Sensor Physics – Part I

Any form of elementary excitation can be used to detect the radiation signal.

An electrical signal can be formed directly by ionization.

Incident radiation quanta impart sufficient energy to individual atomic electrons to form electron-ion pairs (in gases) or electron-hole pairs (in semiconductors and metals).

Other detection mechanisms are

Excitation of optical states (scintillators)

Excitation of lattice vibrations (phonons)

Breakup of Cooper pairs in superconductors

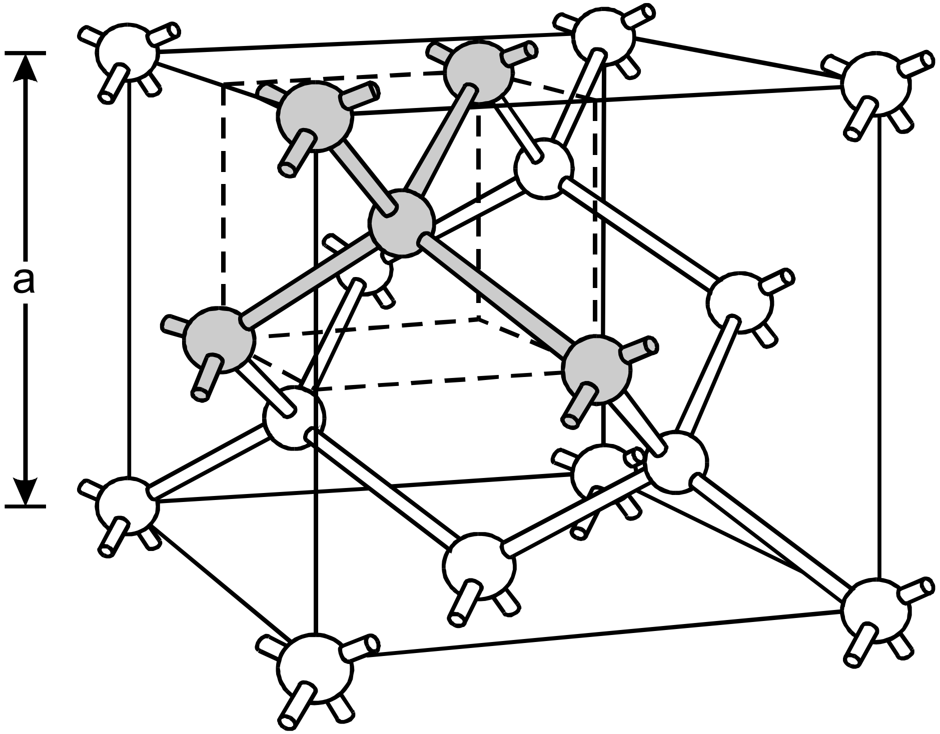
Formation of superheated droplets in superfluid He

Typical excitation energies

Ionization in gases	~30 eV
Ionization in semiconductors	1 – 5 eV
Scintillation	~10 eV
Phonons	meV
Breakup of Cooper Pairs	meV

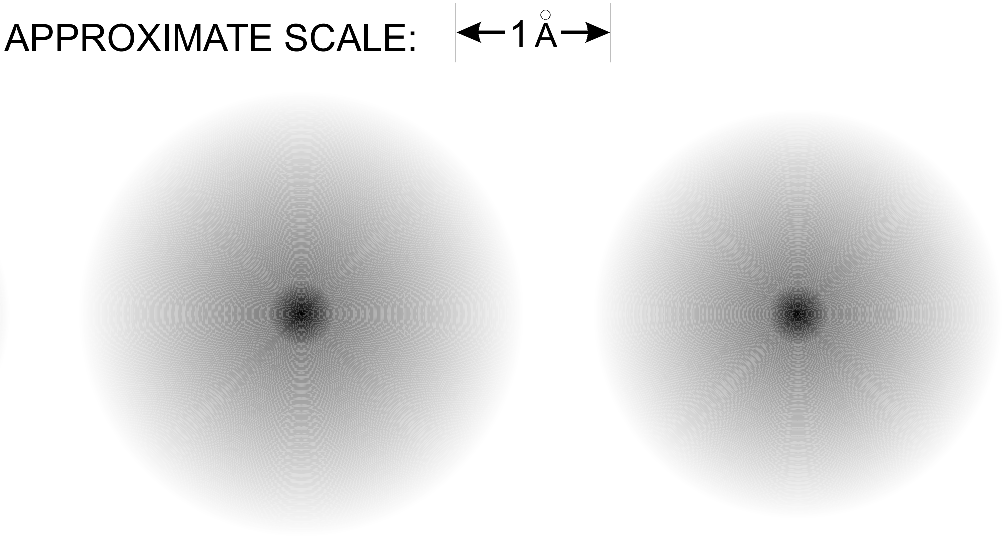
Band Structure in Crystals

Example: Lattice structure of diamond, Si, Ge (“diamond lattice”)



dimension a:	lattice constant	Diamond:	3.56 Å
		Ge:	5.65 Å
		Si:	5.43 Å

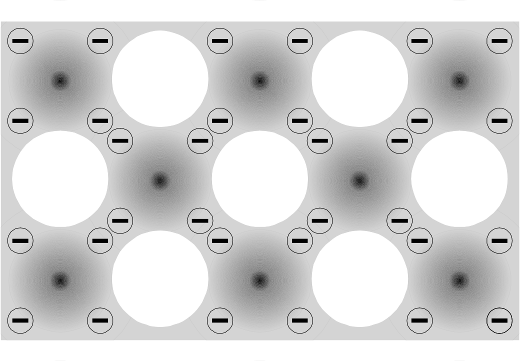
Extent of wavefunctions of typical constituent atoms:



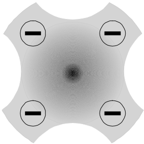
CARBON ($Z = 6$) SILICON ($Z = 14$) GERMANIUM ($Z = 32$)

(following Shockley)

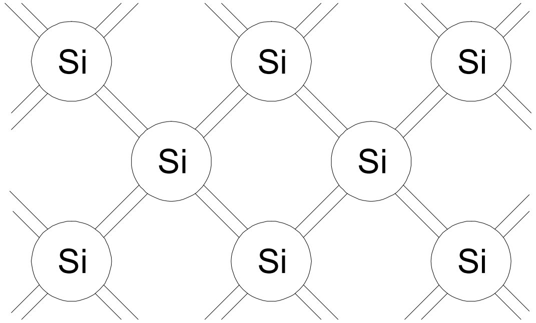
Crystal Bonds



SILICON “CORES” WITH ELECTRON
“CLOUDS” SHOWING VALENCE PAIR BONDS



SILICON ATOM WITH FOUR
VALENCE ELECTRONS



SYMBOLIC PLANE VIEW USING
LINES TO REPRESENT BONDS

When isolated atoms are brought together to form a lattice, the discrete atomic states shift to form energy bands:

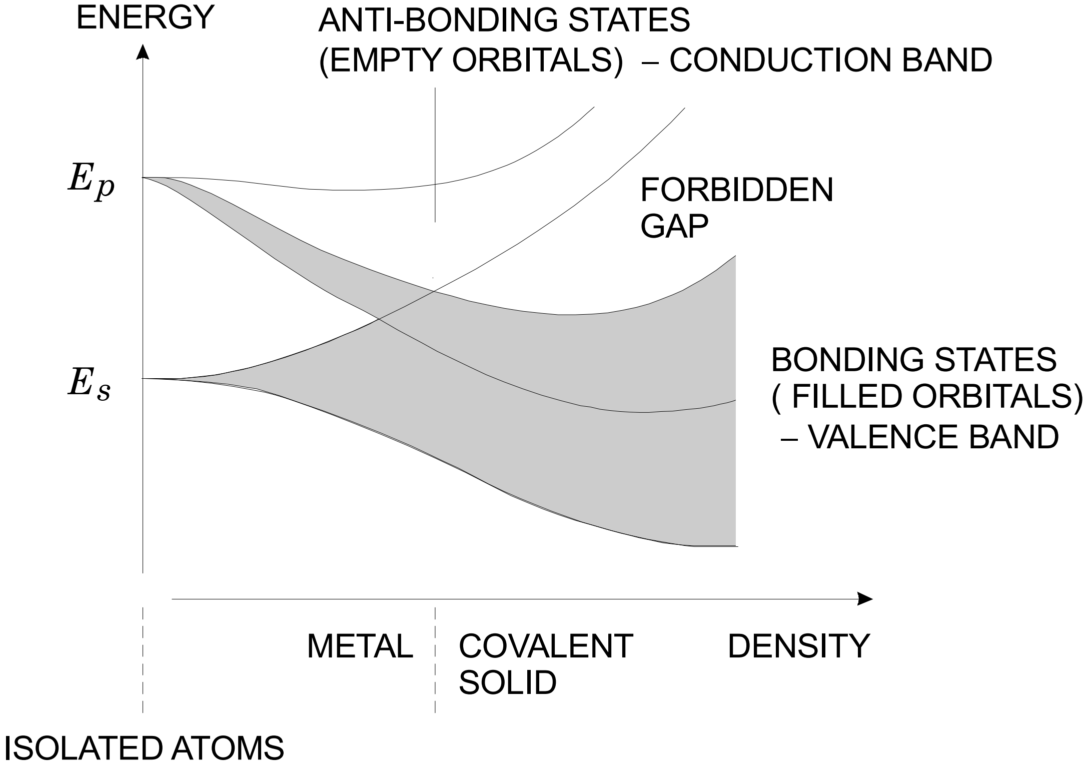
Filled band formed by bonding states: $\Psi = \Psi_a + \Psi_a$

