IEEE NPSS Short Course

Radiation Detection and Measurement

October 27 – 28, 2007

2007 Nuclear Science Symposium and Medical Imaging Conference
Honolulu, Hawaii

Front-End Electronics for Detectors

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These course notes and additional tutorials at
http://www-physics.lbl.gov/~spieler

More detailed discussions in
H. Spieler: Semiconductor Detector Systems, Oxford University Press, 2005

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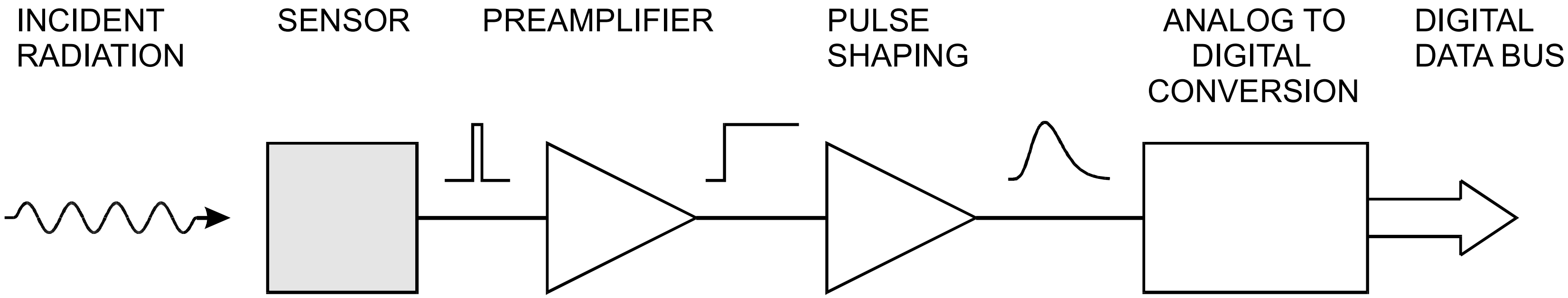
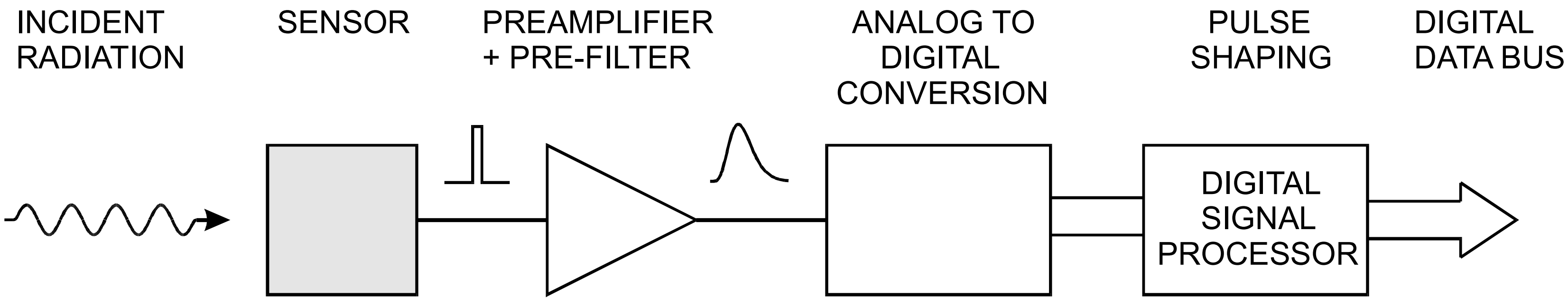
1. Basic Functions of Front-End Electronics



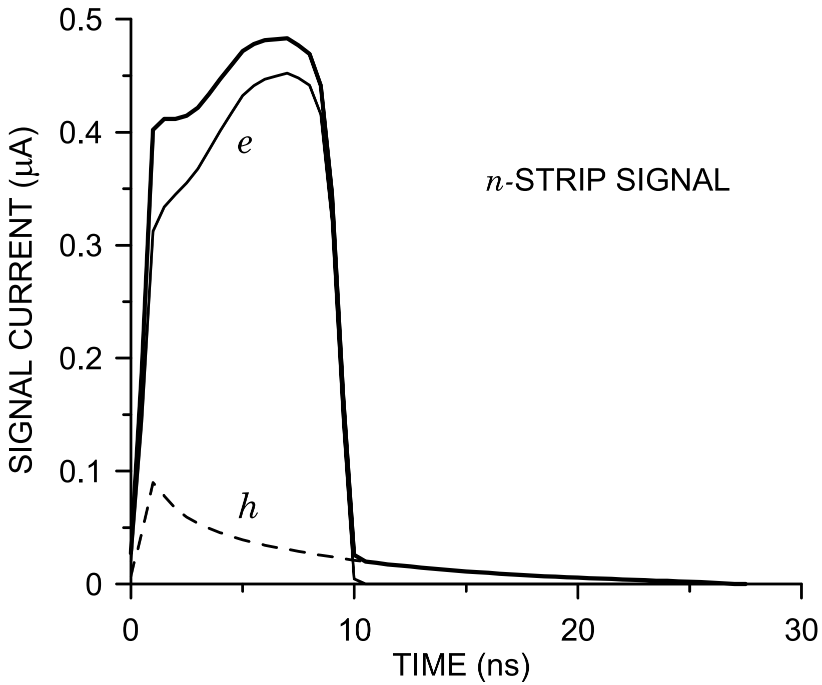
Pulse shaping can also be performed with digital circuitry:

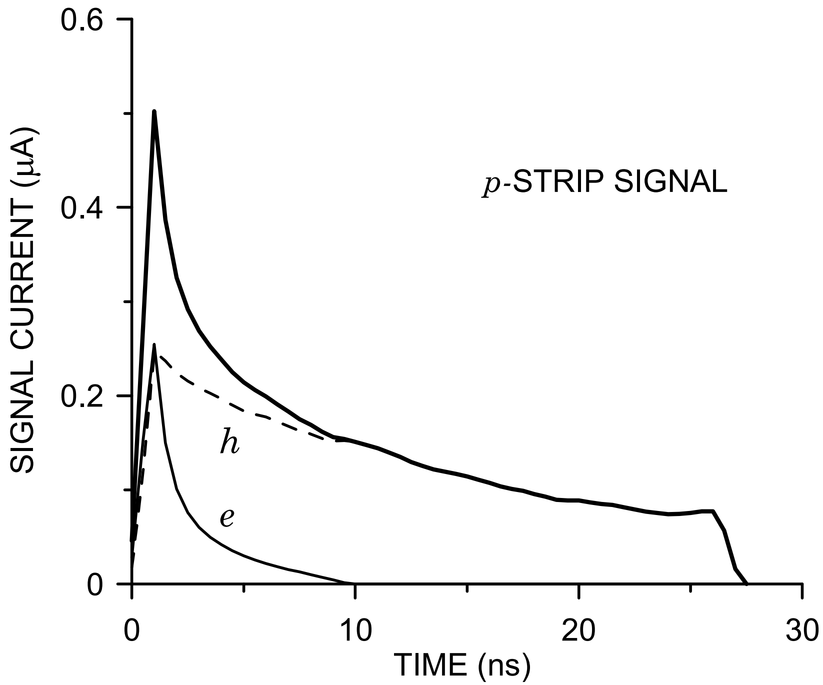


2. Signal Acquisition

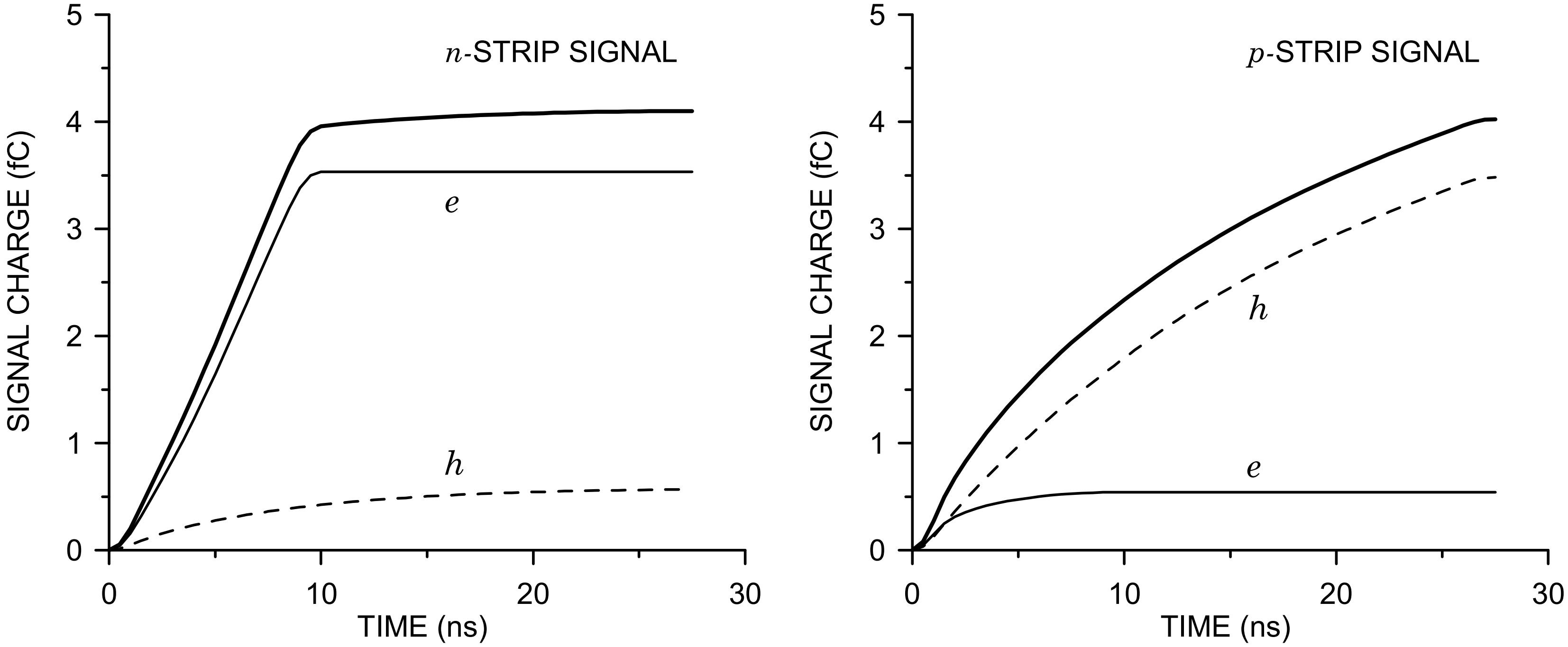
- The primary signal in practically all radiation detectors is a current.

Example: Double-sided silicon strip detector (300 μm thick, 50 μm strip pitch)





Although the signal currents on the p- and n-side are quite different, the integrated current, i.e. the signal charge is the same.

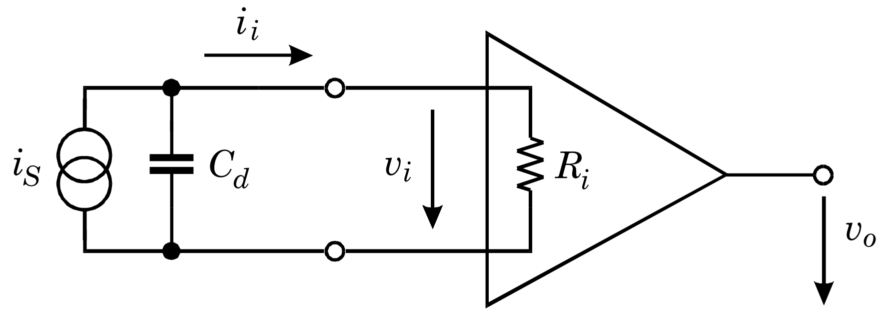


$\Rightarrow$ For energy measurements the signal processing chain must integrate the signal current to yield the charge.

Voltage and Current Mode with Capacitive Sources

Output voltage:

v_o = (voltage gain A_v) × (input voltage v_i)



Operating mode depends on charge collection time t_c and the input time constant $R_i C_d$:

a) $R_i C_d \ll t_c$

detector capacitance discharges rapidly

$\Rightarrow \quad v_o \propto i_s(t)$

current sensitive amplifier

b) $R_i C_d \gg t_c$

detector capacitance discharges slowly

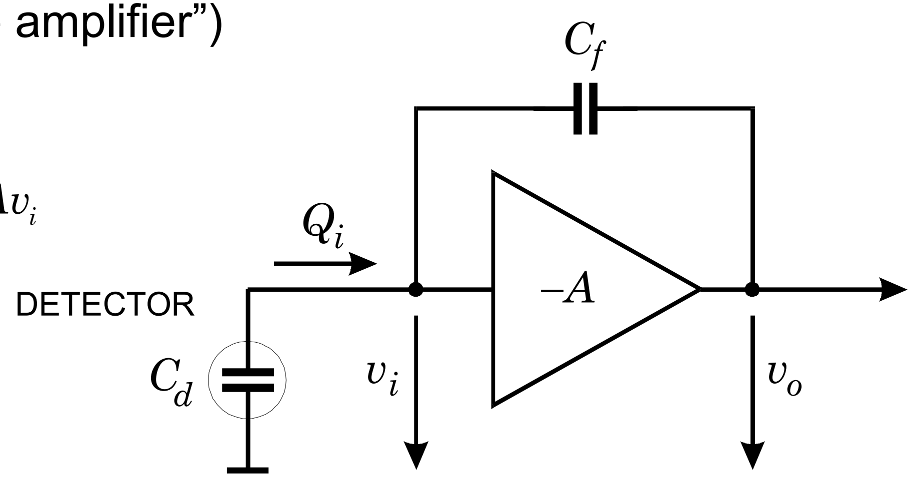
$\Rightarrow \quad v_o \propto \int i_s(t) dt$

voltage sensitive amplifier

Note that in both cases the amplifier is providing voltage gain, so the output signal voltage is determined directly by the input voltage. The difference is that the shape of the input voltage pulse is determined either by the instantaneous current or by the integrated current and the decay time constant.

Goal is to measure signal charge, so it is desirable to use a system whose response is independent of detector capacitance.

Active Integrator ("charge-sensitive amplifier")



Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow v_o = -Av_i$

Input impedance = ∞
(no signal current flows
into amplifier input)

Connect feedback capacitor C_f
between output and input.

Voltage difference across C_f: $\quad v_f = (A+1)v_i$

⇒ Charge deposited on C_f: $\quad Q_f = C_f v_f = C_f (A+1) v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

⇒ Effective input capacitance $\quad C_i = \dfrac{Q_i}{v_i} = C_f (A+1)$ ("dynamic" input capacitance)

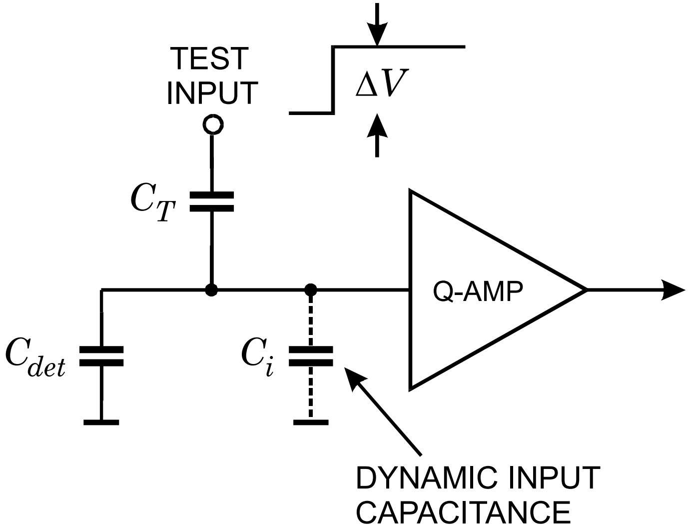
Gain $$A_Q = \frac{dV_o}{dQ_i} = \frac{A \cdot v_i}{C_i \cdot v_i} = \frac{A}{C_i} = \frac{A}{A+1} \cdot \frac{1}{C_f} \approx \frac{1}{C_f} \quad (A >> 1)$$

Set by a well-controlled quantity, the feedback capacitance.

