midband frequency of the shaper.

Therefore,  $R_p \gg \frac{1}{\omega_p C_D}$  as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega C_D)^2} = 2q_e I_D \frac{1}{(\omega C_D)^2}$$

⇒ The noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent with a capacitive signal source.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time  $T$ . The number of electron charges measured is

$$N_e = \frac{I_D T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

*Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?*

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

## b) Parallel Resistance

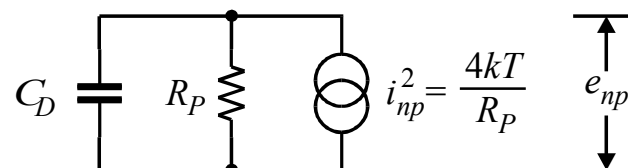
Any shunt resistance  $R_P$  acts as a noise current source. In the specific example shown above, the only shunt resistance is the bias resistor  $R_b$ .

Additional shunt components in the circuit:

1. bias noise current source (infinite resistance by definition)
2. detector capacitance

The noise current flows through both the resistance  $R_P$  and the detector capacitance  $C_D$ .

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$e_{np}^2 = \frac{4kT}{R_P} \left( \frac{R_P \cdot \frac{-\mathbf{i}}{\omega C_D}}{R_P - \frac{\mathbf{i}}{\omega C_D}} \right)^2$$

$$e_{np}^2 = 4kTR_P \frac{1}{1 + (\omega R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^{\infty} e_{np}^2(\omega) d\omega = \int_0^{\infty} \frac{4kTR_P}{1 + (\omega R_P C_D)^2} d\omega = \frac{kT}{C_D},$$

which is independent of  $R_P$ . Commonly referred to as “ $kTC$ ” noise, this contribution is often erroneously interpreted as the “noise of the detector capacitance”.

An ideal capacitor has no thermal noise; all noise originates in the resistor.

So, why is the result independent of  $R_P$ ?

$R_P$  determines the primary noise, but also the noise bandwidth of this subcircuit. As  $R_P$  increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of  $R_P$ .

However,

If one integrates  $e_{np}$  over a bandwidth-limited system (such as our shaper),

$$v_n^2 = \int_0^{\infty} 4kTR_P \left| \frac{G(i\omega)}{1 - i\omega R_P C_D} \right|^2 d\omega,$$

the total noise decreases with increasing  $R_P$ .

### c) Series Resistance

The noise voltage generator associated with the series resistance  $R_S$  is in series with the other noise sources, so it simply contributes

$$e_{nr}^2 = 4kTR_S$$



## d) Amplifier input noise

The amplifier noise voltage sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain, where it appears as a noise voltage generator.

$$e_{na}^2 = e_{nw}^2 + \frac{A_f}{f}$$

$\uparrow$   
 “white noise”

$\uparrow$   
 $1/f$  noise  
 (can also originate in external components)

This noise voltage generator also adds in series with the other sources.

- Amplifiers generally also exhibit input current noise, which is physically present at the input. Its effect is the same as for the detector bias current, so the analysis given in a) can be applied.
- In a well-designed amplifier the noise is dominated by the input transistor (fast, high-gain transistors generally best).  
Noise parameters of transistors are discussed in Chapter 6.

Transistor input noise decreases with transconductance  $\Rightarrow$  increased power

- Minimum device noise limited both by technology and fundamental physics.

## Equivalent Noise Charge

$$Q_n^2 = \left( \frac{e^2}{8} \right) \left[ \left( 2q_e I_D + \frac{4kT}{R_p} + i_{na}^2 \right) \cdot \tau + \left( 4kTR_s + e_{na}^2 \right) \cdot \frac{C_D^2}{\tau} + 4A_f C_D^2 \right]$$

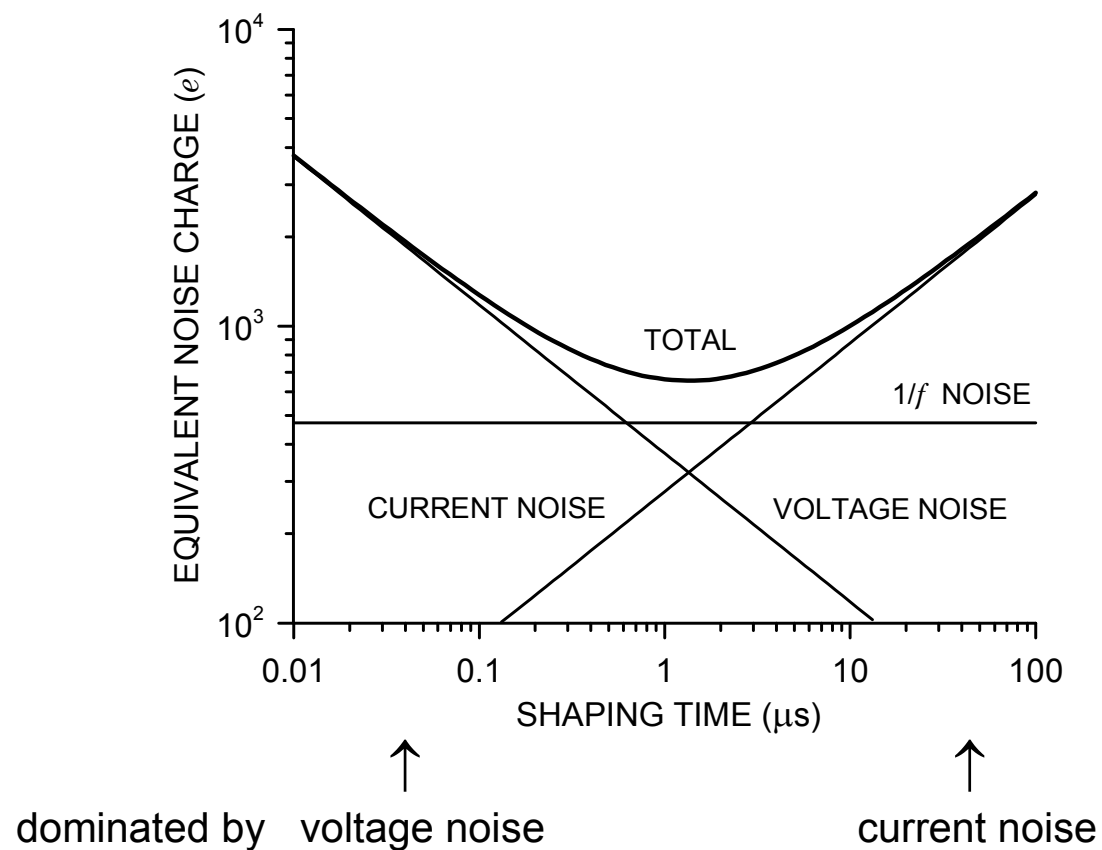
	↑	↑	↑
$e = \exp(1)$	current noise	voltage noise	1/f noise
	$\propto \tau$	$\propto 1/\tau$	independent of $\tau$
	independent of $C_D$	$\propto C_D^2$	$\propto C_D^2$

- Current noise is independent of detector capacitance, consistent with the notion of “counting electrons”.
- Voltage noise increases with detector capacitance (reduced signal voltage)
- $1/f$  noise is independent of shaping time.

In general, the total noise of a  $1/f$  source depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If  $\tau_d$  and  $\tau_i$  are scaled by the same factor, this ratio remains constant.

- Detector leakage current and FET noise decrease with temperature
- ⇒ High resolution Si and Ge detectors for x-rays and gamma rays operate at cryogenic temperatures.

The equivalent noise charge  $Q_n$  assumes a minimum when the current and voltage noise contributions are equal. Typical result:



For a  $CR-RC$  shaper the noise minimum obtains for  $\tau_d = \tau_i = \tau$ .

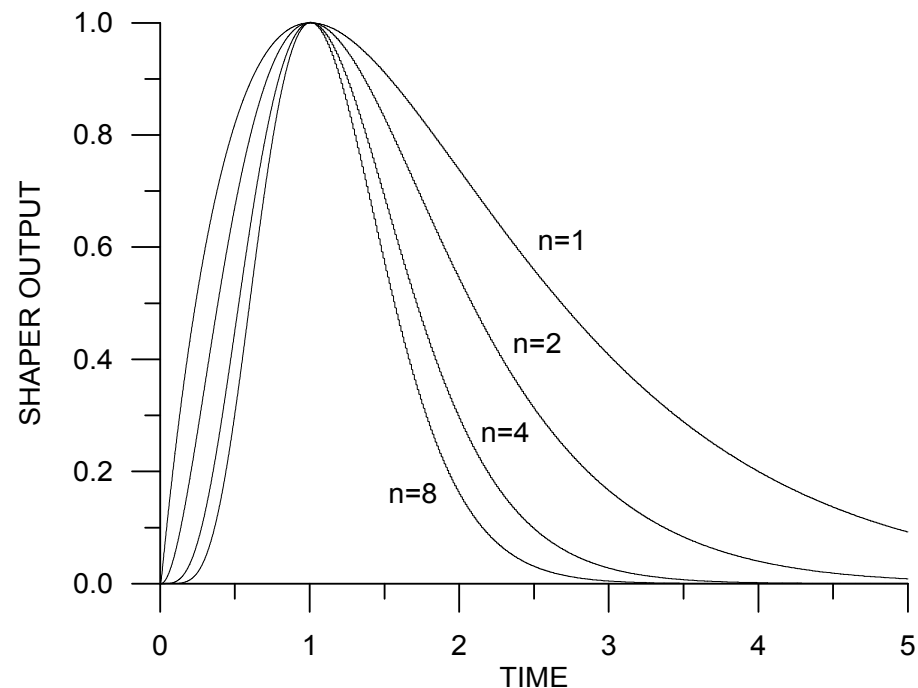
This criterion does not hold for more sophisticated shapers.

## Other Types of Shapers

### Shapers with Multiple Integrators

Start with simple *CR-RC* shaper and add additional integrators ( $n=1$  to  $n=2, \dots n=8$ ).

Change integrator time constants to preserve the peaking time  $\tau_n = \tau_{n=1} / n$



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

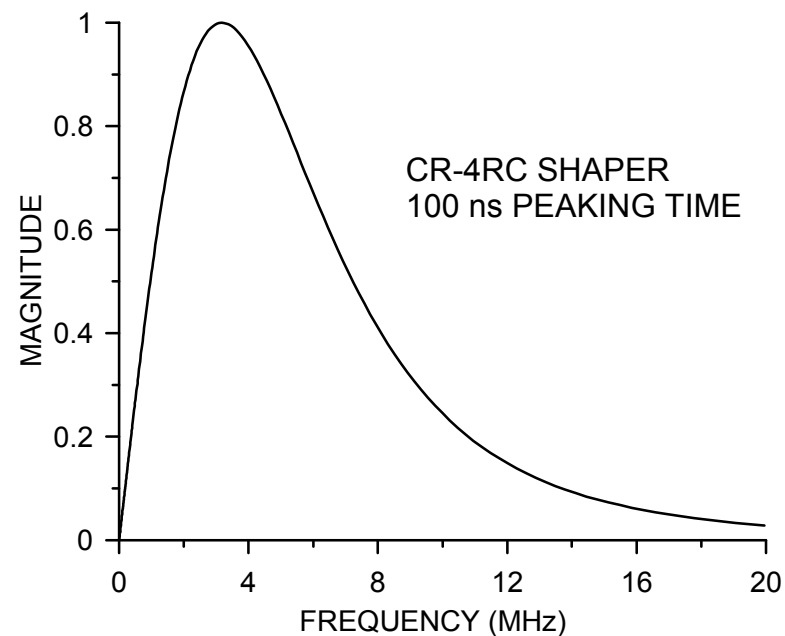
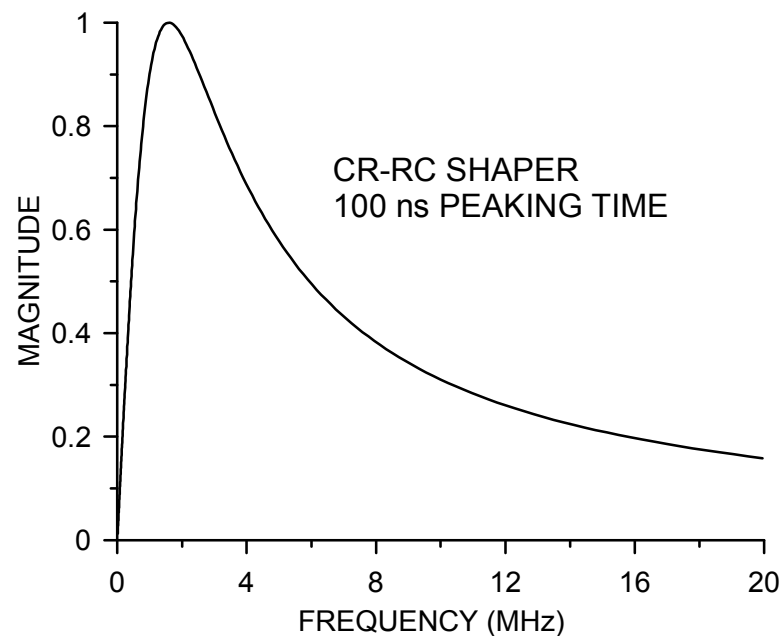
⇒ improved rate capability at the same peaking time

Shapers with the equivalent of 8 *RC* integrators are common. Usually, this is achieved with active filters

(i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

## Time vs. Frequency Response

Two different shapers with the same peaking time



The CR-RC shaper appears narrower, but takes longer to reach the baseline.

## Time-Variant Shapers

Time variant shaper change the filter parameters during the processing of individual pulses.

