Electronics I – Amplifiers, Noise, and Signal Processing

1. Amplifiers

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    Charge-Sensitive Amplifier
    Frequency and Time Response

2. Resolution and Electronic Noise

3. Basic Noise Mechanisms

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    Shot Noise
    "$1/f$" Noise

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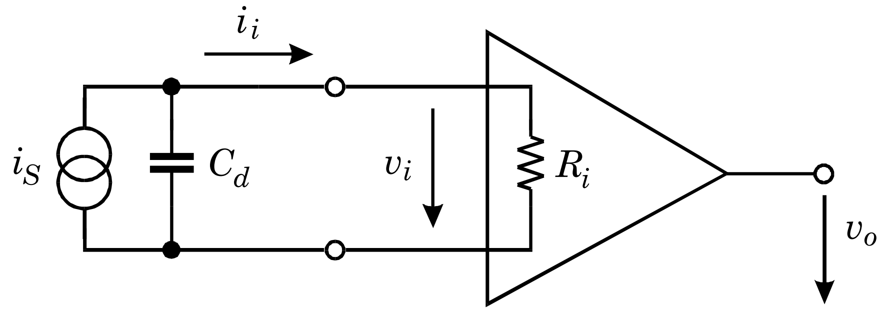
Voltage and Current Mode with Capacitive Sources

Output voltage:

v_o = (voltage gain A_v) × (input voltage v_i).



Operating mode depends on charge collection time t_c and the input time constant R_iC_d :

a) $R_iC_d \ll t_c$

detector capacitance discharges rapidly

$\Rightarrow \quad v_o \propto i_s(t)$

current sensitive amplifier

b) $R_iC_d \gg t_c$

detector capacitance discharges slowly

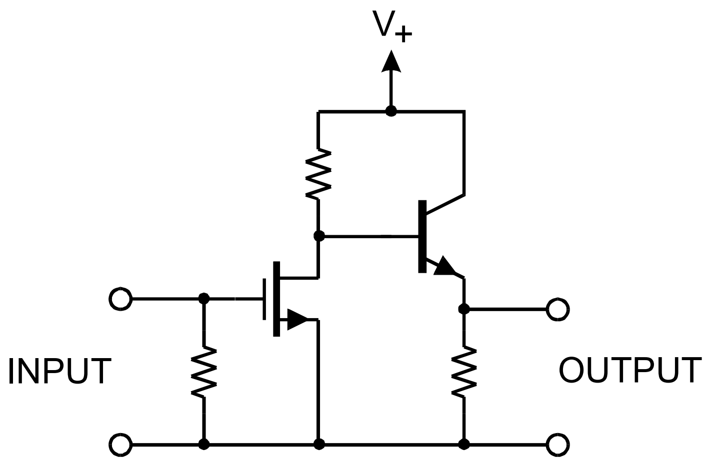
$\Rightarrow \quad v_o \propto \int i_s(t)dt$

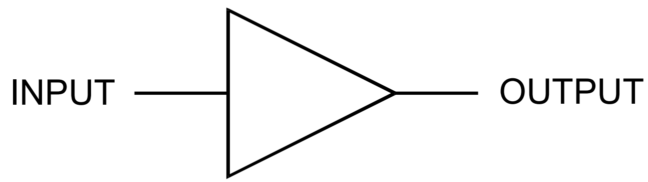
voltage sensitive amplifier

Note that in both cases the amplifier is providing voltage gain, so the output signal voltage is determined directly by the input voltage. The difference is that the shape of the input voltage pulse is determined either by the instantaneous current or by the integrated current and the decay time constant.

Goal is to measure signal charge, so it is desirable to use a system whose response is independent of detector capacitance.

Although an amplifier has a pair of input and a second pair of output connections, since the two have a common connection a simplified representation is commonly used:





Active Integrator ("charge-sensitive amplifier")

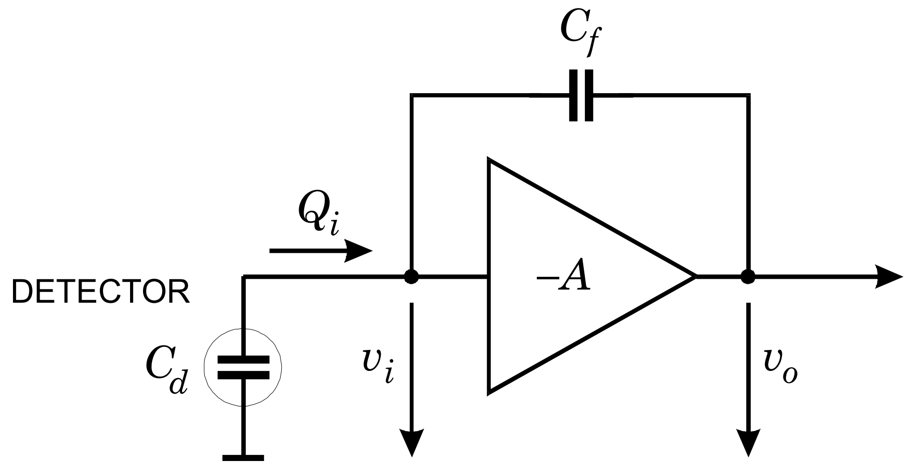
Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow$

$v_o = -Av_i$

Input impedance = ∞ (i.e. no signal current flows into amplifier input)

Connect feedback capacitor C_f between output and input.



Voltage difference across C_f: $\quad v_f = (A+1)v_i$

$\Rightarrow$ Charge deposited on C_f: $\quad Q_f = C_f v_f = C_f (A+1) v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

$\Rightarrow$ Effective input capacitance $\quad C_i = \dfrac{Q_i}{v_i} = C_f (A+1)$ ("dynamic" input capacitance)

Gain $$A_Q = \frac{dV_o}{dQ_i} = \frac{A \cdot v_i}{C_i \cdot v_i} = \frac{A}{C_i} = \frac{A}{A+1} \cdot \frac{1}{C_f} \approx \frac{1}{C_f} \quad (A >> 1)$$

Dependent on a well-controlled quantity, the feedback capacitance.

Q_i is the charge flowing into the preamplifier .... but some charge remains on C_d.

What fraction of the signal charge is measured?

$$\frac{Q_i}{Q_s} = \frac{C_i v_i}{Q_d + Q_i} = \frac{C_i}{Q_s} \cdot \frac{Q_s}{C_i + C_d}$$

$$= \frac{1}{1 + \frac{C_d}{C_i}} \approx 1 \quad (\text{if } C_i >> C_d)$$

Example: $A = 10^3$

$C_f = 1$ pF $\Rightarrow$ $C_i = 1$ nF

$C_{det} = 10$ pF: $Q_i / Q_s = 0.99$

$C_{det} = 500$ pF: $Q_i / Q_s = 0.67$

$\uparrow$

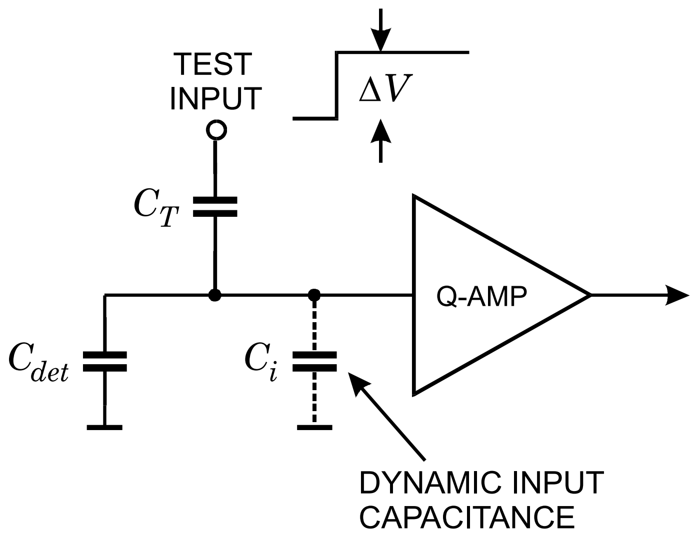
Si Det.: 50 μm thick, 250 mm^2 area

Note: Input coupling capacitor must be $\gg C_i >> C_i$ for high charge transfer efficiency.

Calibration

Inject specific quantity of charge - measure system response

Use voltage pulse (can be measured conveniently with oscilloscope)



$C_i >> C_T$ $\Rightarrow$ Voltage step applied to test input develops over C_T.

$\Rightarrow$ $$Q_T = \Delta V \cdot C_T$$

Accurate expression: $$Q_T = \frac{C_T}{1+\frac{C_T}{C_i}} \cdot \Delta V \approx C_T \left(1 - \frac{C_T}{C_i}\right) \Delta V$$

Typically: $C_T / C_i = 10^{-3} - 10^{-4}$

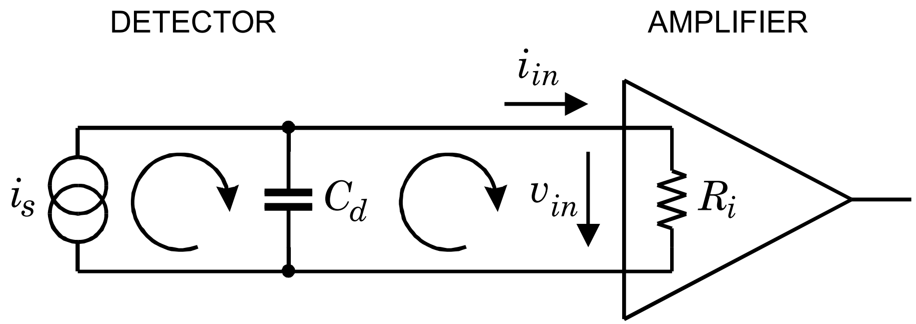
Realistic Charge-Sensitive Preamplifiers

The preceding discussion assumed idealized amplifiers with infinite speed.

In reality, amplifiers may be too slow to follow the instantaneous detector pulse.

Does this incur a loss of charge?

Equivalent Circuit:



↑ charges moving in detector change induced charge on detector electrodes

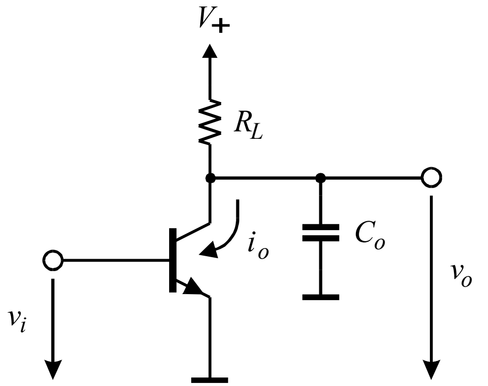
↑ detector capacitance discharges into amplifier

Signal is preserved even if the amplifier responds much more slowly than the detector signal.

However, the response of the amplifier affects the measured pulse shape.

- How do “real” amplifiers affect the measured pulse shape?
- How does the detector affect amplifier response?

A Simple Amplifier

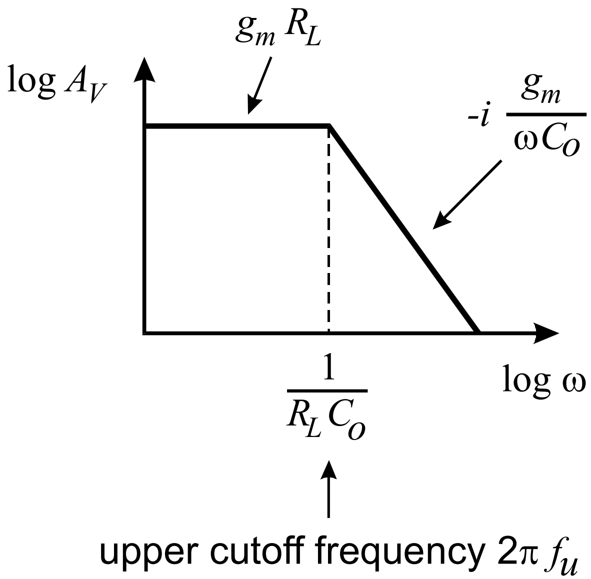


Voltage gain: $$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

$g_m \equiv$ transconductance

$$Z_L = R_L \,//\, C_o$$

$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega\, C_o$$



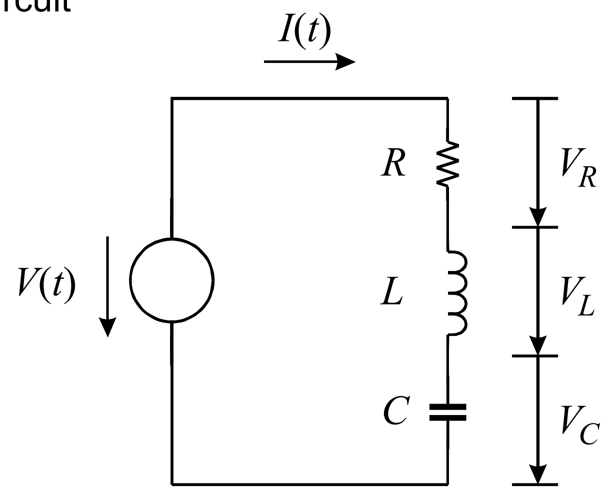
$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\, C_o \right)^{-1}$$

↑ low freq. ↑ high freq.

Appendix 1

Phasors and Complex Algebra in Electrical Circuits

Consider the RLC circuit



$$V = V_R + V_L + V_C$$

$$V = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{dI}{dt}R + L\frac{d^2 I}{dt^2} + \frac{I}{C}$$

Assume that $V(t) = V_0 e^{\mathbf{i}\omega t}$ and $I(t) = I_0 e^{\mathbf{i}(\omega t + \varphi)}$

Then

$$\mathbf{i}\omega V_0 e^{\mathbf{i}\omega t} = \mathbf{i}\omega R I_0 e^{\mathbf{i}(\omega t - \varphi)} - \omega^2 L I_0 e^{\mathbf{i}(\omega t - \varphi)} + \frac{1}{C} I_0 e^{\mathbf{i}(\omega t - \varphi)}$$

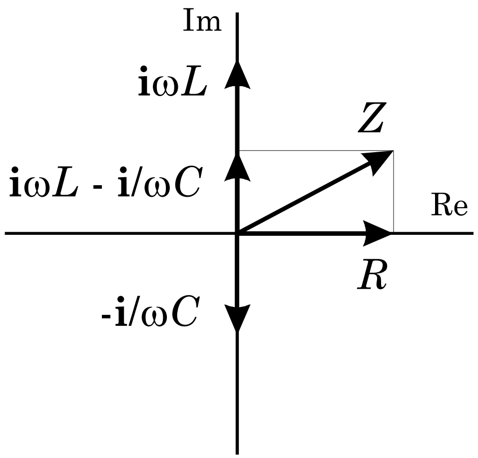
$$\frac{V_0}{I_0} e^{\mathbf{i}\varphi} = R + \mathbf{i}\omega L - \mathbf{i}\frac{1}{\omega C}$$

Thus, we can express the total impedance $Z \equiv (V_0 / I_0)\, e^{\mathbf{i}\varphi}$ of the circuit as a complex number with the magnitude $|Z| = V_0 / I_0$ and phase φ.

In this representation the equivalent resistances (reactances) of L and C are imaginary numbers

$$X_L = \mathbf{i}\omega L \quad \text{and} \quad X_C = -\frac{\mathbf{i}}{\omega C}\ .$$

Plotted in the complex plane:

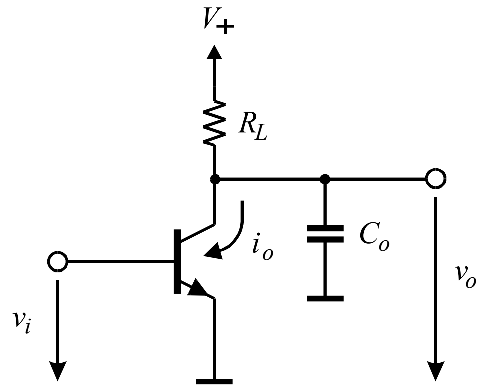


Relative to V_R, the voltage across the inductor V_L is shifted in phase by +90°.

The voltage across the capacitor V_C is shifted in phase by -90°.

Use to represent any element that introduces a phase shift, e.g. an amplifier. A phase shift of +90° appears as $+\mathbf{i}$, -90° as $-\mathbf{i}$.