Calibration

Inject specific quantity of charge - measure system response

Use voltage pulse (can be measured conveniently with oscilloscope)



$C_i \gg C_T$ $\Rightarrow$ Voltage step applied to test input develops over C_T .

$\Rightarrow$ $Q_T = \Delta V \cdot C_T$

Accurate expression:

$$Q_T = \frac{C_T}{1+\frac{C_T}{C_i}} \cdot \Delta V \approx C_T \left(1 - \frac{C_T}{C_i}\right) \Delta V$$

Typically: $C_T / C_i = 10^{-3} - 10^{-4}$

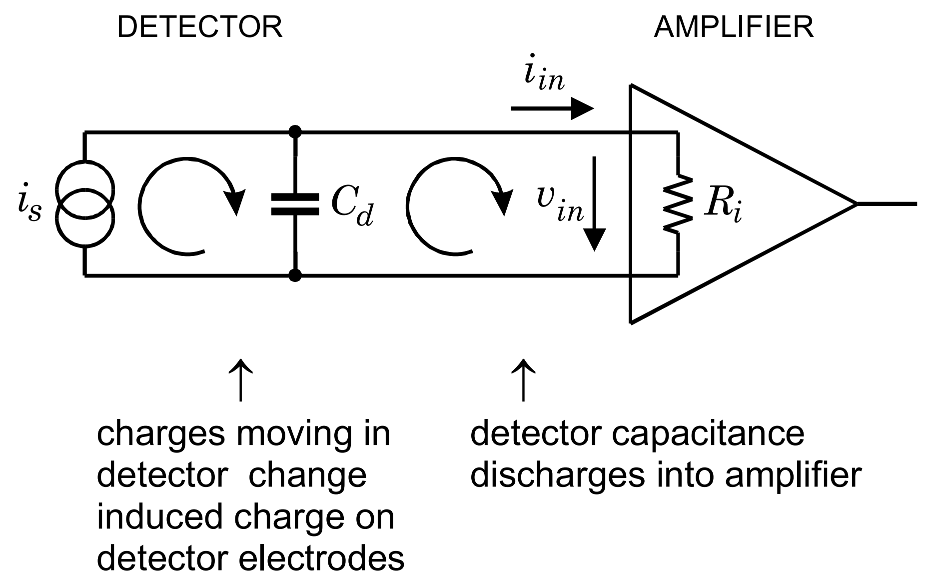
Realistic Charge-Sensitive Preamplifiers

The preceding discussion assumed idealized amplifiers with infinite speed.

In reality, amplifiers may be too slow to follow the instantaneous detector pulse.

Does this incur a loss of charge?

Equivalent Circuit:



Signal is preserved even if the amplifier responds much more slowly than the detector signal.

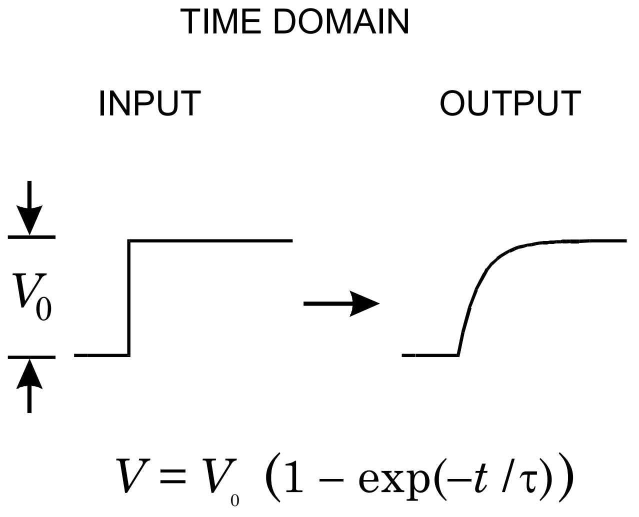
However, the response of the amplifier affects the measured pulse shape.

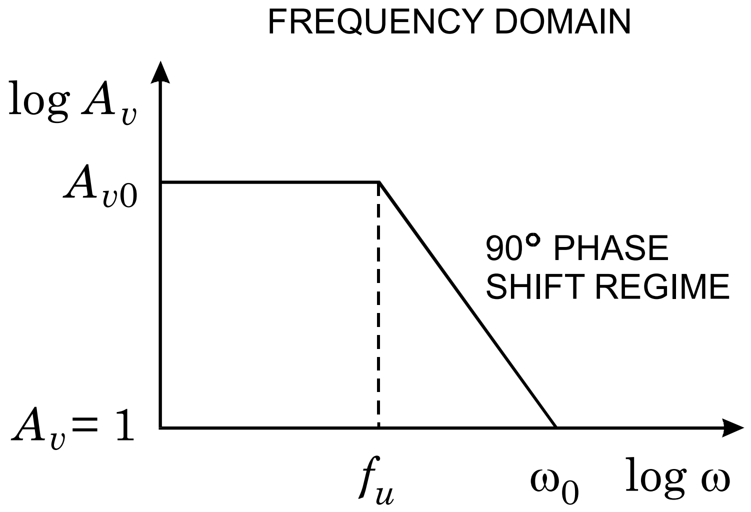
Frequency and Time Response of a Simple Amplifier

A voltage step $v_i(t)$ at the input causes a current step $i_o(t)$ at the output of the transistor.

For the output voltage to change, internal capacitances in the amplifier must first charge up.

⇒ The output voltage changes with a time constant τ.

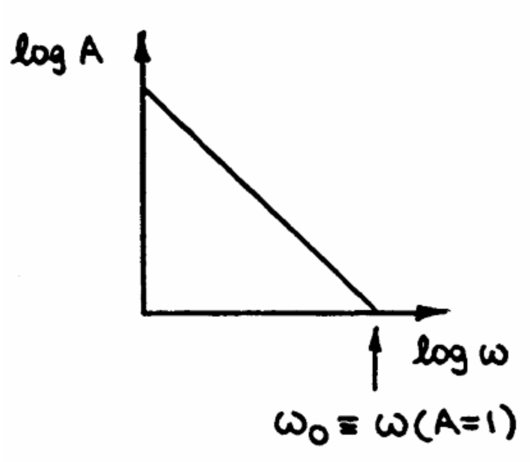




The time constant τ corresponds to the upper cutoff frequency : $\tau = \dfrac{1}{2\pi f_u}$

Input Impedance of a Charge-Sensitive Amplifier

Input impedance $$Z_i = \frac{Z_f}{A+1} \approx \frac{Z_f}{A} \quad (A >> 1)$$



Gain-Bandwidth Product

Feedback amplifiers are typically operated above the upper cutoff frequency, where the gain vs. frequency

$$A = -\mathbf{i}\ \frac{\omega_0}{\omega}$$

Feedback impedance $$Z_f = -\mathbf{i}\ \frac{1}{\omega\ C_f}$$

⇒ Input Impedance $$Z_i = -\frac{\mathbf{i}}{\omega\ C_f} \cdot \frac{1}{-\mathbf{i}\,\frac{\omega_0}{\omega}}$$

$$Z_i = \frac{1}{\omega_0 C_f}$$

Imaginary component vanishes ⇒ *Resistance:* $Z_i \to R_i$

⇒ low frequencies $(f < f_u)$: capacitive input
high frequencies $(f > f_u)$:resistive input