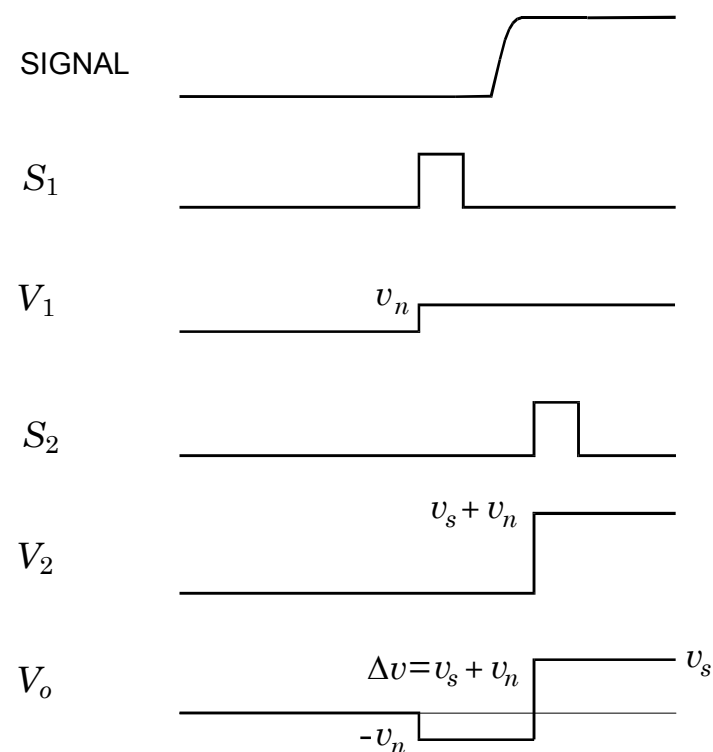
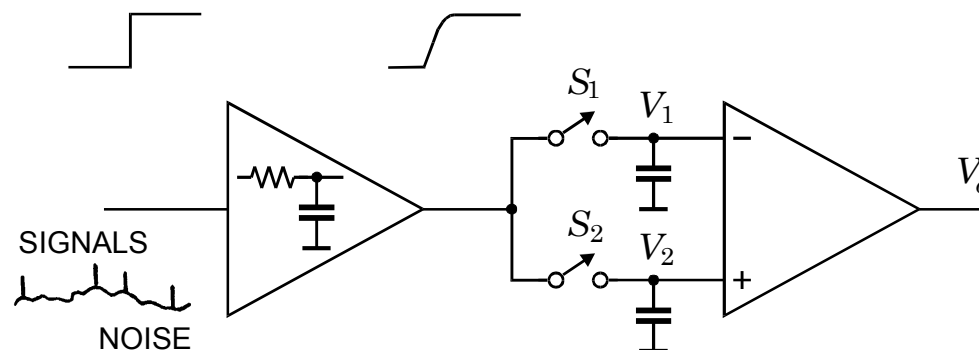
A commonly used time-variant filter is the correlated double-sampler.

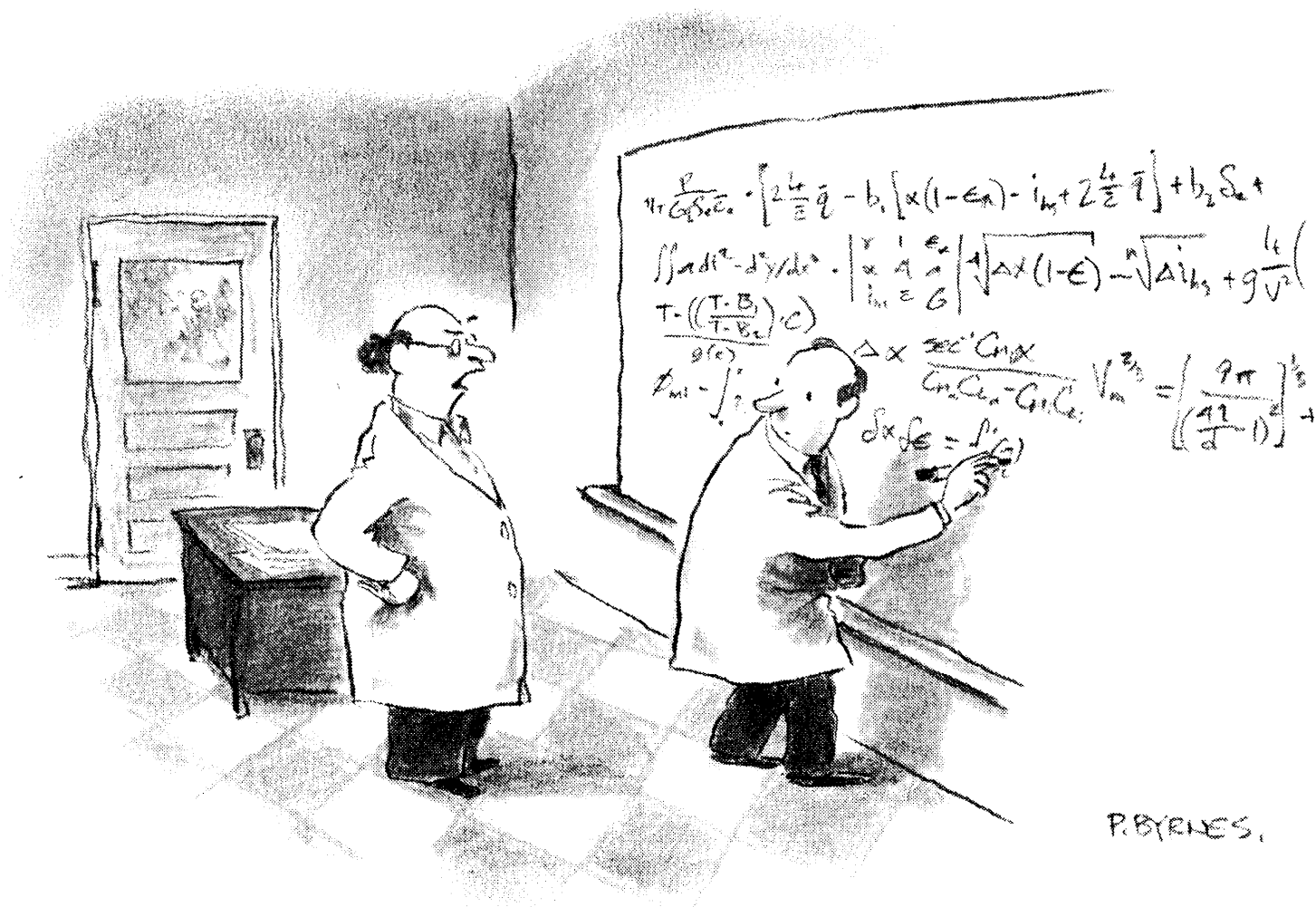
1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal

S/N depends on

1. rise-time of the prefilter
2. time difference between samples

See “Semiconductor Detector Systems” for a detailed noise analysis. (Chapter 4, pp 160-166)





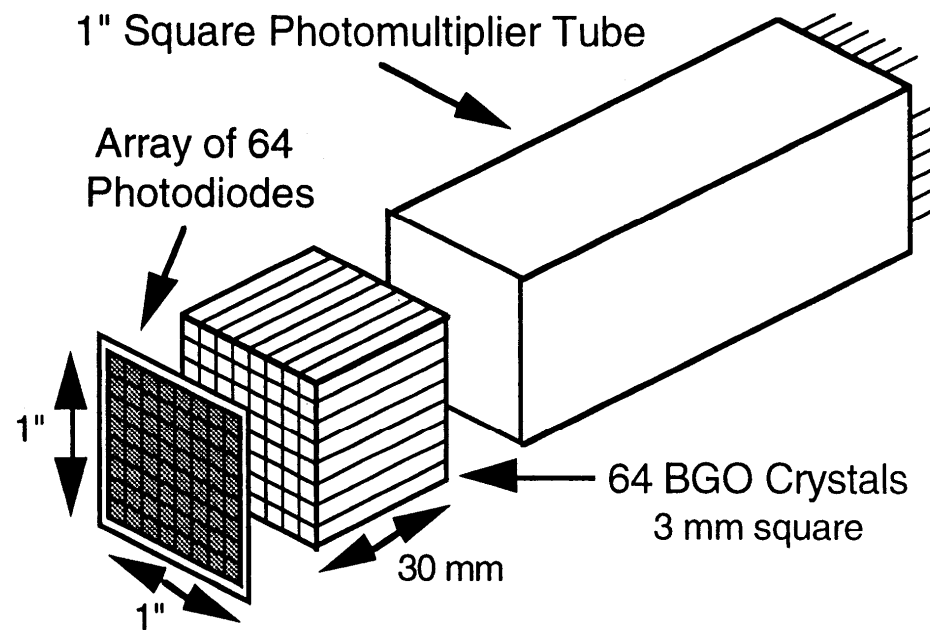
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"Dub."

## Examples: Photodiode Readout

(S. Holland, N. Wang, I. Kipnis, B. Krieger, W. Moses, LBNL)

### Medical Imaging (Positron Emission Tomography)

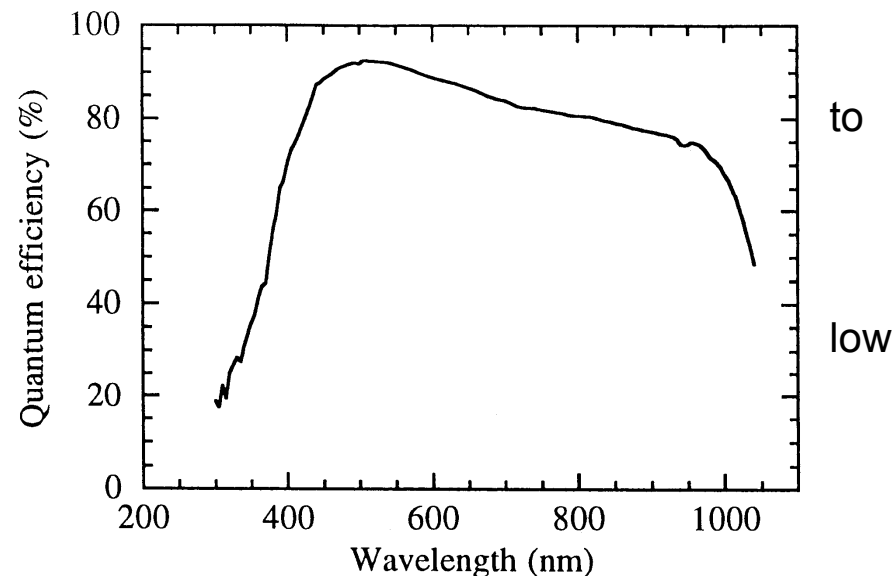


Read out 64 BGO crystals with one PMT (timing, energy) and tag crystal by segmented photodiode array.



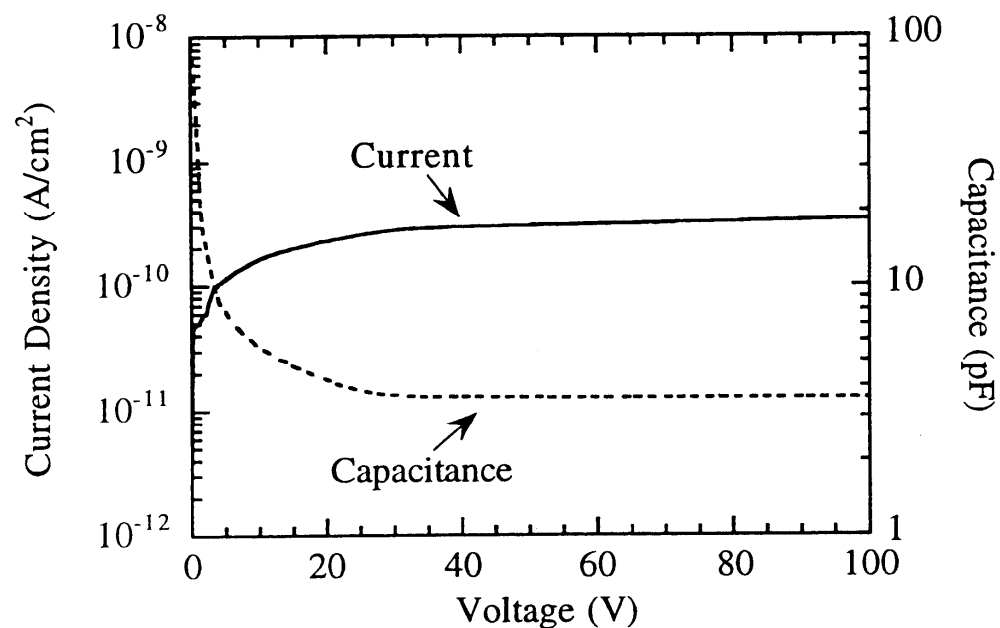
Requires thin dead layer on photodiode  
maximize quantum efficiency.

Thin electrode must be implemented with  
resistance to avoid significant  
degradation of electronic noise.



Furthermore, low reverse bias current  
critical to reduce noise.

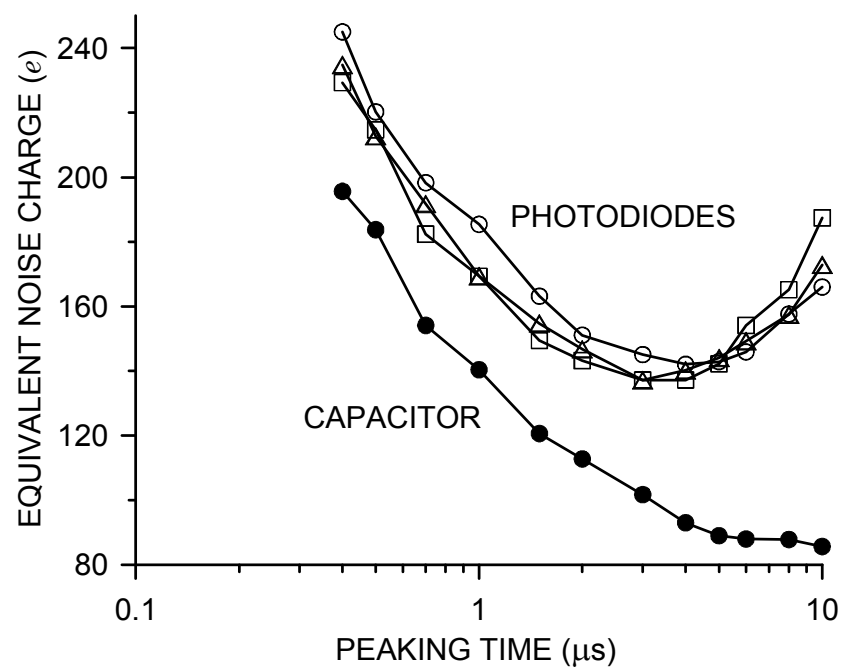
Photodiodes designed and fabricated  
in LBNL Microsystems Lab.