A Simple Amplifier



Voltage gain: $$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

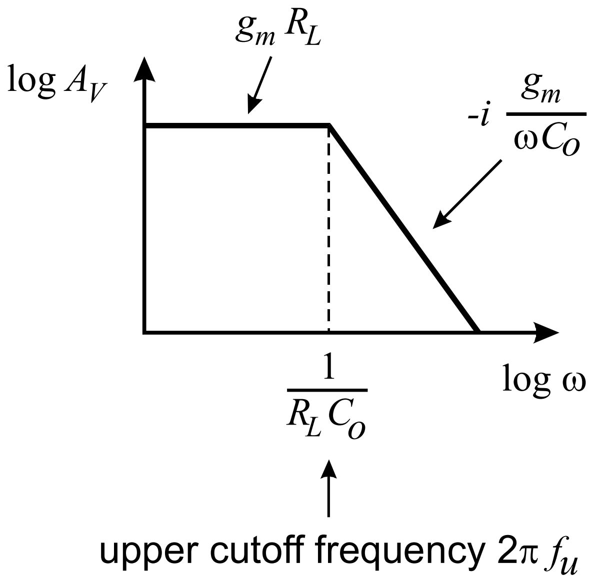
$g_m \equiv$ transconductance

$$Z_L = R_L \, // \, C_o$$

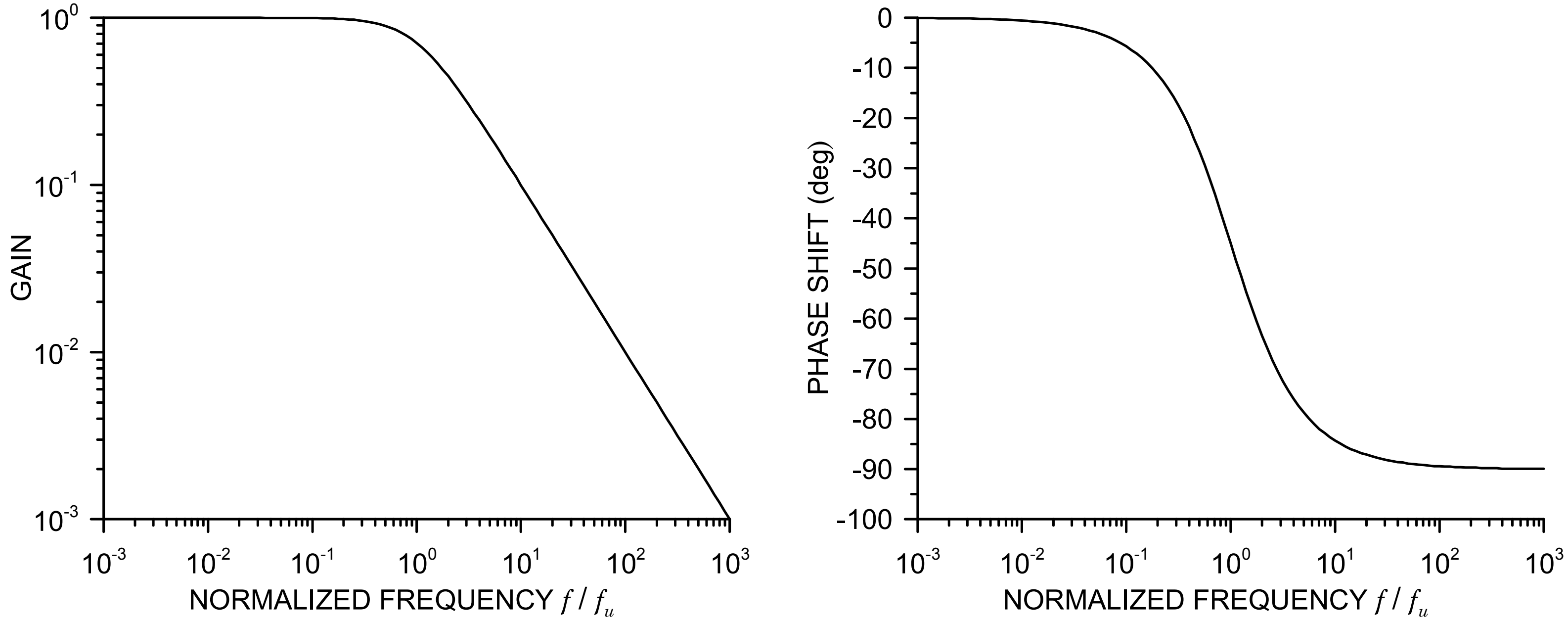
$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega\, C_o$$

$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\, C_o \right)^{-1}$$

low freq. high freq.



Frequency and phase response:



Phase shows change from low-frequency response. For an inverting amplifier add 180°.

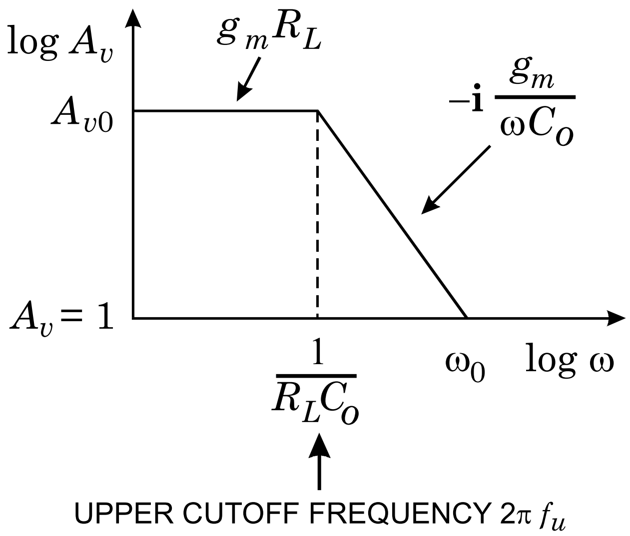
Pulse Response of the Simple Amplifier

A voltage step $v_i(t)$ at the input causes a current step $i_o(t)$ at the output of the transistor.

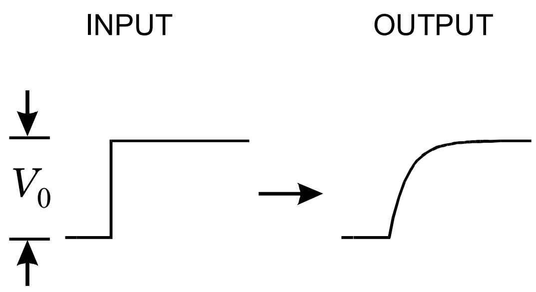
For the output voltage to change, the output capacitance C_o must first charge up.

$\Rightarrow$ The output voltage changes with a time constant $\tau = R_L C_o$

FREQUENCY DOMAIN



TIME DOMAIN



$$V = V_0 \left(1 - \exp(-t/\tau)\right)$$

$$\tau = R_L C_o$$

The time constant τ corresponds to the upper cutoff frequency :

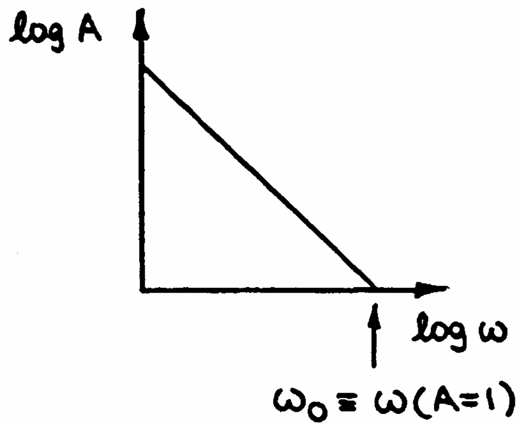
$$\tau = \frac{1}{2\pi f_u}$$

Input Impedance of a Charge-Sensitive Amplifier

Input impedance $$Z_i = \frac{Z_f}{A+1} \approx \frac{Z_f}{A} \quad (A >> 1)$$

Amplifier gain vs. frequency beyond the upper cutoff frequency

$$A = -\mathbf{i}\ \frac{\omega_0}{\omega}$$



Gain-Bandwidth Product

Feedback impedance $$Z_f = -\mathbf{i}\ \frac{1}{\omega\ C_f}$$

⇒ Input Impedance $$Z_i = -\frac{\mathbf{i}}{\omega\ C_f} \cdot \frac{1}{-\mathbf{i}\,\frac{\omega_0}{\omega}}$$

$$Z_i = \frac{1}{\omega_0 C_f}$$

Imaginary component vanishes ⇒ *Resistance:* $Z_i \to R_i$

⇒ low frequencies ($f < f_u$): capacitive input
high frequencies ($f > f_u$): resistive input

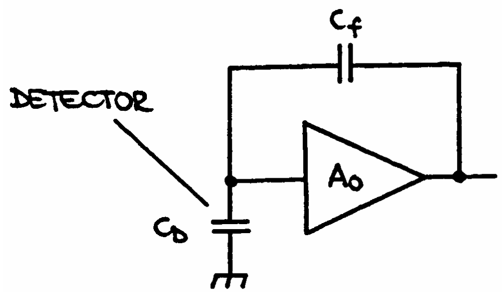
Time Response of a Charge-Sensitive Amplifier

Input resistance and detector capacitance form RC time constant $\tau_i = R_i C_D$

$$\tau_i = \frac{1}{\omega_0 C_f} \cdot C_D$$

$\Rightarrow$ **Rise time increases with detector capacitance.**

Or apply feedback theory:

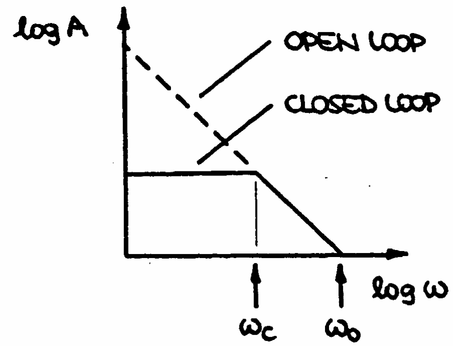


Closed Loop Gain $$A_f = \frac{C_D + C_f}{C_f} \quad (A_f << A_0)$$

$$A_f \approx \frac{C_D}{C_f} \quad (C_D >> C_f)$$

Closed Loop Bandwidth $\omega_C A_f = \omega_0$

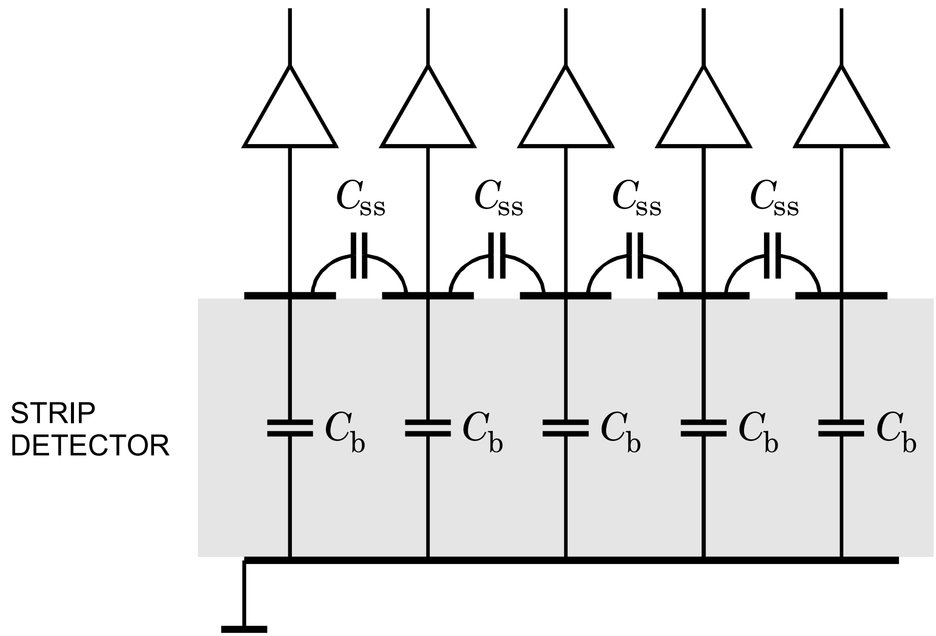
Response Time $$\tau_{amp} = \frac{1}{\omega_C} = C_D \frac{1}{\omega_0 C_f}$$



Same result as from input time constant.

Input impedance is critical in strip or pixel detectors:

Amplifiers must have a low input impedance to reduce transfer of charge through capacitance to neighboring strips



For strip pitches that are smaller than the bulk thickness, the capacitance is dominated by the fringing capacitance to the neighboring strips C_{SS}.

Typically: 1 – 2 pF/cm for strip pitches of 25 – 100 μm on Si.

The backplane capacitance C_b is typically 20% of the strip-to-strip capacitance.

Negligible cross-coupling at shaping times $T_P > (2 \ldots 3) \times R_i C_D$ and if $C_i \gg C_D$.

