III. Electronic Noise

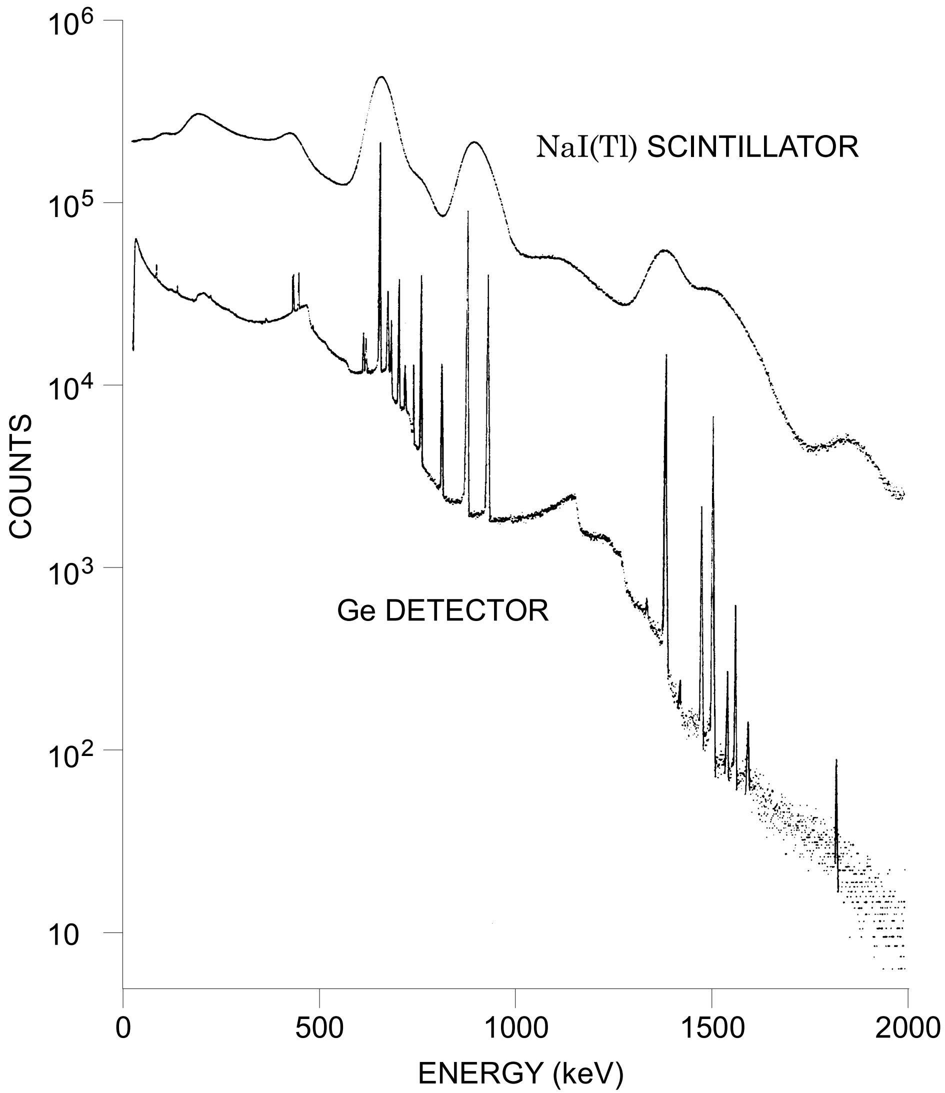
1. Resolution and Electronic Noise

Resolution: the ability to distinguish signal levels

1. Why?

a) Recognize structure in amplitude spectra

Comparison between NaI(Tl) and Ge detectors

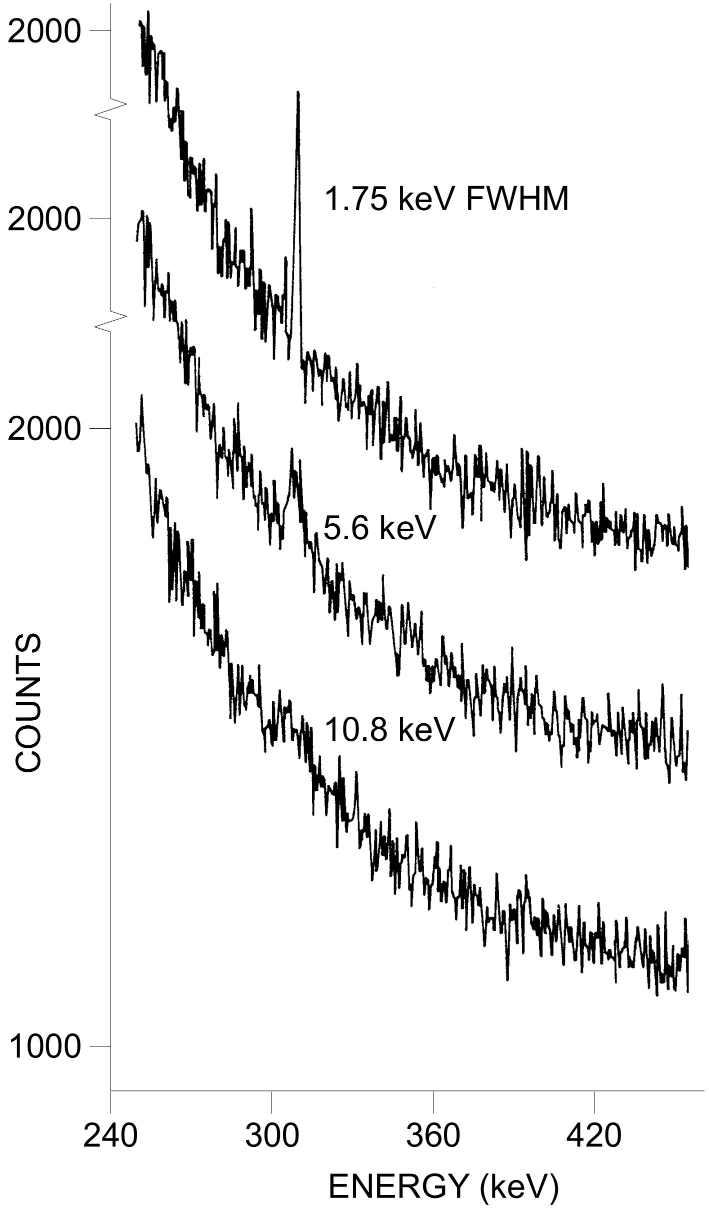


J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446

b) Improve sensitivity

Signal to background ratio improves with better resolution

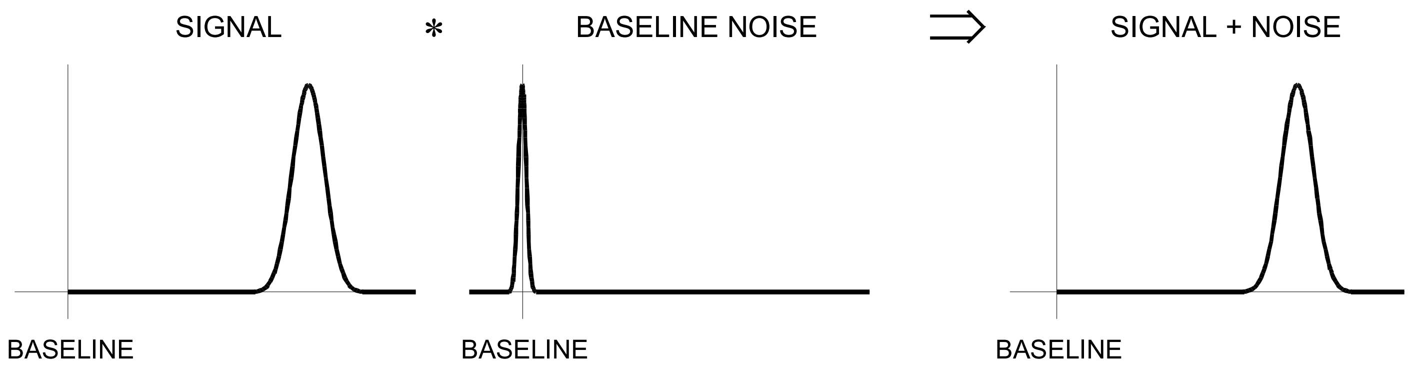
(signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