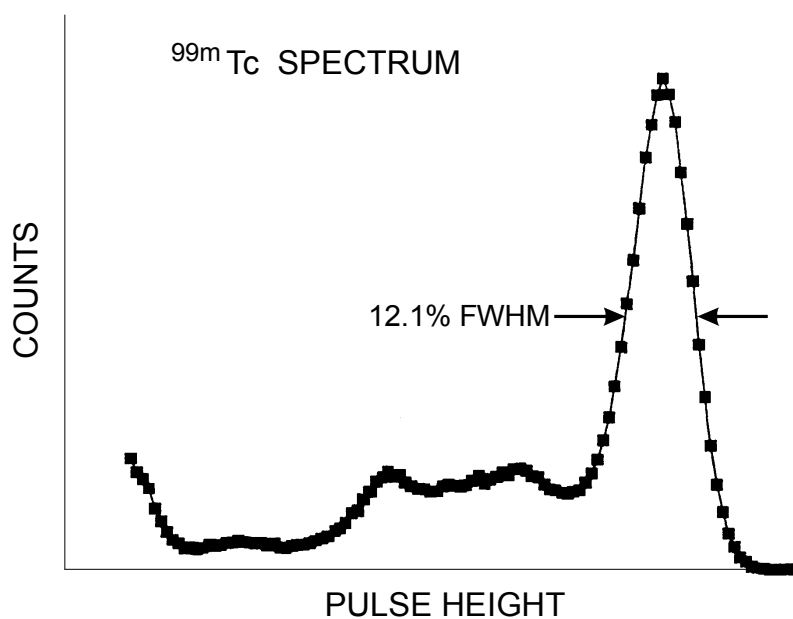
Front-end chip (preamplifier + shaper): 16 channels per chip, die size: 2 x 2 mm<sup>2</sup>,  
1.2 μm CMOS

Continuously adjustable shaping time (0.5 to 50 μs)

Noise vs. shaping time



Energy spectrum with BGO scintillator



Note increase in noise at long shaping times when photodiode is connected - shot noise contribution.

## Examples: Short-Strip Si X-Ray Detector

(B. Ludewigt, C. Rossington, I. Kipnis, B. Krieger, LBNL)

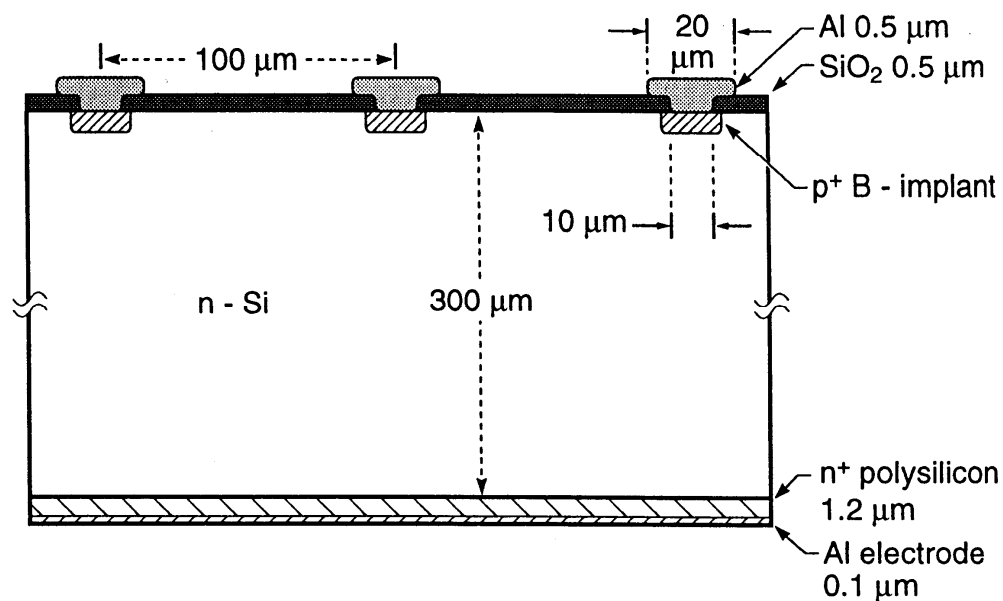
Use detector with multiple strip electrodes not for position resolution, but for segmentation

- ⇒ distribute rate over many channels
- ⇒ reduced capacitance
- ⇒ low noise at short shaping time
- ⇒ higher rate per detector element

For x-ray energies 5 – 25 keV ⇒ photoelectric absorption dominates  
(signal on 1 or 2 strips)

Strip pitch: 100  $\mu\text{m}$

Strip Length: 2 mm  
(matched to ALS beam)

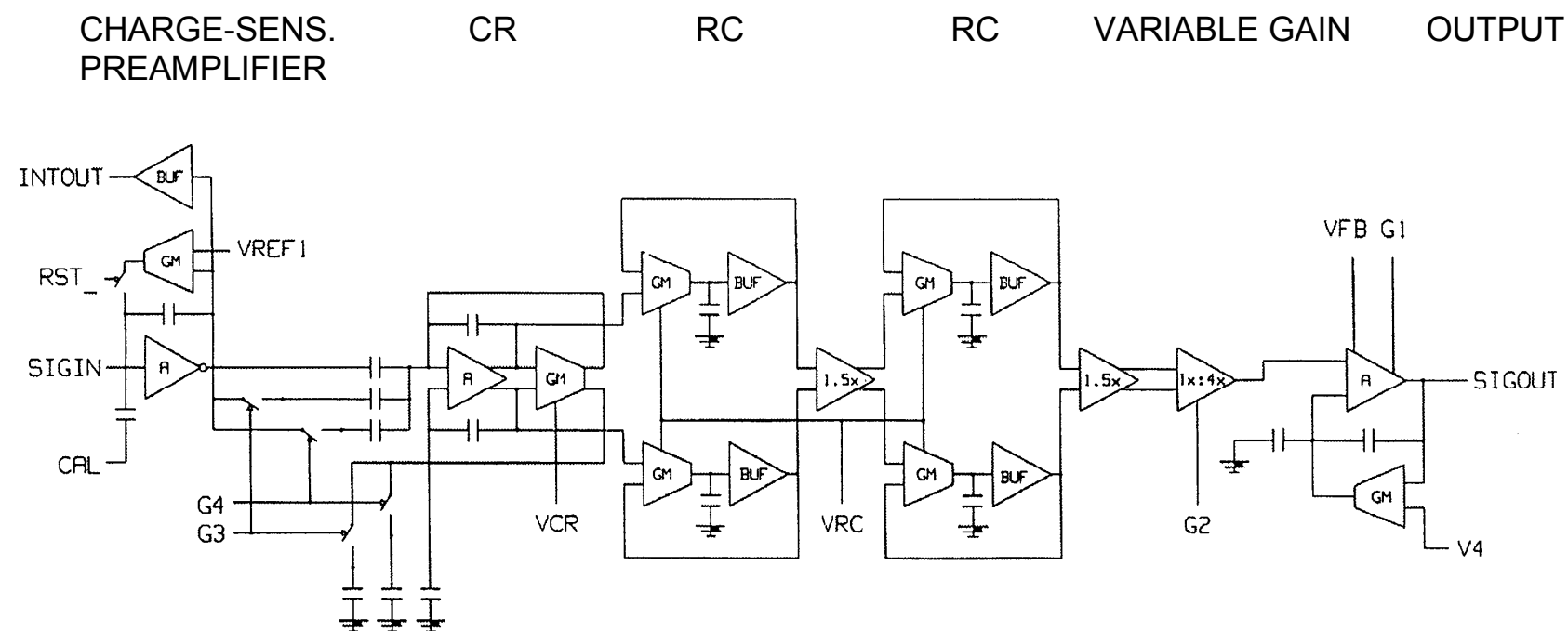


Readout IC tailored to detector

Preamplifier + CR-RC<sup>2</sup> shaper + cable driver to bank of parallel ADCs (M. Maier + H. Yaver)

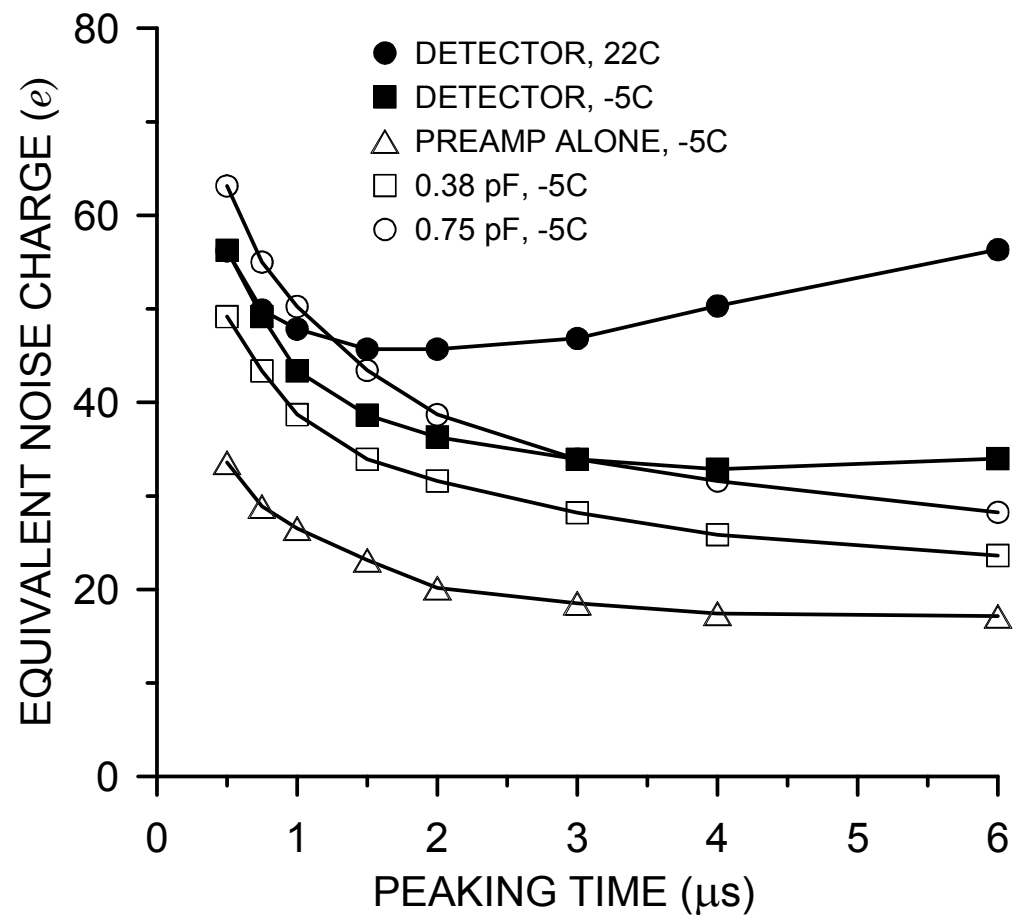
Preamplifier with pulsed reset.

Shaping time continuously variable 0.5 to 20  $\mu$ s.



## Noise Charge vs. Peaking Time

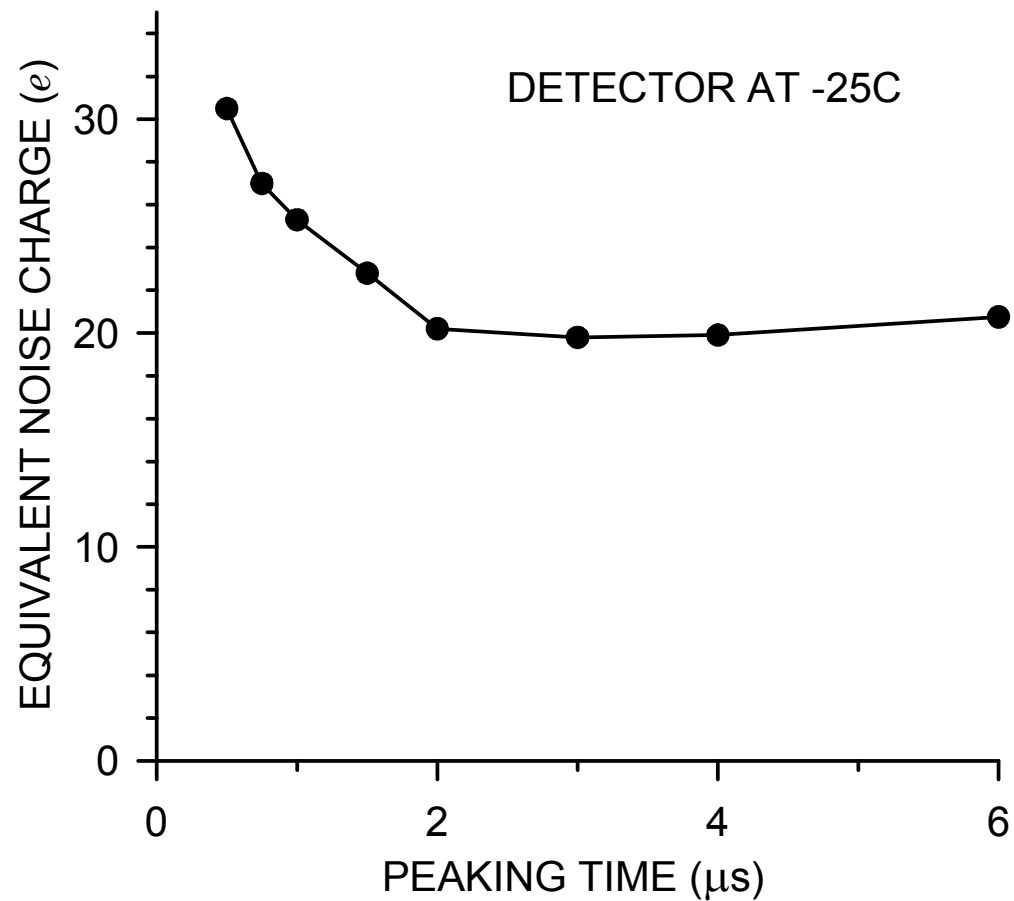
- Open symbols: preamplifier alone and with capacitors connected instead of a detector.
- Connecting the detector increases noise because of added capacitance and detector current (as indicated by increase of noise with peaking time).
- Cooling the detector reduces the current and noise improves.



## Second prototype

Current noise negligible because of cooling.

“Flat” noise vs. shaping time indicates that  $1/f$  noise dominates.





## “Series” and “Parallel” Noise

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

The rest of the world uses equivalent noise voltage and current.