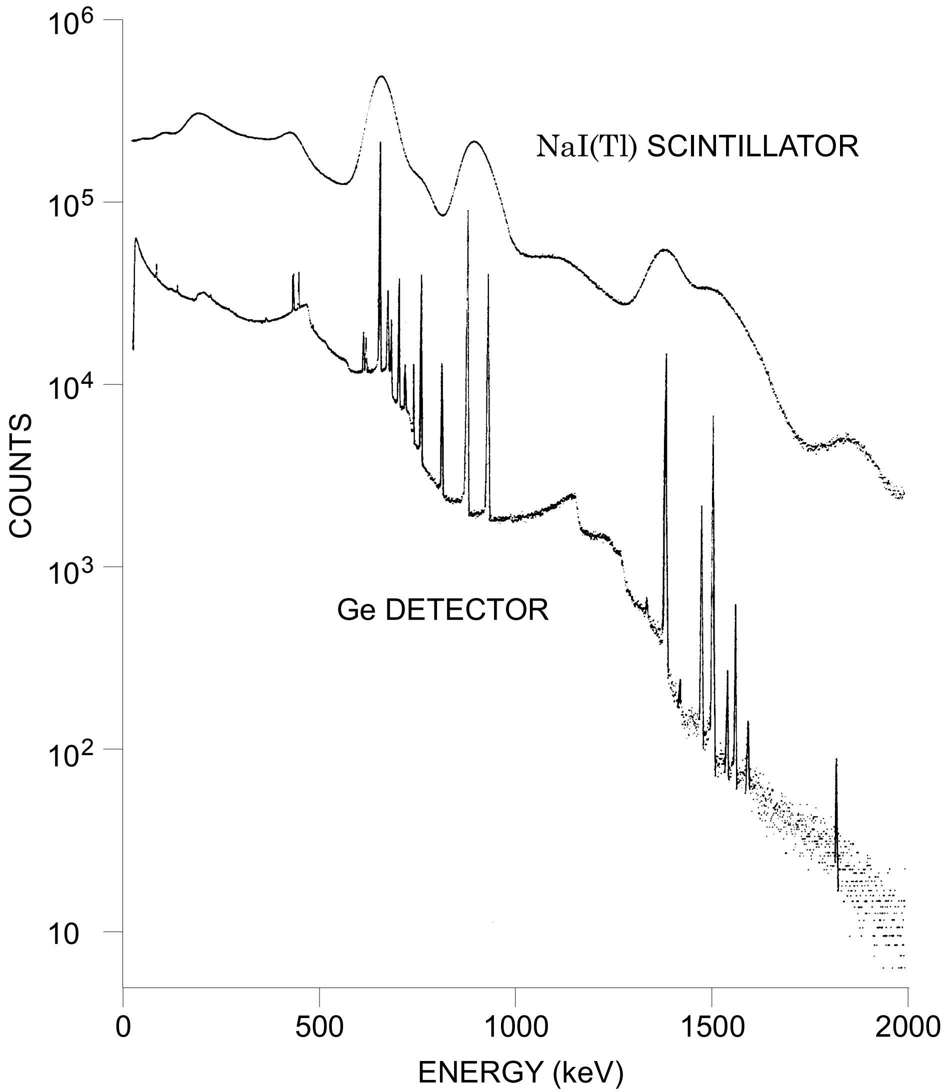
2. Resolution and Electronic Noise

Resolution: the ability to distinguish signal levels

1. Why?

a) Recognize structure in amplitude spectra

Comparison between NaI(Tl) and Ge detectors

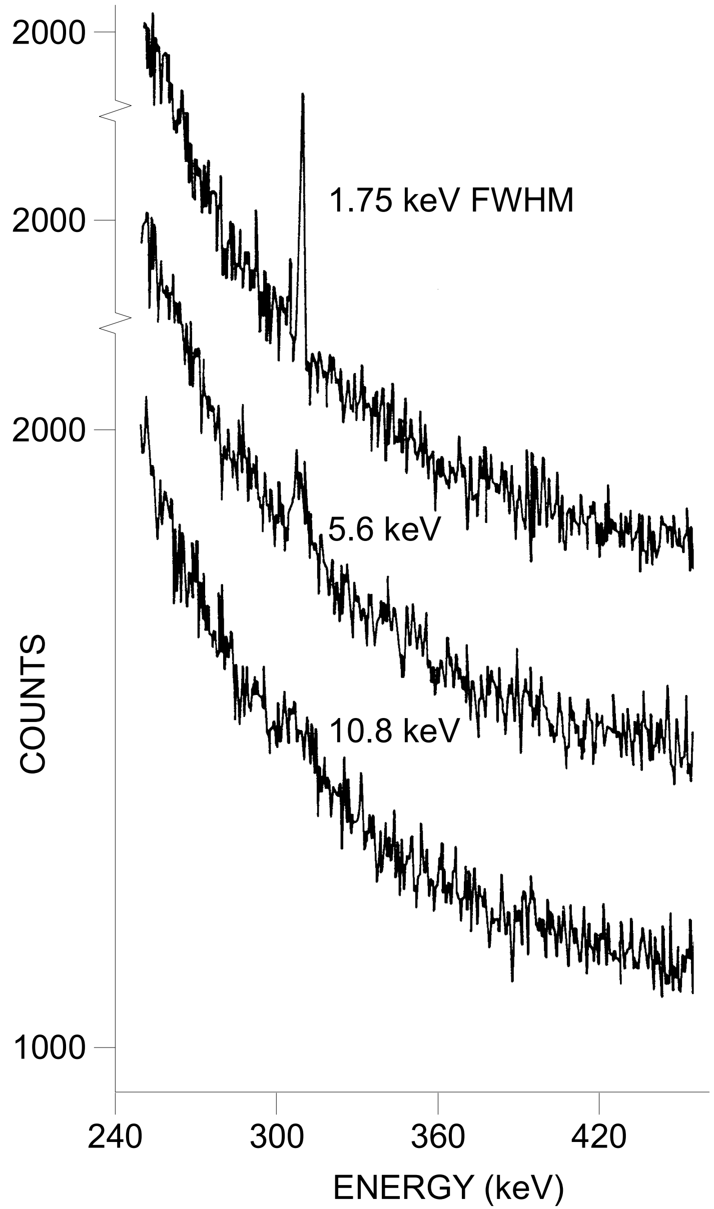


J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446

b) Improve sensitivity

Signal to background ratio improves with better resolution

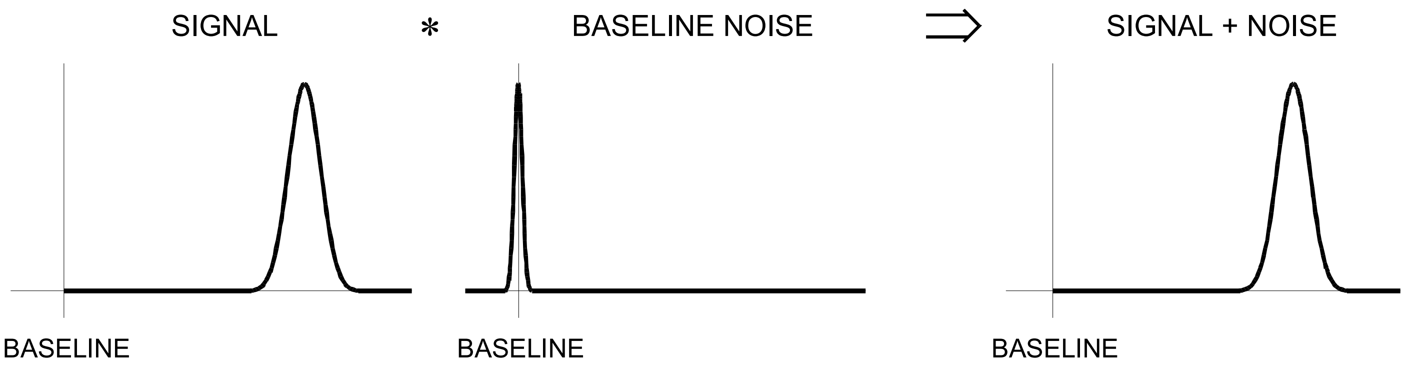
(signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

What determines Resolution?

1. Signal Variance >> Baseline Variance



⇒ Electronic (baseline) noise not important

Examples:
- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

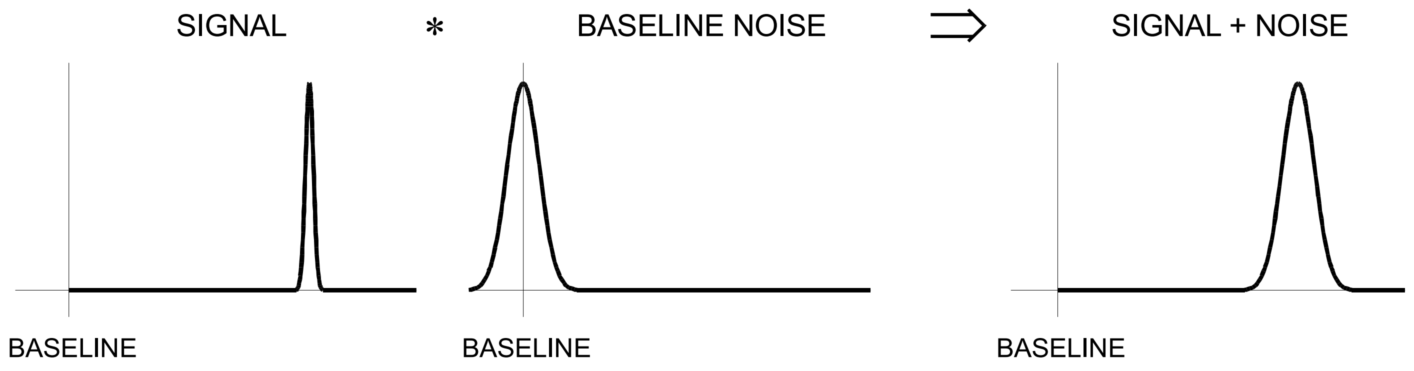
Number of photoelectrons: $N_{pe} \approx 8\cdot 10^4$ [MeV^{-1}] $\times E_\gamma \times QE \approx 2.4\cdot 10^4$

Variance typically: $\sigma_{pe} = N_{pe}^{1/2} \approx 160$ and $\sigma_{pe} / N_{pe} \approx$ 5 - 8%

Signal at PMT anode (assume Gain= 10^4): $Q_{sig} = G_{PMT} N_{pe} \approx 2.4\cdot 10^8$ el and $\sigma_{sig} = G_{PMT}\, \sigma_{pe} \approx 1.2\cdot 10^7$ el

whereas electronic noise easily < 10^4 el

2. Signal Variance << Baseline Variance



$\Rightarrow$ Electronic (baseline) noise critical for resolution

Examples:
- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si : Number of electron-hole pairs $N_{ep} = \dfrac{E_{dep}}{3.6\ \text{eV}}$

Variance $\sigma_{ep} = \sqrt{F \cdot N_{ep}}$ (where F= Fano factor ≈ 0.1)

For 50 keV photons: $\sigma_{ep} \approx 40$ el $\Rightarrow$ $\sigma_{ep} / N_{ep} = 7.5 \cdot 10^{-4}$

Obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

3. Basic Noise Mechanisms and Characteristics

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\ e\ v}{l}$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2 ,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or "1/f" noise

Thermal noise and shot noise are both "white" noise sources, i.e.

power per unit bandwidth (≡ spectral density) is constant: $$\frac{dP_{noise}}{df} = const.$$

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f: $\frac{dP_{noise}}{df} = 4kT$

k = Boltzmann constant

T = absolute temperature

since $P = \frac{V^2}{R} = I^2 R$

R = DC resistance

the spectral noise voltage density $\frac{dV^2_{noise}}{df} \equiv e_n^2 = 4kTR$

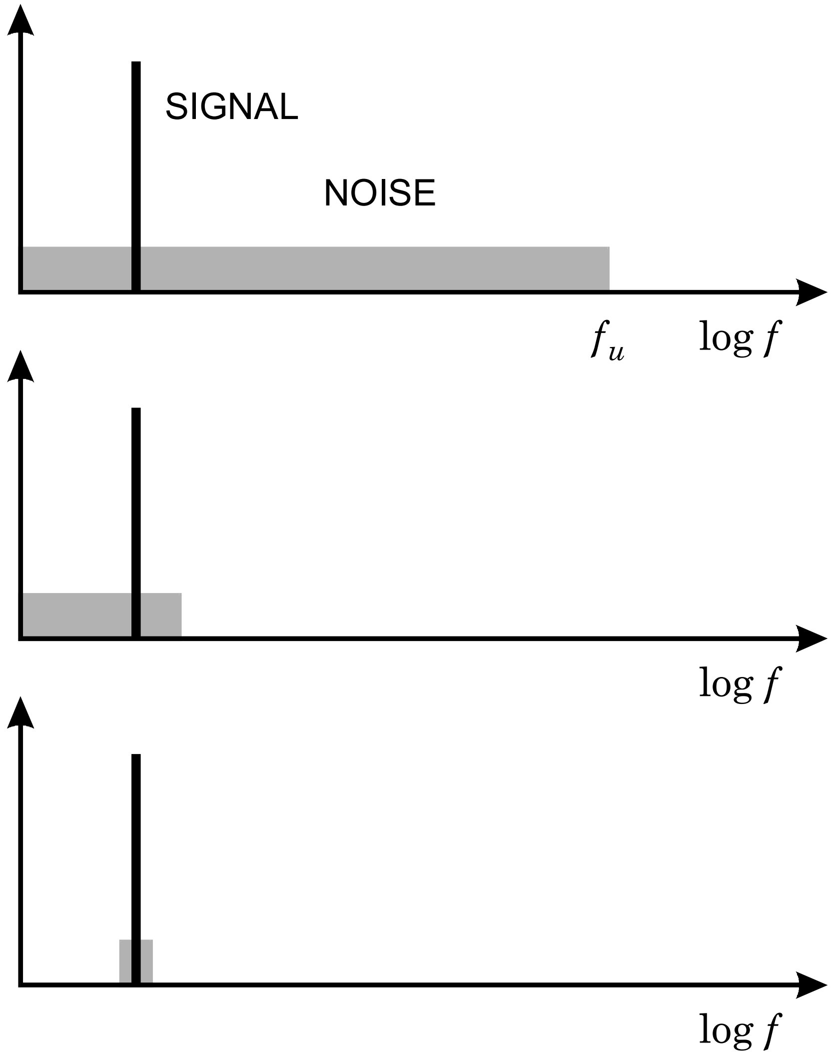
and the spectral noise current density $\frac{dI^2_{noise}}{df} \equiv i_n^2 = \frac{4kT}{R}$

The total noise depends on the bandwidth of the system,
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \; A_v^2(f) \; df$$

Note: Since spectral noise components are not correlated, one must integrate over the noise power.

Total noise increases with bandwidth



Total noise is the integral over the shaded region.

S/N increases as noise bandwidth is reduced until signal components are attenuated significantly.

2. Shot noise

A common example of noise due to number fluctuations is “shot noise”, which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode (emission over a barrier)

Spectral noise current density: $i_n^2 = 2eI$

e = electronic charge
I = DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in “ohmic” conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

Noise Spectral Densities

Spectral Density of Thermal Noise (Johnson Noise)

Two approaches can be used to derive the spectral distribution of thermal noise.

1. The thermal velocity distribution of the charge carriers is used to calculate the time dependence of the induced current, which is then transformed into the frequency domain.

2. Application of Planck's theory of black body radiation.

The first approach clearly shows the underlying physics, whereas the second "hides" the physics by applying a general result of statistical mechnics. However, the first requires some advanced concepts that go well beyond the standard curriculum, so the "black body" approach will be used.

In Planck's theory of black body radiation the energy per mode

$$\overline{E} = \frac{h\nu}{e^{h\nu / kT} - 1}$$

and the spectral density of the radiated power

$$\frac{dP}{d\nu} = \frac{h\nu}{e^{h\nu / kT} - 1}$$

i.e. this is the power that can be extracted in equilibrium.

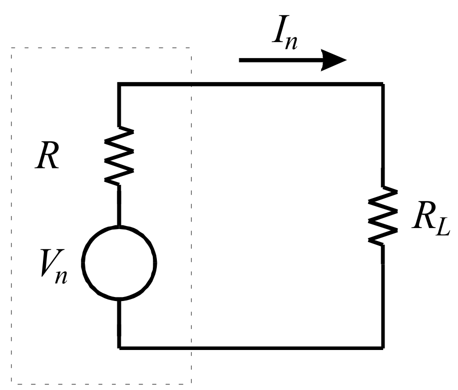
At low frequencies $h\nu \ll kT$:

$$\frac{dP}{d\nu} \approx \frac{h\nu}{\left(1+\frac{h\nu}{kT}\right)-1} = kT \ ,$$

so at low frequencies the spectral density is independent of frequency and for a total bandwidth B the noise power that can be transferred to an external device

$$P_n = kTB \,.$$

To apply this result to the noise of a resistor, consider a resistor R whose thermal noise gives rise to a noise voltage V_n. To determine the power transferred to an external device consider the circuit



The dotted box encloses the equivalent circuit of the resistive noise source.

The power dissipated in the load resistor R_L

$$\frac{V_{nL}^2}{R_L} = I_n^2 R_L = \frac{V_n^2 R_L}{(R+R_L)^2}$$

The maximum power transfer occurs when the load resistance equals the source resistance $R_L = R$, so

$$V_{nL}^2 = \frac{V_n^2}{4} \ .$$

Since the maximum power that can be transferred to R_L is kTB ,

$$\frac{V_{nL}^2}{R} = \frac{V_n^2}{4R} = kTB$$

$$P_n = \frac{V_n^2}{R} = 4kTB$$

and the spectral density of the noise power in the resistor

$$\frac{dP_n}{d\nu} = 4kT \ .$$

Spectral Density of Shot Noise

If an excess electron is injected into a device, it forms a current pulse of duration τ. In a thermionic diode τ is the transit time from cathode to anode, for example. In a semiconductor diode τ is the recombination time. If these times are short with respect to the periods of interest $\tau \ll 1/f$, the current pulse can be represented by a δ pulse. The Fourier transform of a delta pulse yields a "white" spectrum, i.e. the amplitude distribution in frequency is uniform

$$\frac{dI_{n,pk}}{\sqrt{df}} = 2q_e$$

Within an infinitesimally narrow frequency band the individual spectral components are pure sinusoids, so their rms value

$$i_n \equiv \frac{dI_n}{\sqrt{df}} = \frac{2q_e}{\sqrt{2}} = \sqrt{2}q_e$$

If N electrons are emitted at the same average rate, but at different times, they will have the same spectral distribution, but the coefficients will differ in phase. For example, for two currents i_p and i_q with a relative phase φ the total rms current

$$\left\langle i^2 \right\rangle = \left(i_p + i_q e^{i\varphi}\right)\left(i_p + i_q e^{-i\varphi}\right) = i_p^2 + i_q^2 + 2 i_p i_q \cos\varphi$$

For a random phase the third term averages to zero

$$\langle i^2 \rangle = i_p^2 + i_q^2 \;,$$

so if N electrons are randomly emitted per unit time, the individual spectral components simply add in quadrature

$$i_n^2 = 2Nq_e^2$$

The average current

$$I = Nq_e \;,$$

so the spectral noise density

$$i_n^2 \equiv \frac{dI_n^2}{df} = 2q_e I \,.$$

Another derivation utilizes Carson's theorem.

If a single pulse has the amplitude $A(t)$ and its Fourier transform

$$P(f) = \int_{-\infty}^{\infty} A(t) \exp(-\mathbf{i}\omega t) dt ,$$

then a random sequence of pulses occurring at a rate r has the spectral power distribution

$$S(f) = 2r \left| P(f) \right|^2 .$$

Shot noise can be represented as a sequence of delta pulses, whose spectrum is white, so the pulse sequence also has a white spectrum.

Since the rate $r = I / q_e$, the spectral density of shot noise

$$i_n^2 = 2q_e I$$

$\Rightarrow$ The spectral distribution of a DC signal carries information of the signal's origin.

Low Frequency Noise

In a semiconductor, for example, charge can be trapped and then released after a characteristic lifetime τ.

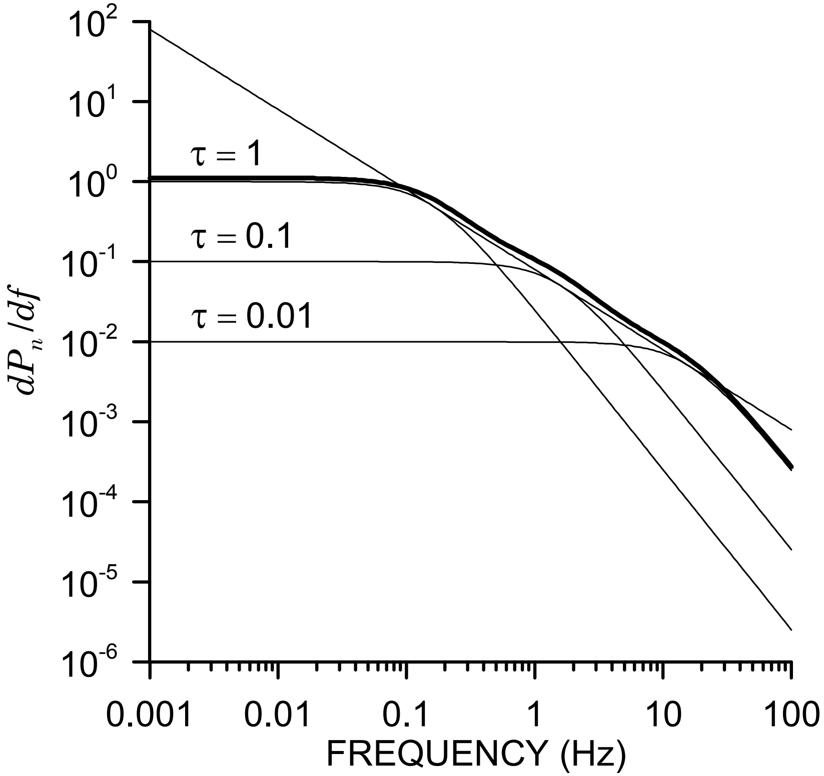
The spectral density for a single lifetime

$$S(f) \propto \frac{\tau}{1+(2\pi f \tau)^2} \ .$$

For $2\pi f \tau \gg 1$: $S(f) \propto \frac{1}{f^2}$.