(Ψ_a = wavefunction of individual atom)

Empty band formed by anti-bonding states: $\Psi = \Psi_a - \Psi_a$

(vanishing occupancy at mid-point between atoms)

Each atom in the lattice contributes its quantum states to each band:



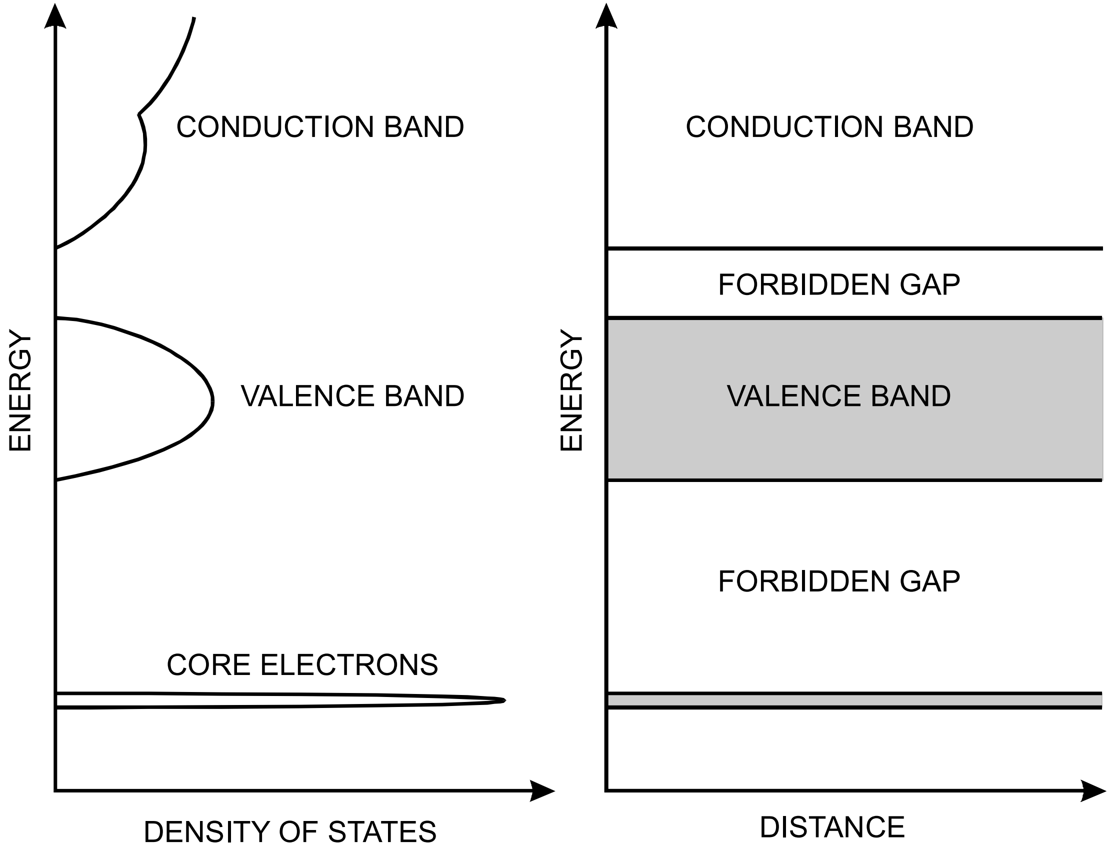
The number of quantum states in the band is equal to the number of states from which the band was formed.

The bands are extended states, i.e. the state contributed by an individual atom extends throughout the crystal.

Energy band structure

Typical band gaps
(valence – conduction band)

Ge	0.7 eV
GaAs	1.4 eV
Si	1.1 eV
Diamond	5.5 eV



(following Shockley)

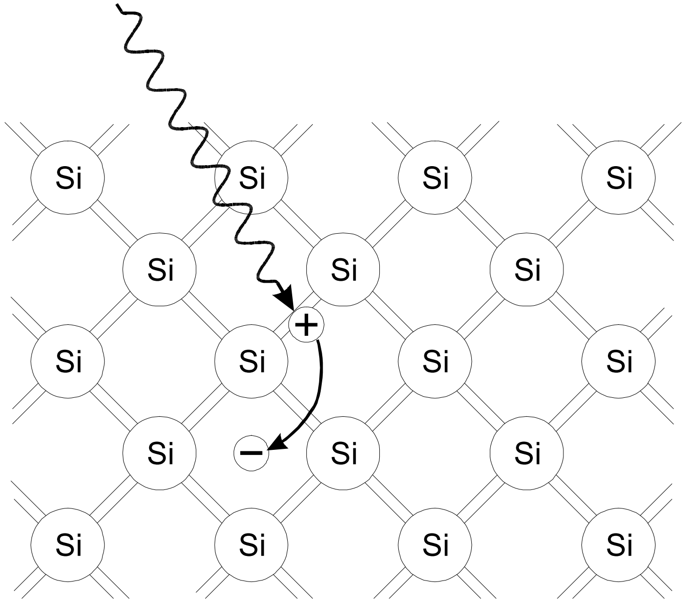
At 0K all electrons occupy bonding states, completely filling the valence band.

If an electric field is applied to the crystal, no current can flow, as this requires that the electrons acquire energy, which they can't, as no higher energy states are available in the valence band.

If energy is imparted to a bond by incident radiation, for example a photon, the bond can be broken,

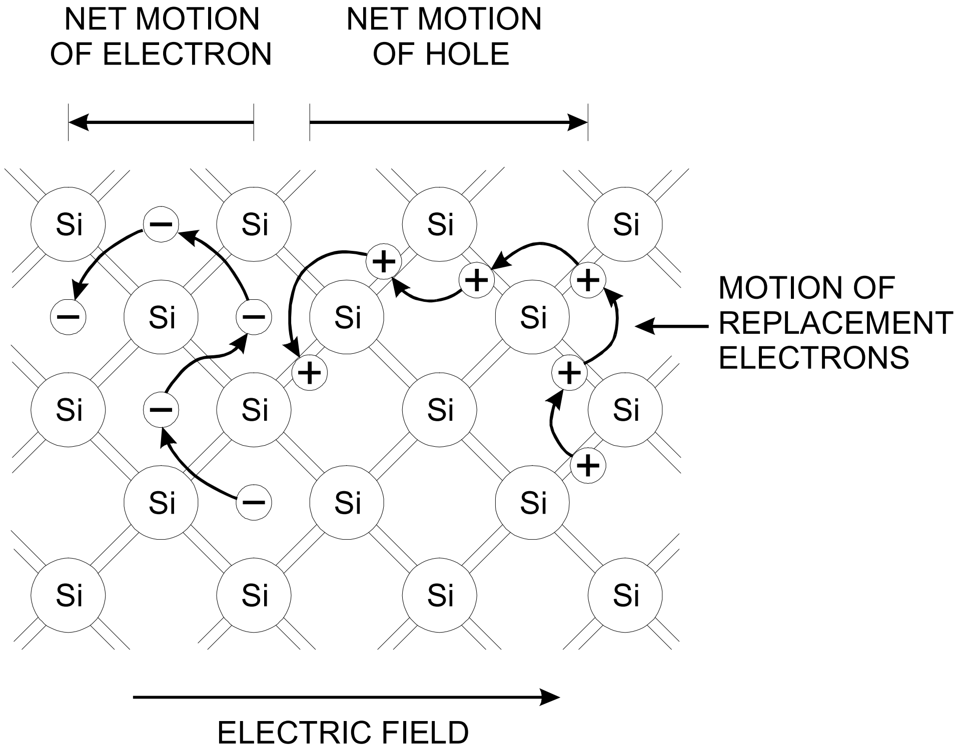
- exciting an electron into the conduction band and
- leaving back a vacant state in the valence band, a "hole".

INCIDENT PHOTON BREAKS BOND



The electron can move freely in its extended state.

The hole can be filled by an electron from a nearby atom, thereby moving to another position.



The motion of the electron and hole can be directed by an electric field.

Holes can be treated as positive charge carriers just like the electrons

However, they tend to move more slowly as hole transport involves sequential transition probabilities (the wavefunction overlap of the hole and its replacement electron).

Ionization energy in solids is proportional to the band gap

small band gap	⇒	~ conductor electric field small DC current >> signal current
large band gap	⇒	insulator high electric field small signal charge + small DC current example: diamond
moderate band gap	⇒	semiconductor high electric field "large" signal charge small DC current, but "*pn*-junction" required. examples: Si, Ge, GaAs

Although phonons have been represented as a penalty that increases the ionization energy, as mentioned above they are another form of elementary excitation that can be used to measure the signal.

More about this on my web page.