What determines Resolution?

a) Signal Variance >> Baseline Variance



⇒ Electronic (baseline) noise not important

Examples:
- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

1 MeV γ-rays absorbed by NaI(Tl) crystal:

Number of photoelectrons: $N_{pe} \approx 8 \cdot 10^4$ [MeV^{-1}] $\times E_\gamma \times QE \approx 2.4 \cdot 10^4$

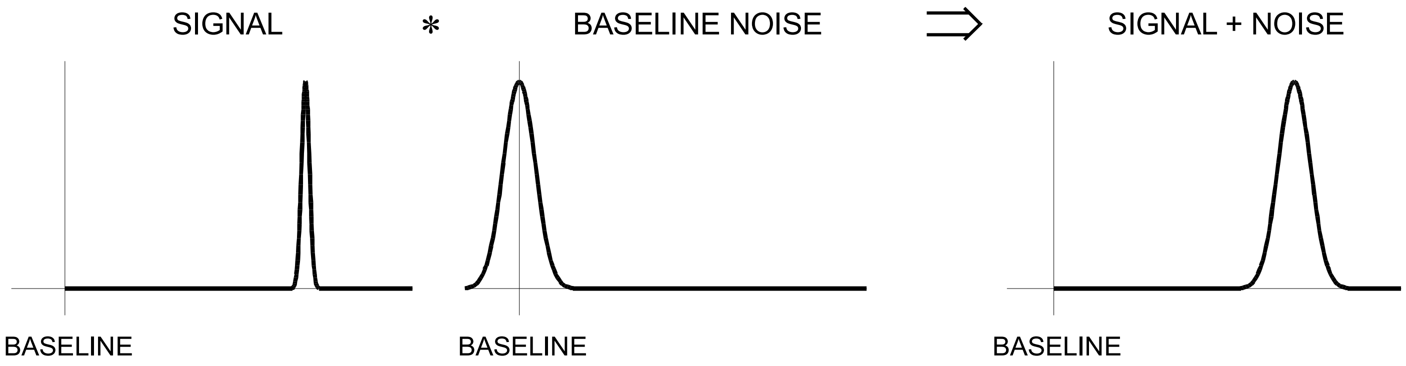
Variance typically: $\sigma_{pe} = N_{pe}^{1/2} \approx 160$ and $\sigma_{pe} / N_{pe} \approx 5$ - 8%

Signal at PMT anode (Gain= 10^4): $Q_{sig} = G_{PMT}\, N_{pe} \approx 2.4 \cdot 10^8$ el

and variance $\sigma_{sig} = G_{PMT}\, \sigma_{pe} \approx 1.2 \cdot 10^7$ el

whereas electronic noise easily $< 10^4$ el

b) Signal Variance << Baseline Variance



$\Rightarrow$ Electronic (baseline) noise critical for resolution

Examples:
- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si : Number of electron-hole pairs $N = \dfrac{E_{dep}}{3.6\text{ eV}}$

Variance $\sigma = \sqrt{F \cdot N}$ (where F= Fano factor $\approx$ 0.1)

For 50 keV photons: $\sigma \approx 40$ el $\Rightarrow$ $\sigma / N = 7.5 \cdot 10^{-4}$

Obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

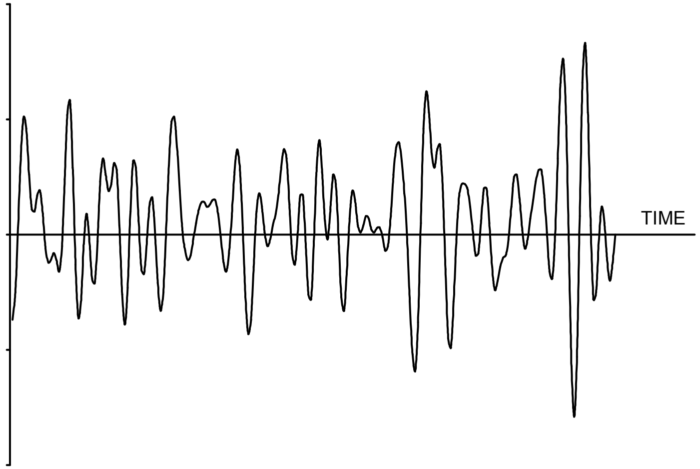
… etc.,

but the (practical) fundamental limit is electronic noise.

Electronic Noise

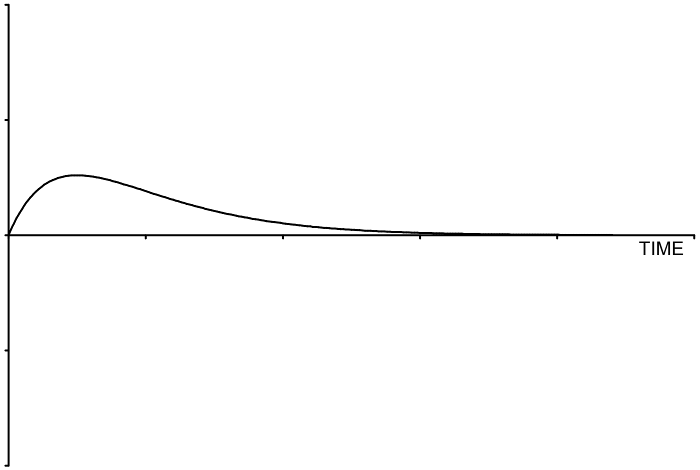
Choose a time when no signal is present.

Amplifier's quiescent output level (baseline):

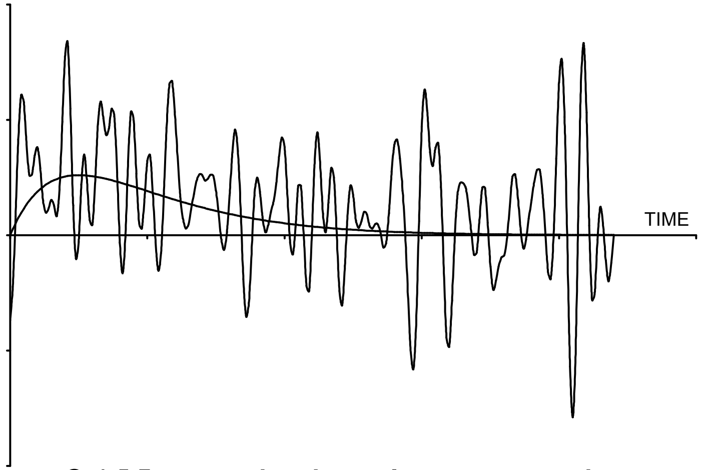


In the presence of a signal, noise + signal add.

Signal



Signal+Noise (S/N = 1)



$S/N \equiv$ peak signal to rms noise

Measurement of peak amplitude yields signal amplitude + noise fluctuation

The preceding example could imply that the fluctuations tend to increase the measured amplitude, since the noise fluctuations vary more rapidly than the signal.

In an optimized system, the time scale of the fluctuation is comparable to the signal peaking time.

Then the measured amplitude fluctuates positive and negative relative to the ideal signal.