However,
several traps with different time constants can yield a "1/f" distribution:

Traps with three time constants of 0.01, 0.1 and 1 s yield a 1/f distribution over two decades in frequency.



Low frequency noise is ubiquitous – must not have 1/f dependence, but commonly called 1/f noise.

Spectral power density: $$\frac{dP_{noise}}{df} = \frac{1}{f^\alpha}$$ (typically α = 0.5 – 2)

4. Charge-Sensitive Amplifier – Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,

  i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.

- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i \gg C_d$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_v$.

- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

Note, that although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.

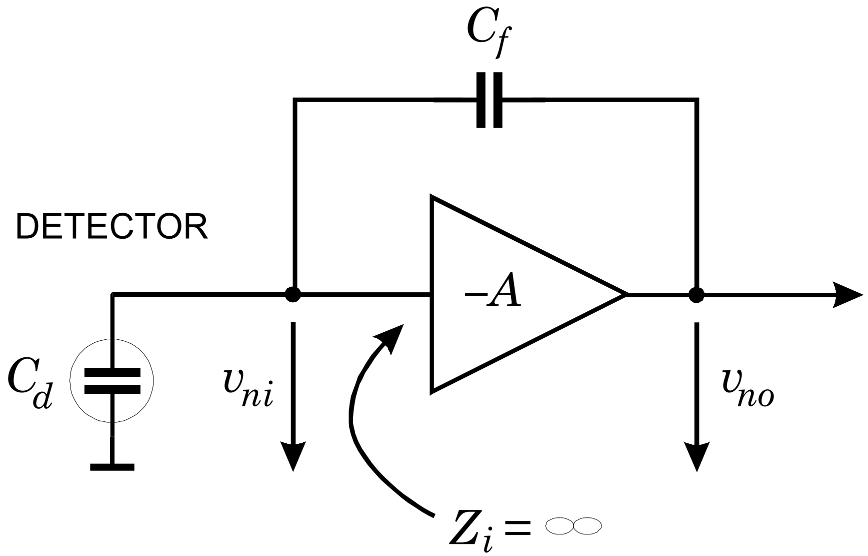
$\Rightarrow$ **S/N at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.

$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_d}}{X_{C_d}} = v_{ni} \frac{\frac{1}{\omega C_f} + \frac{1}{\omega C_d}}{\frac{1}{\omega C_d}}$$

$$v_{no} = v_{ni} \left(1 + \frac{C_d}{C_f} \right)$$



Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni} \left(C_d + C_f \right)$$

Signal-to-noise ratio
$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_d + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage-sensitive amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

Conclusion

In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.

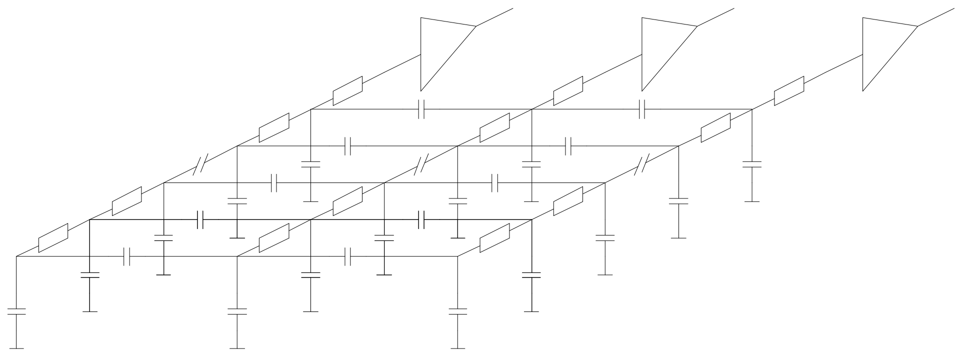
- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

Strip Detector Model for Noise Simulations

Noise coupled from neighbor channels.

Analyze signal and noise in center channel.



Includes:

a) Noise contributions from neighbor channels

b) Signal transfer to neighbor channels

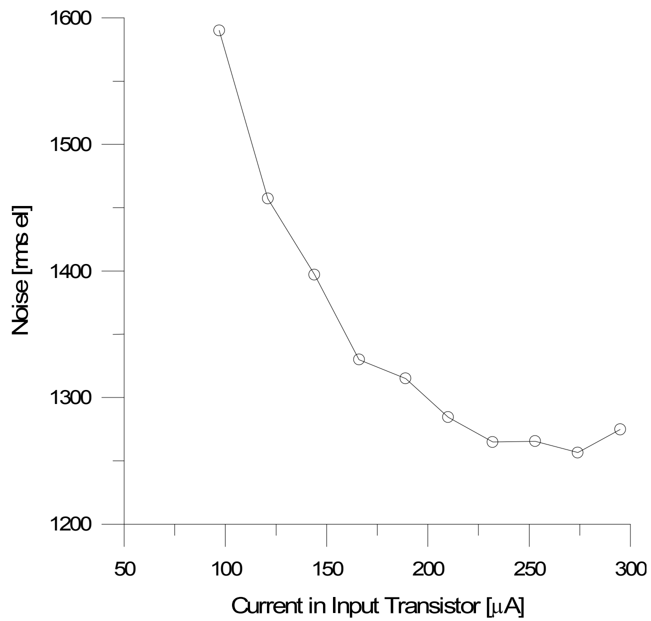
c) Noise from distributed strip resistance

d) Full SPICE model of preamplifier

See Spieler, *Semiconductor Detector Systems* for discussion of noise cross-coupling

Measured Noise of Module:

p-strips on n-bulk, BJT input transistor



Simulation Results: 1460 el (150 μA)

1230 el (300 μA)

$\Rightarrow$ Noise can be predicted with good accuracy.

5. Quantum Noise Limits in Amplifiers

What is the lower limit to electronic noise?

Can it be eliminated altogether, for example by using superconductors and eliminating devices that carry shot noise?

Starting point is the uncertainty relationship

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Consider a narrow frequency band at frequency ω. The energy uncertainty can be given in terms of the uncertainty in the number of signal quanta

$$\Delta E = \hbar \omega \Delta n$$

and the time uncertainty in terms of phase

$$\Delta t = \frac{\Delta \varphi}{\omega},$$

so that

$$\Delta \varphi \Delta n \geq \frac{1}{2}$$

We assume that the distributions in number and phase are Gaussian, so that the equality holds.

Assume a noiseless amplifier with gain G, so that n_1 quanta at the input yield

$$n_2 = G n_1$$

quanta at the output.

Furthermore, the phase at the output φ_2 is shifted by a constant relative to the input.

Then the output must also obey the relationship $$\Delta\varphi_2 \Delta n_2 = \frac{1}{2}$$

However, since $\Delta n_2 = G \Delta n_1$ and $\Delta\varphi_2 = \Delta\varphi_1$:

$$\Delta\varphi_1 \Delta n_1 = \frac{1}{2G} \;,$$

which is smaller than allowed by the uncertainty principle.

This contradiction can only be avoided by assuming that the amplifier introduces noise per unit bandwidth of

$$\frac{dP_{no}}{d\omega} = (G-1)\hbar\omega \ ,$$

which, referred to the input, is

$$\frac{dP_{ni}}{d\omega} = \left(1-\frac{1}{G}\right)\hbar\omega$$

If the noise from the following gain stages is to be small, the gain of the first stage must be large, and then the minimum noise of the amplifier

$$\frac{dP_{ni}}{d\omega} = \hbar\omega$$

At 2 mm wavelength the minimum noise corresponds to about 7K.

This minimum noise limit applies to phase-coherent systems. In systems where the phase information is lost, e.g. bolometers, this limit does not apply.

For a detailed discussion see C.M. Caves, Phys. Rev. D **26** (1982) 1817-1839
H.A. Haus and J.A. Mullen, Phys. Rev. 128 (1962) 2407-2413

6. Pulse Processing

Two conflicting objectives:

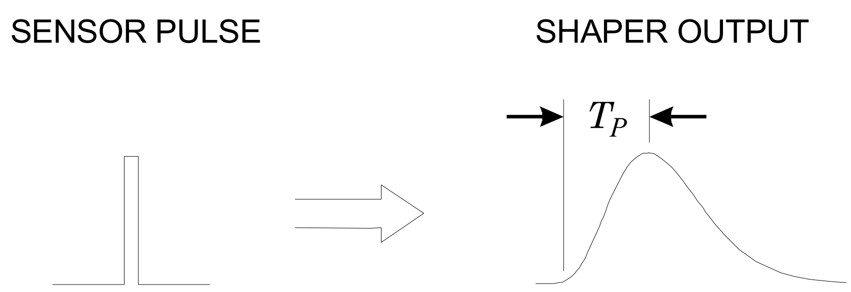
1. Improve Signal-to-Noise Ratio S/N

Restrict bandwidth to match measurement time $\Rightarrow$ Increase pulse width

Typically, the pulse shaper transforms a narrow detector current pulse to

a broader pulse (to reduce electronic noise),

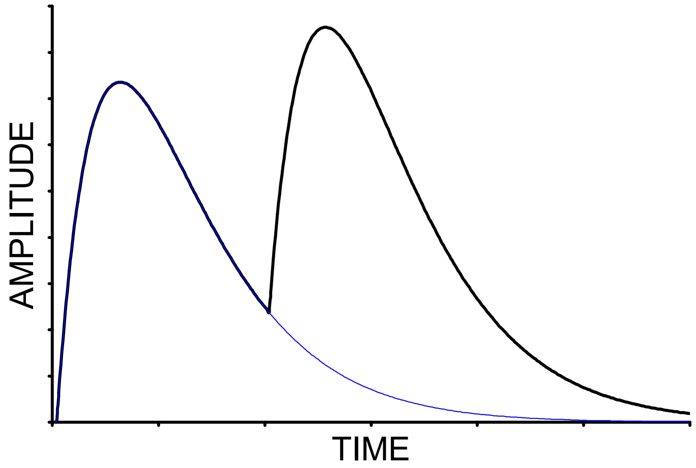
with a gradually rounded maximum at the peaking time T_P
(to facilitate measurement of the peak amplitude)



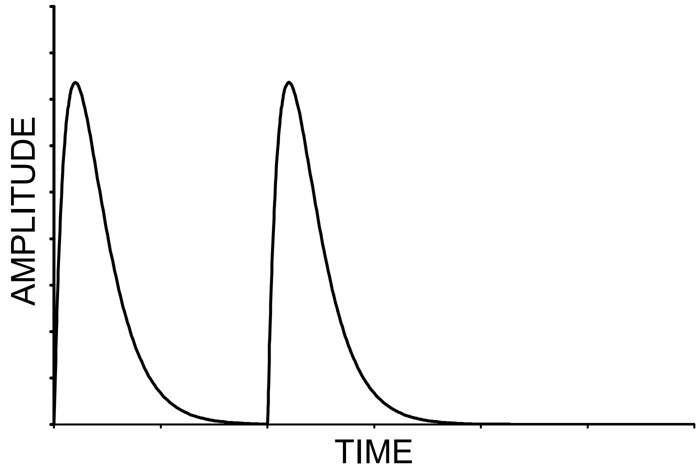
If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or pulse height analysis. The pulse height spectrum is the energy spectrum.

2. Improve Pulse Pair Resolution ⇒ Decrease pulse width

Pulse pile-up distorts amplitude measurement.



Reducing pulse shaping time to 1/3 eliminates pile-up.

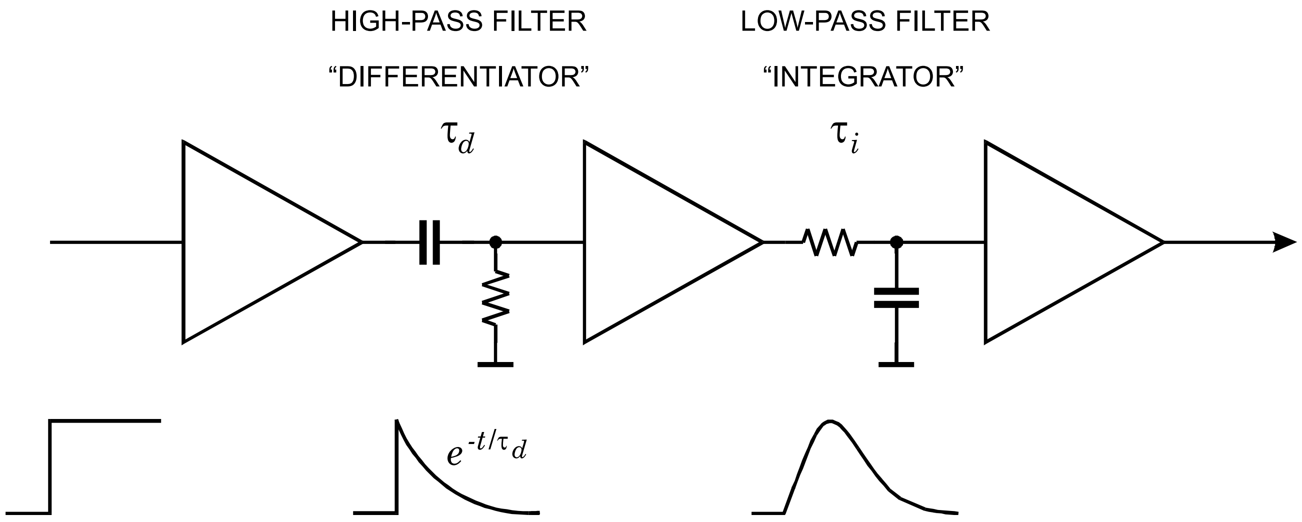


Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

Usually, many considerations combined lead to a "non-textbook" compromise.

- *"Optimum shaping" depends on the application!*
- Shapers need not be complicated – *Every amplifier is a pulse shaper!*

Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

$\Rightarrow$ Useful for estimates, since simple to evaluate

Key elements:

- lower frequency bound
- upper frequency bound
- signal attenuation

important in all shapers.

Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Equivalent Noise Charge

Inject known signal charge into preamp input
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge $\equiv$ Input charge for which $S/N = 1$

Ballistic Deficit

When the rise time of the input pulse to the shaper extends beyond the nominal peaking time, the shaper output is both stretched in time and the amplitude decreases

Shaper output for an input rise time

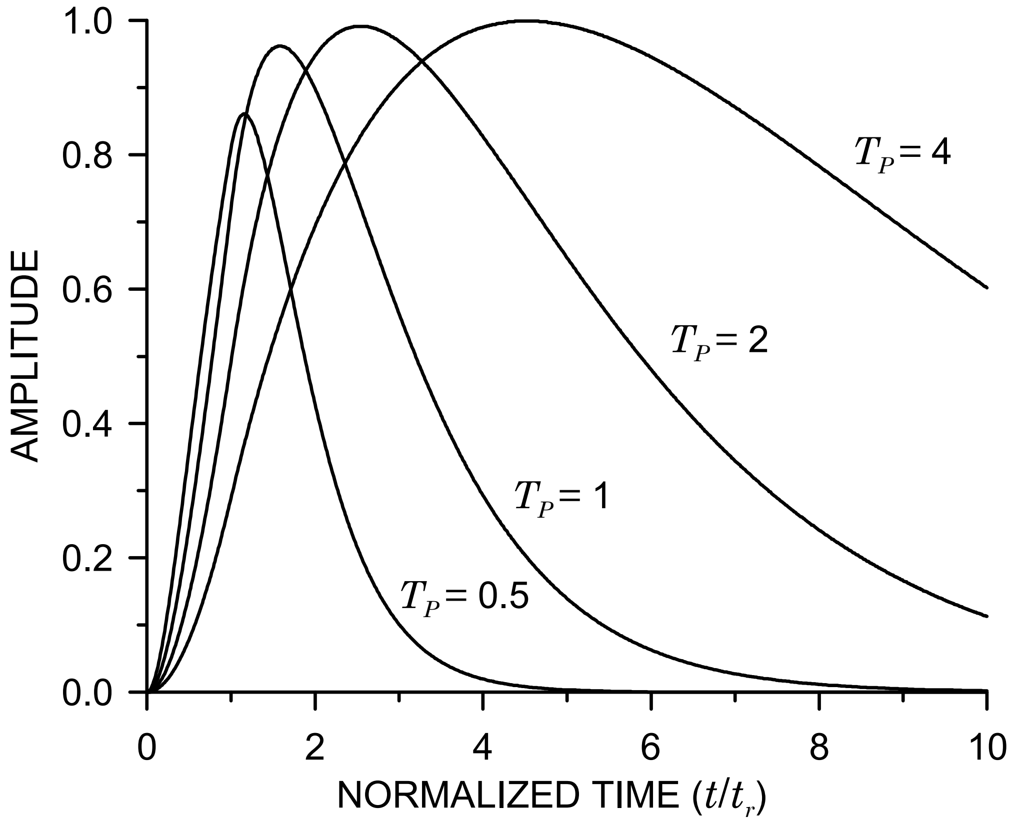
$t_r = 1$

for various values of nominal peaking time.