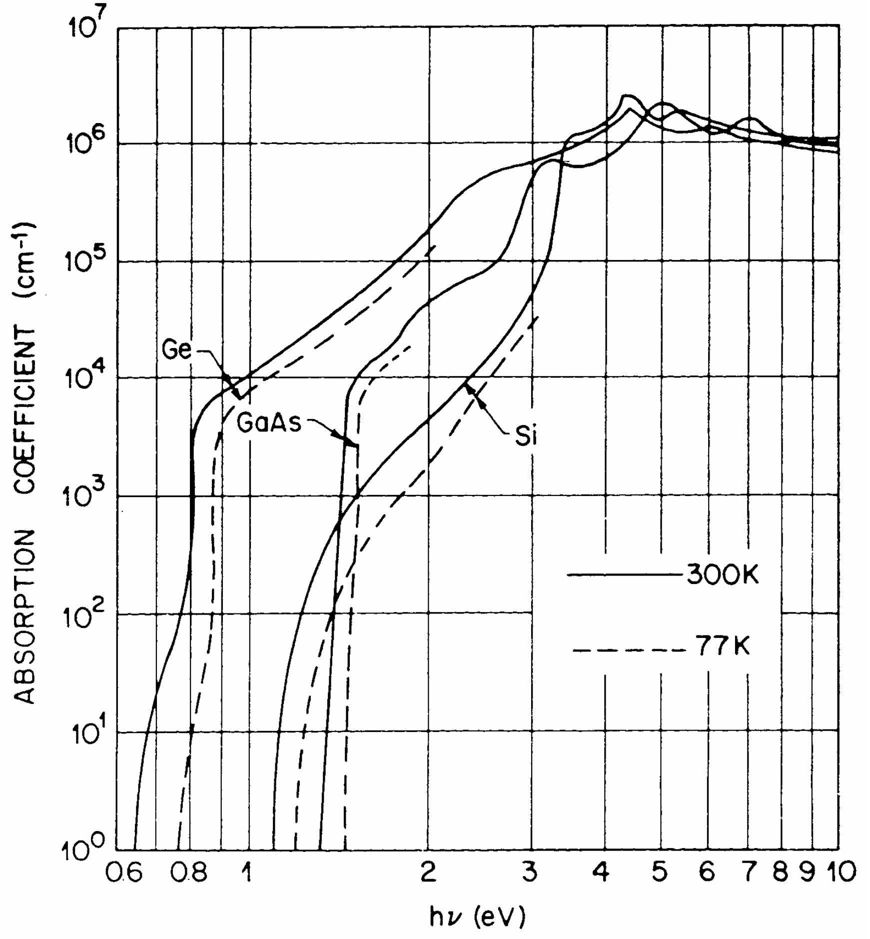
Detector Sensitivity

Example: Ionization signal in semiconductor detectors

a) visible light
(energies near band gap)

Detection threshold = energy required to produce an electron-hole pair ≈ band gap

In indirect bandgap semiconductors (Si), additional momentum required: provided by phonons



(from Sze)

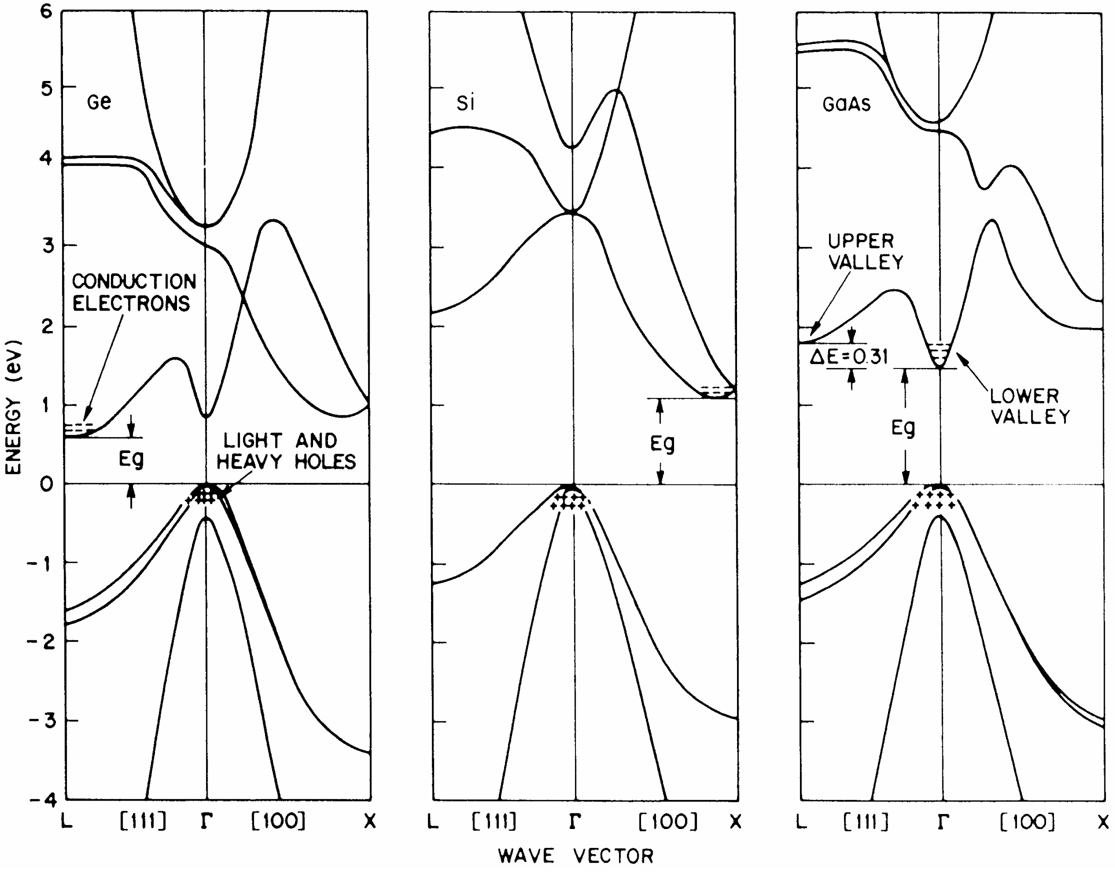
Band Structure

Energy of the conduction and valence band edges vs. wave vector (momentum)
(from Sze, *Physics of Semiconductor Devices*)

Band structure depends on orientation [111] vs. [100] crystal planes.

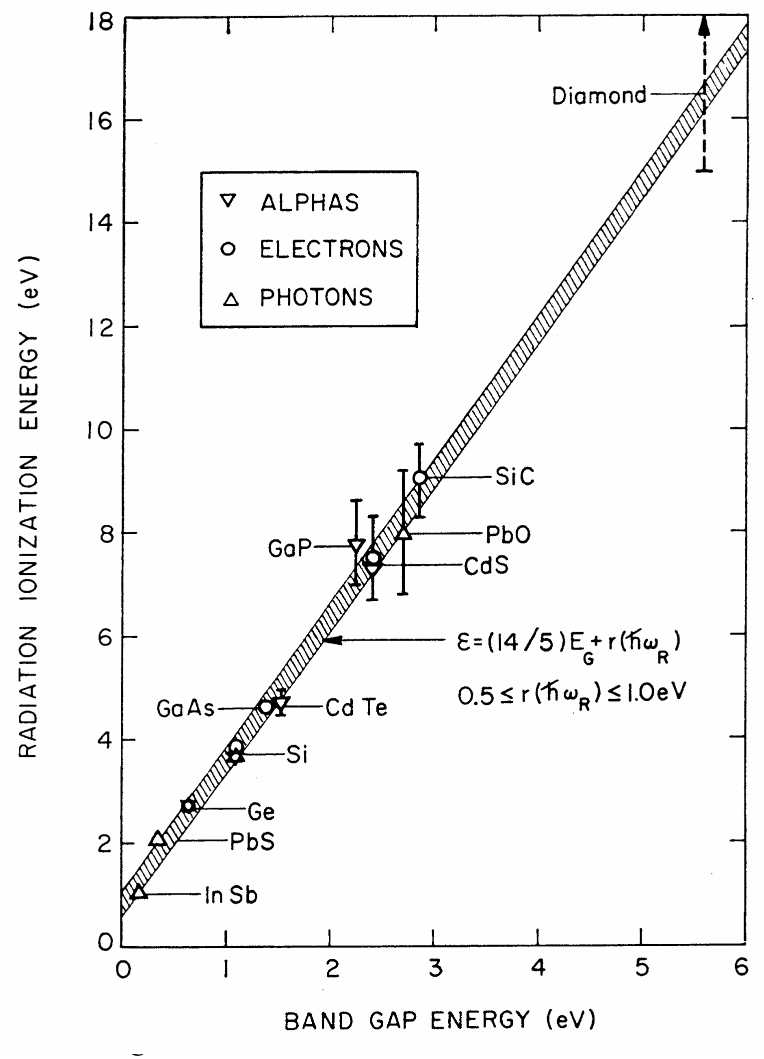
Note that in Si and Ge the minimum of the conduction band is offset from the maximum of the valence band.

⇒ Promotion of an electron from the valence to the conduction band using an energy equal to the minimum gap spacing requires additional momentum transfer



b) high energy quanta ($E \gg E_g$)

It is experimentally observed that the energy required to form an electron-hole pair exceeds the bandgap.



.A. Klein, J. Applied Physics **39** (1968) 2029

Why?