Time Response of a Charge-Sensitive Amplifier

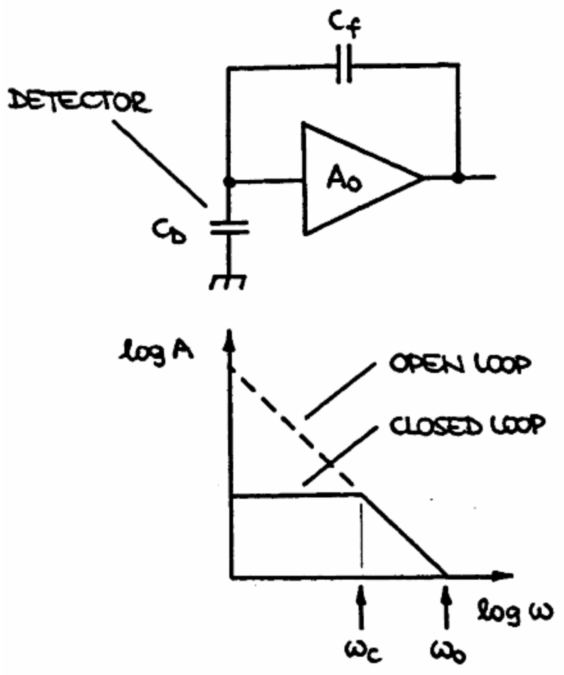
Input resistance and detector capacitance form RC time constant

$$\tau_i = R_i C_D$$

$$\tau_i = \frac{1}{\omega_0 C_f} \cdot C_D$$

$\Rightarrow$ **Rise time increases with detector capacitance.**

Or apply feedback theory:



Closed Loop Gain $\quad A_f = \frac{C_D + C_f}{C_f} \quad (A_f << A_0)$

$$A_f \approx \frac{C_D}{C_f} \quad (C_D >> C_f)$$

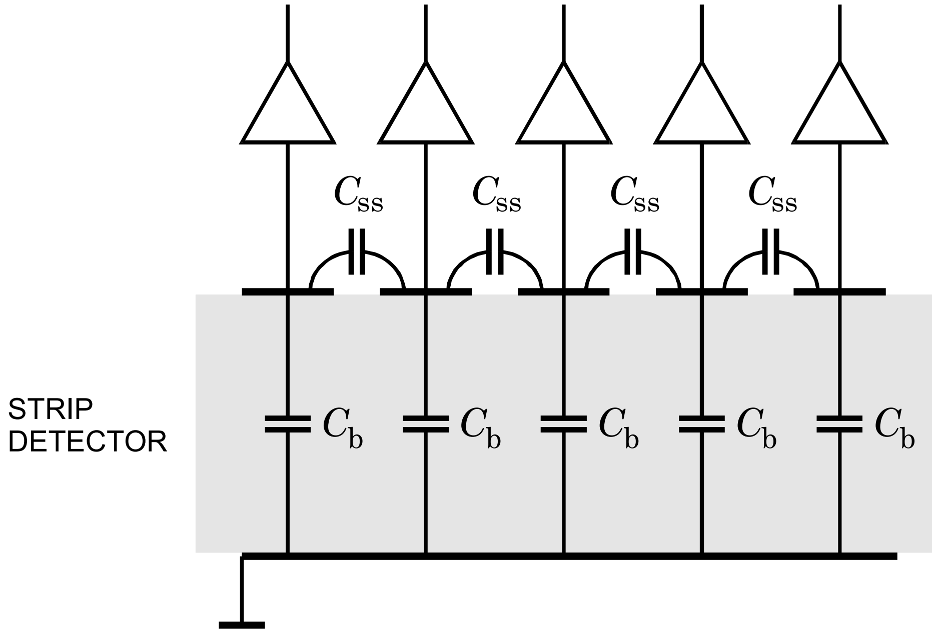
Closed Loop Bandwidth $\quad \omega_C A_f = \omega_0$

Response Time $\quad \tau_{amp} = \frac{1}{\omega_C} = C_D \frac{1}{\omega_0 C_f}$

Same result as from input time constant.

Input impedance is critical in strip or pixel detectors:

Amplifiers must have a low input impedance to reduce transfer of charge through capacitance to neighboring strips



For strip pitches that are smaller than the bulk thickness, the capacitance is dominated by the fringing capacitance to the neighboring strips C_{SS}.

Typically: 1 – 2 pF/cm for strip pitches of 25 – 100 μm on Si.

The backplane capacitance C_b is typically 20% of the strip-to-strip capacitance.

Negligible cross-coupling at shaping times $T_P > (2 \ldots 3) \times R_i C_D$ and if $C_i \gg C_D$.