Note that the shaper with $T_P = 0.5$ peaks at $t = 1.15 t_r$

and

attains only 86% of the pulse height achieved at longer shaping times.

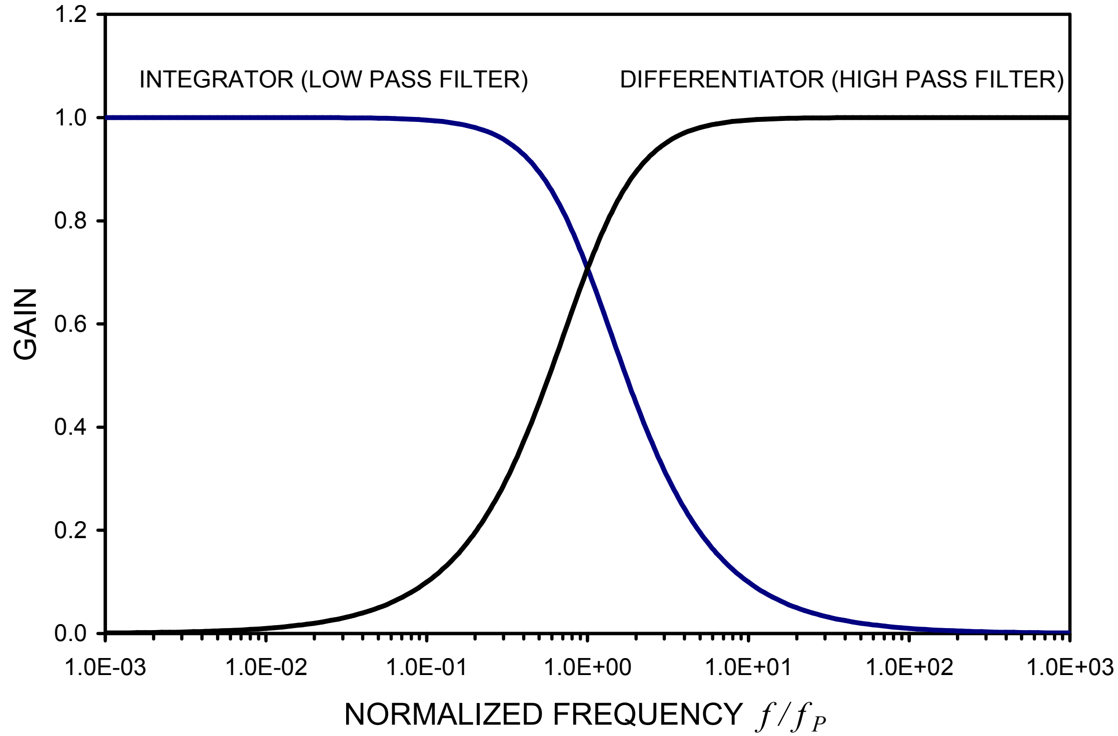


Dependence of Equivalent Noise Charge on Shaping Time

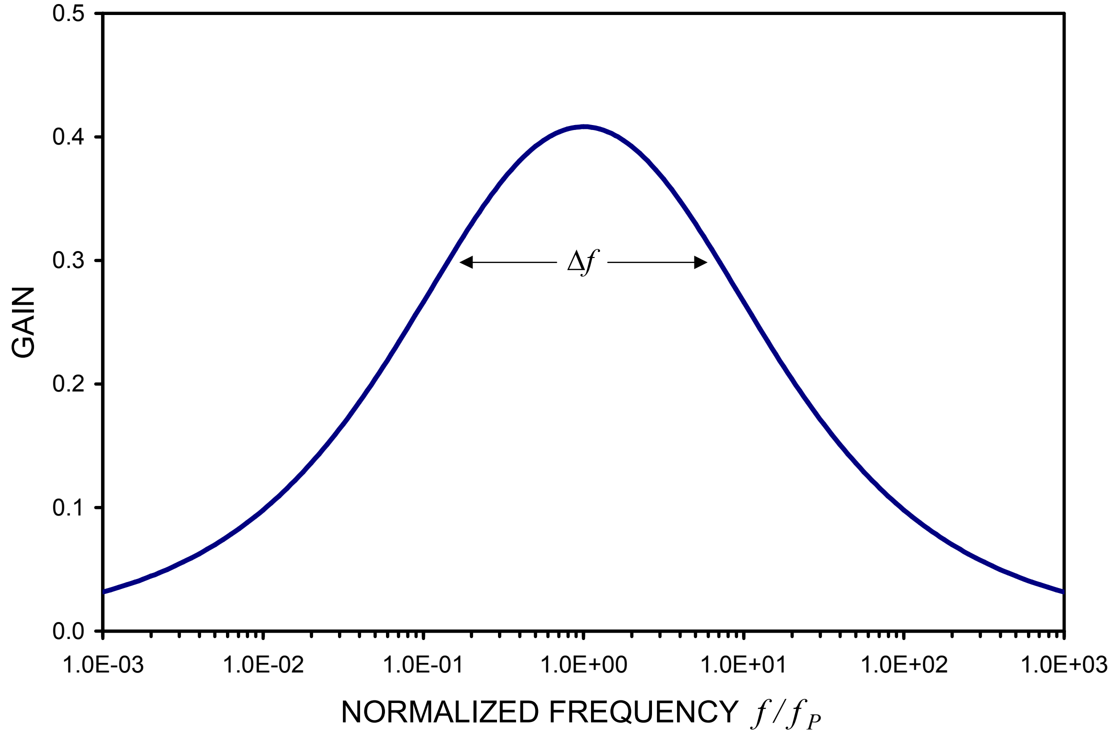
Assume that differentiator and integrator time constants are equal $\tau_i = \tau_d \equiv \tau$.

$\Rightarrow$ Both cutoff frequencies equal: $f_U = f_L \equiv f_P = 1/2\pi\tau$.

Frequency response of individual pulse shaping stages



Combined frequency response

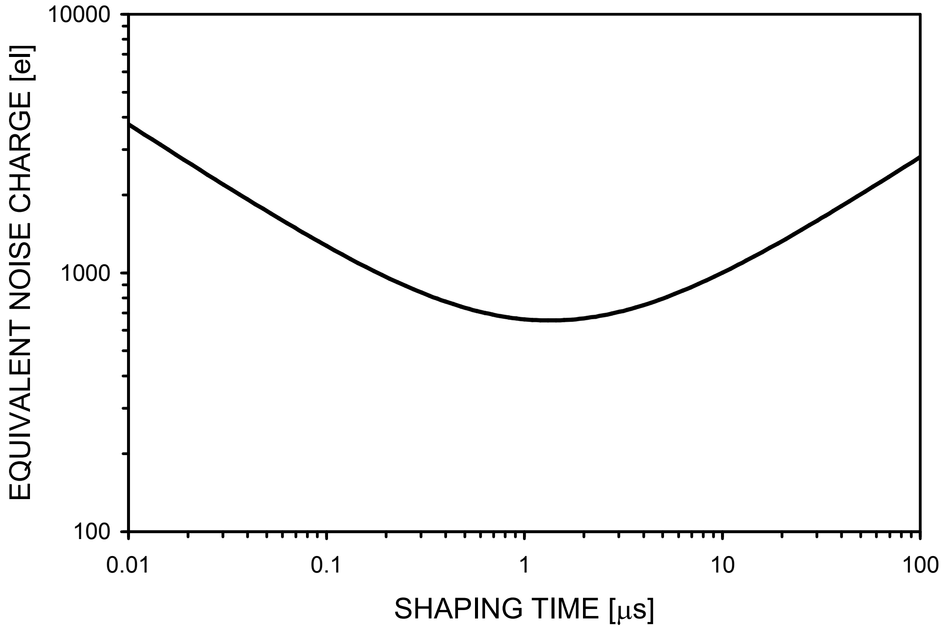


Logarithmic frequency scale $\Rightarrow$ shape of response independent of τ.

However, bandwidth Δf decreases with increasing time constant τ.

$\Rightarrow$ for white noise sources expect noise to decrease with bandwidth, i.e. decrease with increasing time constant.

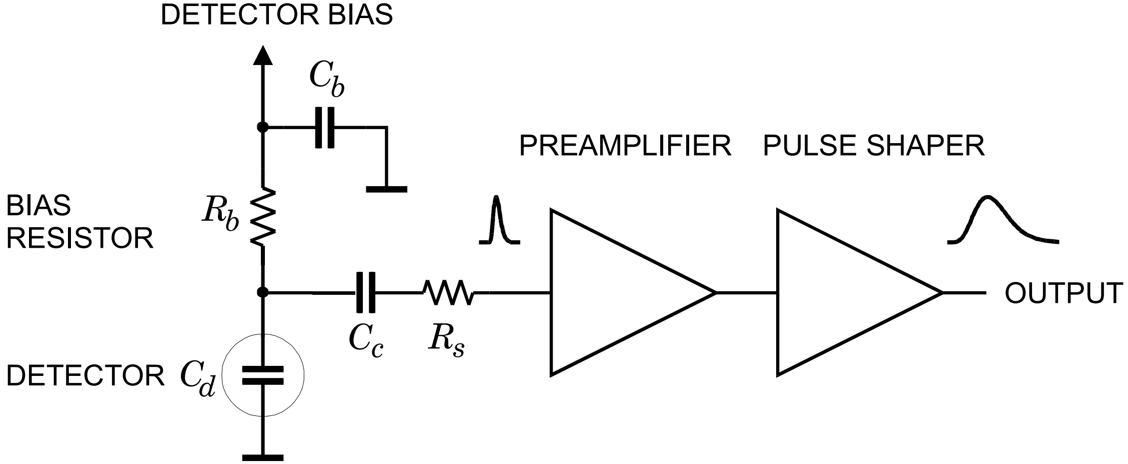
Result of typical noise measurement vs. shaping time



Noise sources (thermal and shot noise) have a flat (“white”) frequency distribution.

Why doesn’t the noise decrease monotonically with increasing shaping time (decreasing bandwidth)?

7. Analytical Analysis of a Detector Front-End



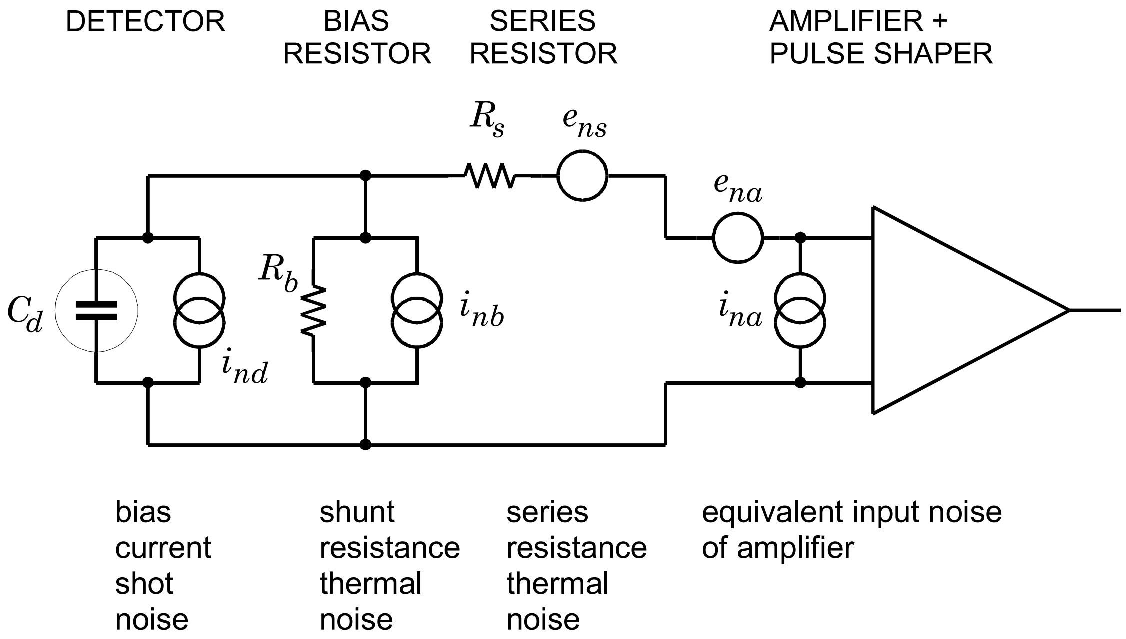
Detector bias voltage is applied through the resistor R_B. The bypass capacitor C_B serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the "far end" of the bias resistor to ground, so that R_B appears to be in parallel with the detector.

The coupling capacitor C_C in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why a capacitor serving this role is also called a "blocking capacitor").

The series resistor R_S represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients ("input protection")

Equivalent circuit for noise analysis



In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage appearing at the amplifier input.

Resistors can be modeled either as voltage or current generators.

- Resistors in parallel with the input act as current sources
- Resistors in series with the input act as voltage sources.

Steps in the analysis:

1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources
2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.
3. Determine the output signal for a known signal charge and calculate equivalent noise charge (signal charge for S/N= 1)

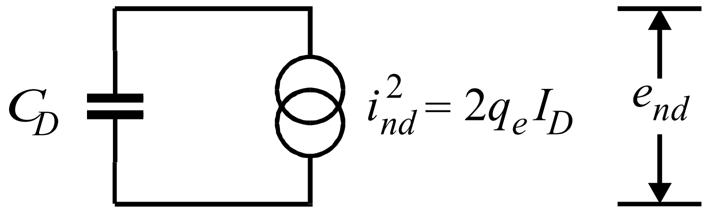
First, assume a simple CR-RC shaper with

equal differentiation and integration time constants $\tau_d = \tau_i = \tau$,

which in this special case is equal to the peaking time.

Noise Contributions

1. Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite
2. The shunt resistance R_P is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

Does this assumption make sense?

If R_P is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_P C_D \gg T_P \approx \frac{1}{\omega_P}$$

where ω_P is the midband frequency of the shaper. Therefore, $R_P \gg \dfrac{1}{\omega_P C_D}$ as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega\, C_D)^2} = 2q_e I_D \frac{1}{(\omega\, C_D)^2}$$

⇒ The noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time T. The number of electron charges measured is

$$N_e = \frac{I_D T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

2. Parallel Resistance

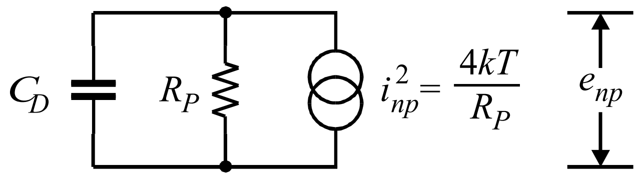
Any shunt resistance R_P acts as a noise current source. In the specific example shown above, the only shunt resistance is the bias resistor R_b.

Additional shunt components in the circuit:

1. bias noise current source (infinite resistance by definition)
2. detector capacitance

The noise current flows through both the resistance R_P and the detector capacitance C_D.

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$e_{np}^2 = \frac{4kT}{R_P}\left(\frac{R_P \cdot \dfrac{-\mathbf{i}}{\omega\, C_D}}{R_P - \dfrac{\mathbf{i}}{\omega\, C_D}}\right)^2$$

$$e_{np}^2 = 4kTR_P \frac{1}{1 + (\omega\, R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^\infty e_{np}^2(\omega)d\omega = \int_0^\infty \frac{4kTR_P}{1+(\omega R_P C_D)^2}\, d\omega = \frac{kT}{C_D} ,$$

which is independent of R_P. Commonly referred to as "kTC" noise, this contribution is often erroneously interpreted as the "noise of the detector capacitance".

An ideal capacitor has no thermal noise; all noise originates in the resistor.

So, why is the result independent of R_P?

R_P determines the primary noise, but also the noise bandwidth of this subcircuit. As R_P increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of R_P.

However,
If one integrates e_{np} over a bandwidth-limited system (such as our shaper),

$$v_n^2 = \int_0^\infty 4kTR_P \left| \frac{G(i\omega)}{1 - i\omega R_P C_D} \right|^2 d\omega$$

the total noise decreases with increasing R_P.

3. Series Resistance

The noise voltage generator associated with the series resistance R_S is in series with the other noise sources, so it simply contributes

$$e_{nr}^2 = 4kTR_S$$

4. Amplifier input noise

The amplifier noise voltage sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain, where it appears as a noise voltage generator.

$$e_{na}^2 = \underbrace{e_{nw}^2}_{\text{“white noise”}} + \underbrace{\frac{A_f}{f}}_{1/f \text{ noise (can also originate in external components)}}$$

This noise voltage generator also adds in series with the other sources.

- Amplifiers generally also exhibit input current noise, which is physically present at the input. Its effect is the same as for the detector bias current, so the analysis given in 1. can be applied.
- In a well-designed amplifier the noise is dominated by the input transistor (fast, high-gain transistors generally best). Noise parameters of transistors are discussed in the Appendix.

  Transistor input noise decreases with transconductance

  $\Rightarrow$ increased power
- Minimum device noise limited both by technology and fundamental physics.