When particle deposits energy one must conserve both

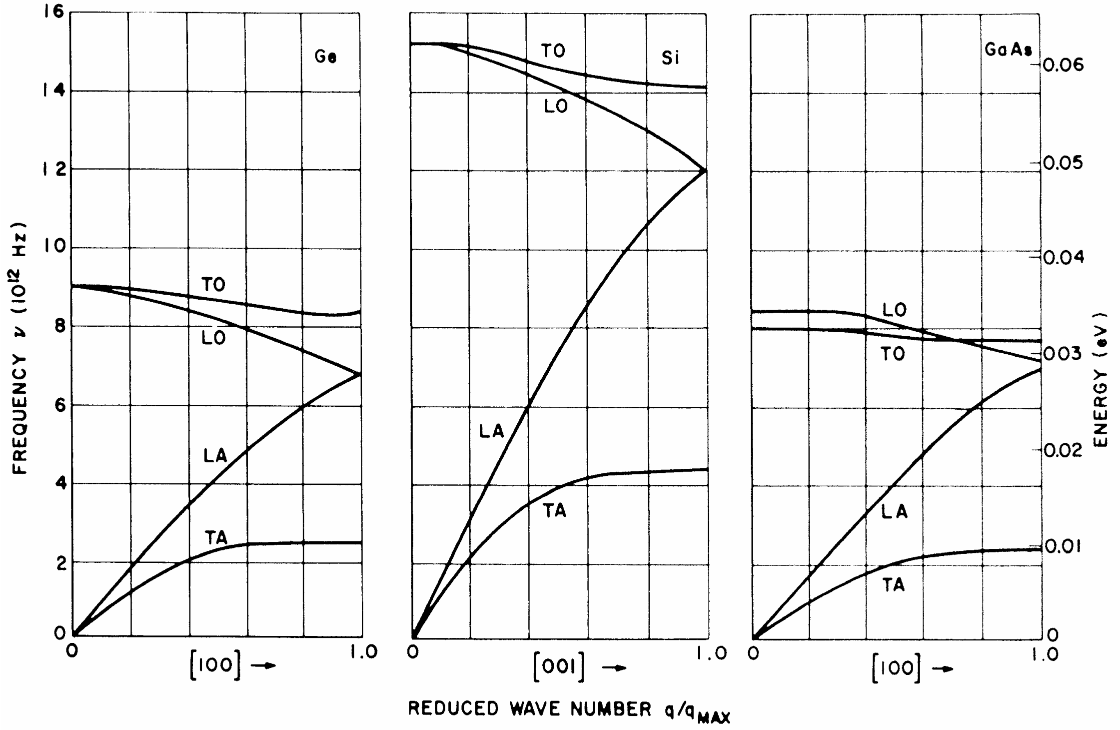
energy and momentum

momentum conservation not fulfilled by transition across gap

⇒ excite lattice vibrations (phonons)

Phonon energy vs. momentum (wavevector *k*)

In a semiconductor ionization detector ~60% of the deposited energy goes into phonon excitation.



Instead of detecting electron-hole pairs, detect heat or phonons

Energy scale: 10 meV ⇒ lower energy threshold

Another possibility: Breakup of Cooper pairs in superconductors

The energy gap 2Δ (order 1 meV) is equivalent to the band gap in semiconductors.

Signal Fluctuations in a Scintillation Detector

Example: Scintillation Detector - a typical NaI(Tl) system (from Derenzo)

Resolution of energy measurement determined by statistical variance of produced signal quanta.

$$\frac{\Delta E}{E} = \frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Resolution determined by smallest number of quanta in chain, i.e. number of photoelectrons arriving at first dynode.

In this example

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{3000}} = \text{ 2\% rms = 5\% FWHM}$$

Typically 7 – 8% obtained, due to non-uniformity of light collection and gain.

511 keV gamma ray

⇓

25000 photons in scintillator

⇓

15000 photons at photocathode

⇓

3000 photoelectrons at first dynode

⇓

$3 \cdot 10^9$ electrons at anode

2 mA peak current

Fluctuations in the Signal Charge: the Fano Factor

The mean ionization energy exceeds the bandgap for two reasons

1. Conservation of momentum requires excitation of lattice vibrations
2. Many modes are available for the energy transfer with an excitation energy less than the bandgap.

Two types of collisions are possible:

a) Lattice excitation, i.e. phonon production (with no formation of mobile charge).

b) Ionization, i.e. formation of a mobile charge pair.

Same treatment as in following derivation can be applied to scintillators:

lattice excitation ≙ excitation of non-radiative states
Ionization ≙ excitation of non-radiative states
(emission of scintillation photons)

or to liquid ionization chambers.

Fano Factor in Semiconductors

The incident photon energy goes into two modes:

1. ionization (excitation of electrons into the conduction band)
   characterized by the bandgap energy E_g

2. phonons (lattice vibrations),
   characterized by the average phonon energy $\overline{E}_{ph}$

The total energy

$$E_0 = n_{ph}\overline{E}_{ph} + n_g E_g \qquad (1)$$

Since the absorbed energy is fixed, any variation in energy going into phonon modes must be balanced by an opposite change in ionization, so

$$\delta n_{ph}\overline{E}_{ph} = \delta n_g E_g \qquad (2)$$

Thus, provided the number of phonon excitations is large, so that Gaussian statistics apply

$$\left(\delta n_g\right)^2 = \left(\frac{\overline{E}_{ph}}{E_g}\right)^2 \cdot \left(\delta n_{ph}\right)^2 = \left(\frac{\overline{E}_{ph}}{E_g}\right)^2 \cdot n_{ph} \qquad (3)$$

Assume that the ratio of the total energy required to form an electron-hole pair to the bandgap E_i / E_g is constant. Then for a given number of electron-hole pairs n_g the total absorbed energy

$$E_0 = \left(\frac{E_i}{E_g} \right) n_g E_g \tag{4}$$

and eqn 1 can be rewritten as

$$n_{ph} = n_g \frac{E_g}{\overline{E}_{ph}} \left(\frac{E_i}{E_g} - 1 \right) \tag{5}$$

Inserting this into eqn 3 yields

$$\left(\delta n_g \right)^2 = \left[\frac{\overline{E}_{ph}}{E_g} \left(\frac{E_i}{E_g} - 1 \right) \right] \cdot n_g \tag{6}$$

Simple statistics would imply that $\left(\delta n_g \right)^2 = n_g$, so eqn 6 shows that the variance in the number of electron-hole pairs is reduced by the Fano factor

$$F = \frac{\overline{E}_{ph}}{E_g} \left(\frac{E_i}{E_g} - 1 \right) \tag{7}$$

Since $E_i / E_g \approx 3$ and $\overline{E}_{ph} \ll E_g$, the Fano factor $F < 1$.

In Silicon $E_{ph} = 0.037$ eV $E_g = 1.1$ eV $E_i = 3.6$ eV

for which the above expression yields $F = 0.08$, in reasonable agreement with the measured value $F = 0.1$.

⇒ The variance of the signal charge is smaller than naively expected:

$$\sigma_Q \approx 0.3\sqrt{N_Q}$$

A similar treatment can be applied if the degrees of freedom are much more limited and Poisson statistics are necessary.

However, when applying Poisson statistics to the situation of a fixed energy deposition, which imposes an upper bound on the variance, one can not use the usual expression for the variance $\mathrm{var}\ N = \overline{N}$ Instead, the variance is $\overline{(N-\overline{N})^2} = F\overline{N}$ as shown by Fano [1] in the original paper.

An accurate calculation of the Fano factor requires a detailed accounting of the energy dependent cross sections and the density of states of the phonon modes. This is discussed by Alkhazov [2] and van Roosbroeck [3].

References:
1. U. Fano, Phys. Rev. **72** (1947) 26
2. G.D. Alkhazov et al., NIM **48** (1967) 1
3. W. van Roosbroeck, Phys. Rev. **139** (1963) A1702

Intrinsic Resolution of Semiconductor Detectors

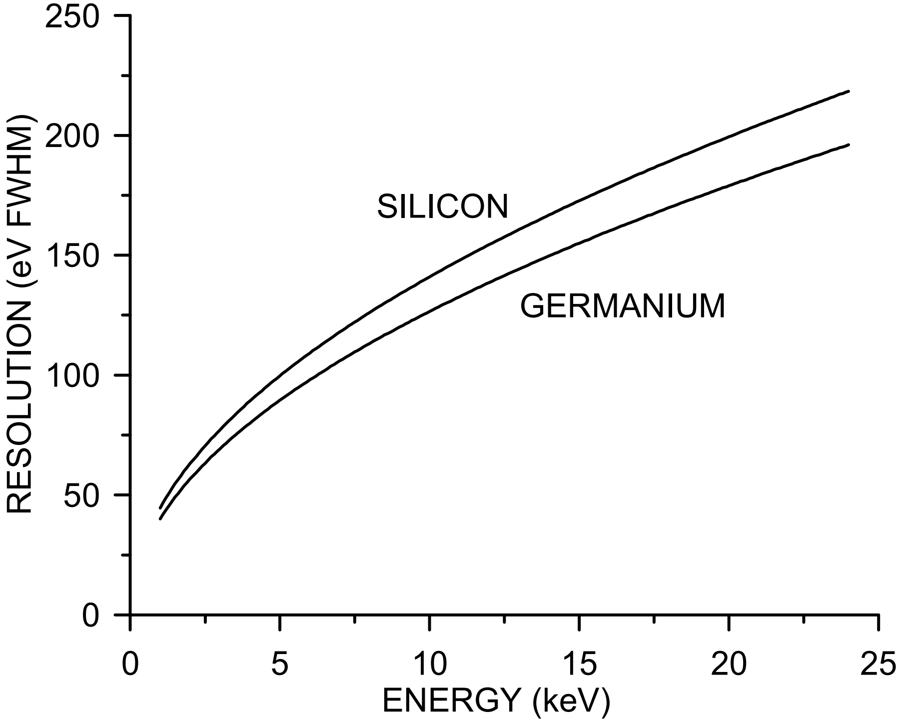
$$\Delta E_{FWHM} = 2.35 \cdot \varepsilon_i \sqrt{FN_Q} = 2.35 \cdot \varepsilon_i \sqrt{F \frac{E}{E_i}} = 2.35 \cdot \sqrt{FEE_i}$$

Si: $E_i = 3.6$ eV $F = 0.1$

Ge: $E_i = 2.9$ eV $F = 0.1$

Detectors with good efficiency for this energy range have sufficiently small capacitance to allow electronic noise of ~100 eV FWHM, so the variance of the detector signal is a significant contribution.

At energies >100 keV the detector sizes required tend to increase the electronic noise to dominant levels.