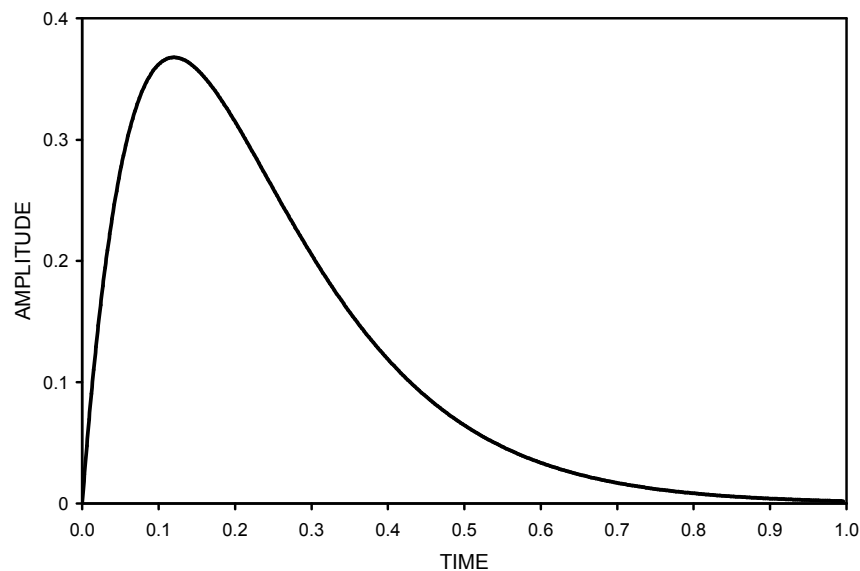
Since they are physically meaningful, use of these widely understood terms is preferable (and makes more sense).



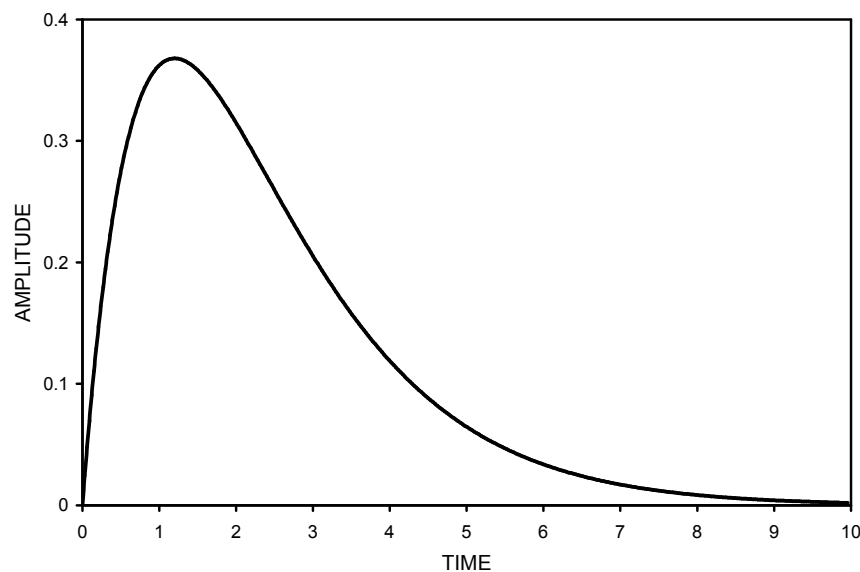
## Scaling of Filter Noise Parameters

Pulse shape is the same when shaping time is changed.

shaping time =  $\tau$



shaping time =  $10\tau$



Shaper can be characterized by a “shape factor” which multiplied by the shaping time sets the noise bandwidth.

The expression for the equivalent noise charge

$$Q_n^2 = \left( \frac{e^2}{8} \right) \left[ \left( 2q_e I_D + \frac{4kT}{R_p} + i_{na}^2 \right) \cdot \tau + \left( 4kTR_S + e_{na}^2 \right) \cdot \frac{C_D^2}{\tau} + 4A_f C_D^2 \right]$$

↑

$e = \exp(1)$

↑

current noise  
 $\propto \tau$   
independent of  $C_D$

↑

voltage noise  
 $\propto 1/\tau$   
 $\propto C_D^2$

↑

1/f noise  
independent of  $\tau$   
 $\propto C_D^2$

can be put in a more general form that applies to all type of pulse shapers:

$$Q_n^2 = i_n^2 T_s F_i + C^2 e_n^2 \frac{F_v}{T_s} + F_{vf} A_f C^2$$

- The current and voltage terms are combined and represented by  $i_n^2$  and  $e_n^2$ .
- The shaper is characterized by a shape and characteristic time (e.g. the peaking time).
- A specific shaper is described by the “shape factors”  $F_i$ ,  $F_v$ , and  $F_{vf}$ .
- The effect of the shaping time is set by  $T_s$ .

## Detector Noise Summary

Two basic noise mechanisms: input noise current  $i_n$   
 input noise voltage  $e_n$

Equivalent Noise Charge:  $Q_n^2 = i_n^2 T_s F_i + C^2 e_n^2 \frac{F_v}{T_s}$

$T_s$  Characteristic shaping time (*e.g.* peaking time)

$F_i$ ,  $F_v$  "Shape Factors" that are determined by the shape of the pulse.

$C$  Total capacitance at the input (detector capacitance + input capacitance of preamplifier + stray capacitance + ... )

Note that  $F_i < F_v$  for higher order shapers.

Typical values of  $F_i$ ,  $F_v$

CR-RC shaper  $F_i = 0.924$   $F_v = 0.924$

CR-(RC)<sup>4</sup> shaper  $F_i = 0.45$   $F_v = 1.02$

CR-(RC)<sup>7</sup> shaper  $F_i = 0.34$   $F_v = 1.27$

CAFE chip  $F_i = 0.4$   $F_v = 1.2$

Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).

Minimum noise obtains when the current and voltage noise contributions are equal.

### Current noise

- detector bias current increases with detector size, strongly temperature dependent
- noise from resistors shunting the input increases as resistance is decreased
- input transistor – low for FET, higher for BJTs

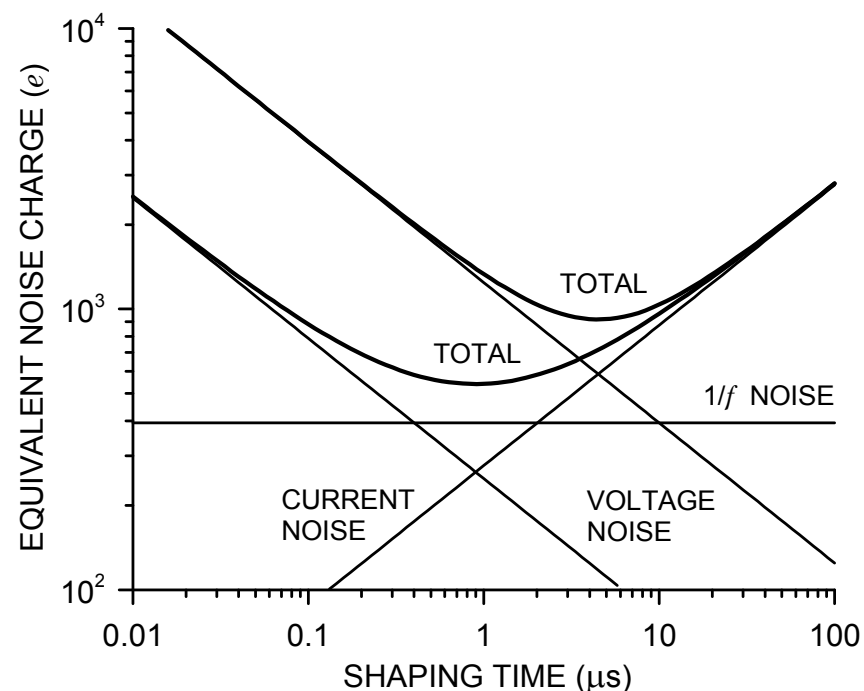
### Voltage noise

- input transistor – noise decreases with increased current
- series resistance, e.g. detector electrode, protection circuits

FETs commonly used as input devices – improved noise performance when cooled ( $T_{opt} \approx 130$  K)

Bipolar transistors advantageous at short shaping times (<100 ns).

When collector current is optimized, bipolar transistor equivalent noise charge is independent of shaping time (see Chapter 6).



Equivalent Noise Charge vs. Detector Capacitance ( $C = C_d + C_a$ )

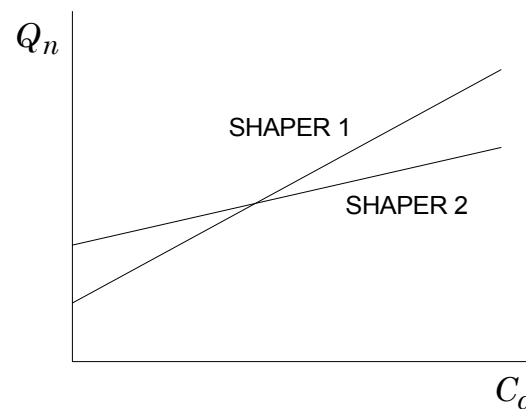
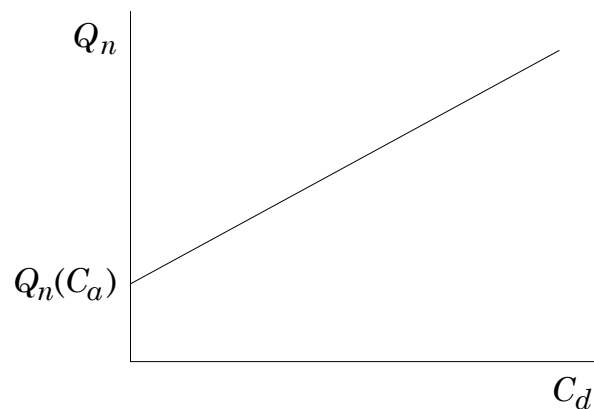
$$Q_n = \sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}$$

$$\frac{dQ_n}{dC_d} = \frac{2C_d e_n^2 F_v \frac{1}{T}}{\sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}}$$

If current noise  $i_n^2 F_i T$  is negligible, i.e. **voltage noise dominates**:  $\frac{dQ_n}{dC_d} \approx 2e_n \cdot \sqrt{\frac{F_v}{T}}$

Zero intercept:  $Q_n|_{C_d=0} = C_a e_n \sqrt{F_v / T}$

↑            ↑  
input      shaper  
stage



# Noise vs. Power Dissipation

## Analog front-end:

Equivalent Noise Charge: 
$$Q_n^2 \approx i_n^2 T_S + e_n^2 C_d^2 \frac{1}{T_S}$$

$T_S$  Shaping Time

$i_n$  Spectral noise current density  $i_n^2 = 2eI_{bias} \propto$  strip length

$C_d$  Detector capacitance  $\propto$  strip length

$e_n$  Amplifier spectral noise voltage density  $e_n^2 \approx \frac{1}{g_m}$

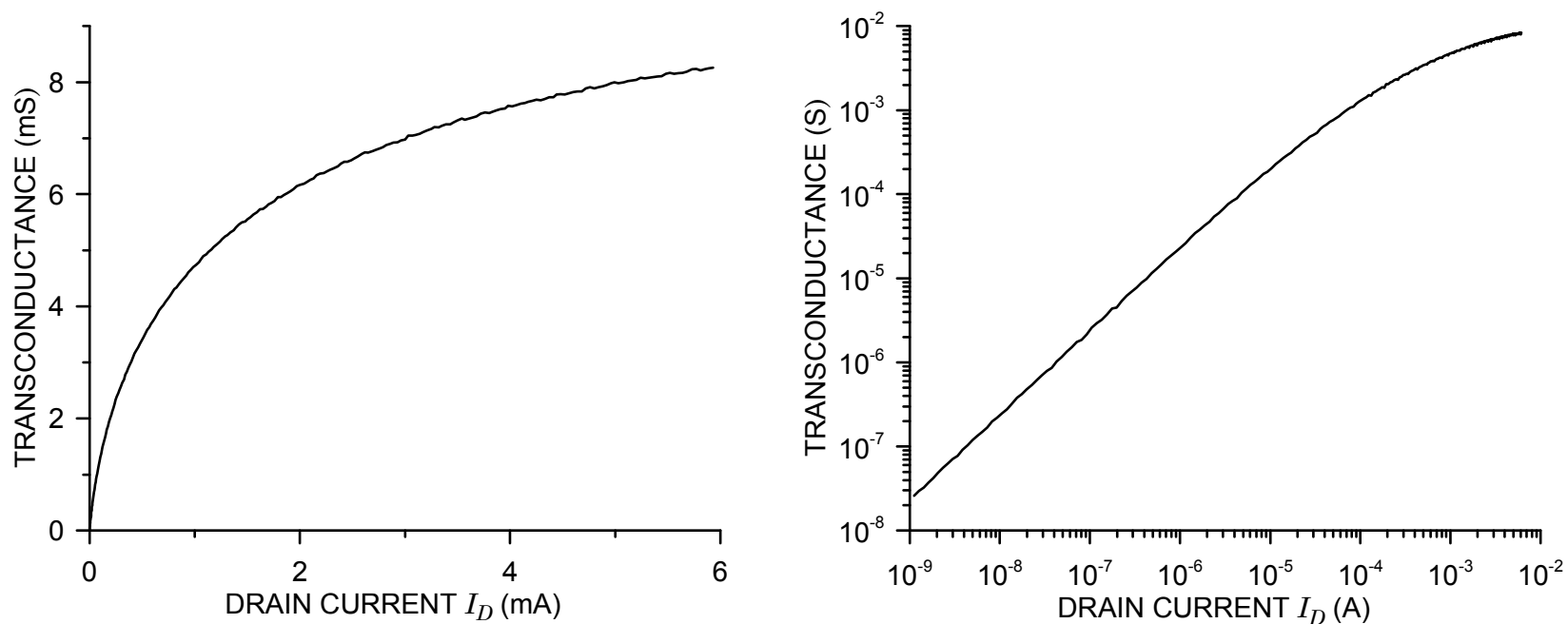
Amplifier spectral noise voltage density  $e_n$  depends on transconductance  $g_m$  of the input transistor.

How does transconductance depend on the current (power) of the input transistor?

In analog circuitry the current draw is driven by the requirements of noise and speed.