Equivalent Noise Charge

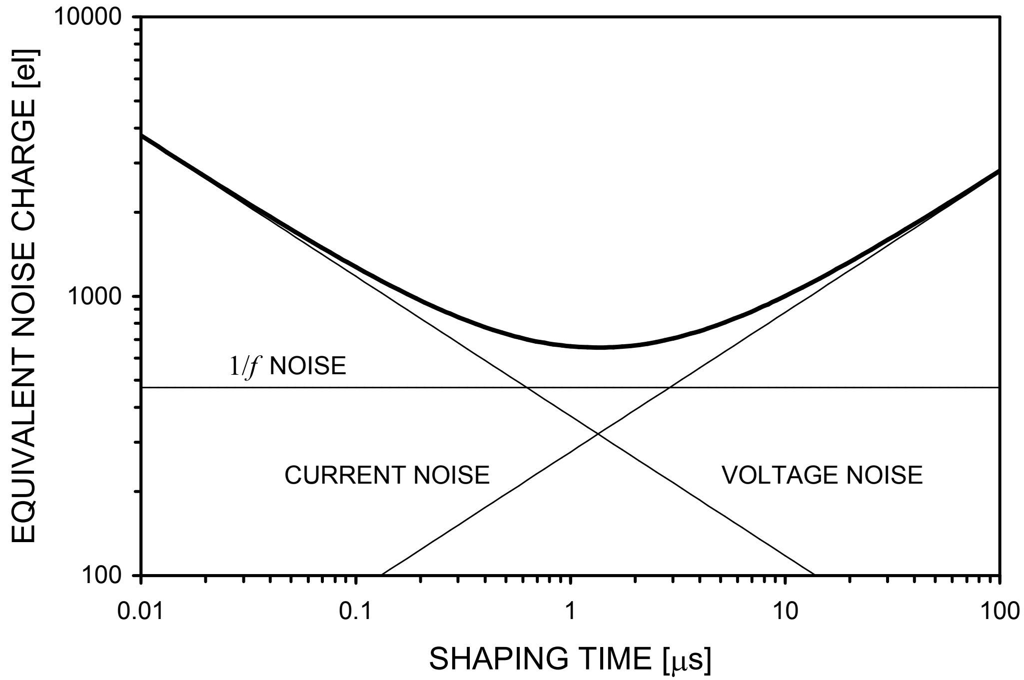
$$Q_n^2 = \left(\frac{\mathbf{e}^2}{8}\right)\left[\left(2q_e I_D + \frac{4kT}{R_P} + i_{na}^2\right)\cdot\tau + \left(4kTR_S + e_{na}^2\right)\cdot\frac{C_D^2}{\tau} + 4A_f C_D^2\right]$$

	↑	↑	↑
$\mathbf{e} = \exp(1)$	current noise	voltage noise	$1/f$ noise
	$\propto \tau$	$\propto 1/\tau$	independent of τ
	independent of C_D	$\propto C_D^2$	$\propto C_D^2$

- Current noise is independent of detector capacitance, consistent with the notion of "counting electrons".
- Voltage noise increases with detector capacitance (reduced signal voltage)
- $1/f$ noise is independent of shaping time.
  In general, the total noise of a $1/f$ source depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If τ_d and τ_i are scaled by the same factor, this ratio remains constant.
- detector leakage current and FET noise decrease with temperature

  ⇒ high resolution Si and Ge detectors for x-rays and gamma rays operate at cryogenic temperatures.

The equivalent noise charge Q_n assumes a minimum when the current and voltage noise contributions are equal. Typical result:



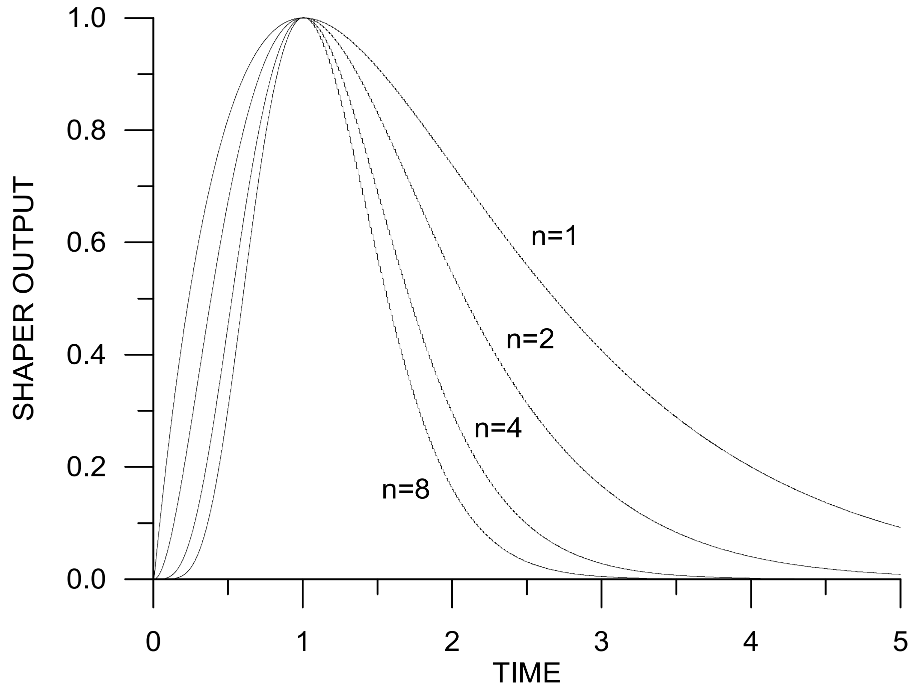
dominated by voltage noise current noise

For a CR-RC shaper the noise minimum obtains for $\tau_d = \tau_i = \tau$.

This criterion does not hold for more sophisticated shapers.

8. Other Types of Shapers

Shapers with Multiple Integrators



Start with simple *CR-RC* shaper and add additional integrators
($n=1$ to $n=2$, ... $n=8$).

Change integrator time constants to preserve the peaking time $\tau_n = \tau_{n=1} / n$

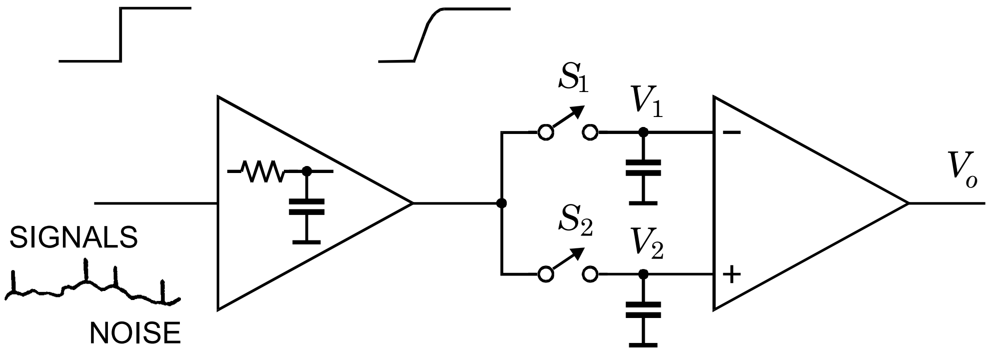
Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

$\Rightarrow$ improved rate capability at the same peaking time

Shapers with the equivalent of 8 *RC* integrators are common.
Usually, this is achieved with active filters

(i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

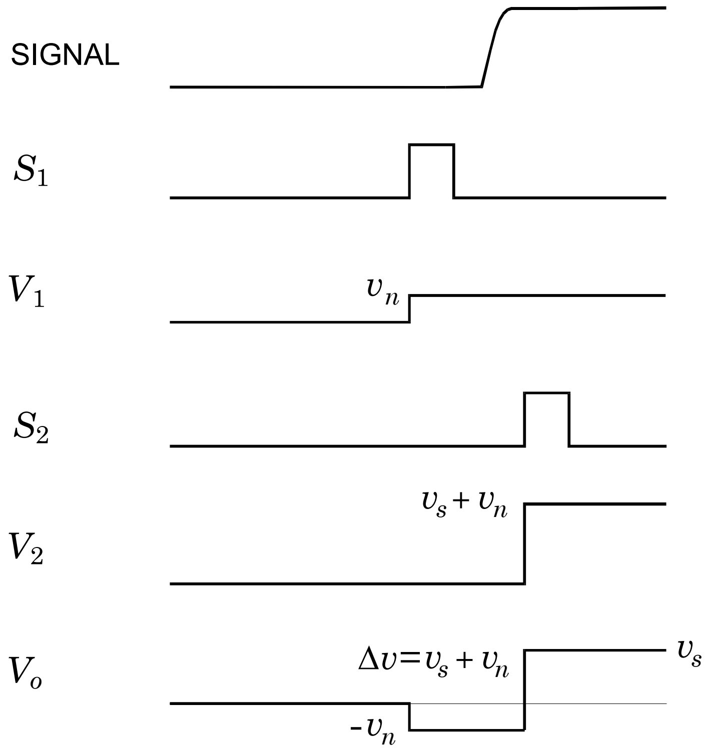
Time-Variant Shapers



Time variant shaper change the filter parameters during the processing of individual pulses.

A commonly used time-variant filter is the correlated double-sampler.

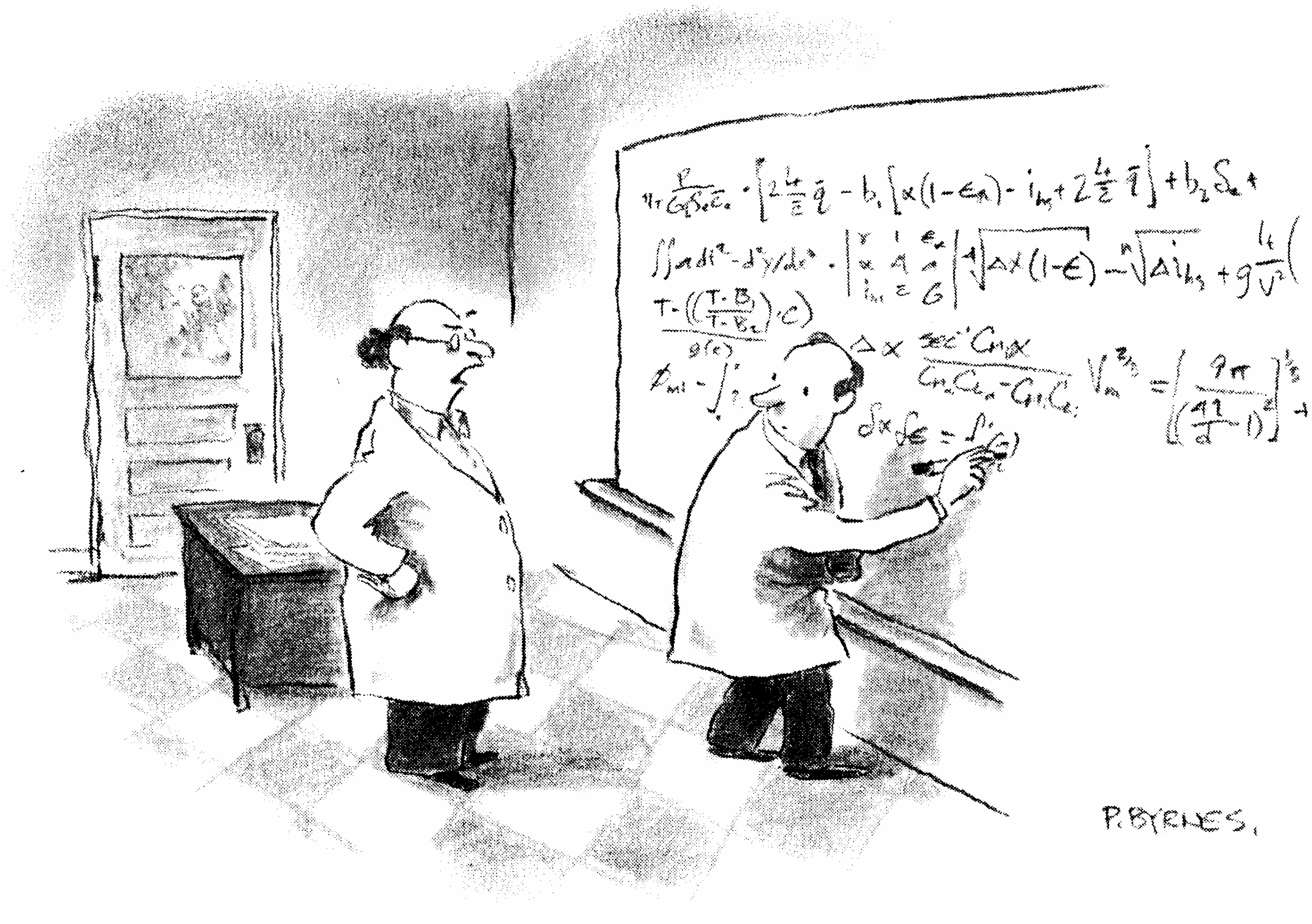
1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal



S/N depends on

1. time constant of prefilter
2. time difference between samples

See "Semiconductor Detector Systems" for a detailed noise analysis.

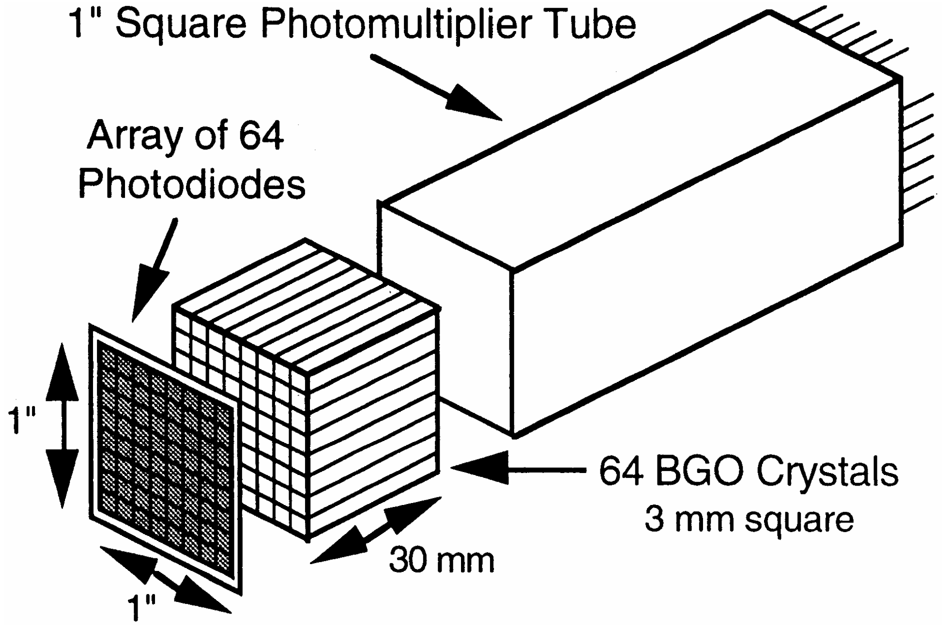


"Duh."

9. Examples: Photodiode Readout

(S. Holland, N. Wang, I. Kipnis, B. Krieger, W. Moses, LBNL)

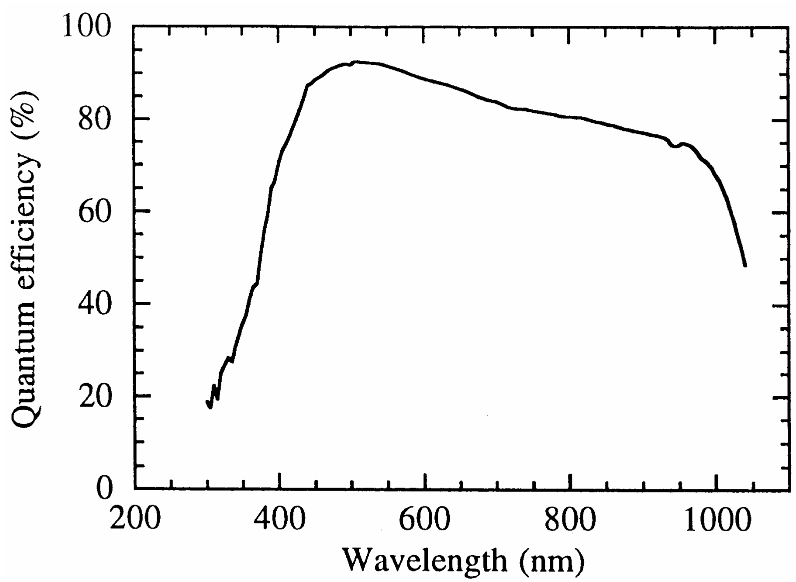
Medical Imaging (Positron Emission Tomography)



Read out 64 BGO crystals with one PMT (timing, energy) and tag crystal by segmented photodiode array.