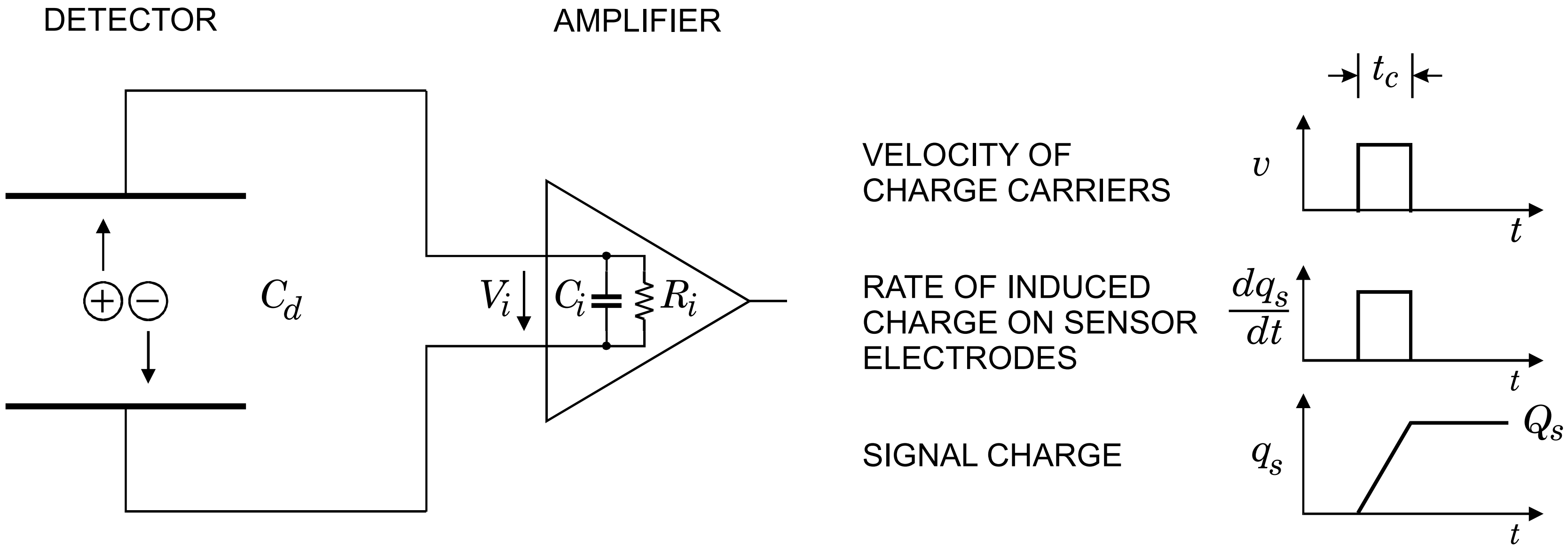
3. Signal Formation

Semiconductor Detectors are Ionization Chambers:

Detection volume with electric field

Energy deposited → positive and negative charge pairs

Charges move in field → external electrical signal current



DETECTOR

AMPLIFIER

VELOCITY OF CHARGE CARRIERS

RATE OF INDUCED CHARGE ON SENSOR ELECTRODES

SIGNAL CHARGE

If $R_i \cdot (C_d + C_i) \gg$ collection time t_c the peak voltage at the amplifier input $V_s = \dfrac{Q_s}{C_{\mathrm{det}} + C_i}$

Ionization chambers can be made with any medium that allows charge collection to a pair of electrodes.

Medium can be
- gas
- liquid
- solid

Crude comparison of relevant properties

	gas	**liquid**	**solid**
density	low	moderate	high
atomic number Z	low	moderate	moderate
ionization energy ε_i	moderate	moderate	low
signal speed	moderate	moderate	fast

Desirable properties:

- low ionization energy ⇒
  1. increased charge yield dq/dE
  2. superior resolution $\dfrac{\Delta E}{E} \propto \dfrac{1}{\sqrt{N}} \propto \dfrac{1}{\sqrt{E/E_i}} \propto \sqrt{E_i}$
- high field in detection volume ⇒
  1. fast response
  2. improved charge collection efficiency (reduced trapping)

Formation of a High-Field Region

To form a current that can be measured in the external circuit, the signal charge carriers must be brought into motion. This is done by establishing a field in the detection volume. Increasing the field will sweep the charge more rapidly from the detection volume.

The conduction band is only empty at 0K.

As the temperature is increased, thermal excitation can promote electrons across the band gap into the conduction band.

Pure Si: carrier concentration ~ 10^{10} cm^{-3} at 300K (resistivity ≈ 400 $\text{k}\Omega\cdot\text{cm}$)

Since the Si lattice contains $5\cdot 10^{22}$ atoms/cm^3, many states are available in the conduction band to allow carrier motion.

In reality, crystal imperfections and minute impurity concentrations limit Si carrier concentrations to ~10^{11} cm^{-3} at 300K.

This is too high for use in a simple crystal detector.

A crystal detector is feasible with diamond,
but the charge yield is smaller due to the larger band gap.

High-field region with low DC current in semiconductors
is most easily achieved utilizing a *pn*-junction.

⇒ Introduction of impurities to control conductivity.

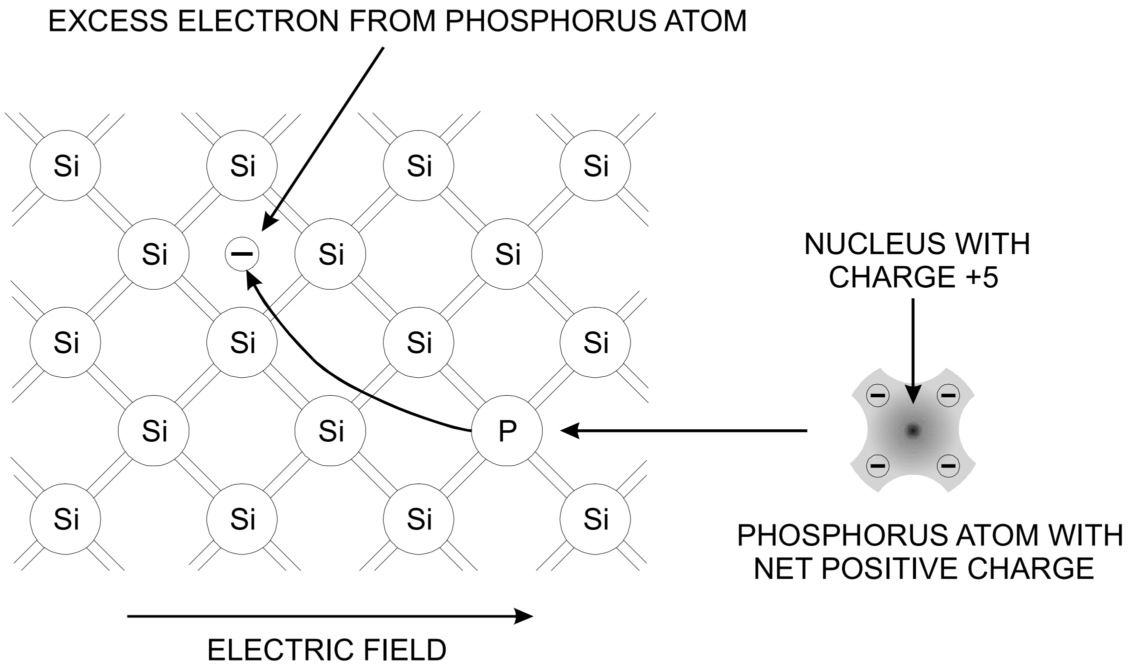
Doping

The conductivity of semiconductors can be controlled by introducing special impurities.

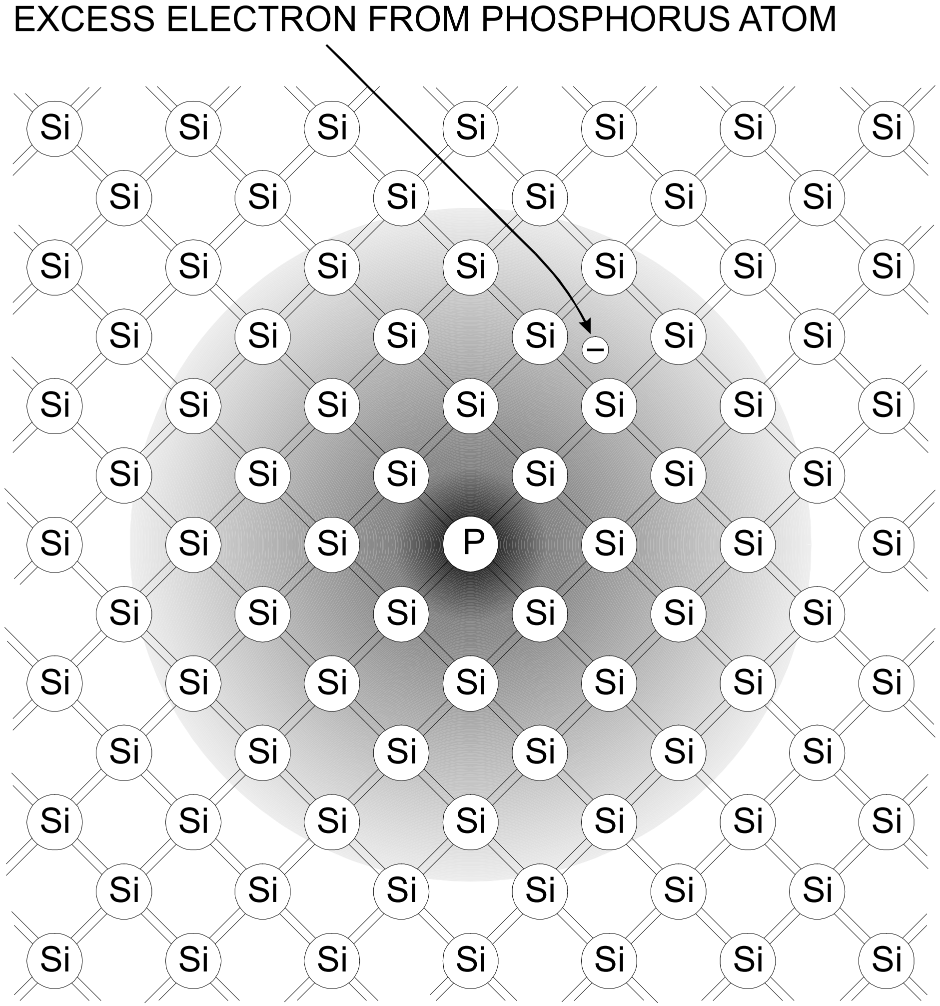
required concentrations: ~10^{12} – 10^{18} cm^{-3}

Replacing a silicon atom (group 4 in periodic table, i.e. 4 valence electrons) by an atom with 5 valence electrons, e.g. P, As, Sb, leaves one valence electron without a partner.