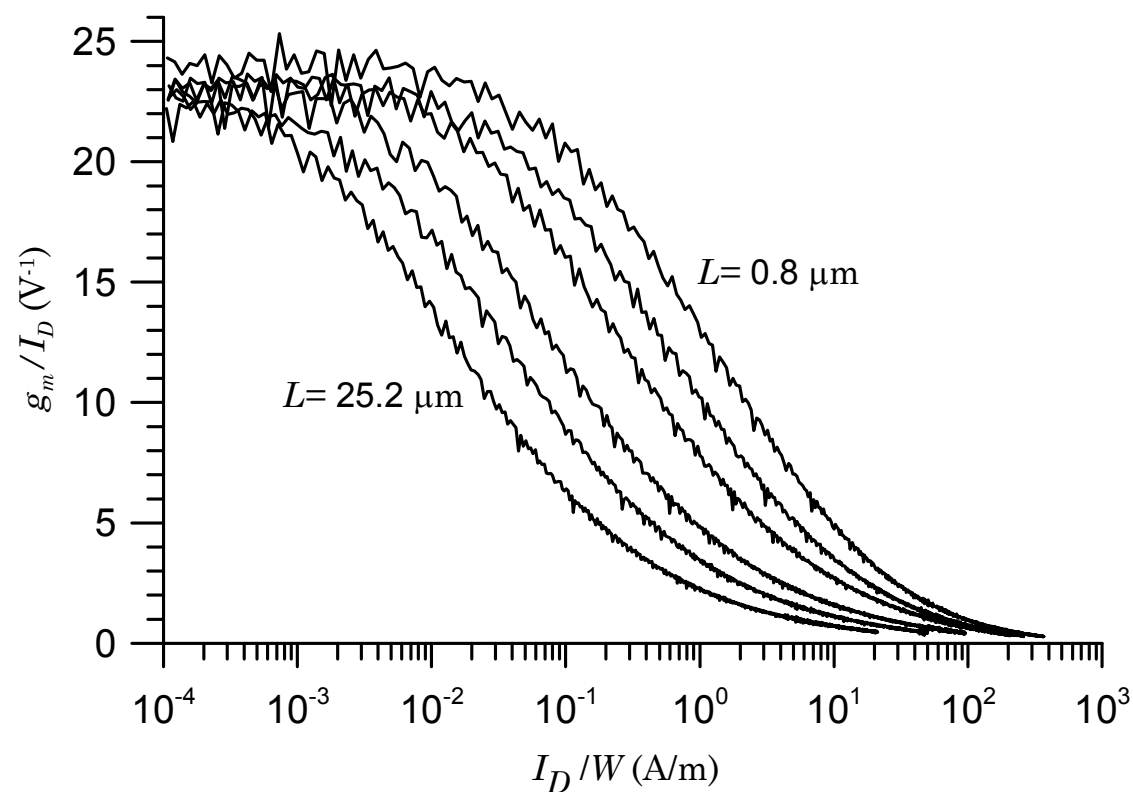
Both depend on transconductance  $g_m = \frac{dI_C}{dV_{BE}}$  (BJT) or  $g_m = \frac{dI_D}{dV_{GS}}$  (FET).

FET transconductance is a non-linear function of current ( $W=100$ ,  $L=0.8 \mu\text{m}$ ):



Power efficiency depends on transconductance per unit current  $g_m / I_D$ .

## Measurements on 0.8 $\mu\text{m}$ CMOS process



For a given device the x-values are proportional to device current.

e.g. for  $W = 100 \mu\text{m}$ ,  $I_D / W = 10$  corresponds to a current of 1 mA.

Traditional detector front-ends were designed to minimize noise, but accepting a 3 to 5-fold increase in noise reduces power by orders of magnitude!



## Scaling of transistor size to optimize power

Procedure:

- For a small device select the current density for a given  $g_m / I_D$  from plot on previous page.
- Then increase device width while scaling the device current proportionally (maintain current density).

⇒ increase transconductance (reduce noise)

- Minimum noise when

$$C_{FET} = C_{det} \quad (\text{capacitive matching})$$

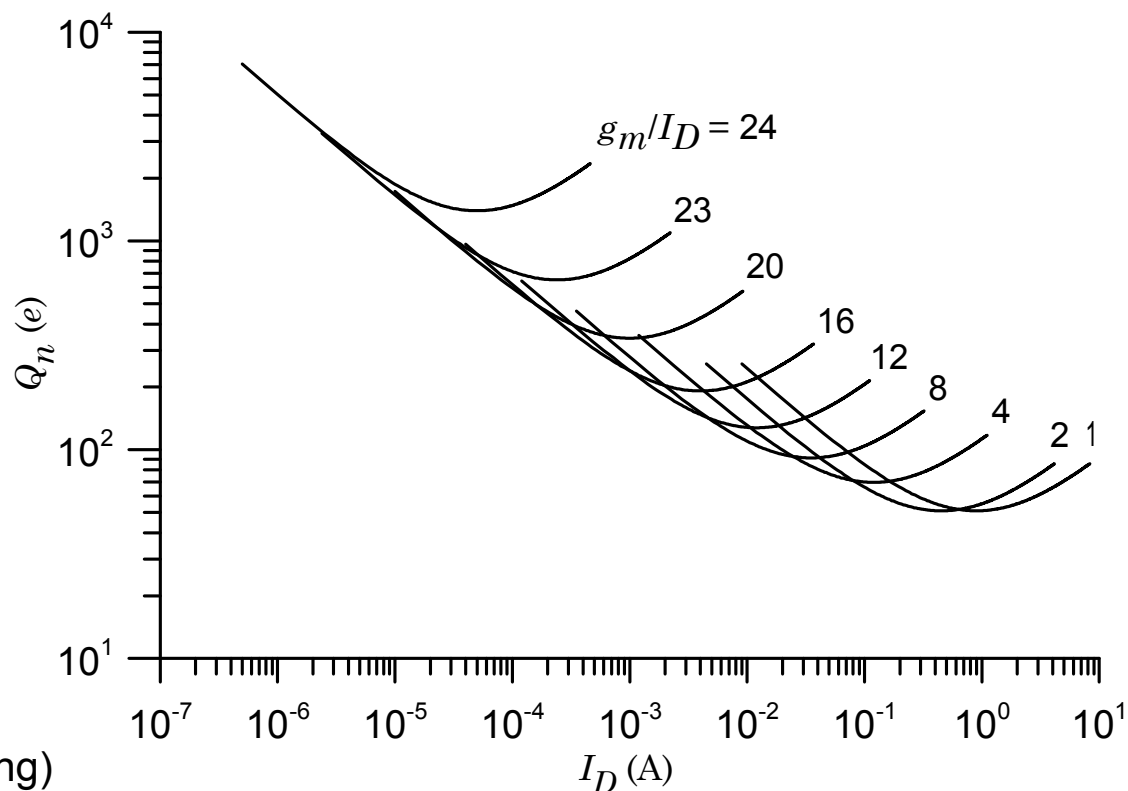
For larger device widths the increase in capacitance overrides the reduction in noise.

**This yields minimum noise, but is not most power efficient!**

For  $g_m / I_D = 24$ , minimum noise of 1400  $e$  at 50  $\mu\text{A}$ , but

for  $g_m / I_D = 20$  a noise level of 1000  $e$  is obtained at 30  $\mu\text{A}$ .

Given noise level can be achieved at low and high current.



Under optimum scaling to maintain signal-to-noise ratio,  
input transistor power ( $\approx$  preamp power) scales with  $(S/N)^2$ .

### Power Reduction

1. Segmentation reduces detector capacitance  
 $\Rightarrow$  lower noise for given power
2. Segmentation reduces the hit rate per channel  
 $\Rightarrow$  longer shaping time, reduce voltage noise
3. Segmentation reduces the leakage current per channel  
(smaller detector volume)  
 $\Rightarrow$  reduced shot noise, increased radiation resistance

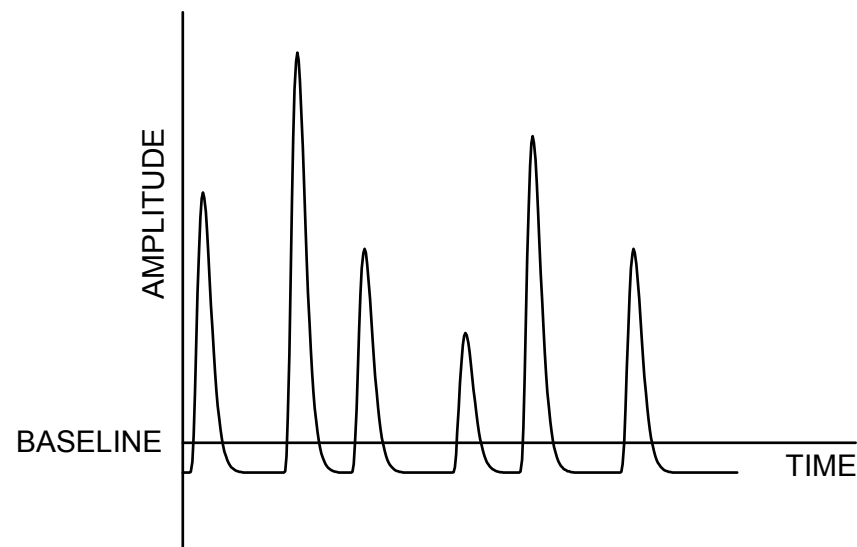
Segmentation key concept in large-scale detector systems.  
(also to increase radiation resistance)

### 3. Some Other Aspects of Pulse Shaping

#### Baseline Restoration

Any series capacitor in a system prevents transmission of a DC component.

A sequence of unipolar pulses has a DC component that depends on the duty factor, i.e. the event rate.



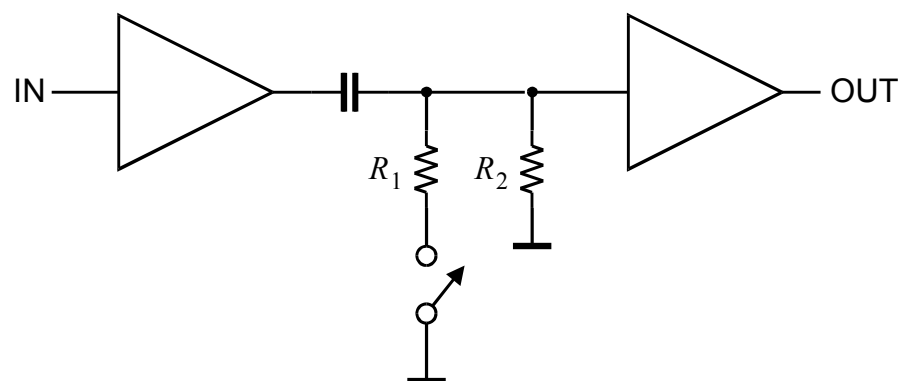
⇒ The baseline shifts to make the overall transmitted charge equal zero.

Random rates lead to random fluctuations of the baseline shift ⇒ spectral broadening

- These shifts occur whenever the DC gain is not equal to the midband gain.  
The baseline shift can be mitigated by a baseline restorer (BLR).

Principle of a baseline restorer:

Connect signal line to ground during the absence of a signal to establish the baseline just prior to the arrival of a pulse.



$R_1$  and  $R_2$  determine the charge and discharge time constants.

The discharge time constant (switch opened) must be much larger than the pulse width.

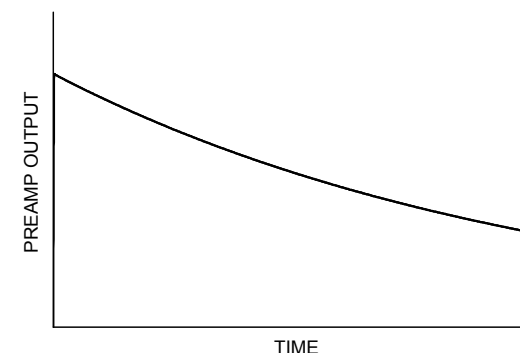
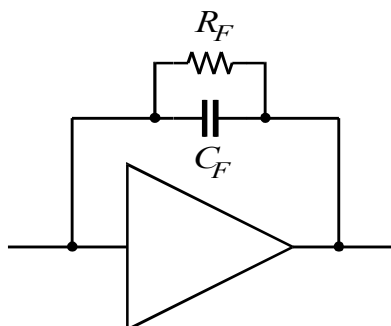
Originally performed with diodes (passive restorer), baseline restoration circuits now tend to include active loops with adjustable thresholds to sense the presence of a signal (gated restorer).

Asymmetric charge and discharge time constants improve performance at high count rates.

- This is a form of time-variant filtering. Care must be exercised to reduce noise and switching artifacts introduced by the BLR.
- Good pole-zero cancellation (next topic) is crucial for proper baseline restoration.

## Tail (Pole Zero) Cancellation

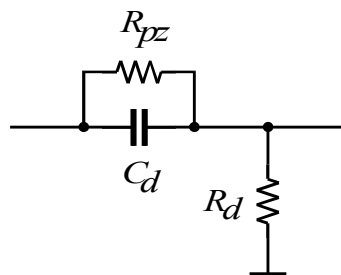
Feedback capacitor in charge sensitive preamplifier must be discharged. Commonly done with resistor.



Output no longer a step, but decays exponentially  
Exponential decay superimposed on shaper output.