Measurements taken at 4 different times:

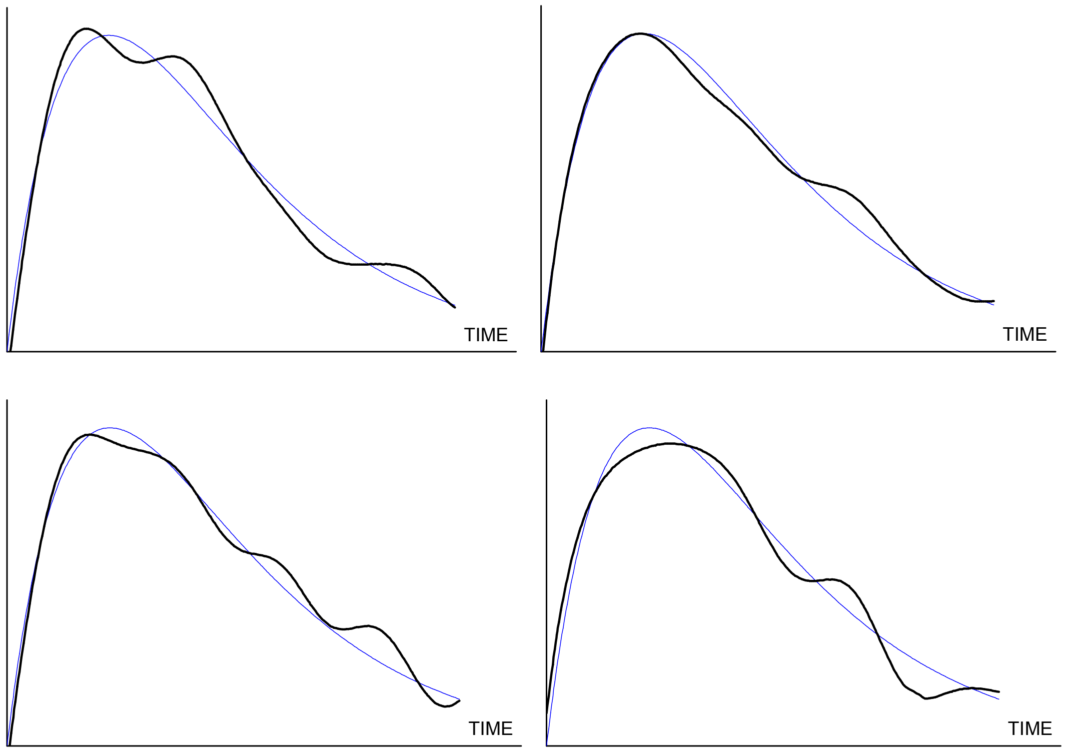
noiseless signal superimposed for comparison

$S/N = 20$

Noise affects

Peak signal

Time distribution



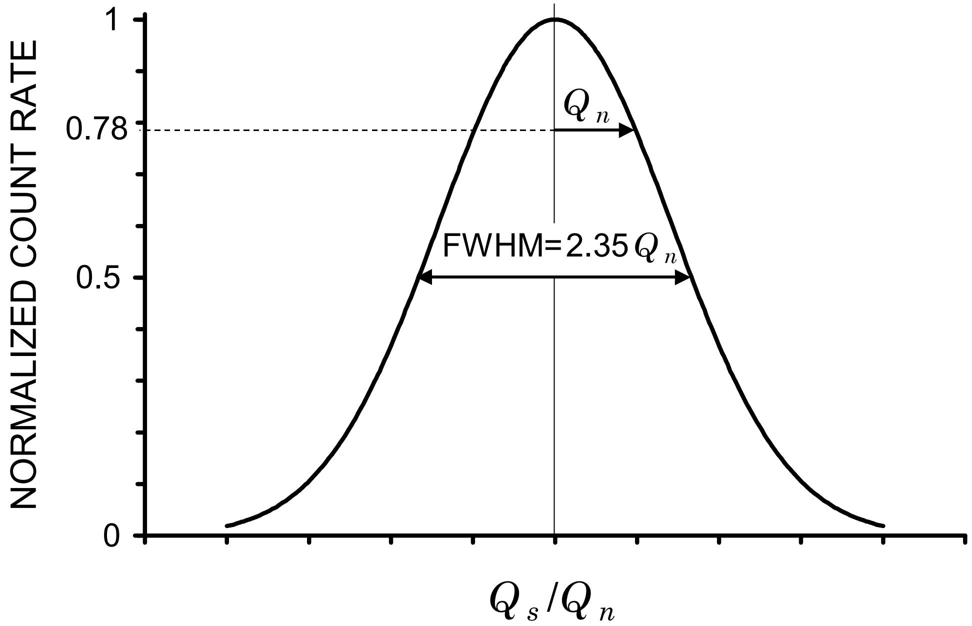
Electronic noise is purely random.

⇒ amplitude distribution is Gaussian

⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has Gaussian distribution



Measuring Resolution

Inject an input signal with known charge using a pulse generator set to approximate the detector signal shape.

Measure the pulse height spectrum. peak centroid ⇒ signal magnitude
peak width ⇒ noise (FWHM= 2.35 Q_n)

2. Basic Noise Mechanisms and Characteristics

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\ e\ v}{l}$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2 ,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
excess or "1/f" noise

Thermal noise and shot noise are both "white" noise sources, i.e.

power per unit bandwidth (≡ spectral density) is constant: $\dfrac{dP_{noise}}{df} = const.$

Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Noise power density vs. frequency f: $\frac{dP_{noise}}{df} = 4kT$

k = Boltzmann constant

T = absolute temperature

R = DC resistance

since $P = \frac{V^2}{R} = I^2 R$

the spectral noise voltage density $\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$

and the spectral noise current density $\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$

The total noise depends on the bandwidth of the system.
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

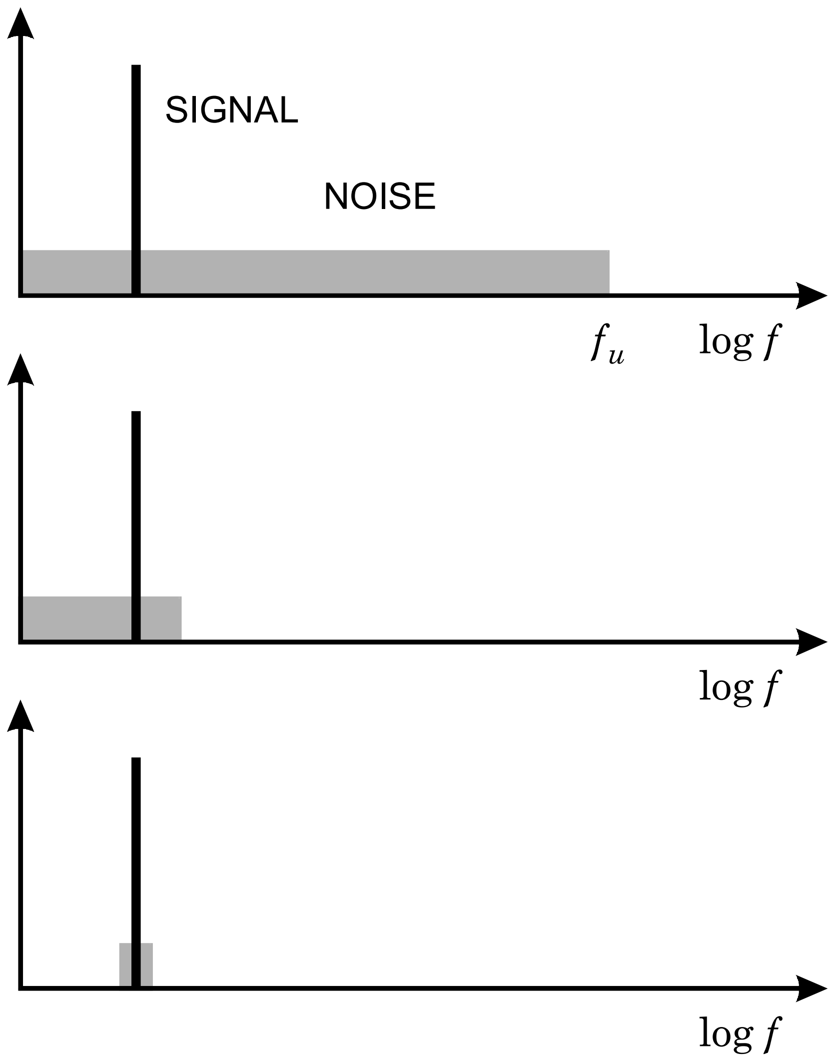
$$v_{on}^2 = \int_0^\infty e_n^2 \, A_v^2(f) \, df$$

Note: Since spectral noise components are not correlated, one must integrate over the noise power (proportional to voltage or current squared).