Since the impurity contributes an excess electron to the lattice, it is called a donor.



The wavefunction of the dopant atom extends over many neighbors.

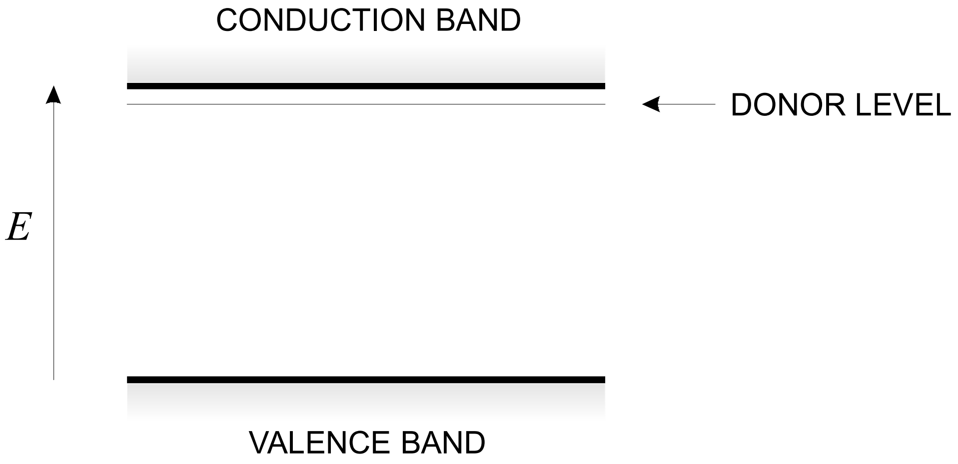


(following Shockley)

The excess electron is only loosely bound, as the Coulomb force is reduced by the dielectric constant ε of the medium (ε =12 in Si).

$$E_i(lattice) \;\propto\; \frac{E_i(atom)}{\varepsilon^2}$$

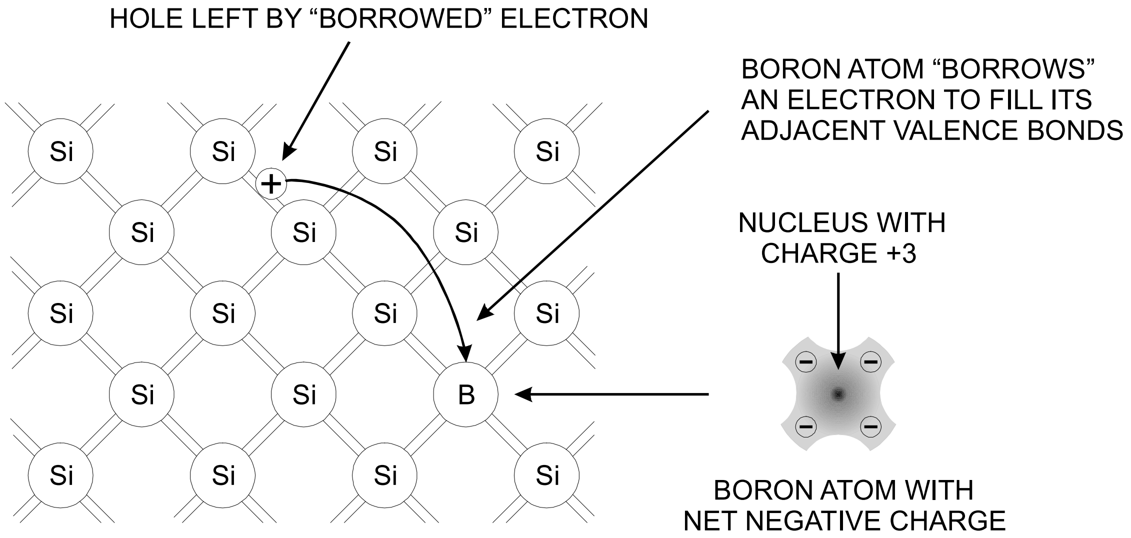
The bound level of this unpaired electron is of order 0.01 eV below the conduction band (e.g. for P: E_c - 0.045 eV).



⇒ substantial ionization probability at room temperature (E= 0.026 eV) – “donor”

⇒ electrons in conduction band

Conversely, introducing a group 3 atom (B, Al, Ga, In) leaves a Si valence electron without a partner.



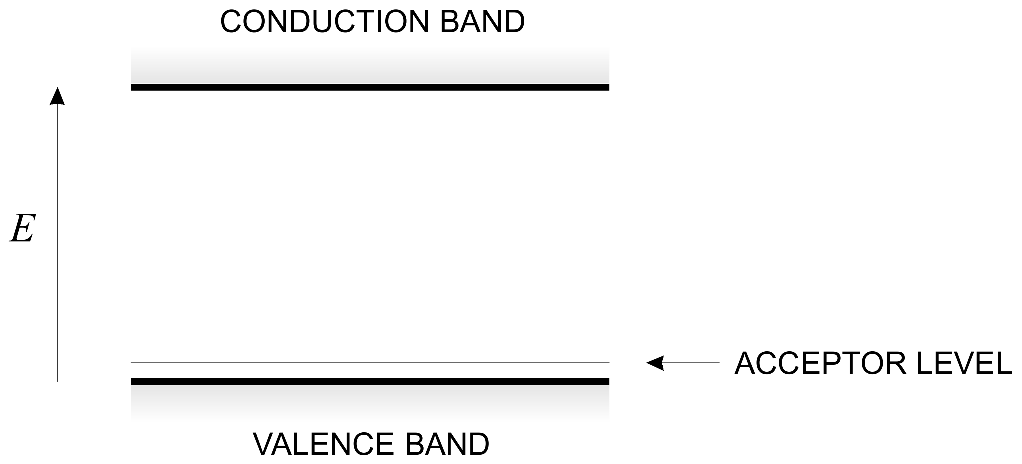
(following Shockley)

To close its shell the B atom “borrows” an electron from a lattice atom in the vicinity.

This type of dopant is called an “acceptor”.

The “borrowed” electron is bound, but somewhat less than other valence electrons since the B nucleus only has charge 3.

This introduces a bound state close to the valence band, also of order 0.01 eV from the band edge.



For example, a B atom in Si forms a state at E_v + 0.045 eV.

Again, as this energy is comparable to kT at room temperature, electrons from the valence band can be excited to fill a substantial fraction of these states.

The electrons missing from the valence band form mobile charge states called “holes”, which behave similarly to an electron in the conduction band, i.e. they can move freely throughout the crystal.

Since the charge carriers in the donor region are electrons, i.e. negative, it is called “n-type”.
Conversely, as the charge carriers in the acceptor region are holes, i.e. positive, it is called “p-type”.

pn-Junction

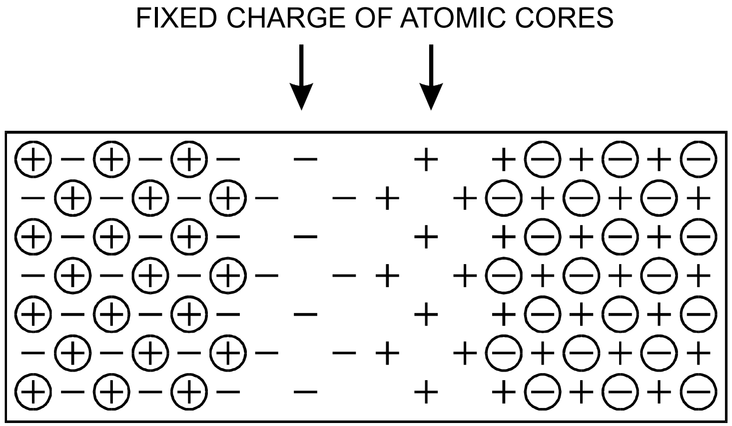
Consider a crystal suitably doped that a donor region and an acceptor adjoin each other, a “pn-junction”.

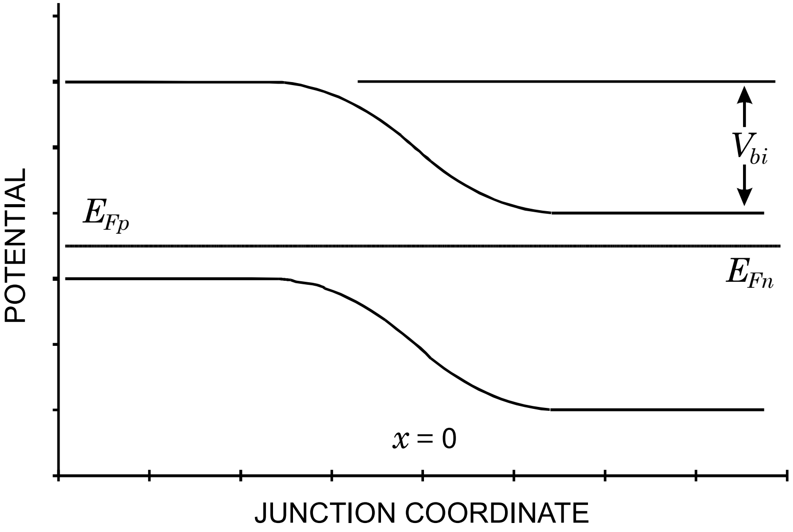
Thermal diffusion will drive holes and electrons across the junction.