⇒ undershoot

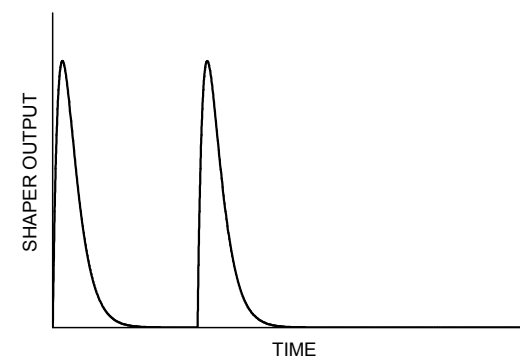
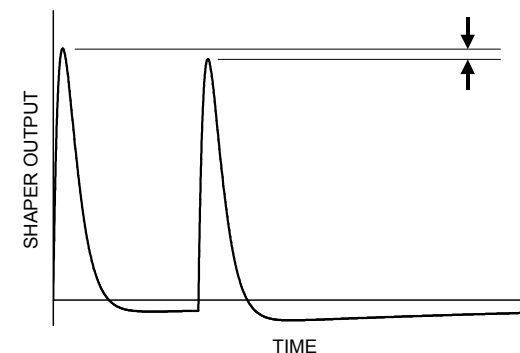
⇒ loss of resolution  
due to baseline  
variations



Add  $R_{pz}$  to differentiator:

“zero” cancels “pole” of preamp when  $R_F C_F = R_{pz} C_d$

Technique also used to compensate for “tails” of detector pulses: “tail cancellation”



## Bipolar vs. Unipolar Shaping

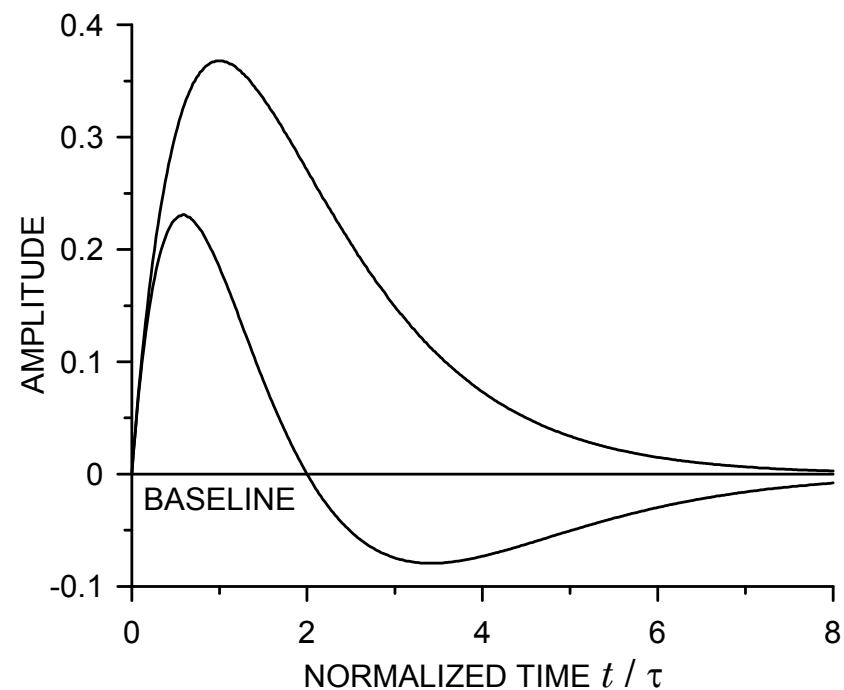
Unipolar pulse + 2<sup>nd</sup> differentiator

→ Bipolar pulse

Electronic resolution with bipolar shaping  
typ. 25 – 50% worse than for  
corresponding unipolar shaper.

However ...

- Bipolar shaping eliminates baseline shift  
(as the DC component is zero).
- Pole-zero adjustment less critical
- Added suppression of low-frequency noise.
- Not all measurements require optimum noise performance.  
Bipolar shaping is much more convenient for the user (important in large systems!)  
– often the method of choice.

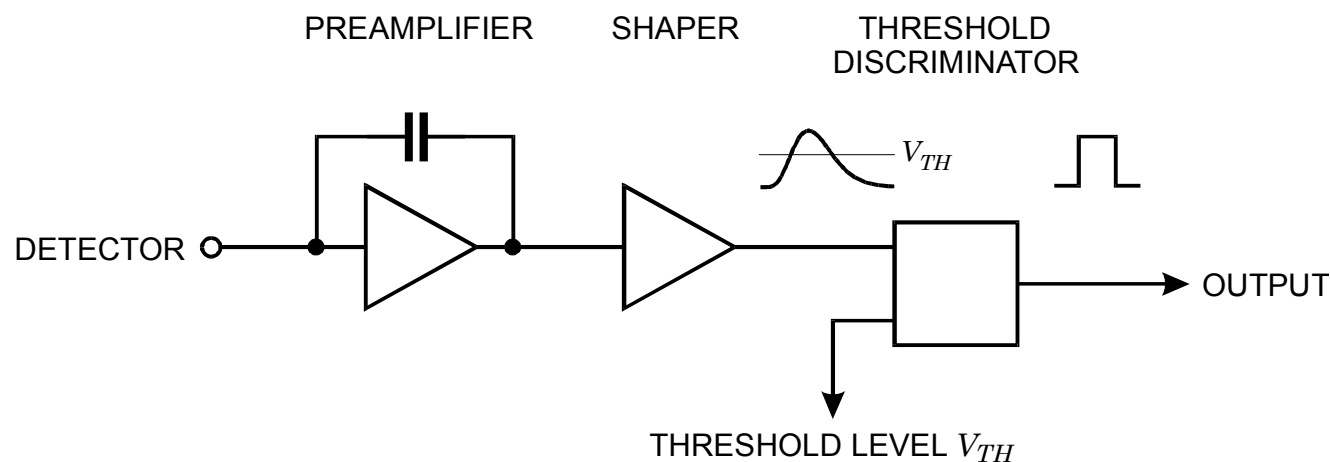


## 4. Threshold Discriminator Systems

The simplest form of a digitized readout is a threshold discriminator system, which produces a normalized (digital) output pulse when the input signal exceeds a certain level.

Noise affects not only the resolution of amplitude measurements, but also the determines the minimum detectable signal threshold.

Consider a system that only records the presence of a signal if it exceeds a fixed threshold.



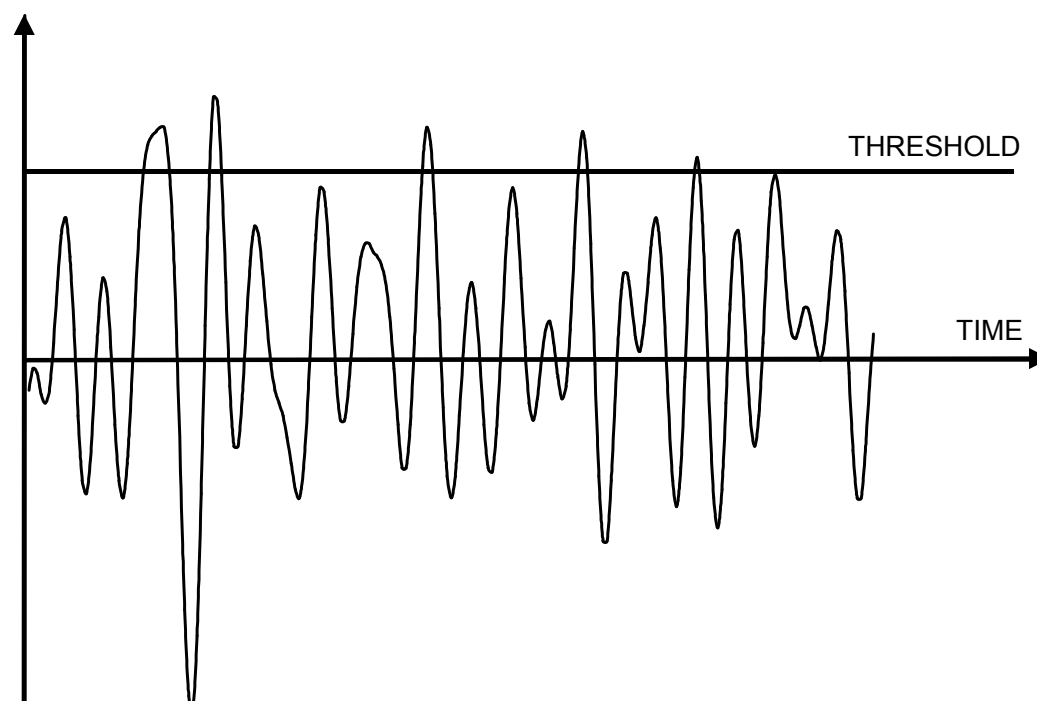
How small a detector pulse can still be detected reliably?

Consider the system at times when no detector signal is present.

Noise will be superimposed on the baseline.

Some noise pulses will exceed the threshold.

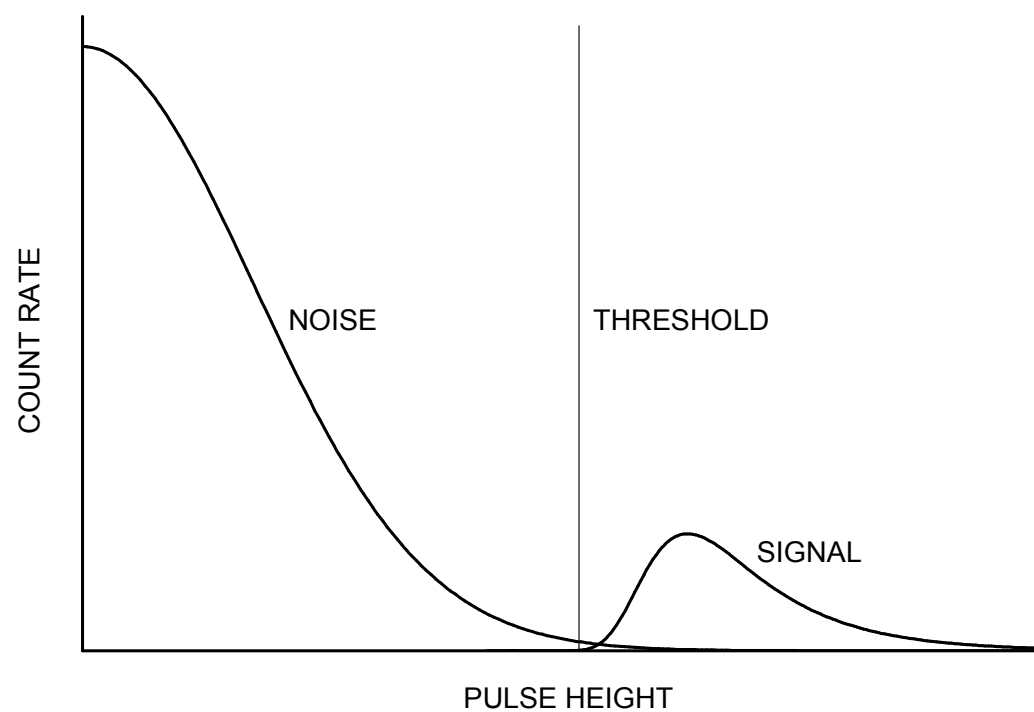
This is always true since the amplitude spectrum of Gaussian noise extends to infinity





The threshold must be set

1. high enough to suppress noise hits
2. low enough to capture the signal



With the threshold level set to 0 relative to the baseline, all of the positive excursions will be recorded.

Assume that the desired signals are occurring at a certain rate.

If the detection reliability is to be >99%, for example, then the rate of noise hits must be less than 1% of the signal rate.

The rate of noise hits can be reduced by increasing the threshold.

**If the system were sensitive to pulse magnitude alone**, the integral over the Gaussian distribution (the error function) would determine the factor by which the noise rate  $f_{n0}$  is reduced.

$$\frac{f_n}{f_{n0}} = \frac{1}{Q_n \sqrt{2\pi}} \int_{Q_T}^{\infty} e^{-(Q/2Q_n)^2} dQ ,$$

where  $Q$  is the equivalent signal charge,

$Q_n$  the equivalent noise charge and

$Q_T$  the threshold level.

However, since the pulse shaper broadens each noise impulse, **the time dependence is equally important**. For example, after a noise pulse has crossed the threshold, a subsequent pulse will not be recorded if it occurs before the trailing edge of the first pulse has dropped below threshold.

## Combined probability function

Both the amplitude and time distribution are Gaussian.

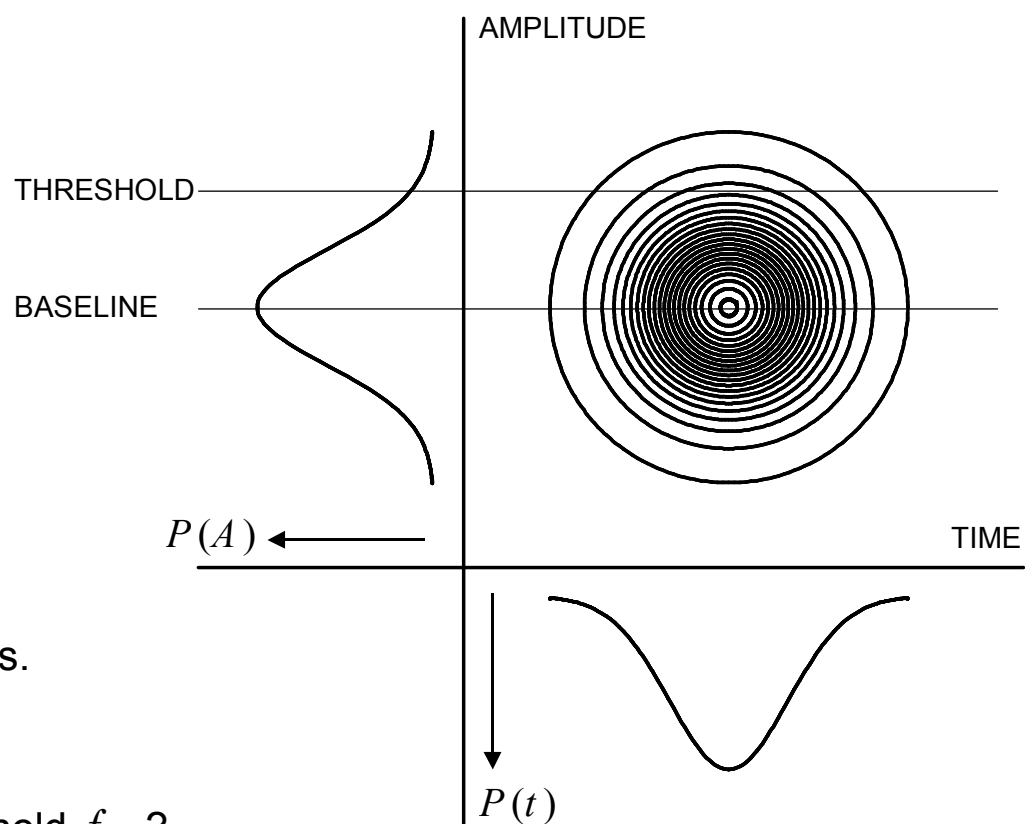
The rate of noise hits is determined by integrating the combined probability density function in the regime that exceeds the threshold.

This yields

$$f_n = f_{n0} \cdot e^{-Q_T^2/2Q_n^2}$$

Of course, one can just as well use the corresponding voltage levels.

What is the noise rate at zero threshold  $f_{n0}$  ?



For a system with the frequency response  $A(f)$  the frequency of zeros

$$f_0^2 = 4 \cdot \frac{\int_0^{\infty} f^2 A^2(f) df}{\int_0^{\infty} A^2(f) df}$$

(Rice, Bell System Technical Journal, **23** (1944) 282 and **24** (1945) 46)

Since we are interested in the number of positive excursions exceeding the threshold,  $f_{n0}$  is  $\frac{1}{2}$  the frequency of zero-crossings.