Total noise increases with bandwidth.

Total noise is the integral over the shaded region.

S/N increases as noise bandwidth is reduced until signal components are attenuated significantly.



Shot noise

A common example of noise due to number fluctuations is “shot noise”, which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode
(emission over a barrier)

Spectral noise current density: $i_n^2 = 2eI$ e = electronic charge
I = DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in “ohmic” conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

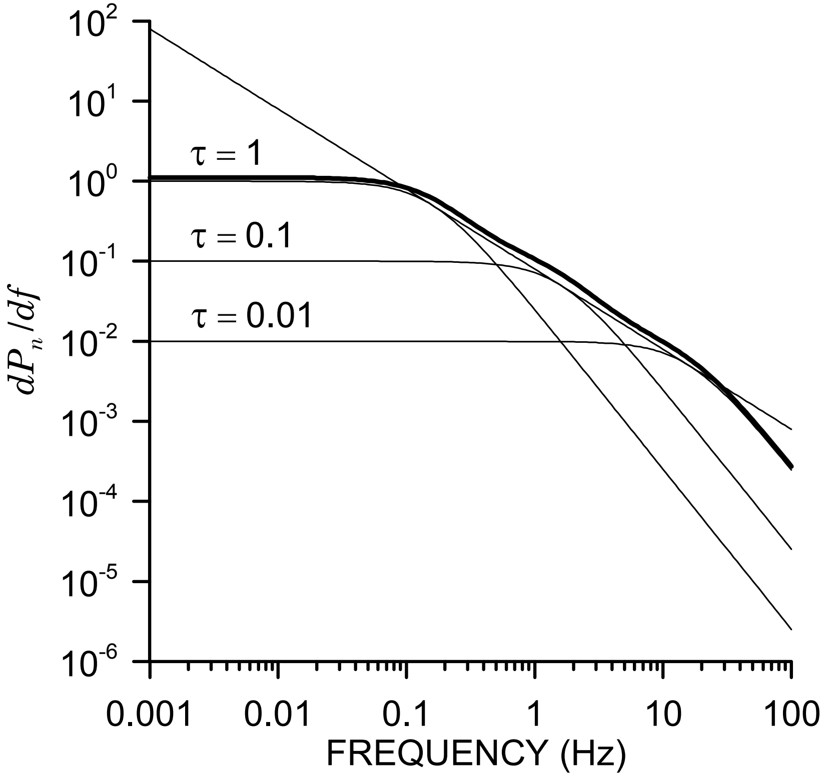
Low Frequency (“$1/f$”) Noise

Charge can be trapped and then released after a characteristic lifetime τ.

The spectral density for a single lifetime

$$S(f) \propto \frac{\tau}{1+(2\pi f\tau)^2} \ .$$

For $2\pi f\tau \gg 1$: $S(f) \propto \dfrac{1}{f^2}$.



However,
several traps with different time constants can yield a “$1/f$“ distribution:

Traps with three time constants of 0.01, 0.1 and 1 s yield a $1/f$ distribution over two decades in frequency.

Low frequency noise is ubiquitous – must not have $1/f$ dependence, but commonly called $1/f$ noise.

Spectral power density: $\dfrac{dP_{noise}}{df} = \dfrac{1}{f^{\alpha}}$ (typically α = 0.5 – 2)

Noise Bandwidth vs. Signal Bandwidth

Consider an amplifier with the frequency response $A(f)$. This can be rewritten

$A(f) \equiv A_0 G(f)$,

where A_0 is the maximum gain and $G(f)$ describes the frequency response.

For example, for the simple amplifier described above

$$A_v = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\, C_o \right)^{-1} = g_m R_L \; \frac{1}{1 + \mathbf{i}\omega R_L C_o}$$

and using the above convention $\quad A_0 \equiv g_m R_L \quad \text{and} \quad G(f) \equiv \dfrac{1}{1 + \mathbf{i}\;(2\pi f\; R_L C_o)}$

If a “white” noise source with spectral density e_{ni} is present at the input, the total noise voltage at the output is

$$v_{no} = \sqrt{\int_0^\infty e_{ni}^2 \left| A_0 G(f) \right|^2 df} = e_{ni}\, A_0 \sqrt{\int_0^\infty G^2(f) df} \equiv e_{ni}\, A_0 \sqrt{\Delta f_n}$$

Δf_n is the “noise bandwidth”.

Note that, in general, the noise bandwidth and the signal bandwidth are not the same.

If the upper cutoff frequency is determined by a single RC time constant, as in the "simple amplifier",

the signal bandwidth $$\Delta f_s = f_u = \frac{1}{2\pi\, RC}$$

and the noise bandwidth $$\Delta f_n = \frac{1}{4RC} = \frac{\pi}{2} f_u \quad .$$

Noise Bandwidth and Low Frequency (1/*f*) Noise

For a spectral noise density $P_{nf} = \frac{S_f}{f}$

and a corresponding voltage density $e_{nf}^2 = \frac{A_f}{f}$

the total noise integrated in a frequency band f_1 to f_2 is

$$v_{nf}^2 = \int_{f_1}^{f_2} \frac{A_f}{f}\, df = A_f \log\left(\frac{f_2}{f_1}\right)$$

Thus, for a $1/f$ spectrum the total noise depends on the ratio of the upper to lower cutoff frequency.

Since this is a power distribution, the voltage or current spectral density changes 10-fold over a 100-fold span in frequency.

Frequently, the $1/f$ noise corner is specified: frequency where $1/f$ noise intercepts white noise.

Higher white noise level reduces corner frequency, so lower noise corner does not equate to lower $1/f$ noise.

"Noiseless" Resistance Example – Dynamic Resistance

In many instances a resistance is formed by the slope of a device's current-voltage characteristic, rather than by a static ensemble of electrons agitated by thermal energy.

Example: forward-biased semiconductor diode

Diode current vs. voltage $$I = I_0 (e^{q_e V/kT} - 1)$$

The differential resistance $$r_d = \frac{dV}{dI} = \frac{kT}{q_e I}$$

i.e. at a given current the diode presents a resistance, e.g. 26 Ω at I = 1 mA and T = 300 K.

Note that two diodes can have different charge carrier concentrations, but will still exhibit the same dynamic resistance at a given current, so the dynamic resistance is not uniquely determined by the number of carriers, as in a resistor.

There is no thermal noise associated with this "dynamic" resistance, although the current flow carries shot noise.

Correlated Noise

Generally, noise power is additive.: $P_{n,tot} = P_{n1} + P_{n1} + \ldots$

However, in a coherent system (i.e. a system that preserves phase), the power often results from the sum of voltages or currents, which is sensitive to relative phase.

For two correlated noise sources N_1 and N_2 the total noise

$$N = N_1^2 + N_2^2 + 2CN_1N_2$$

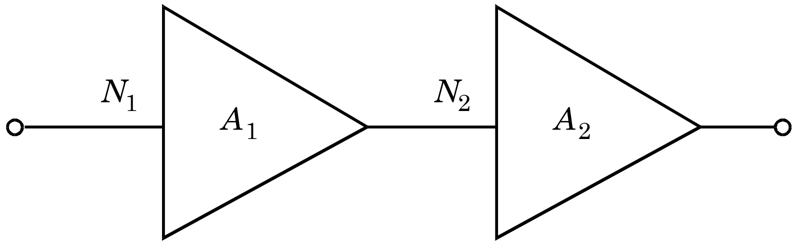
where the correlation coefficient C can range from –1 (anti-correlated, i.e. identical, but 180° out of phase) to +1 (fully correlated).

For uncorrelated noise components $C = 0$ and then individual current or voltage noise contributions add in quadrature, e.g.

$$V_{n,tot} = \sqrt{\sum_i V_{ni}^2}$$

Noise in Amplifier Chains

Consider a chain of two amplifiers (or amplifying devices), with gains A_1 and A_1, and input noise levels N_1 and N_2 .