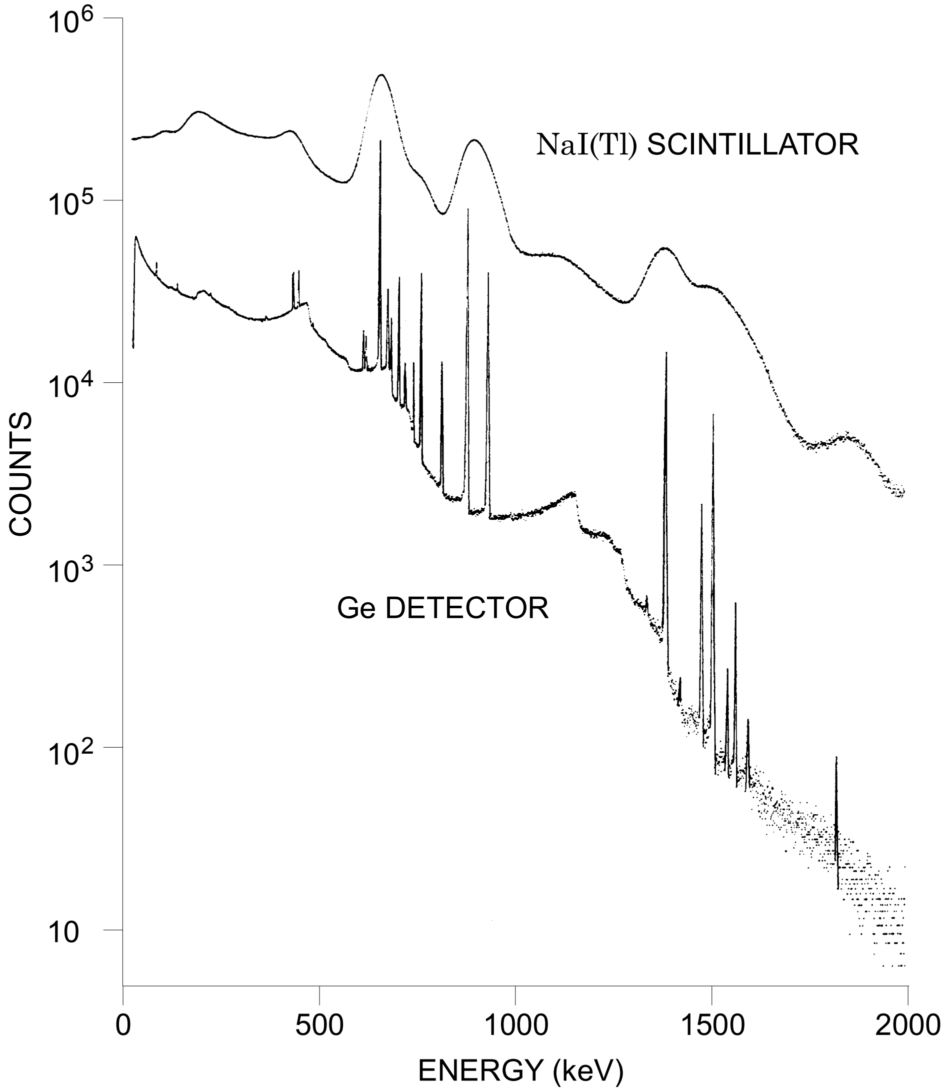
2. Resolution and Electronic Noise

Resolution: the ability to distinguish signal levels

1. Why?

a) Recognize structure in amplitude spectra

Comparison between NaI(Tl) and Ge detectors

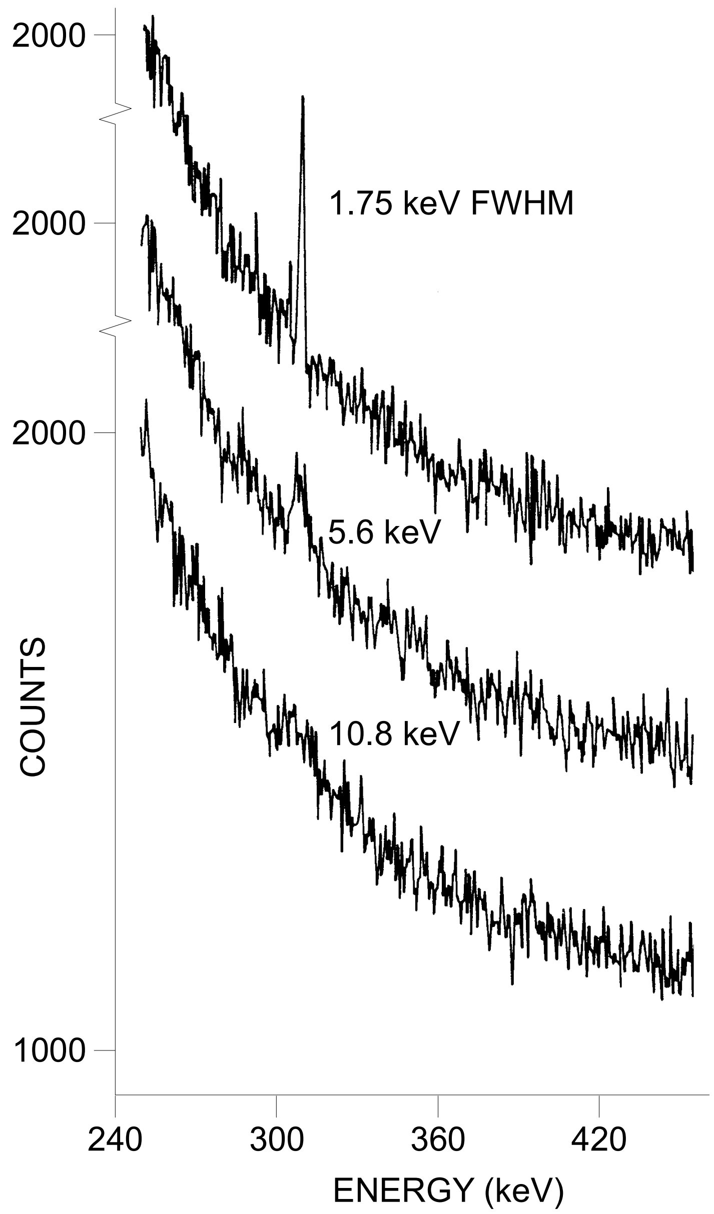


J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446

b) Improve sensitivity

Signal to background ratio improves with better resolution

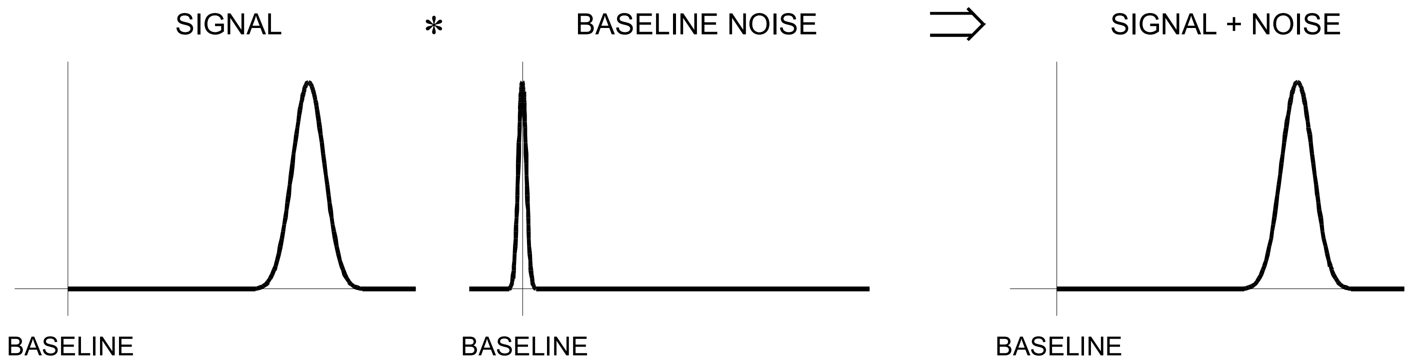
(signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

What determines Resolution?

1. Signal Variance >> Baseline Variance



⇒ Electronic (baseline) noise not important

Examples:
- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

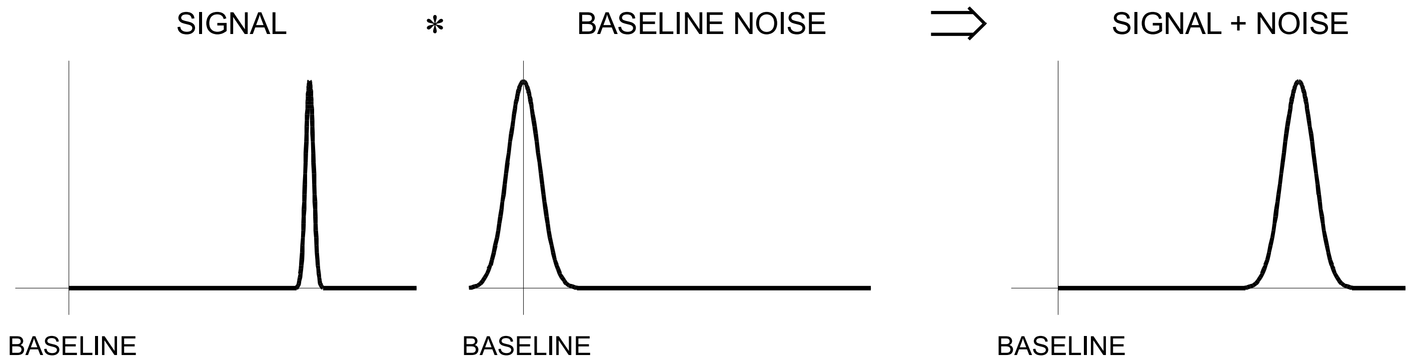
Number of photoelectrons: $N_{pe} \approx 8 \cdot 10^4$ [MeV^{-1}] $\times E_\gamma \times QE \approx 2.4 \cdot 10^4$

Variance typically: $\sigma_{pe} = N_{pe}^{1/2} \approx 160$ and $\sigma_{pe} / N_{pe} \approx 5$ - 8%

Signal at PMT anode (assume Gain= 10^4): $Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8$ el and
$\sigma_{sig} = G_{PMT}\, \sigma_{pe} \approx 1.2 \cdot 10^7$ el

whereas electronic noise easily < 10^4 el

2. Signal Variance << Baseline Variance



⇒ Electronic (baseline) noise critical for resolution

Examples:
- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si : Number of electron-hole pairs $N_{ep} = \dfrac{E_{dep}}{3.6\text{ eV}}$

Variance $\sigma_{ep} = \sqrt{F \cdot N_{ep}}$ (where F= Fano factor ≈ 0.1)

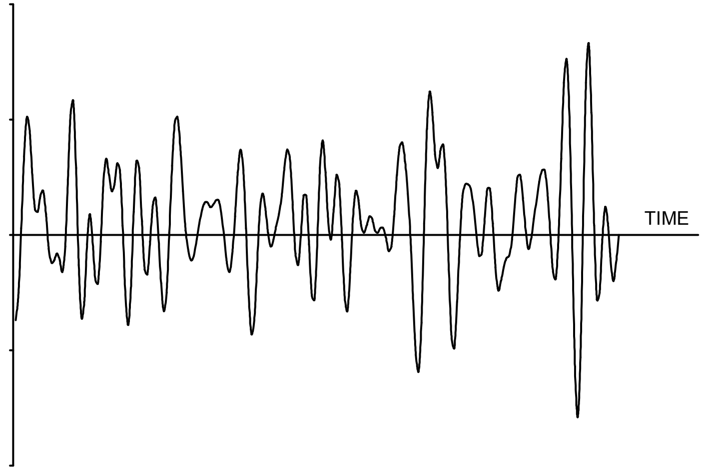
For 50 keV photons: $\sigma_{ep} \approx$ 40 el ⇒ σ_{ep} / N_{ep} = $7.5 \cdot 10^{-4}$

Obtainable noise levels are 10 to 1000 el.