For an ideal band-pass filter with lower and upper cutoff frequencies  $f_1$  and  $f_2$  the noise rate

$$f_0 = 2 \sqrt{\frac{1}{3} \frac{f_2^3 - f_1^3}{f_2 - f_1}}$$

For a *CR-RC* filter with  $\tau_i = \tau_d$  the ratio of cutoff frequencies of the noise bandwidth is

$$\frac{f_2}{f_1} = 4.5$$

so to a good approximation one can neglect the lower cutoff frequency and treat the shaper as a low-pass filter, *i.e.*  $f_1 = 0$ .

Then

$$f_0 = \frac{2}{\sqrt{3}} f_2$$

An ideal bandpass filter has infinitely steep slopes, so the upper cutoff frequency  $f_2$  must be replaced by the noise bandwidth.

The noise bandwidth of an  $RC$  low-pass filter with time constant  $\tau$  is  $\Delta f_n = \frac{1}{4\tau}$

Setting  $f_2 = \Delta f_n$  yields the frequency of zeros  $f_0 = \frac{1}{2\sqrt{3} \tau}$

and the frequency of noise hits vs. threshold

$$f_n = f_{n0} \cdot e^{-Q_{th}^2/2Q_n^2} = \frac{f_0}{2} \cdot e^{-Q_{th}^2/2Q_n^2} = \frac{1}{4\sqrt{3} \tau} \cdot e^{-Q_{th}^2/2Q_n^2}$$

Thus, the required threshold-to-noise ratio for a given frequency of noise hits  $f_n$  is

$$\frac{Q_T}{Q_n} = \sqrt{-2 \log(4\sqrt{3} f_n \tau)} \approx \sqrt{-2 \log\left(\frac{f_n}{f_P}\right)},$$

where  $f_P$  is the peaking frequency of the shaper

Note that product of noise rate and shaping time  $f_n \tau$  determines the required threshold-to-noise ratio, i.e. for a given threshold-to-noise ratio the noise rate is higher at short shaping times

⇒ The noise rate for a given threshold-to-noise ratio is proportional to bandwidth.

⇒ To obtain the same noise rate, a fast system requires a larger threshold-to-noise ratio than a slow system with the same noise level.

## Efficiency vs. Occupancy

Frequently a threshold discriminator system is used in conjunction with other detectors that provide additional information, for example the time of a desired event.

In a collider detector the time of beam crossings is known, so the output of the discriminator is sampled at specific times.

The number of recorded noise hits then depends on

1. the sampling frequency (e.g. bunch crossing frequency)  $f_S$
2. the width of the sampling interval  $\Delta t$ , which is determined by the time resolution of the system.

The product  $f_S \Delta t$  determines the fraction of time the system is open to recording noise hits, so the rate of recorded noise hits is  $f_S \Delta t f_n$ .

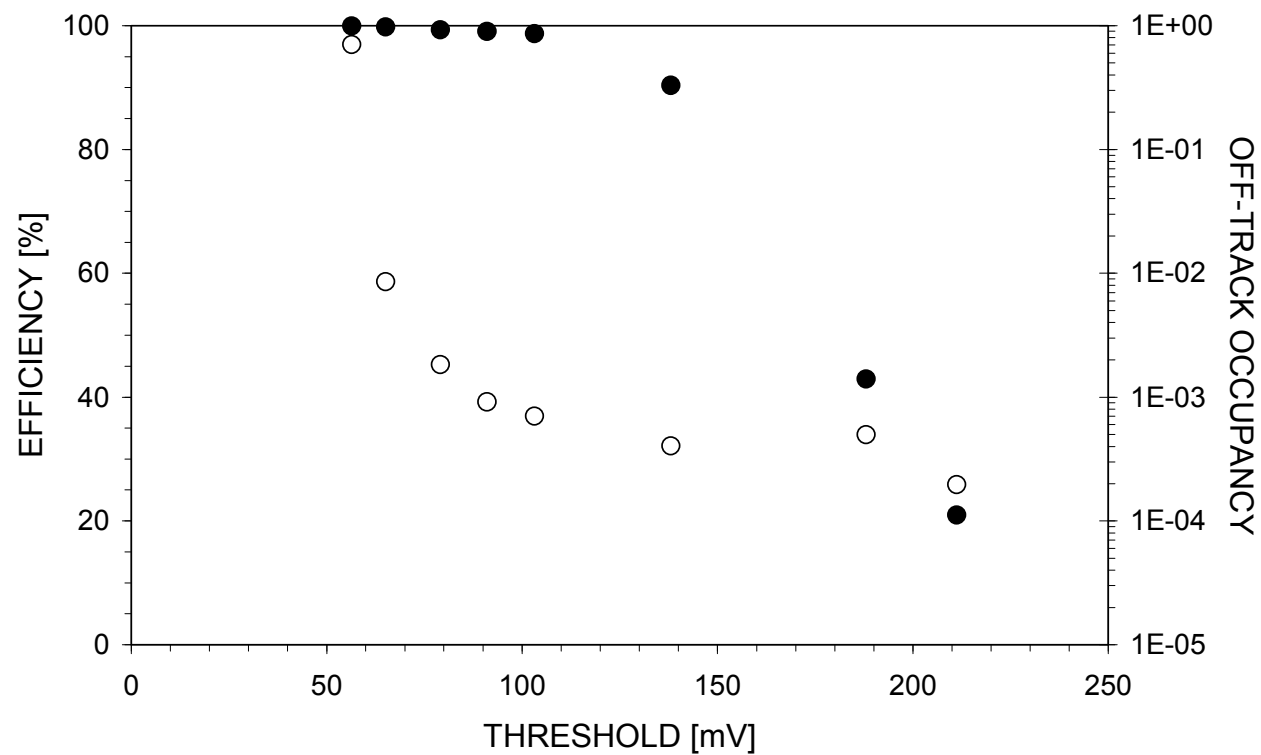
Often it is more interesting to know the probability of finding a noise hit in a given interval, i.e. the occupancy of noise hits, which can be compared to the occupancy of signal hits in the same interval.

This is the situation in a storage pipeline, where a specific time interval is read out after a certain delay time (e.g. trigger latency)

The occupancy of noise hits in a time interval  $\Delta t$ : 
$$P_n = \Delta t \cdot f_n = \frac{\Delta t}{2\sqrt{3} \tau} \cdot e^{-Q_T^2/2Q_n^2}$$

i.e. the occupancy falls exponentially with the square of the threshold-to-noise ratio.

Example of noise occupancy (open circles) and efficiency (solid circles) vs. threshold in a practical detector module:



Note that an extended overlap region of high efficiency and low noise occupancy is desired.



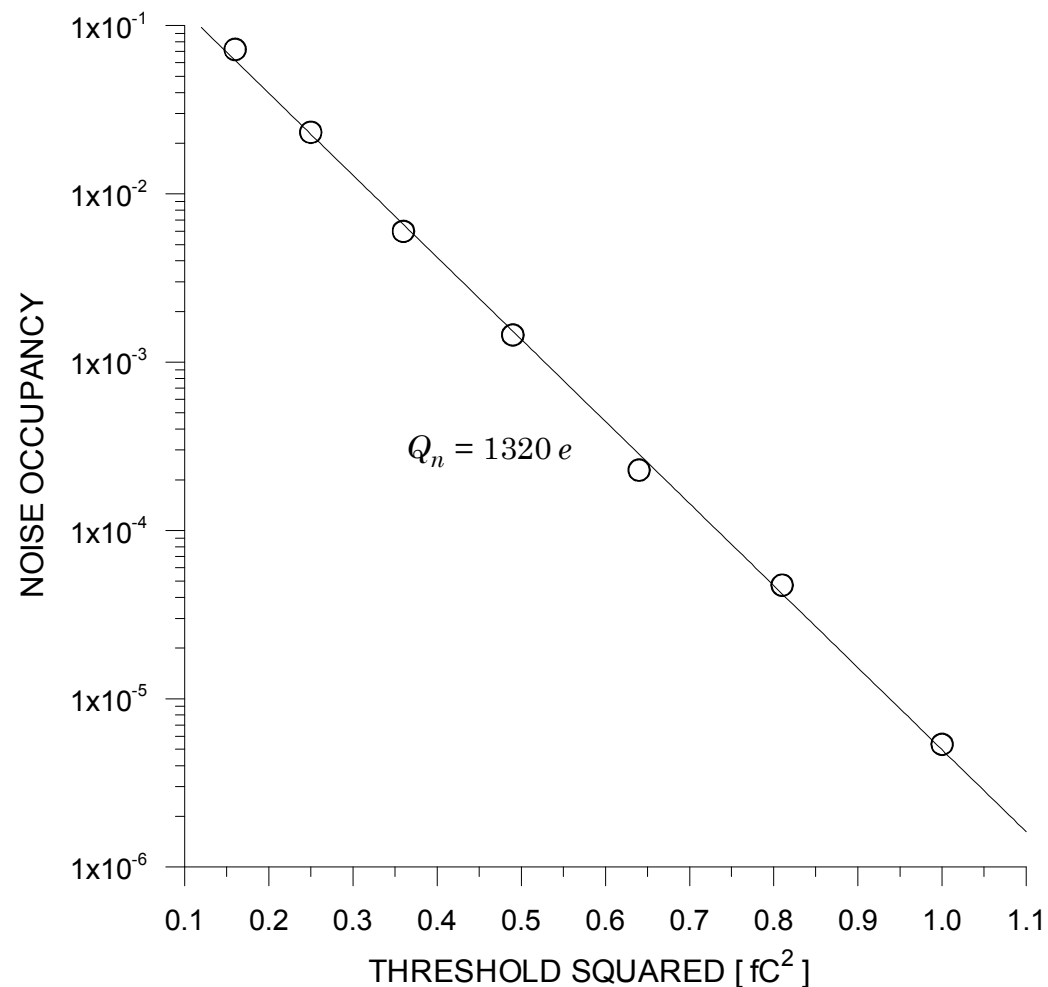
The dependence of occupancy on threshold can be used to measure the noise level.

$$\log P_n = \log \left( \frac{\Delta t}{2\sqrt{3} \tau} \right) - \frac{1}{2} \left( \frac{Q_T}{Q_n} \right)^2,$$

so the *slope* of  $\log P_n$  vs.  $Q_T^2$  yields the noise level.

This analysis is *independent of the details of the shaper*, which affect only the offset.

Measured online during a beam test at KEK in Japan



## 5. Timing Measurements

Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

- Timing measurements optimize determination of time of occurrence.
- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

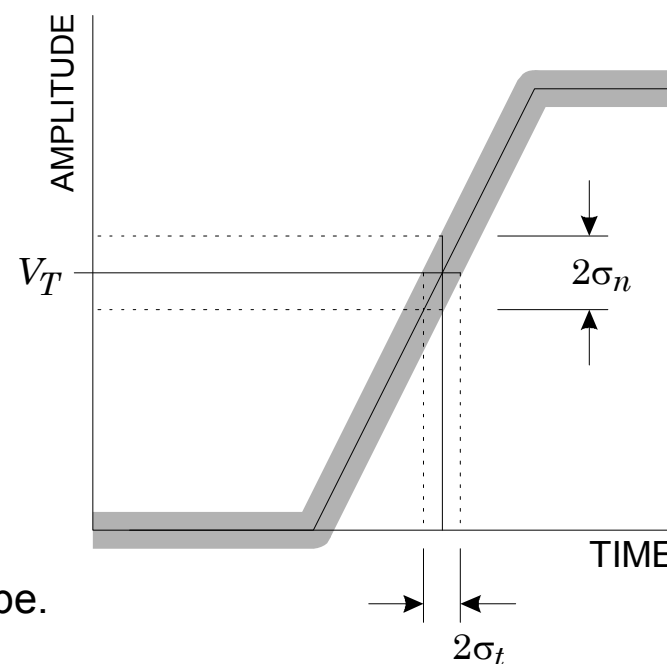
The instantaneous signal level is modulated by noise.

⇒ time of threshold crossing fluctuates

$$\sigma_t = \frac{\sigma_n}{\left. \frac{dV}{dt} \right|_{V_T}} \approx \frac{t_r}{S/N}$$

$t_r$  = rise time

Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.



## Pulse Shaping

Consider a system whose bandwidth is determined by a single  $RC$  integrator.

The time constant of the  $RC$  low-pass filter determines the

- rise time (and hence  $dV / dt$ )
- amplifier bandwidth (and hence the noise)

Time dependence: 
$$V_o(t) = V_0(1 - e^{-t/\tau})$$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2 \tau$$

In terms of bandwidth

$$t_r = 2.2 \tau = \frac{2.2}{2\pi f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

For a cascade of amplifiers: 
$$t_r \approx \sqrt{t_{r1}^2 + t_{r2}^2 + \dots + t_{rn}^2}$$

## Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude  $V_0$  and a rise time  $t_c$  passing through an amplifier chain with a rise time  $t_{ra}$ .

1. Amplifier rise time  $\gg$  Signal rise time:

$$\text{Noise} \propto \sqrt{f_u} \propto \sqrt{\frac{1}{t_{ra}}}$$

$$\frac{dV}{dt} \propto \frac{1}{t_{ra}} \propto f_u$$

increase in bandwidth  $\Rightarrow$  improvement in  $dV/dt$  outweighs increase in noise.

2. Amplifier rise time  $\ll$  Signal rise time

increase in noise without increase in  $dV/dt$

Optimum: The amplifier rise time should be chosen to match the signal rise time.

Differentiation time constant: choose greater than rise time constant

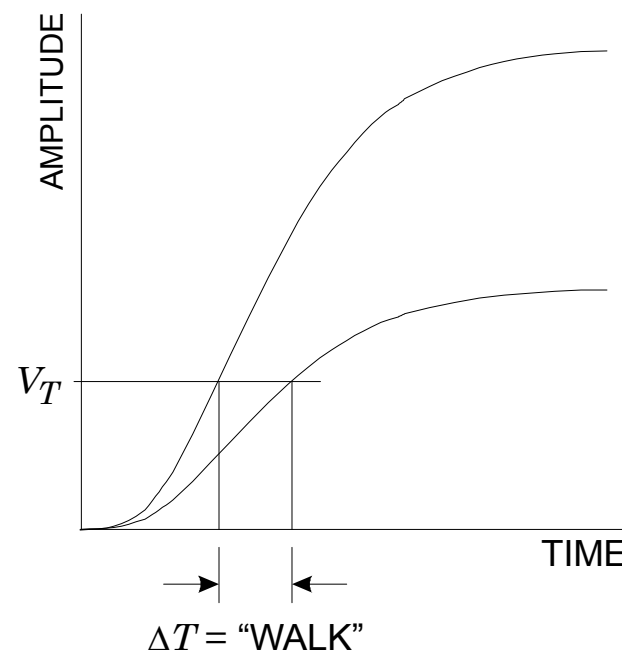
$$(\tau_{diff} = 10\tau_{int} \text{ incurs 20\% loss in pulse height})$$

## Time Walk

For a fixed trigger level the time of threshold crossing depends on pulse amplitude.

⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)



If the rise time is known, “time walk” can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

## Lowest Practical Threshold

Single  $RC$  integrator has maximum slope at  $t=0$ :  $\frac{d}{dt}(1 - e^{-t/\tau}) = \frac{1}{\tau} e^{-t/\tau}$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small  $t$  the slope at the output of a single  $RC$  integrator is linear, so initially the pulse can be approximated by a ramp  $\alpha t$ .

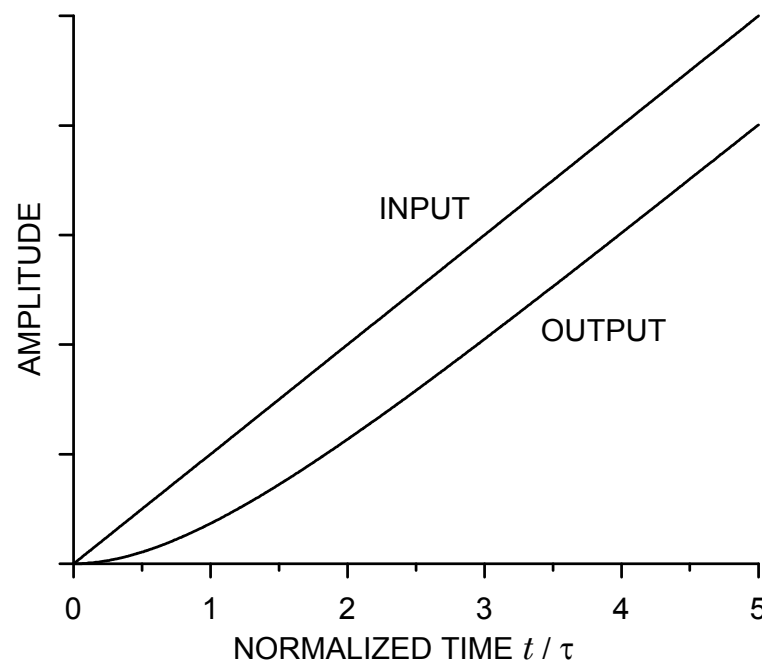
Response of the following integrator

$$V_i = \alpha t \rightarrow V_o = \alpha(t - \tau) - \alpha \tau e^{-t/\tau}$$

$\Rightarrow$  The output is delayed by  $\tau$   
and curvature is introduced at small  $t$ .

Output attains 90% of input slope after  
 $t = 2.3\tau$ .

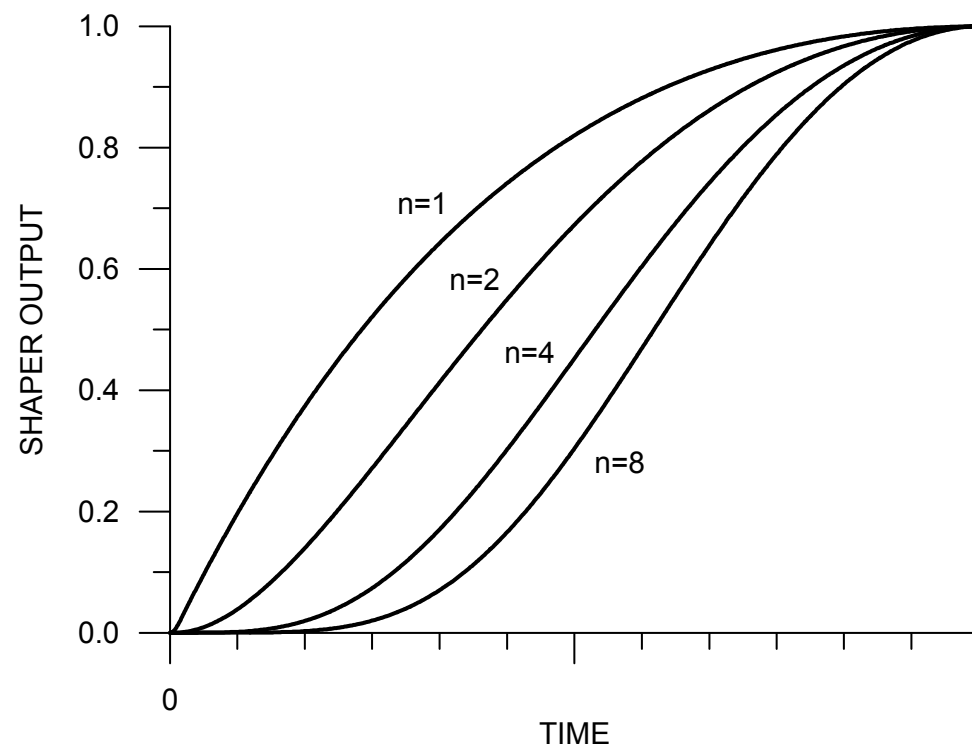
Delay for  $n$  integrators =  $n\tau$



Additional *RC* integrators introduce more curvature at the beginning of the pulse.

Output pulse shape for multiple *RC* integrators

(normalized to preserve the peaking time,  $\tau_n = \tau_{n-1} / n$ )



Increased curvature at beginning of pulse limits the minimum threshold for good timing.

⇒ One dominant time constant best for timing measurements

Unlike amplitude measurements, where multiple integrators are desirable to improve pulse symmetry and count rate performance.

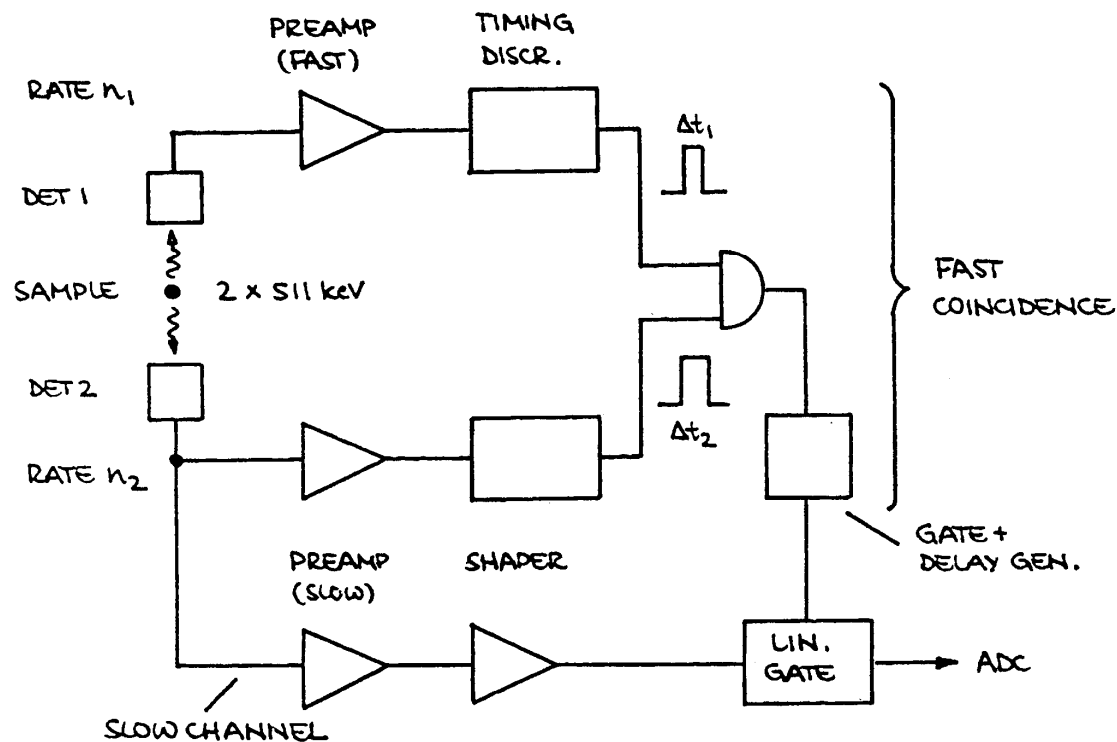
## Example: $\gamma - \gamma$ coincidence (as used in positron emission tomography)

Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.

- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output if the two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.





This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

### Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are  $n_1$  and  $n_2$ , and the timing pulse widths are  $\Delta t_1$  and  $\Delta t_2$ , the probability of a pulse from the first source occurring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate  $n_2$ , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$

$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

i.e. in the arrangement shown above, the chance coincidence rate increases with the **square** of the source strength.

Example:

$$n_1 = n_2 = 10^6 \text{ s}^{-1}$$

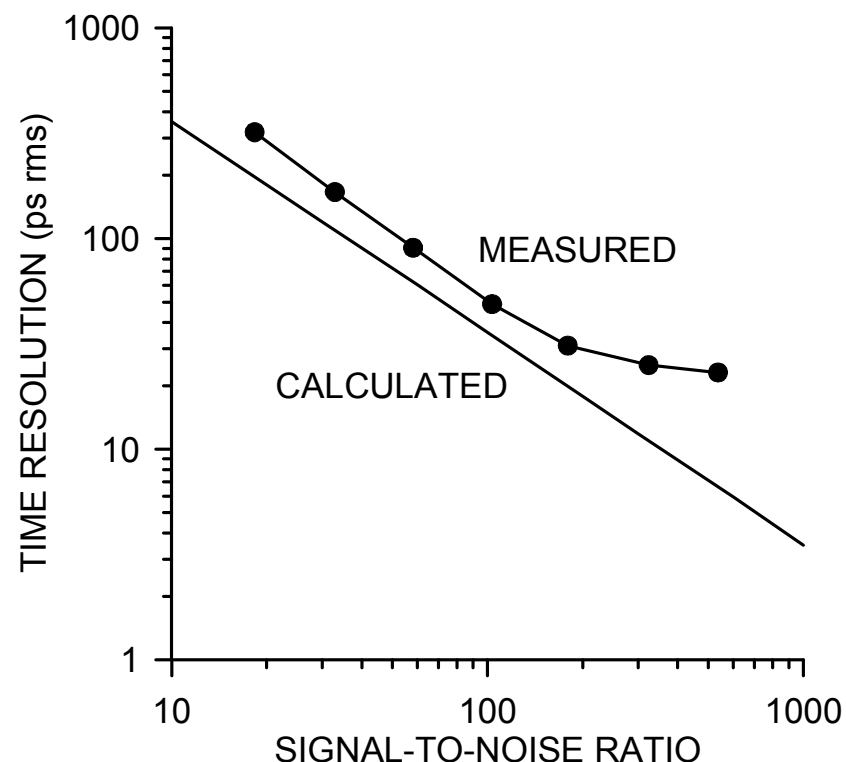
$$\Delta t_1 = \Delta t_2 = 5 \text{ ns} \Rightarrow n_c = 10^4 \text{ s}^{-1}$$

## Fast Timing: Comparison between theory and experiment

Time resolution  $\propto 1/(S/N)$

At  $S/N < 100$  the measured curve lies above the calculation because the timing discriminator limited the rise time.

At high  $S/N$  the residual jitter of the time digitizer limits the resolution.



For more details on fast timing with semiconductor detectors, see

H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.

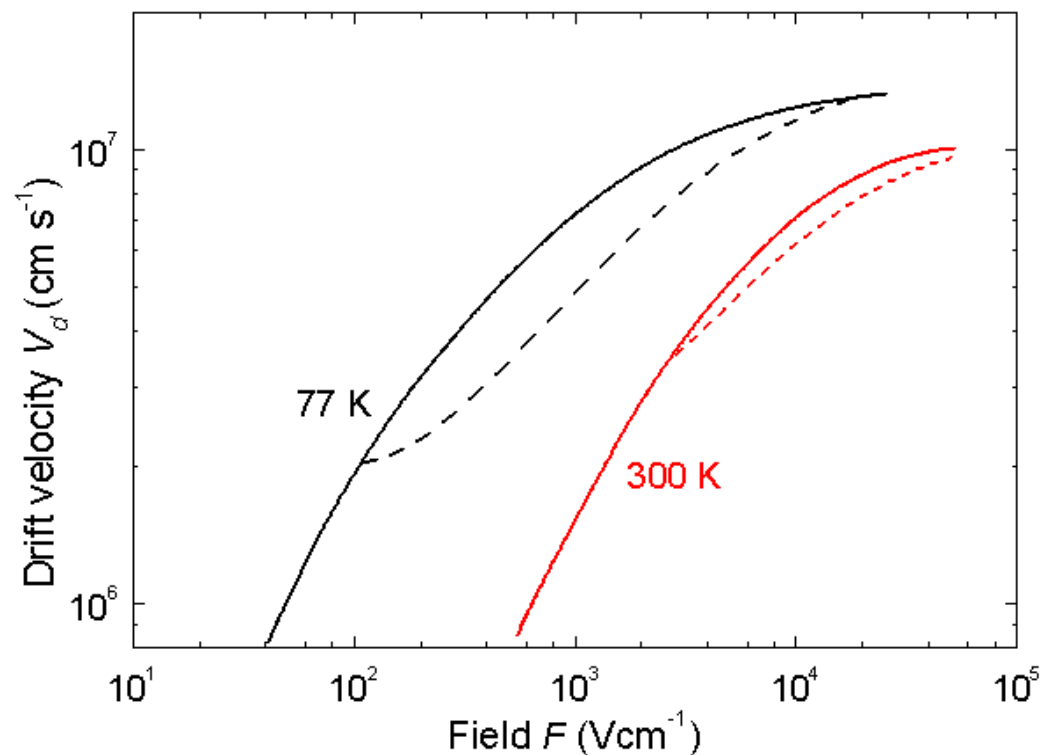
## Improved Timing at Low Temperatures?

Carrier mobility increases at low temperatures.

Electron drift velocity in Si:  
(low doping levels)

solid:  $E \parallel (111)$

dashed:  $E \parallel (100)$



Jacobini et al. Solid State Elec 20/2 (1977) 77-89

At low fields ~10-fold increase, but saturation velocity at 77K only increases 30%, so at the high fields optimal for timing only modest improvement.

Ionization coefficient  $\alpha(77\text{K})/\alpha(300\text{K}) \approx 2$ , so maximum bias voltage reduced (breakdown).