A signal S is applied to the first amplifier, so the input signal-to-noise ratio is S/N_1.

At the output of the first amplifier the signal is A_1S and the noise A_1N.

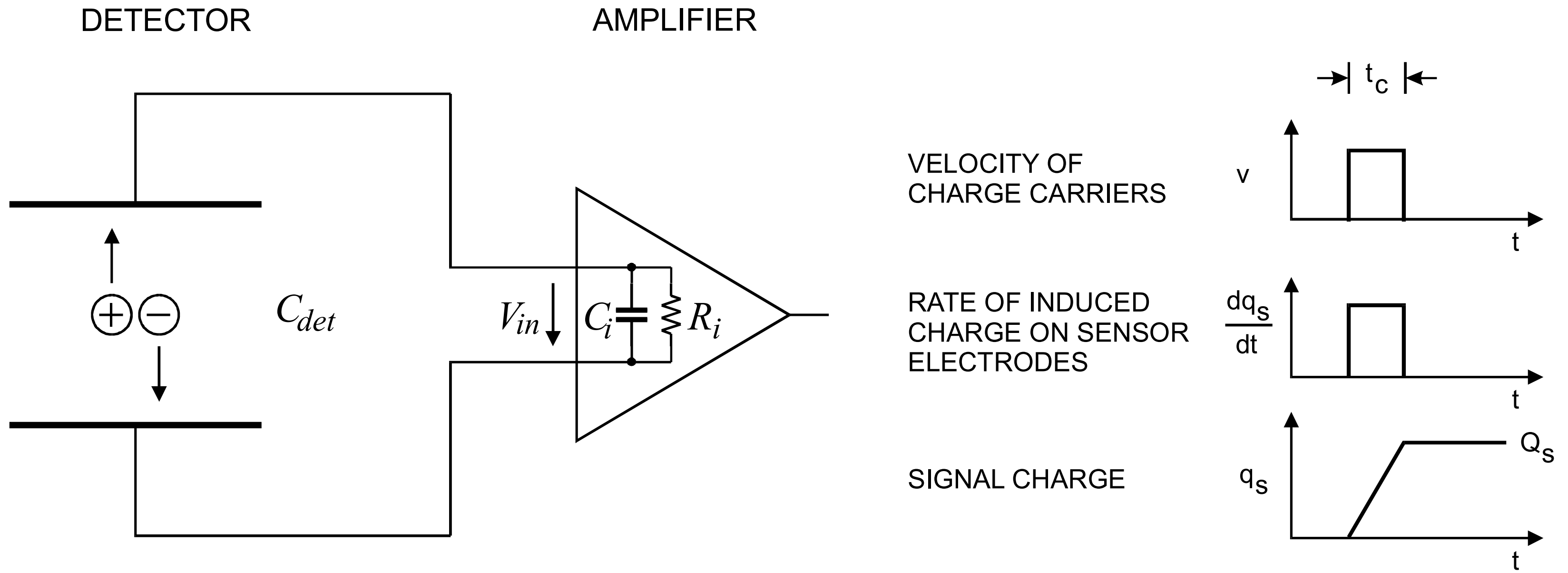
Both are amplified by the second amplifier, but in addition the second amplifier contributes its noise, so the signal-to-noise ratio at the output of the second amplifier

$$\left(\frac{S}{N}\right)^2 = \frac{(SA_1A_2)^2}{(N_1A_1A_2)^2 + (N_2A_2)^2} = \frac{S^2}{N_1^2 + \left(\frac{N_2}{A_1}\right)^2} = \left(\frac{S}{N_1}\right)^2 \frac{1}{1 + \left(\frac{N_2}{A_1N_1}\right)^2}$$

The overall sign-to-noise ratio is reduced, but the noise contribution from the second-stage can be negligible, provided the gain of the first stage is sufficiently high.

$\Rightarrow$ In a well-designed system the noise is dominated by the first gain stage.

3. Signal-to-Noise Ratio vs. Detector Capacitance



if $R_i \times (C_{det} + C_i) \gg$ collection time,

peak voltage at amplifier input $V_{in} = \dfrac{Q_s}{C} = \dfrac{\int i_s dt}{C} = \dfrac{Q_s}{C_{det} + C_i}$

↑

Magnitude of voltage depends on total capacitance at input!

The peak amplifier signal V_S is inversely proportional to the **total capacitance at the input**, i.e. the sum of

1. detector capacitance,
2. input capacitance of the amplifier, and
3. stray capacitances.

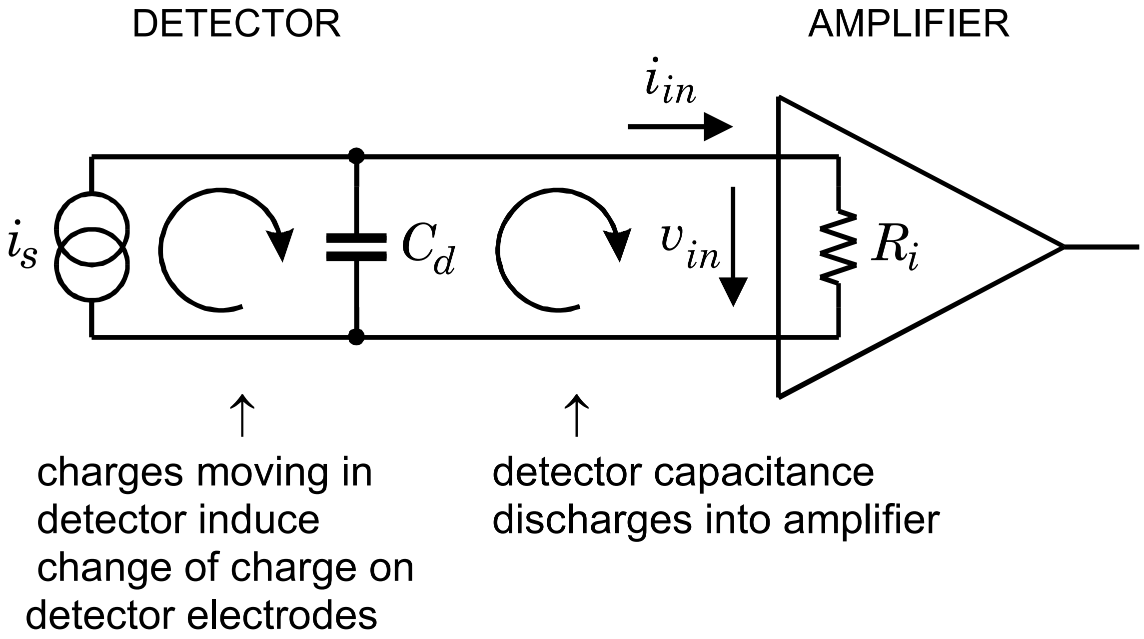
Assume an amplifier with a noise voltage v_n at the input.

Then the signal-to-noise ratio

$$\frac{S}{N} = \frac{V_S}{v_n} \propto \frac{1}{C}$$

- However, S/N does not become infinite as $C \to 0$
  (then front-end operates in current mode)
- The result that $S/N \propto 1/C$ generally applies to systems that measure signal charge.

Equivalent Circuit for General Analysis



The speed of the amplifier does not have to match the speed of the sensor signal.

Initially charge is integrated on the sensor capacitance.

As the amplifier responds, the signal is transferred to the amplifier.

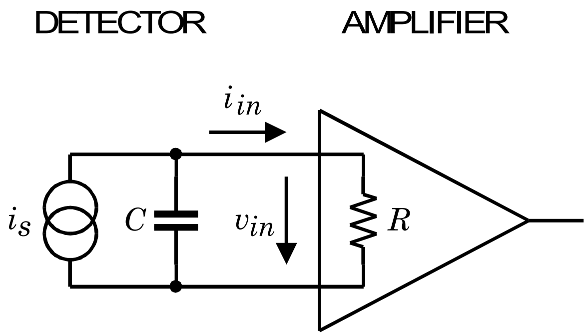
Assume an amplifier with constant noise.