Baseline Fluctuations (Electronic Noise)

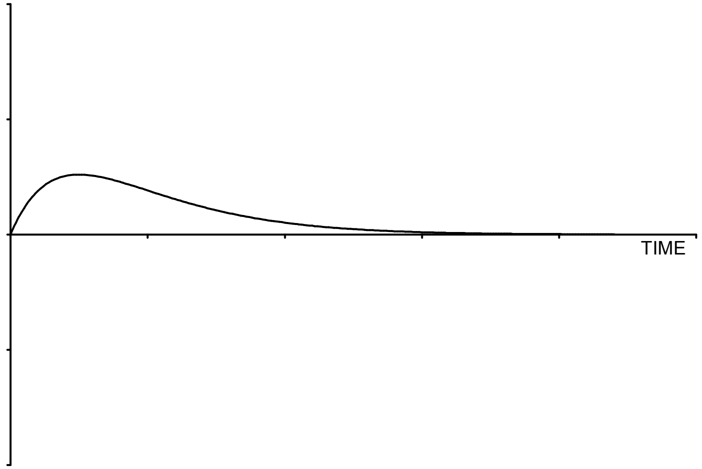
Choose a time when no signal is present.

Amplifier's quiescent output level (baseline):

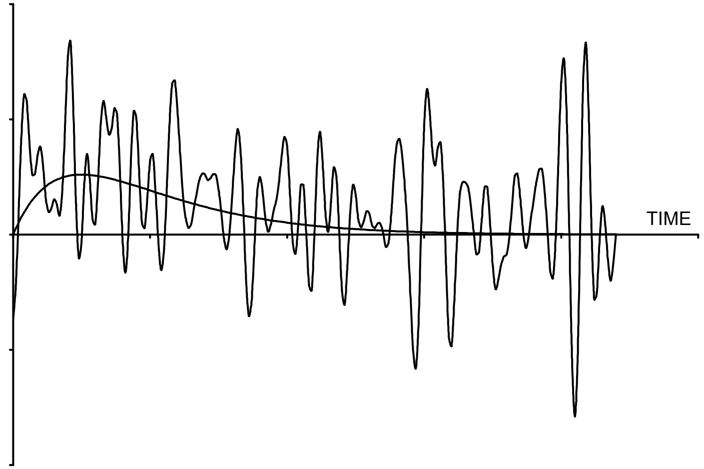


In the presence of a signal,
noise + signal add.

Signal:



Signal+Noise (S/N = 1)



$S/N \equiv$ peak signal to rms noise

Measurement of peak amplitude yields signal amplitude + noise fluctuation.

The preceding example could imply that the fluctuations tend to increase the measured amplitude, since the noise fluctuations vary more rapidly than the signal.

In an optimized system, the time scale of the fluctuation is comparable to the signal peaking time.

Then the measured amplitude fluctuates positive and negative relative to the ideal signal.

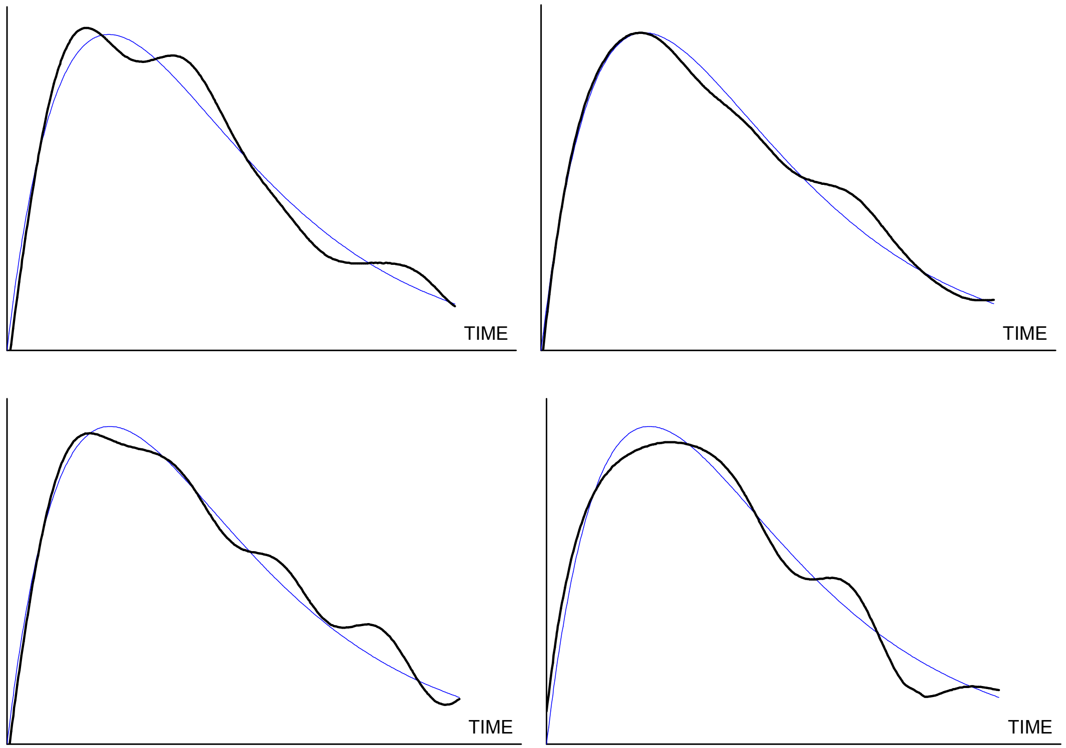
Measurements taken at 4 different times:

Noiseless signal superimposed for comparison

S/N = 20

Noise affects

- Peak signal
- Time distribution



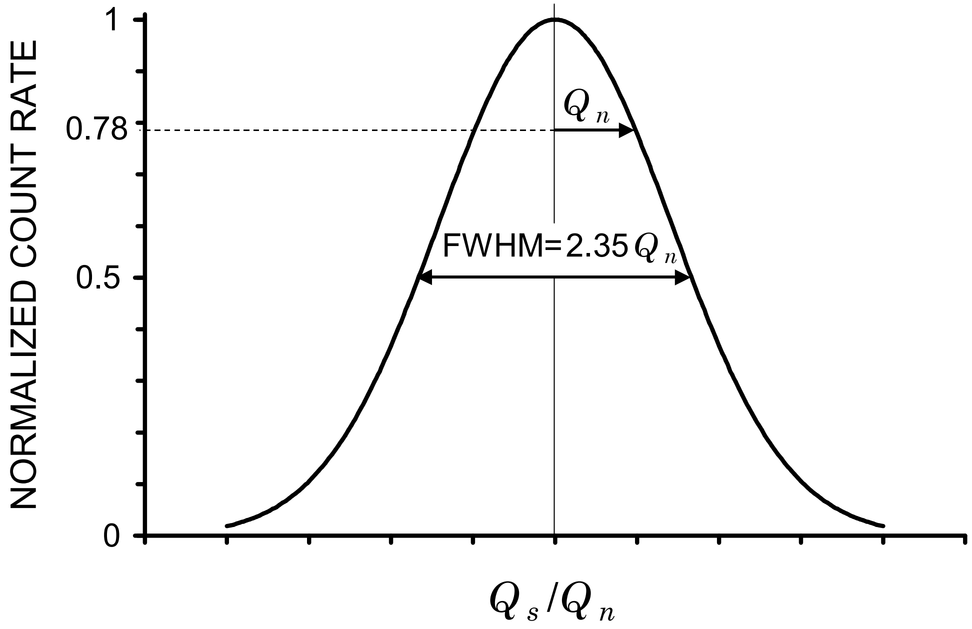
Electronic noise is purely random.

⇒ amplitude distribution is Gaussian

⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has Gaussian distribution



Measuring Resolution

Inject an input signal with known charge using a pulse generator set to approximate the detector signal shape.