Although the p and n regions were originally electrically neutral, as electrons diffuse from the n to the p region, they uncover their respective donor atoms, leaving a net positive charge in the n region.

This positive space charge exerts a restraining force on the electrons that diffused into the p region, i.e. diffusion of electrons into the p region builds up a potential. The diffusion depth is limited when the space charge potential exceeds the available energy for thermal diffusion.

The corresponding process also limits the diffusion of holes into the n-region.





The diffusion of holes and electrons across the junction leads to a region free of mobile carriers – the “depletion region”, bounded by conductive regions, which are n- and p-doped, respectively.

Strictly speaking, the depletion region is not completely devoid of mobile carriers, as the diffusion profile is a gradual transition.

Nevertheless, since the carrier concentration is substantially reduced, it is convenient to treat the depletion zone as an abrupt transition between bulk and 0 carrier concentration.

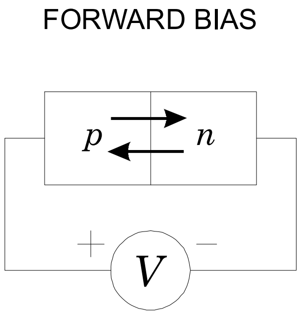
Furthermore, the formation of the two adjacent space charge regions builds up a potential barrier between the n and p regions, which impedes the further flow of charge.

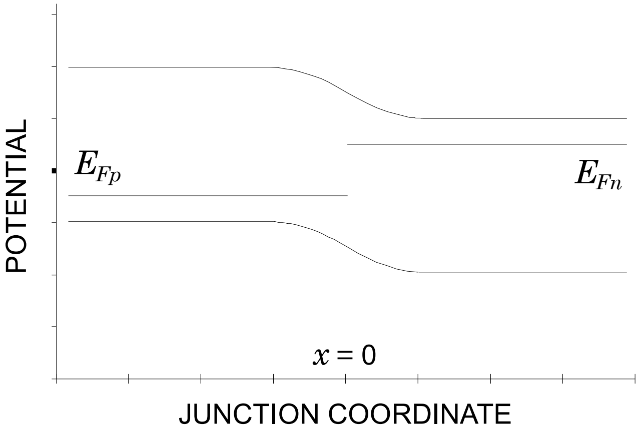
The magnitude of this potential barrier is typically 50 – 90%
of the band-gap, depending on relative doping levels.

This represents the situation in thermal equilibrium.

By application of an external potential, two distinctly different non-equilibrium modes can be established.

a) positive potential applied to the p region
negative potential applied to the n region



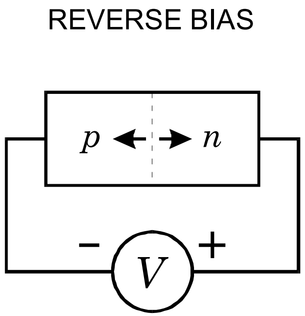


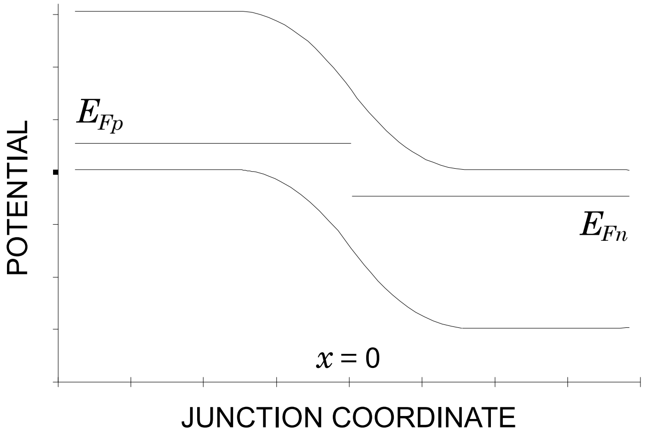
The externally applied voltage reduces the potential barrier, allowing increased charge transfer across the junction.

⇒ “forward bias”

Electrons flowing from the n-region across the junction are replenished from the external voltage supply and large current flow is possible.

b) negative potential applied to the p region
positive potential applied to the n region





This arrangement increases the potential barrier across the junction, impeding the flow of current.

⇒ “reverse bias”

Potential across junction is increased ⇒ wider depletion region

The p-n junction is asymmetric with respect to current flow (diode).

a) forward bias

positive supply connection → p contact
negative supply connection → n contact

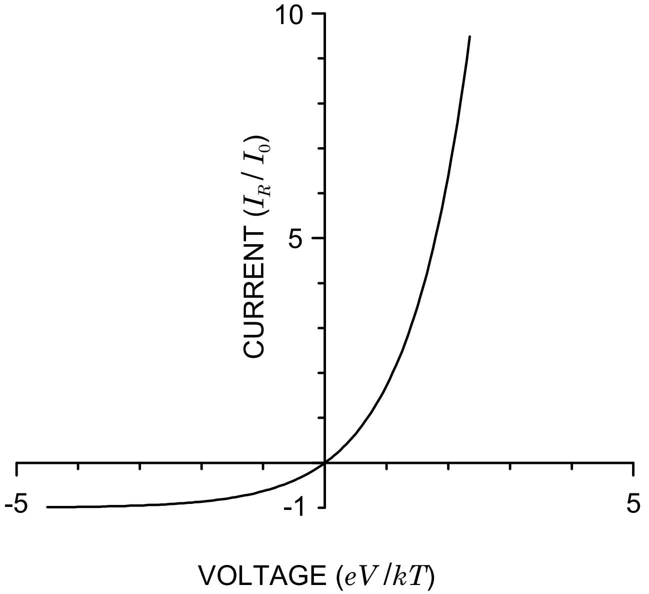
⇒ large current flow

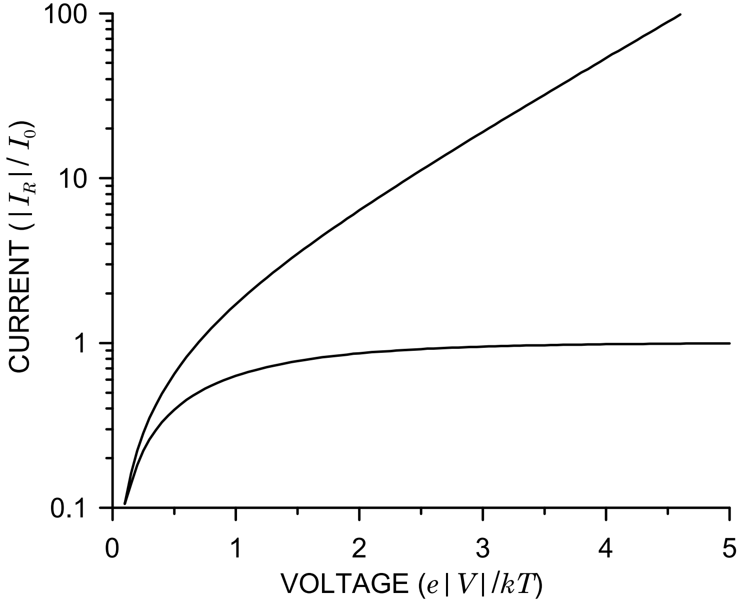
Diode current vs. voltage $I = I_0(e^{q_e V/kT} - 1)$
(Shockley equation)

b) reverse bias

positive supply connection → n contact
negative supply connection → p contact

⇒ small current flow





- Since the depletion region is a volume with an electric field, it by itself could be used as a radiation detector.

- The width of the depletion region is increased by reverse bias.

Depletion width and electric field in p-n junction

Assume a reverse bias voltage V_b and that the potential changes only in the direction perpendicular to the n-p interface. Poisson's equation is then

$$\frac{d^2V}{dx^2} + \frac{Nq_e}{\varepsilon} = 0 \qquad (1)$$

where N is the dopant concentration and q_e the electronic charge.

Consider an abrupt junction where charge densities on the n and p sides are $N_d\, q_e$ and $N_a\, q_e$, respectively.