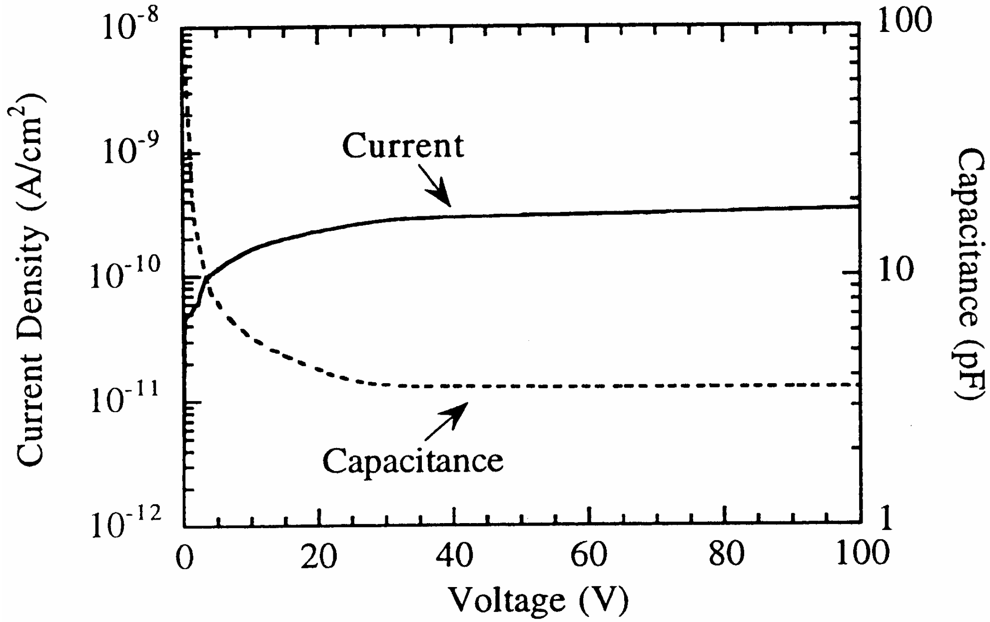
Requires thin dead layer on photodiode to maximize quantum efficiency.

Thin electrode must be implemented with low resistance to avoid significant degradation of electronic noise.



Furthermore, low reverse bias current critical to reduce noise.

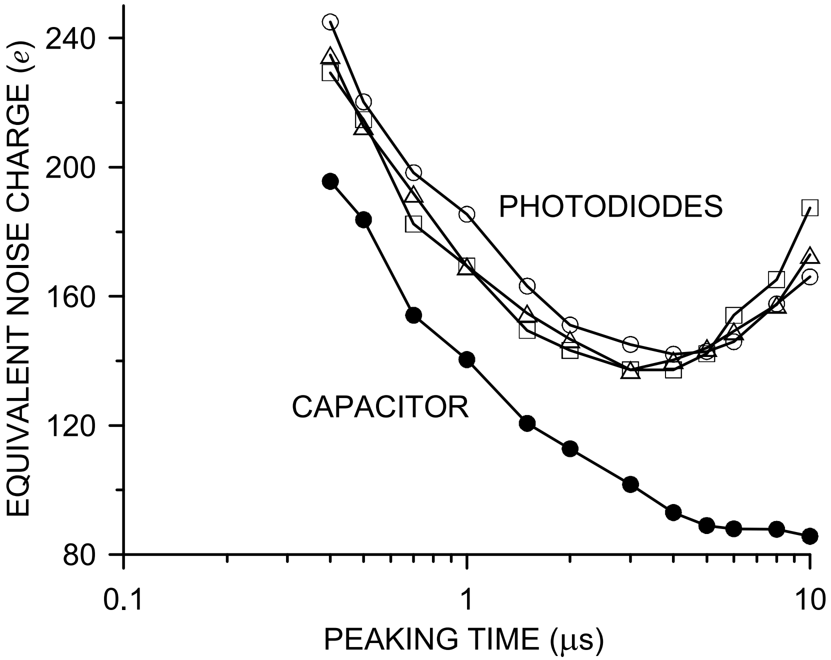
Photodiodes designed and fabricated in LBNL Microsystems Lab.



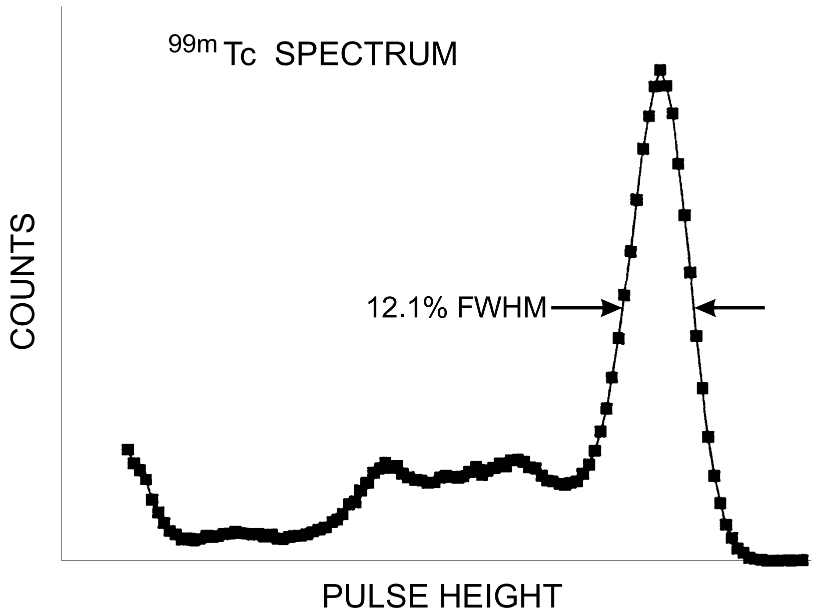
Front-end chip (preamplifier + shaper): 16 channels per chip, die size: 2 x 2 mm^2, 1.2 μm CMOS

continuously adjustable shaping time (0.5 to 50 μs)

Noise vs. shaping time



Energy spectrum with BGO scintillator



Note increase in noise at long shaping times when photodiode is connected - shot noise contribution.

Examples: Short-Strip Si X-Ray Detector

(B. Ludewigt, C. Rossington, I. Kipnis, B. Krieger, LBNL)

Use detector with multiple strip electrodes not for position resolution,

but for segmentation $\Rightarrow$ distribute rate over many channels

$\Rightarrow$ reduced capacitance

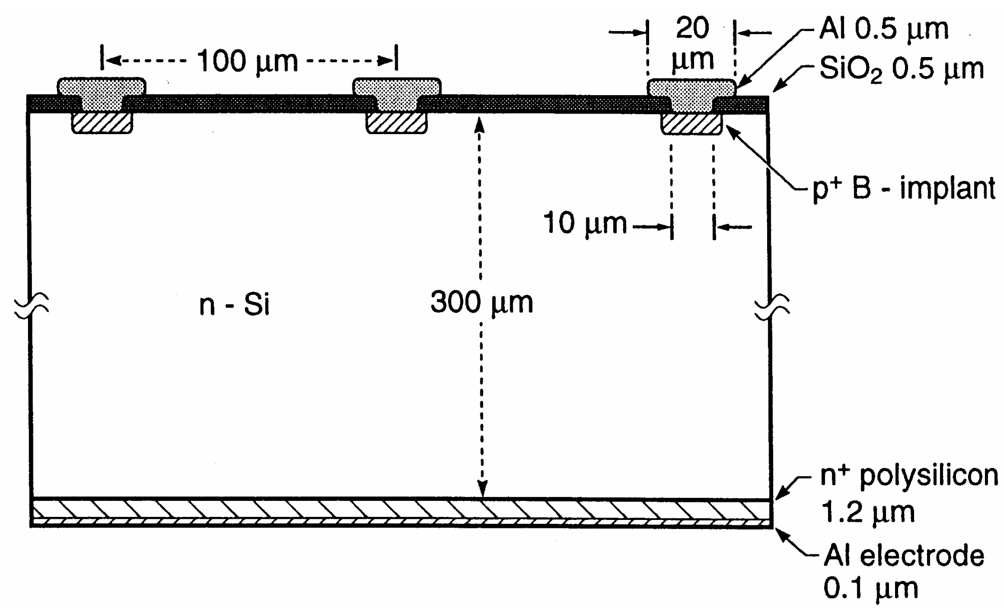
$\Rightarrow$ low noise at short shaping time

$\Rightarrow$ higher rate per detector element

For x-ray energies 5 – 25 keV $\Rightarrow$ photoelectric absorption dominates (signal on 1 or 2 strips)

Strip pitch: 100 μm

Strip Length: 2 mm (matched to ALS)



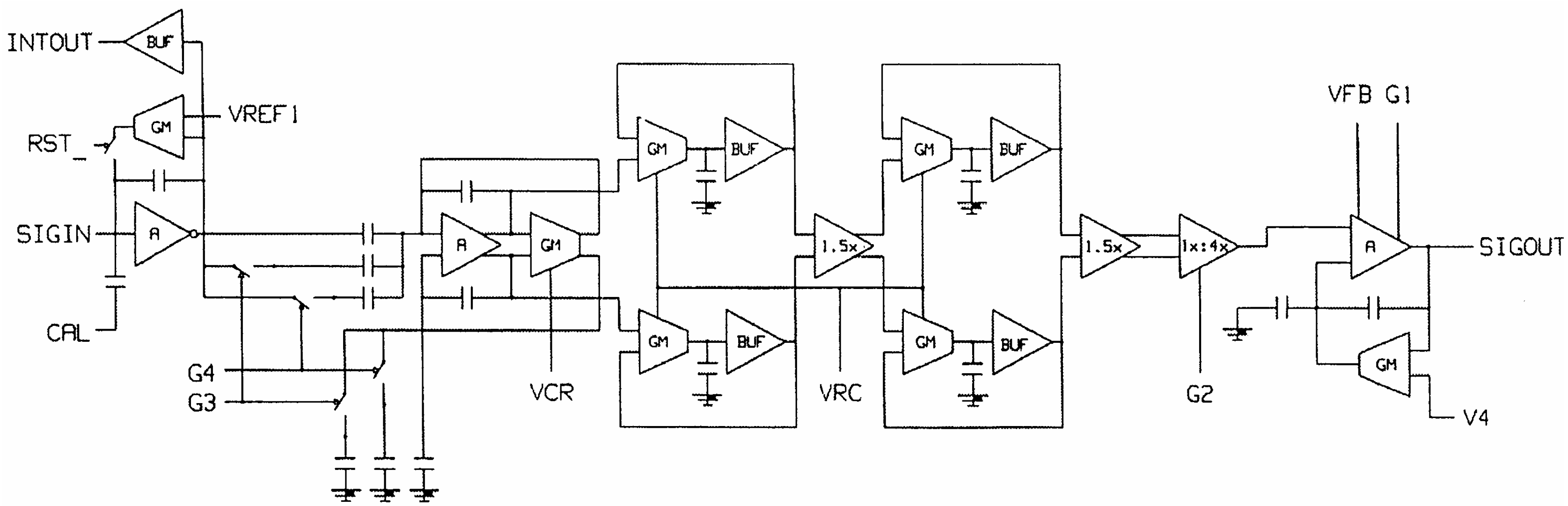
Readout IC tailored to detector

Preamplifier + CR-RC^2 shaper + cable driver to bank of parallel ADCs (M. Maier + H. Yaver)

Preamplifier with pulsed reset.

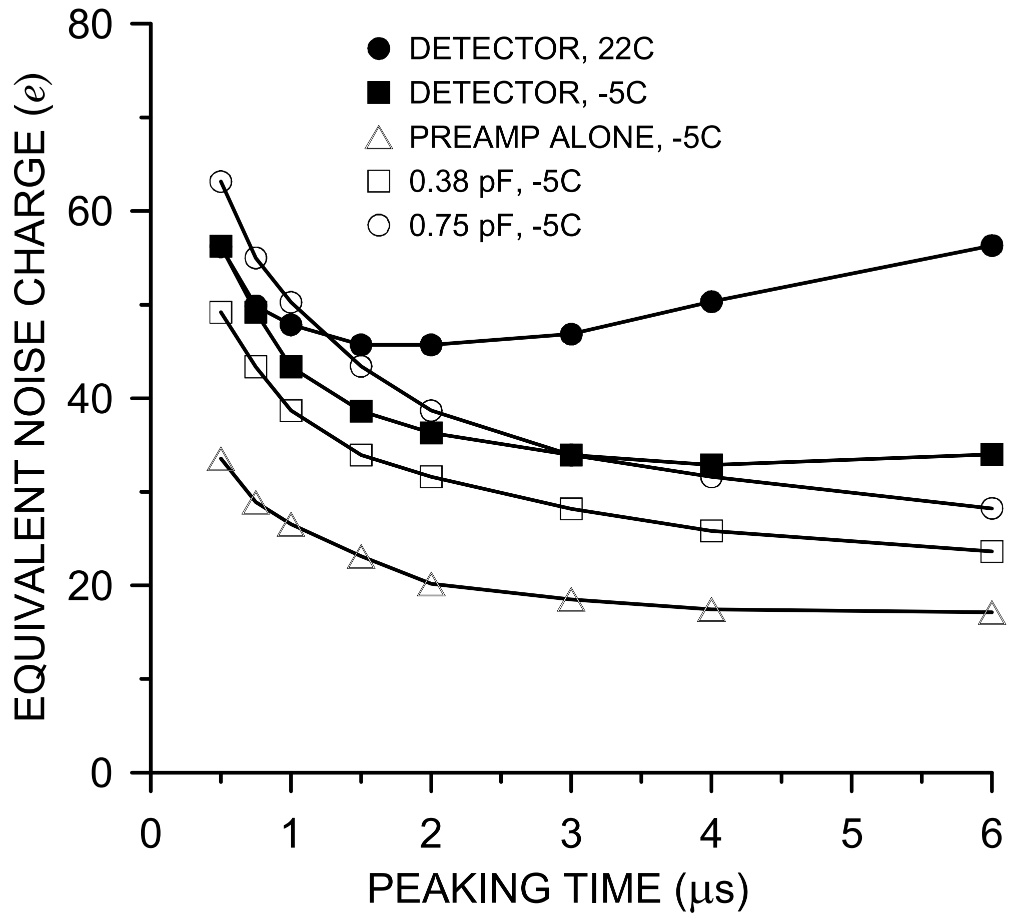
Shaping time continuously variable 0.5 to 20 μs.

CHARGE-SENS. PREAMPLIFIER | CR | RC | RC | VARIABLE GAIN | OUTPUT



Noise Charge vs. Peaking Time

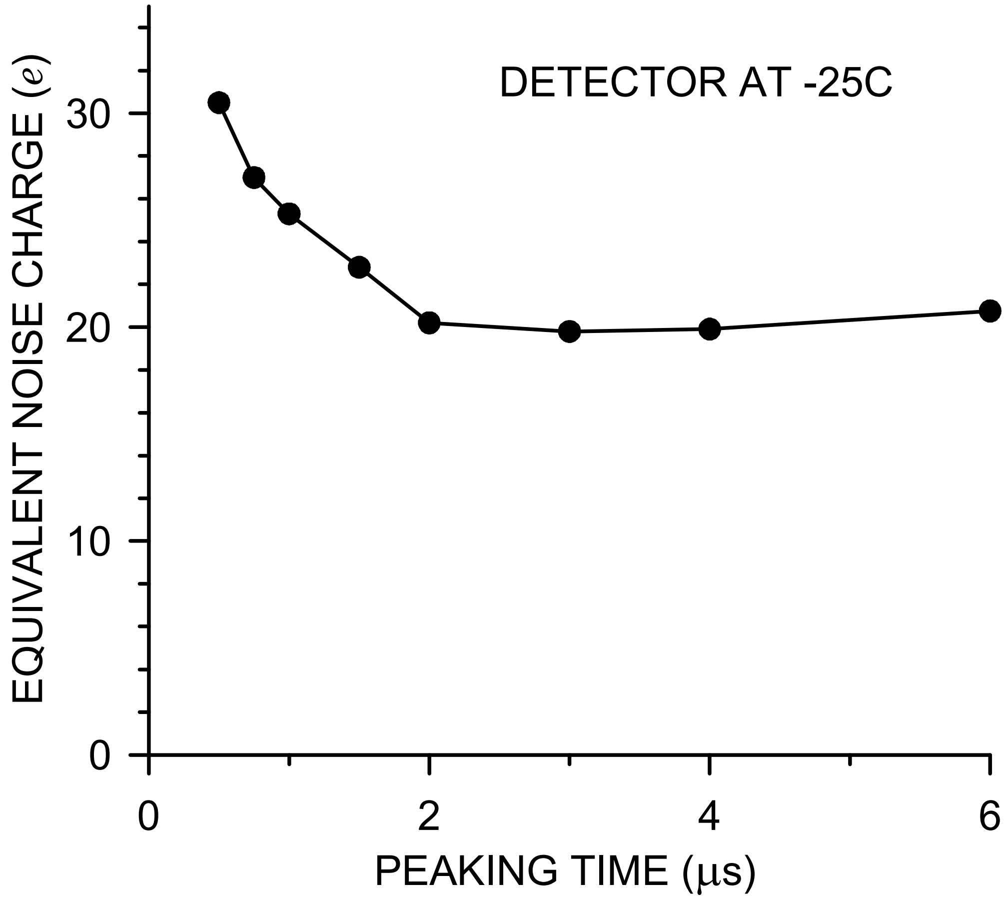
- Open symbols: preamplifier alone and with capacitors connected instead of a detector.
- Connecting the detector increases noise because of added capacitance and detector current (as indicated by increase of noise with peaking time).
- Cooling the detector reduces the current and noise improves.