Then signal-to-noise ratio (and the equivalent noise charge) depend on the signal magnitude.

The pulse shape registered by amplifier depends on the input time constant RC_{det}.

Assume a rectangular detector current pulse of duration T and magnitude i_S.

Equivalent circuit



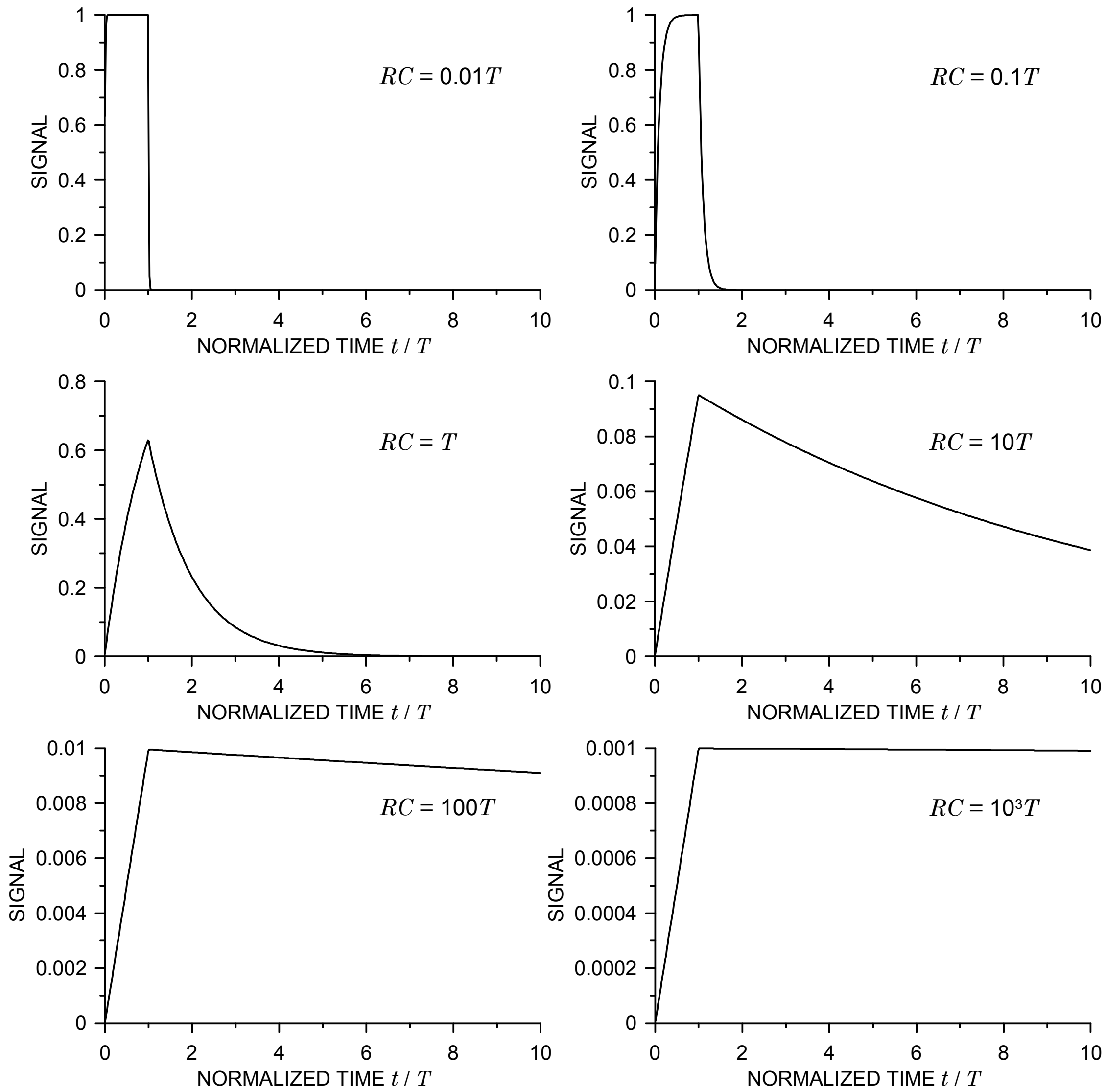
Input current to amplifier

$$0 \le t < T: \qquad i_{in}(t) = i_S \left(1 - e^{-t/RC} \right)$$

$$T \le t \le \infty: \qquad i_{in}(t) = i_S \left(e^{T/RC} - 1 \right) \cdot e^{-t/RC}$$

At short time constants $RC \ll T$ the amplifier pulse approximately follows the detector current pulse.

As the input time constant RC increases, the amplifier signal becomes longer and the peak amplitude decreases, although the integral, i.e. the signal charge, remains the same.



At long time constants the detector signal current is integrated on the detector capacitance and the resulting voltage sensed by the amplifier

$$V_{in} = \frac{Q_{det}}{C} = \frac{\int i_s dt}{C}$$

Then the peak amplifier signal is inversely proportional to the **total capacitance at the input**,

i.e. the sum of detector capacitance,

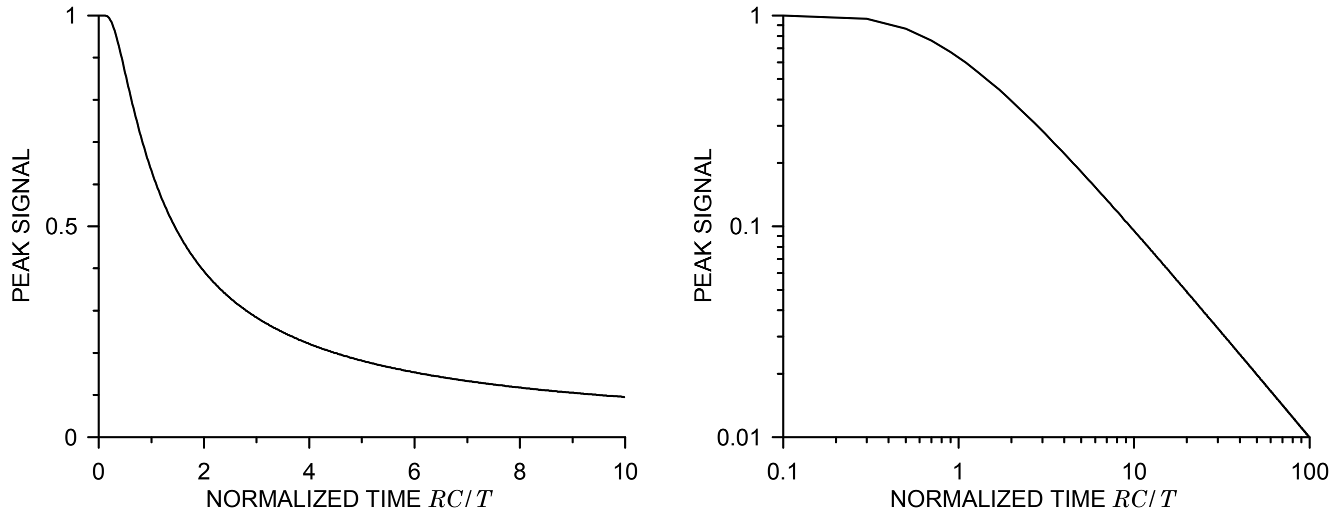
input capacitance of the amplifier, and

stray capacitances.

Maximum signal vs. capacitance

At small time constants the amplifier signal approximates the detector current pulse and is independent of capacitance.

At large input time constants (RC/T > 5) the maximum signal falls linearly with capacitance.



⇒ For input time constants large compared to the detector pulse duration the signal-to-noise ratio decreases with detector capacitance.

Caution when extrapolating to smaller capacitances:

If S/N = 1 at RC/T = 100, decreasing the capacitance to 1/10 of its original value (RC/T = 10), increases S/N to 10.
However, if initially RC/T = 1, the same 10-fold reduction in capacitance (to RC/T = 0.1) only yields S/N = 1.6.

Noise vs. Detector Capacitance – Charge-Sensitive Amplifier

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,
- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i \gg C_d$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_v$.
- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

Although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier.