If the limits of the depletion region are x_n on the n-side and x_p on the p-side, after two successive integrations one obtains on the n-side

$$\frac{dV}{dx} = -\frac{q_e N_d}{\varepsilon}(x - x_n) \tag{2}$$

and

$$V = -\frac{q_e N_d}{\varepsilon}\frac{x^2}{2} + \frac{q_e N_d x x_n}{\varepsilon} + V_j \tag{3}$$

where V_j is the potential at the metallurgical junction. For $x = x_n$

$$V(x_n) = V_b = \frac{q_e N_d x_n^2}{2\varepsilon} + V_j \tag{4}$$

and the contribution of the n-region to the total reverse bias potential becomes

$$V_b - V_j = \frac{q_e N_d x_n^2}{2\varepsilon} \; . \tag{5a}$$

Correspondingly, in the p-region

$$V_j = \frac{q_e N_a x_p^2}{2\varepsilon} \tag{5b}$$

and the total potential becomes

$$V_b = \frac{q_e}{2\varepsilon}(N_d x_n^2 + N_a x_p^2). \tag{6}$$

Due to overall charge neutrality

$$N_d x_n = N_a x_p \tag{7}$$

and

$$V_b = \frac{q_e}{2\varepsilon}\left(1+\frac{N_a}{N_d}\right) N_a x_p^2 = \frac{q_e}{2\varepsilon}\left(1+\frac{N_d}{N_a}\right) N_d x_n^2 \; . \tag{8}$$

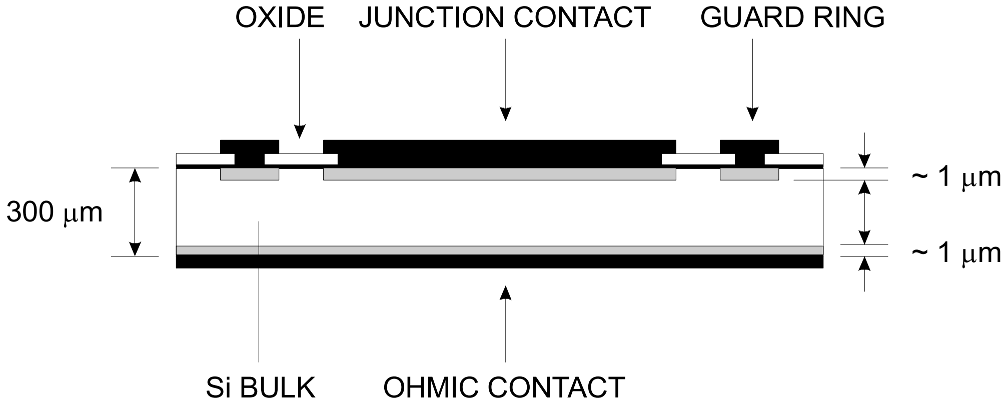
The depletion widths on the n- and p-side of the junction are

$$x_n = \sqrt{\frac{2\varepsilon V_b}{q_e N_d (1+N_d/N_a)}} \; ; \quad x_p = \sqrt{\frac{2\varepsilon V_b}{q_e N_a (1+N_a/N_d)}} \tag{9}$$

and the total depletion width becomes

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon V_b}{q_e}\frac{N_a + N_d}{N_a N_d}} \; . \tag{10}$$

Detector diodes are usually asymmetrically doped. The starting material (bulk) is lightly doped and the junction is formed by diffusing or ion-implanting a highly doped layer.



The external connection to the lightly doped bulk is made by an additional highly doped layer of the same type (non-rectifying, “ohmic” contact).

- The depletion region then extends predominantly into the lightly doped bulk.

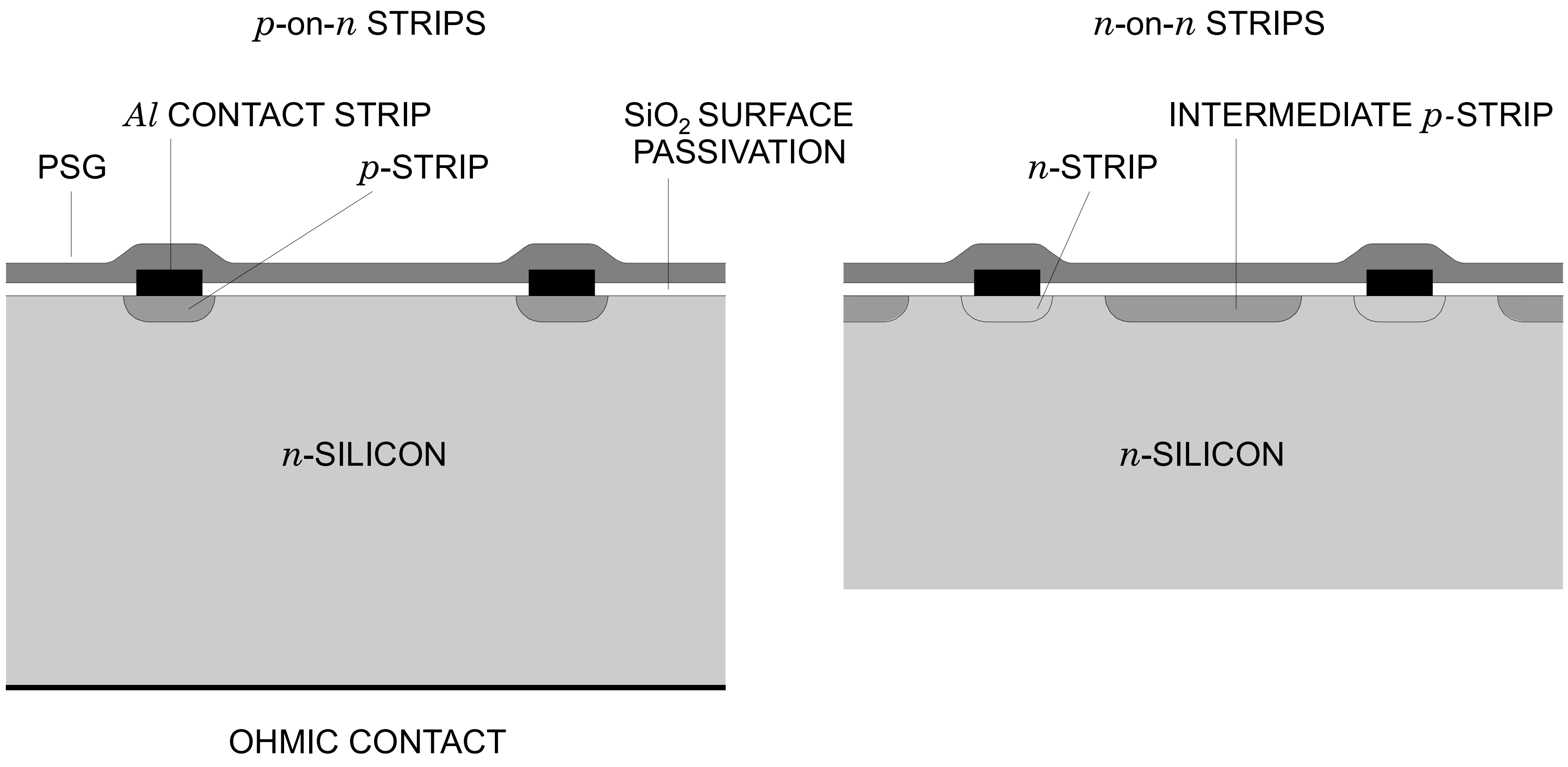
Other details:

The guard ring isolates the wafer edge (saw cut) from the active region.

In the gap between the detector electrode and the guard ring it is critical to provide a neutral interface at the silicon surface to prevent formation of a conductive path.

This is best accomplished by oxide passivation (SiO_2).

Strip and pixel detectors utilize a similar structure, except that the pn-junction is segmented:



When, for example, $N_a \gg N_d$, the depletion region extends predominantly into the n-side and the total depletion width is

$$W \approx x_n = \sqrt{\frac{2\varepsilon V_b}{q_e N_d}} \;. \qquad (11)$$

The doping concentration is commonly expressed in terms of resistivity

$$\rho = (\mu q_e N)^{-1},$$

because this is a readily measurable quantity. The parameter μ describes the relationship between the applied field and carrier velocity (to be discussed later).

Using resistivity the depletion width becomes

$$W = \sqrt{2\varepsilon \mu_n \rho_n V_b} \;. \qquad (12)$$

Note that this introduces an artificial distinction between the n- and p-regions, because the mobilities μ for electrons and holes are different.

> *Since the mobility of holes is approximately 1/3 that of electrons, p-type material of a given doping concentration will have 3 times the resistivity of n-type material of the same concentration.*

As discussed earlier, even in the absence of an external voltage electrons and holes to diffuse across the junction, establishing a "built-in" reverse bias voltage V_{bi}. If we take this inherent bias voltage into account and set for the bias voltage $V_b \to V_b + V_{bi}$, one obtains for the one-sided junction

$$W \approx x_1 = \sqrt{\frac{2\varepsilon(V_b + V_{bi})}{q_e N_d}} = \sqrt{2\varepsilon\mu_n\rho_n(V_b + V_{bi})}.$$

For example, in n-type silicon (V_b in volts and ρ in Ω·cm): $W = 0.5\ \mu m \text{ x } \sqrt{\rho(V_b + V_{bi})}$

and in p-type material: $W = 0.3\ \mu m \text{ x } \sqrt{\rho(V_b + V_{bi})}$

The depleted junction volume is free of mobile charge and thus forms a capacitor, bounded by the conducting p- and n-type semiconductor on each side.

The capacitance is $C = \varepsilon \frac{A}{W} = A\sqrt{\frac{\varepsilon\, q_e N}{2(V_b + V_{bi})}}$

For bias voltages $V_b \gg V_{bi}$ $C \propto \frac{1}{\sqrt{V_b}}$

In technical units $\frac{C}{A} = \frac{\varepsilon}{W} \approx 1\ [\text{pF/cm}]\ \frac{1}{W}$

A diode with 100 μm thickness has about 1 pF/mm^2.

The capacitance vs. voltage characteristic of a diode can be used to determine the doping concentration of the detector material.

$$\frac{C}{A} = \sqrt{\frac{\varepsilon\ q_e N}{2(V_b + V_{bi})}}$$

In a plot of $(A/C)^2$ vs. the detector bias voltage V_b the slope of the voltage dependent portion yields the doping concentration N.

Example: Si pad detector, A= 1 cm^2, 100 μm thick

$$\frac{1}{N} = \frac{d(1/C^2)}{dV}\left(\frac{\varepsilon q_e}{2}\right) = \frac{1}{5\cdot 10^{12}}$$