Measure the pulse height spectrum.

peak centroid ⇒ signal magnitude
peak width ⇒ noise (FWHM= 2.35 Q_n)

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

Basic Noise Mechanisms and Characteristics

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\ e\ v}{l}$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2 ,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or "1/f" noise

Thermal noise and shot noise are both "white" noise sources, i.e.

power per unit bandwidth (≡ spectral density) is constant: $$\frac{dP_{noise}}{df} = const.$$

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f: $\frac{dP_{noise}}{df} = 4kT$

$k =$ Boltzmann constant

$T =$ absolute temperature

$R =$ DC resistance

since $P = \frac{V^2}{R} = I^2 R$

the spectral noise voltage density $\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$

and the spectral noise current density $\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$

The total noise depends on the bandwidth of the system,
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

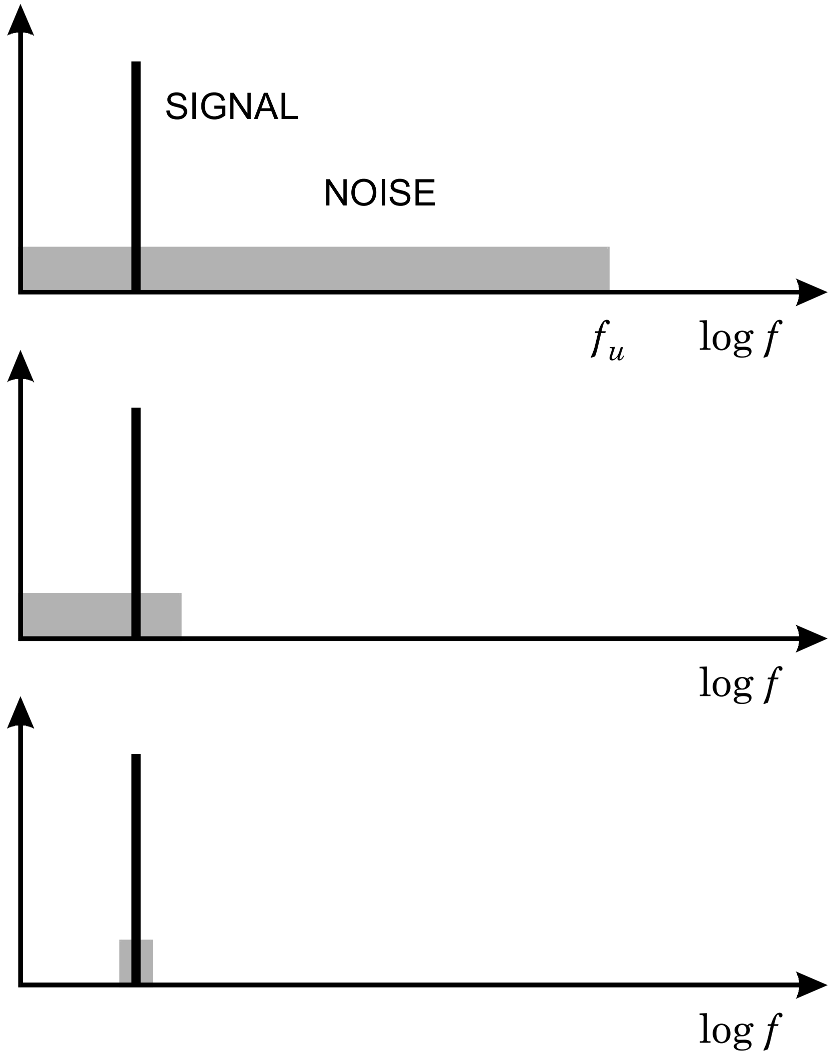
$$v_{on}^2 = \int_0^\infty e_n^2 \, A_v^2(f) \, df$$

Note: Since spectral noise components are not correlated, one must integrate over the noise power.

Total noise increases with bandwidth.

Total noise is the integral over the shaded region.

S/N increases as noise bandwidth is reduced until signal components are attenuated significantly.



2. Shot noise

A common example of noise due to number fluctuations is “shot noise”, which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode (emission over a barrier)

Spectral noise current density: $i_n^2 = 2q_e I$

$q_e =$ electronic charge
$I =$ DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in “ohmic” conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

Low Frequency (“$1/f$”) Noise

In a semiconductor, for example, charge can be trapped and then released after a characteristic lifetime τ.

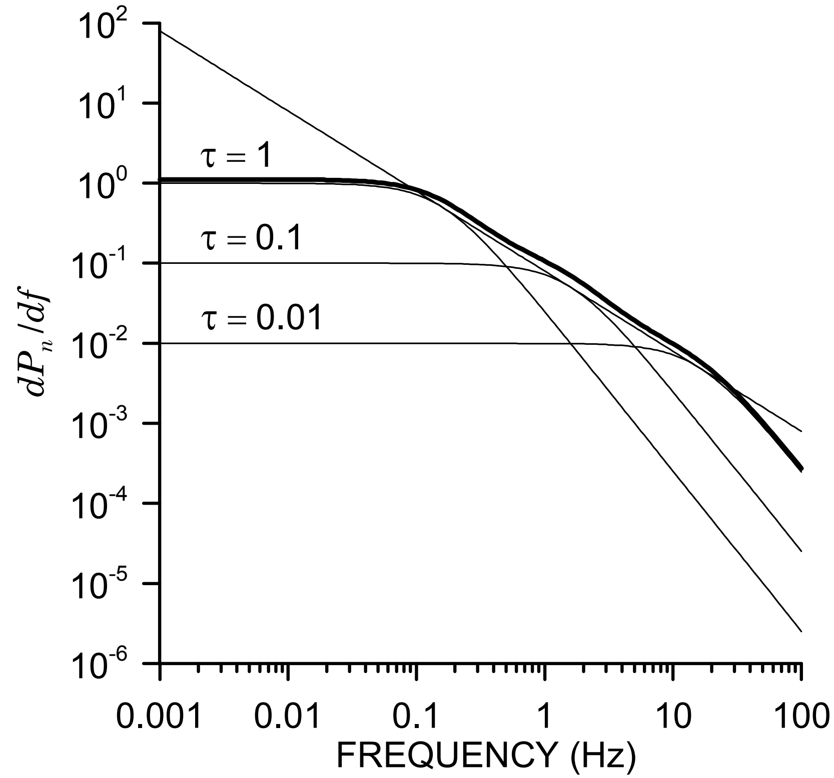
The spectral density for a single lifetime

$$S(f) \propto \frac{\tau}{1+(2\pi f\tau)^2} \; .$$

For $2\pi f\tau \gg 1$: $S(f) \propto \frac{1}{f^2}$.

However,
several traps with different time constants can yield a “$1/f$“ distribution:

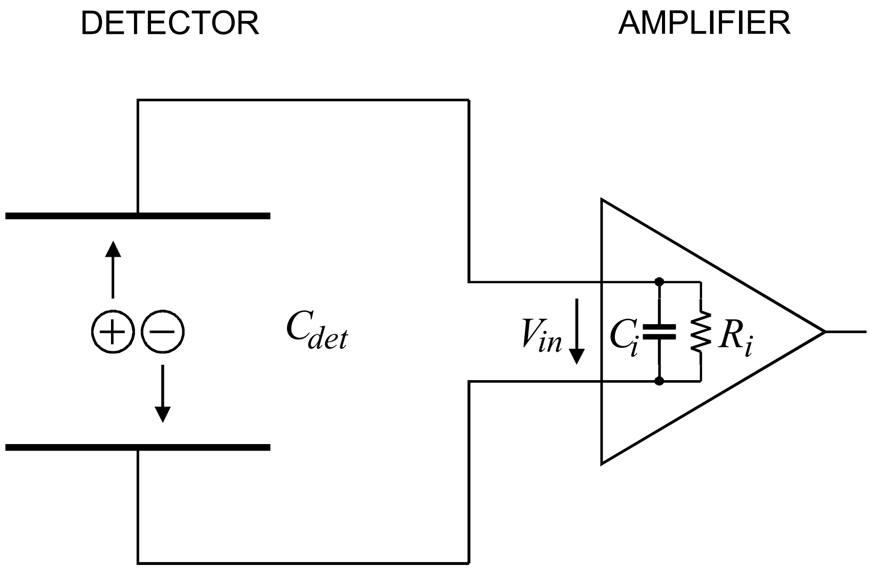
Traps with three time constants of 0.01, 0.1 and 1 s yield a $1/f$ distribution over two decades in frequency.