Second prototype

Current noise negligible because of cooling.

“Flat” noise vs. shaping time indicates that $1/f$ noise dominates.



Numerical expression for the noise of a CR-RC shaper
(amplifier current noise negligible)

(note that some units are "hidden" in the numerical factors)

$$Q_n^2 = 12\, I_B\, \tau \;+\; 6 \cdot 10^5 \frac{\tau}{R_P} \;+\; 3.6 \cdot 10^4\, e_n^2\, \frac{C^2}{\tau} \quad \text{[rms electrons}^2\text{]}$$

where

τ	shaping time constant [ns]
I_B	detector bias current + amplifier input current [nA]
R_P	input shunt resistance [kΩ]
e_n	equivalent input noise voltage spectral density [nV/√Hz]
C	total input capacitance [pF]

Q_n= 1 *el* corresponds to 3.6 eV in Si
2.9 eV in Ge

“Series” and “Parallel” Noise

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

The rest of the world uses equivalent noise voltage and current.

Since they are physically meaningful, use of these widely understood terms is preferable.

The expression for the equivalent noise charge

$$Q_n^2 = \left(\frac{\mathbf{e}^2}{8}\right)\left[\left(2q_e I_D + \frac{4kT}{R_P} + i_{na}^2\right)\cdot\tau + \left(4kTR_S + e_{na}^2\right)\cdot\frac{C_D^2}{\tau} + 4A_f C_D^2\right]$$

$\mathbf{e} = \exp(1)$

	current noise	voltage noise	$1/f$ noise
	$\propto \tau$	$\propto 1/\tau$	independent of τ
	independent of C_D	$\propto {C_D}^2$	$\propto {C_D}^2$

can be put in a more general form that applies to all type of pulse shapers:

$$Q_n^2 = i_n^2 \; T_s \; F_i \; + \; C^2 \; e_n^2 \; \frac{F_v}{T_s} + \; F_{vf} \; A_f \; C^2$$

- The current and voltage terms are combined and represented by i_n^2 and e_n^2.
- The shaper is characterized by a shape and characteristic time (e.g. the peaking time).
- A specific shaper is described by the “shape factors” F_i, F_v, and F_{vf}.
- The effect of the shaping time is set by T_S.

10. Detector Noise Summary

Two basic noise mechanisms: input noise current i_n
input noise voltage e_n

Equivalent Noise Charge:

$$Q_n^2 = i_n^2 \, T_s \, F_i + C^2 \, e_n^2 \, \frac{F_v}{T_s}$$

Where T_s Characteristic shaping time (*e.g.* peaking time)

F_i , F_v "Shape Factors" that are determined by the shape of the pulse.

C Total capacitance at the input node (detector capacitance + input capacitance of preamplifier + stray capacitance + …)

Typical values of F_i , F_v

CR-RC shaper	F_i = 0.924	F_v = 0.924
CR-(RC)^4 shaper	F_i = 0.45	F_v = 1.02
CR-(RC)^7 shaper	F_i = 0.34	F_v = 1.27
CAFE chip	F_i = 0.4	F_v = 1.2

Note that $F_i < F_v$ for higher order shapers.

Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).

Minimum noise obtains when the current and voltage noise contributions are equal.

Current noise

- detector bias current
  increases with detector size, strongly temperature dependent

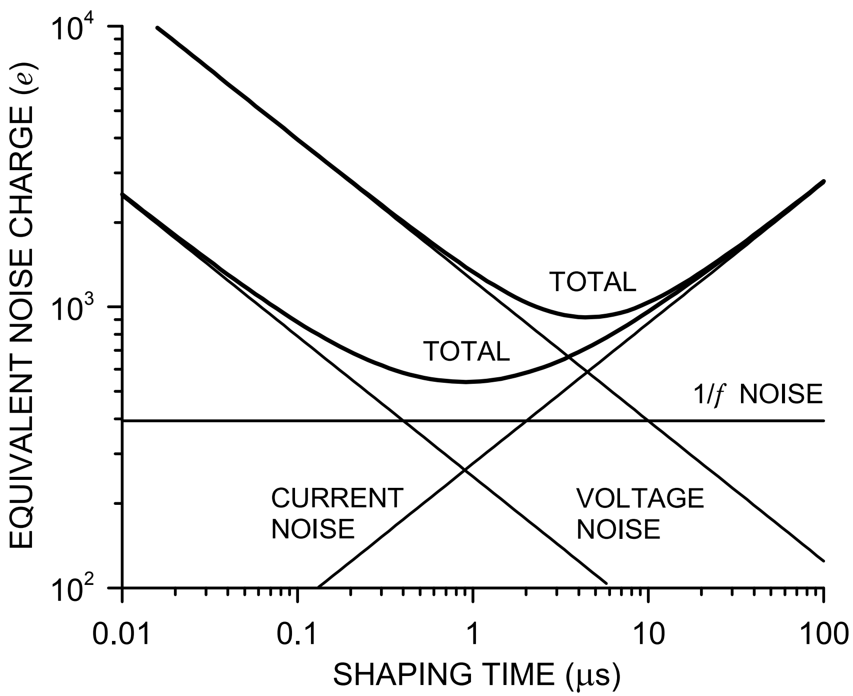
- resistors shunting the input
  increases as resistance is decreased

- input transistor – low for FET, higher for BJTs

Voltage noise

- input transistor –
  noise decreases with increased current

- series resistance
  e.g. detector electrode, protection circuits



FETs commonly used as input devices – improved noise performance when cooled ($T_{opt} \approx$ 130 K)

Bipolar transistors advantageous at short shaping times (<100 ns).
When collector current is optimized, bipolar transistor equivalent noise charge is independent of shaping time (see Appendix).

Equivalent Noise Charge vs. Detector Capacitance ($C = C_d + C_a$)

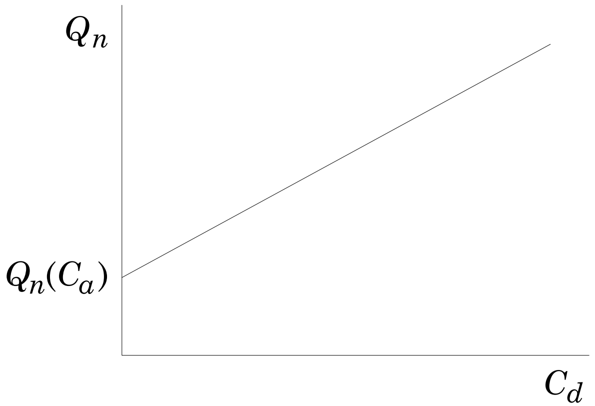
$$Q_n = \sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}$$

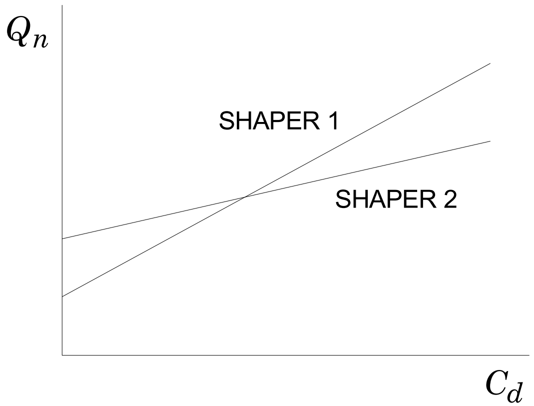
$$\frac{dQ_n}{dC_d} = \frac{2C_d e_n^2 F_v \frac{1}{T}}{\sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}}$$

If current noise $i_n^2 F_i T$ is negligible, i.e. **voltage noise dominates:**

$$\frac{dQ_n}{dC_d} \approx \underbrace{2e_n}_{\text{input stage}} \cdot \underbrace{\sqrt{\frac{F_v}{T}}}_{\text{shaper}}$$

Zero intercept: $Q_n\big|_{C_d=0} = C_a e_n \sqrt{F_v / T}$





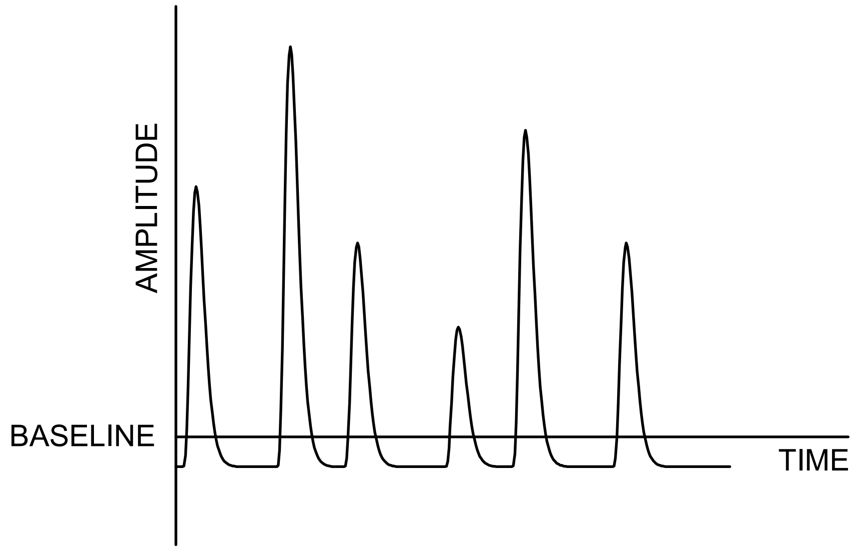
11. Some Other Aspects of Pulse Shaping

Baseline Restoration

Any series capacitor in a system prevents transmission of a DC component.

A sequence of unipolar pulses has a DC component that depends on the duty factor, i.e. the event rate.

⇒ The baseline shifts to make the overall transmitted charge equal zero.



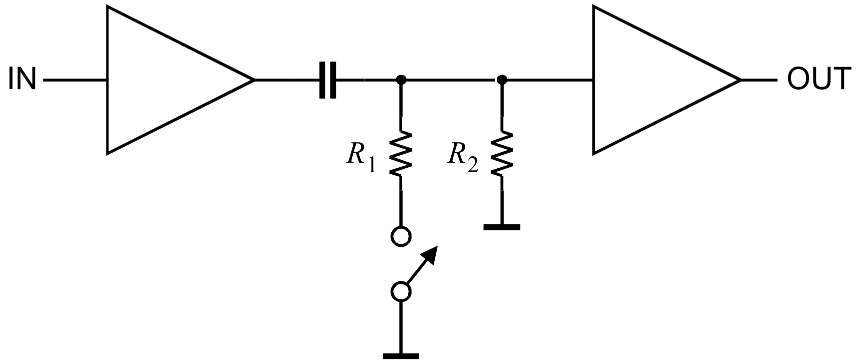
Random rates lead to random fluctuations of the baseline shift ⇒ spectral broadening

- These shifts occur whenever the DC gain is not equal to the midband gain

  The baseline shift can be mitigated by a baseline restorer (BLR).

Principle of a baseline restorer:

Connect signal line to ground during the absence of a signal to establish the baseline just prior to the arrival of a pulse.



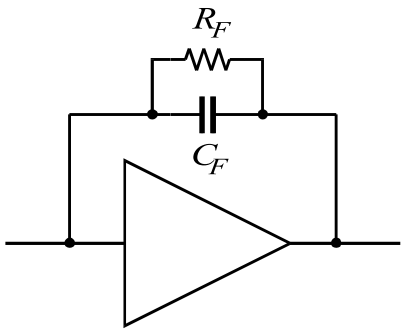
R_1 and R_2 determine the charge and discharge time constants.
The discharge time constant (switch opened) must be much larger than the pulse width.

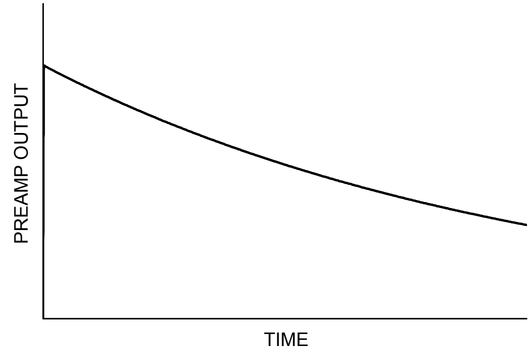
Originally performed with diodes (passive restorer), baseline restoration circuits now tend to include active loops with adjustable thresholds to sense the presence of a signal (gated restorer).
Asymmetric charge and discharge time constants improve performance at high count rates.

- This is a form of time-variant filtering. Care must be exercised to reduce noise and switching artifacts introduced by the BLR.
- Good pole-zero cancellation (next topic) is crucial for proper baseline restoration.

Tail (Pole Zero) Cancellation

Feedback capacitor in charge sensitive preamplifier must be discharged. Commonly done with resistor.



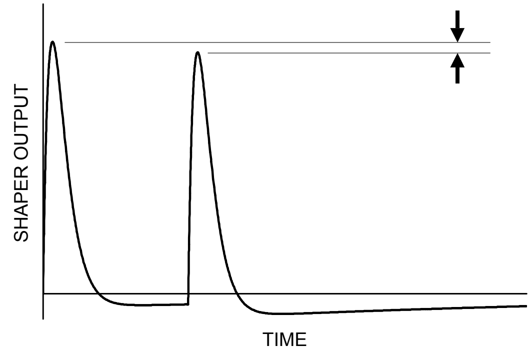


Output no longer a step, but decays exponentially

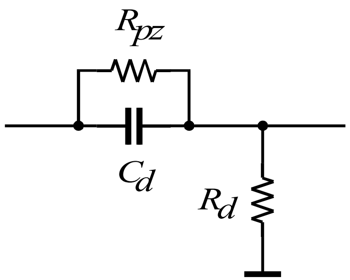
Exponential decay superimposed on shaper output.

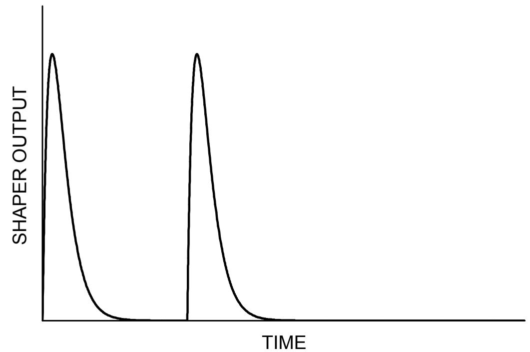
⇒ undershoot

⇒ loss of resolution
due to baseline
variations



Add R_{pz} to differentiator:

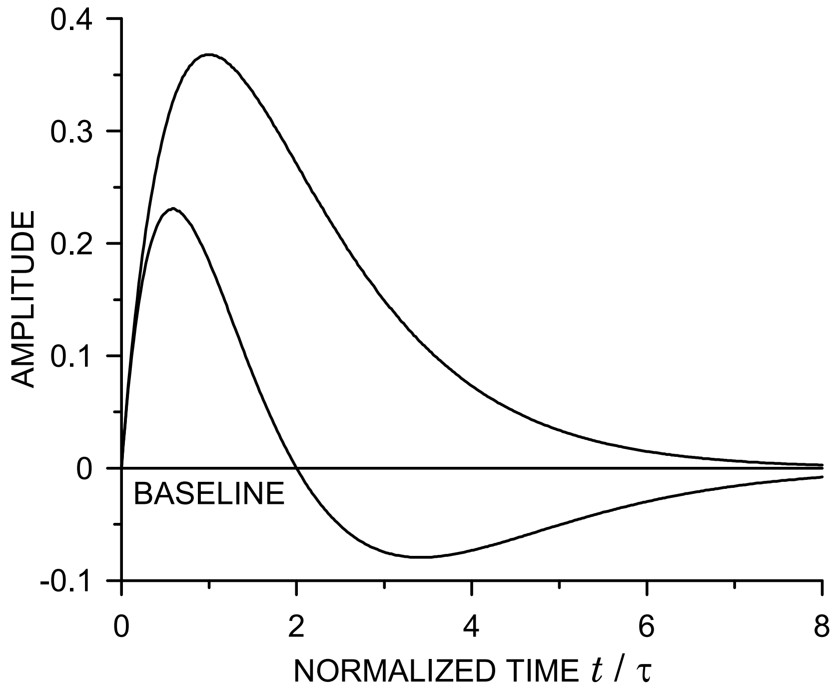




“zero” cancels “pole” of preamp when $R_F C_F = R_{pz} C_d$

Technique also used to compensate for “tails” of detector pulses: “tail cancellation”

Bipolar vs. Unipolar Shaping



Unipolar pulse + 2nd differentiator

→ Bipolar pulse

Electronic resolution with bipolar shaping typ. 25 – 50% worse than for corresponding unipolar shaper.

However …

- Bipolar shaping eliminates baseline shift (as the DC component is zero).
- Pole-zero adjustment less critical
- Added suppression of low-frequency noise (see Part 7).
- Not all measurements require optimum noise performance. Bipolar shaping is much more convenient for the user (important in large systems!) – often the method of choice.