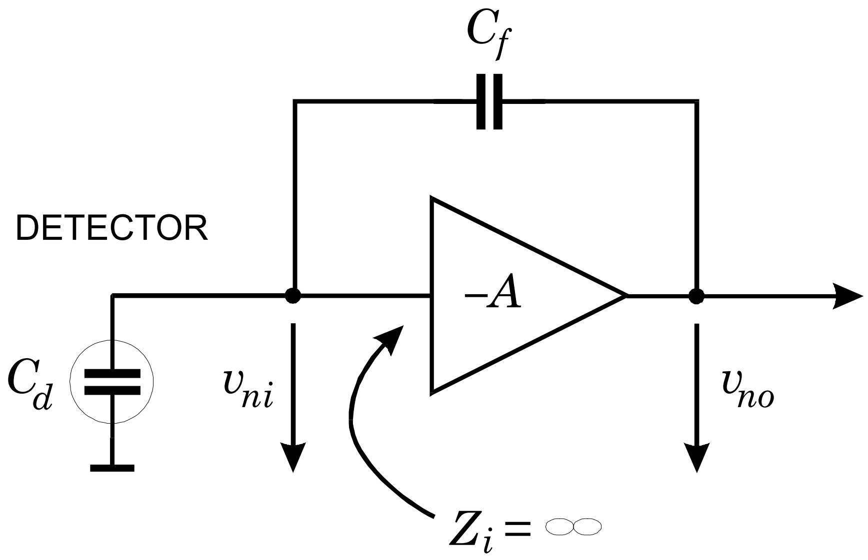
Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input to which the loop responds in the same manner as to a detector signal.

$\Rightarrow$ S/N at the amplifier output depends on feedback.

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.

$$v_{no} = v_{ni}\frac{X_{C_f} + X_{C_d}}{X_{C_d}} = v_{ni}\frac{\dfrac{1}{\omega C_f} + \dfrac{1}{\omega C_d}}{\dfrac{1}{\omega C_d}}$$



$$v_{no} = v_{ni}\left(1 + \frac{C_d}{C_f}\right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni}\left(C_d + C_f\right)$$

Signal-to-noise ratio

$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_d + C_f)} = \frac{1}{C}\frac{Q_s}{v_{ni}}$$

Same result as for voltage amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

Conclusion

In general

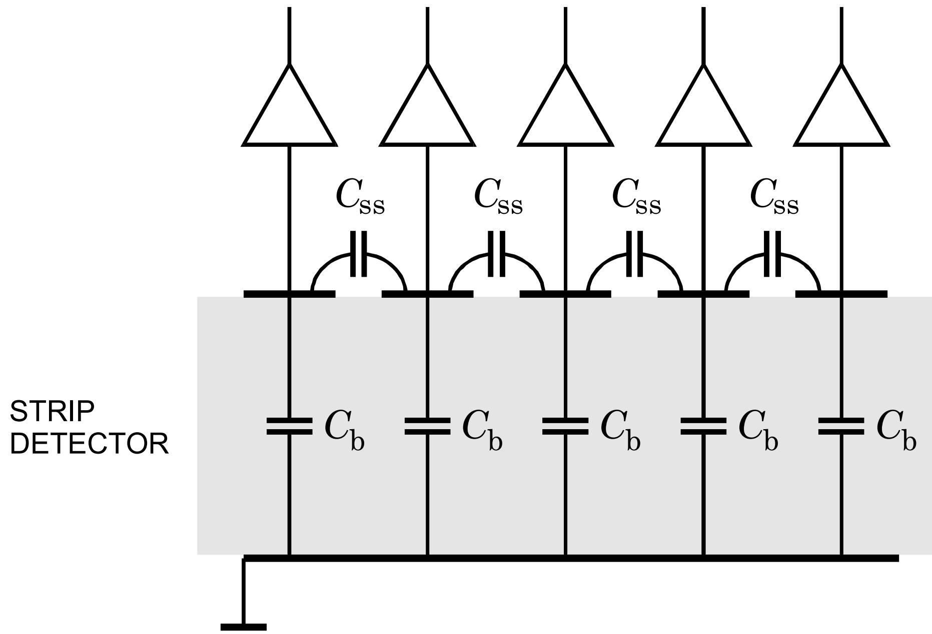
- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.

- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

4. Complex Sensors

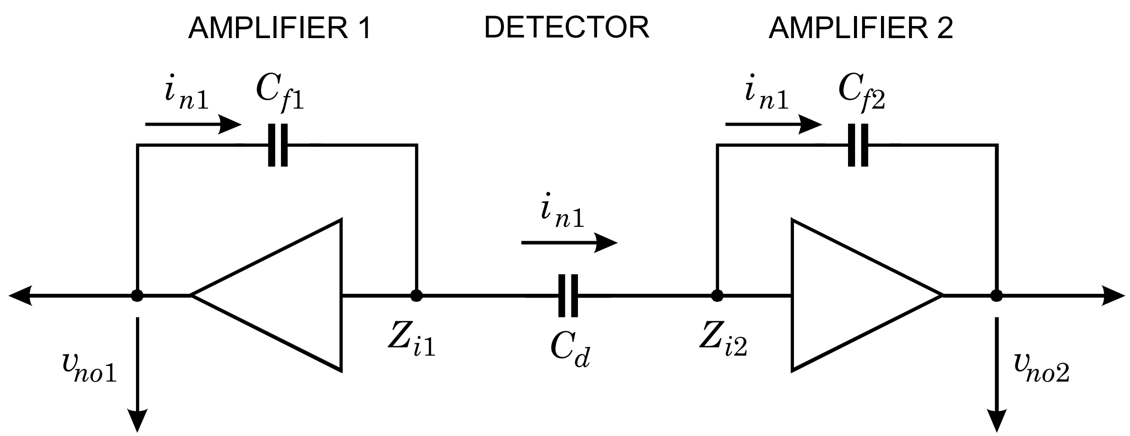
Cross-coupled noise



Noise at the input of an amplifier is cross-coupled to its neighbors.

Principle of Noise Cross-Coupling

Consider a capacitance connecting two amplifier inputs, e.g. amplifiers at opposite electrodes of a detector with capacitance C_d:



First, assume that amplifier 2 is noiseless.

The noise voltage v_{no1} causes a current flow i_{n1} to flow through the feedback capacitance C_{f1} and the detector capacitance C_d into the input of amplifier 2.

Note that for a signal originating at the output of amplifier 1, its input impedance Z_{i1} is high (∞ for an idealized amplifier), so all of current i_{n1} flows into amplifier 2.

Amplifier 2 presents a low impedance to the noise current i_{n1}, so its magnitude

$$i_{n1} = \frac{v_{no1}}{X_{C_{f1}} + X_{C_d}} = \frac{v_{no1}}{\dfrac{1}{\omega C_{f1}} + \dfrac{1}{\omega C_d}}.$$

The voltage at the output of amplifier is the product of the input current times the feedback impedance,

$$v_{no12} = \frac{i_{n1}}{\omega C_{f2}} = \frac{v_{no1}}{\dfrac{1}{\omega C_{f1}} + \dfrac{1}{\omega C_d}} \cdot \frac{1}{\omega C_{f2}} = \frac{v_{no1}}{\dfrac{C_{f2}}{C_{f1}} + \dfrac{C_{f2}}{C_d}}.$$

For identical amplifiers $C_{f1} = C_{f2}$. Furthermore, $C_{f2} \ll C_d$, so the additional noise from amplifier 1 at the output of amplifier 2 is

$$v_{no12} = v_{no1}.$$

This adds in quadrature to the noise of amplifier 2. Since both amplifiers are same, $v_{no1} = v_{no2}$, so cross-coupling increases the noise by a factor $\sqrt{2}$.

Cross-Coupling in Strip Detectors

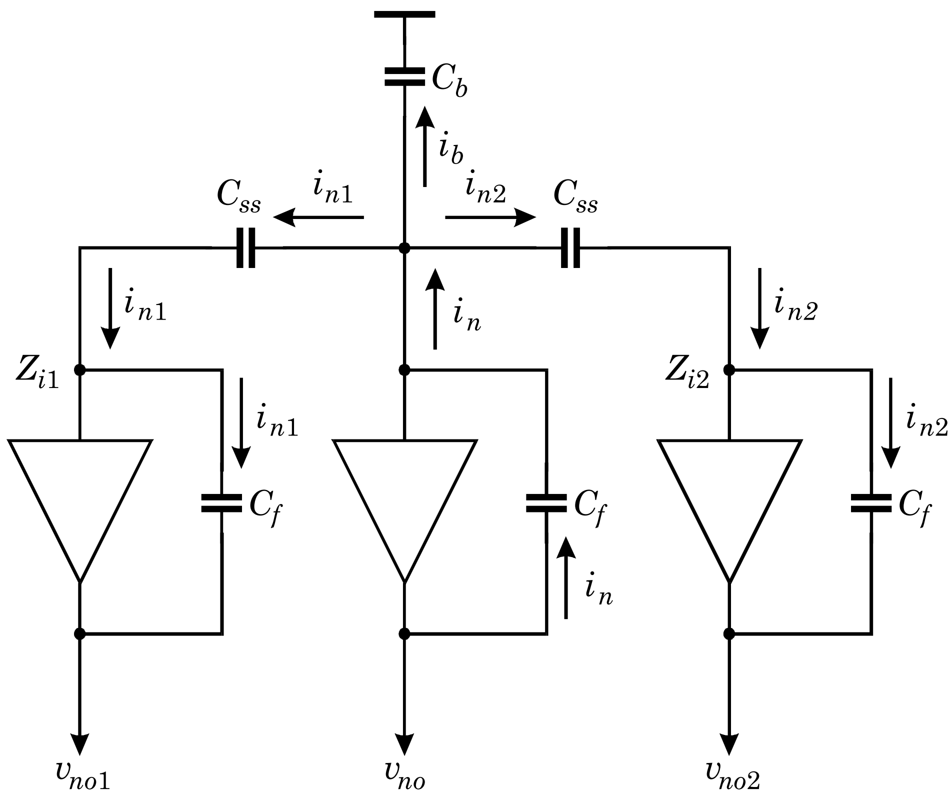
The backplane capacitance C_b attenuates the signal transferred through the strip-to-strip capacitance C_{ss}.

The additional noise introduced into the neighbor channels

$$v_{no1} = v_{no2} \approx \frac{v_{no}}{2} \frac{1}{1 + 2C_b / C_{ss}}$$

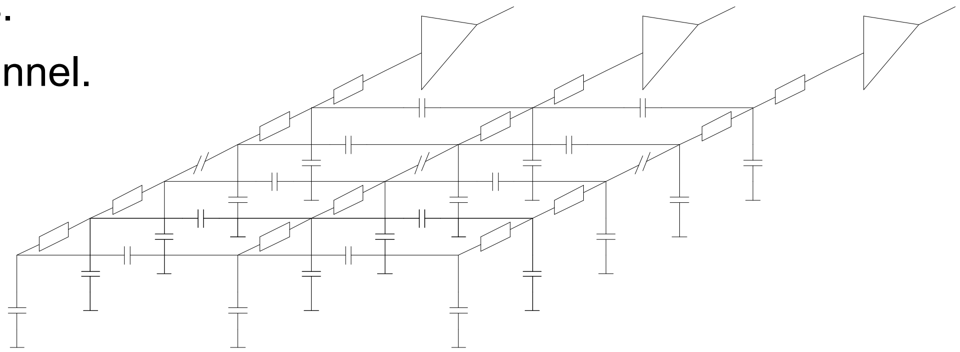
For $C_b = 0$, $v_{no1} = v_{no2} = v_{no} / 2$ and the total noise increases by a factor

$$\sqrt{1 + 0.5^2 + 0.5^2} = 1.22$$



For a backplane capacitance $C_b = C_{ss} / 10$ the noise increases by 16%.

Strip Detector Model for Noise Simulations



Noise coupled from neighbor channels.

Analyze signal and noise in center channel.

Includes:

a) Noise contributions from
neighbor channels

b) Signal transfer to
neighbor channels

c) Noise from distributed strip resistance

d) Full SPICE model of preamplifier

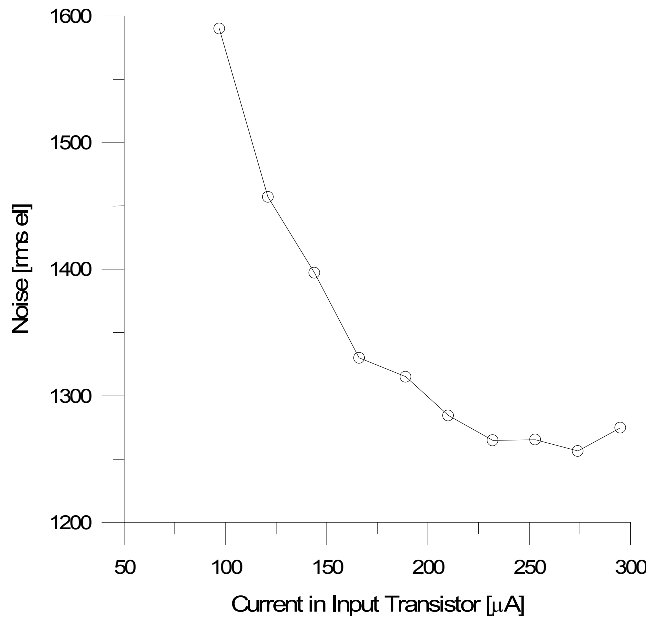
Measured Noise of Module:

p-strips on n-bulk, BJT input transistor

Simulation Results: 1460 el (150 μA)

1230 el (300 μA)

$\Rightarrow$ Noise can be predicted with good accuracy.



5. Quantum Noise Limits in Amplifiers

What is the lower limit to electronic noise?

Can it be eliminated altogether, for example by using superconductors and eliminating devices that carry shot noise?

Starting point is the uncertainty relationship

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Consider a narrow frequency band at frequency ω. The energy uncertainty can be given in terms of the uncertainty in the number of signal quanta

$$\Delta E = \hbar \omega \Delta n$$

and the time uncertainty in terms of phase

$$\Delta t = \frac{\Delta \varphi}{\omega},$$

so that

$$\Delta \varphi \cdot \Delta n \geq \frac{1}{2}$$

We assume that the distributions in number and phase are Gaussian, so that the equality holds.

Assume a noiseless amplifier with gain G, so that n_1 quanta at the input yield

$$n_2 = Gn_1$$

quanta at the output.

Furthermore, the phase at the output φ_2 is shifted by a constant relative to the input.

Then the output must also obey the relationship $\Delta\varphi_2 \Delta n_2 = \dfrac{1}{2}$

However, since $\Delta n_2 = G\Delta n_1$ and $\Delta\varphi_2 = \Delta\varphi_1$:

$$\Delta\varphi_1 \Delta n_1 = \frac{1}{2G} \;,$$

which is smaller than allowed by the uncertainty principle.

This contradiction can only be avoided by assuming that the amplifier introduces noise per unit bandwidth of

$$\frac{dP_{no}}{d\omega} = (G-1)\hbar\omega \ ,$$

which, referred to the input, is

$$\frac{dP_{ni}}{d\omega} = \left(1 - \frac{1}{G}\right)\hbar\omega$$

If the noise from the following gain stages is to be small, the gain of the first stage must be large, and then the minimum noise of the amplifier

$$\frac{dP_{ni}}{d\omega} = \hbar\omega$$

At 2 mm wavelength the minimum noise corresponds to about 7K.

This minimum noise limit applies to phase-coherent systems. In systems where the phase information is lost, e.g. bolometers, this limit does not apply.

For a detailed discussion see C.M. Caves, Phys. Rev. D **26** (1982) 1817-1839
H.A. Haus and J.A. Mullen, Phys. Rev. 128 (1962) 2407-2413