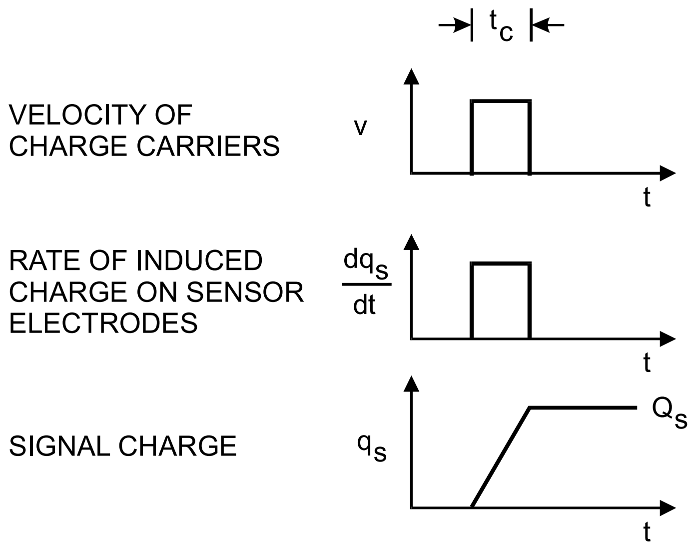


Low frequency noise is ubiquitous – must not have $1/f$ dependence, but commonly called $1/f$ noise.

Spectral power density: $$\frac{dP_{noise}}{df} = \frac{1}{f^{\alpha}}$$ (typically α = 0.5 – 2)

Signal-to-Noise Ratio vs. Detector Capacitance





if $R_i \times (C_{det} + C_i) \gg$ collection time,

peak voltage at amplifier input $V_{in} = \dfrac{Q_s}{C} = \dfrac{\int i_s dt}{C} = \dfrac{Q_s}{C_{det} + C_i}$

↑

Magnitude of voltage depends on total capacitance at input!

The peak amplifier signal V_S is inversely proportional to the **total capacitance at the input**, i.e. the sum of

1. detector capacitance,
2. input capacitance of the amplifier, and
3. stray capacitances.

Assume an amplifier with a noise voltage v_n at the input.

Then the signal-to-noise ratio

$$\frac{S}{N} = \frac{V_S}{v_n} \propto \frac{1}{C}$$

- However, S/N does not become infinite as $C \to 0$
  (then front-end operates in current mode)

- The result that $S/N \propto 1/C$ generally applies to systems that measure signal charge.

Charge-Sensitive Amplifier – Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,

  i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.

- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i \gg C_d$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_v$.

- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

Note, that although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.

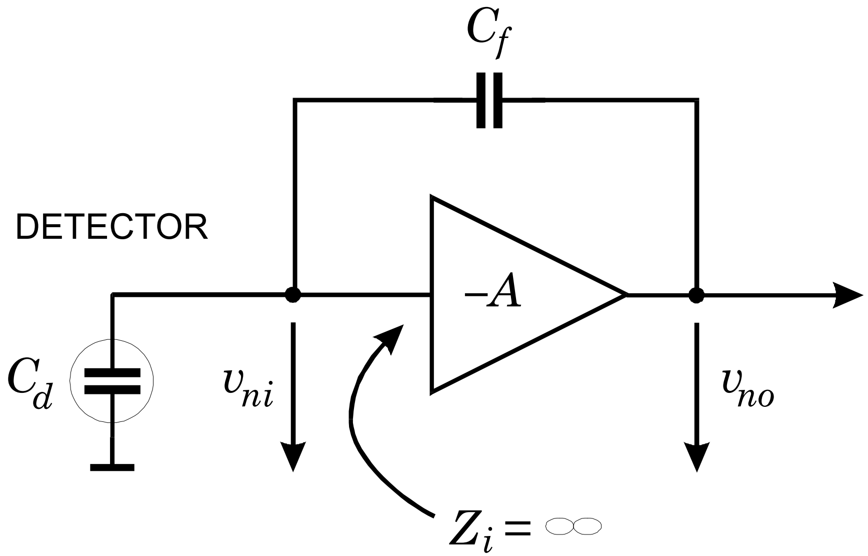
$\Rightarrow$ ***S*/*N*** **at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.

$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_d}}{X_{C_d}} = v_{ni} \frac{\frac{1}{\omega C_f} + \frac{1}{\omega C_d}}{\frac{1}{\omega C_d}}$$

$$v_{no} = v_{ni} \left(1 + \frac{C_d}{C_f} \right)$$



Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni} \left(C_d + C_f \right)$$

Signal-to-noise ratio $$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_d + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage-sensitive amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

Conclusion

In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.
- S/N cannot be *improved* by feedback.

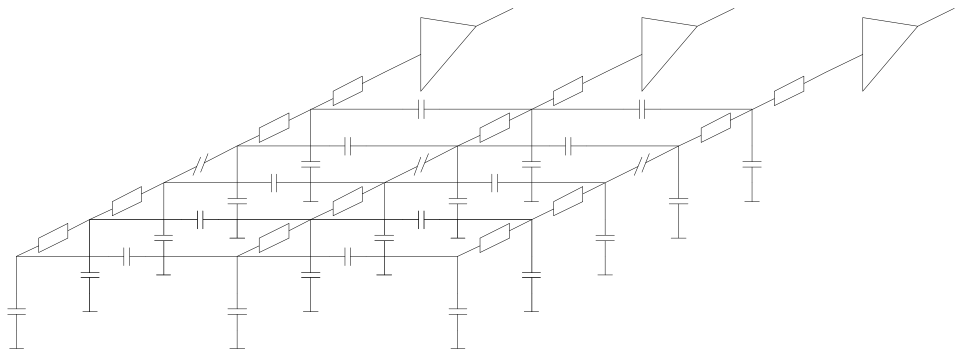
Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

Multi-electrode detectors (strips, pixels) require more complex analysis.

Strip Detector Model for Noise Simulations

Noise coupled from neighbor channels.

Analyze signal and noise in center channel.



Includes:
a) Noise contributions from neighbor channels
b) Signal transfer to neighbor channels
c) Noise from distributed strip resistance
d) Full SPICE model of preamplifier

See Spieler, *Semiconductor Detector Systems* for discussion of noise cross-coupling

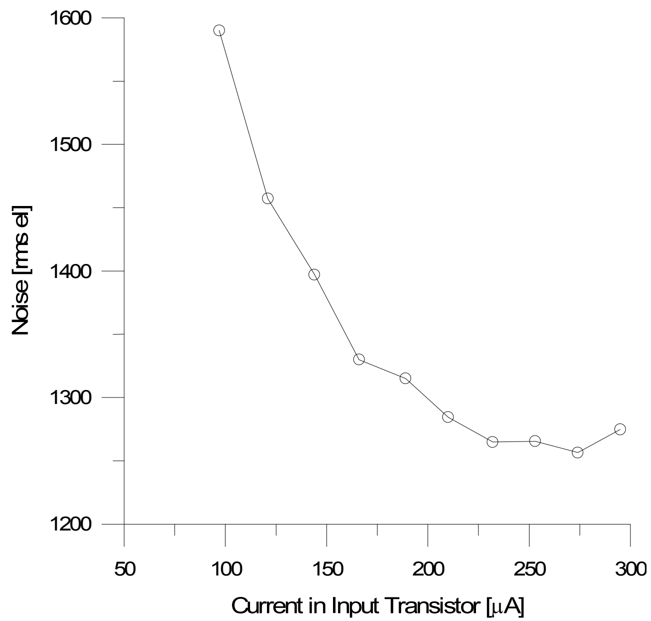
Measured Noise of Module:

p-strips on n-bulk, BJT input transistor

Simulation Results: 1460 el (150 μA)

1230 el (300 μA)

⇒ Noise can be predicted with good accuracy.



3. Pulse Processing

Two conflicting objectives:

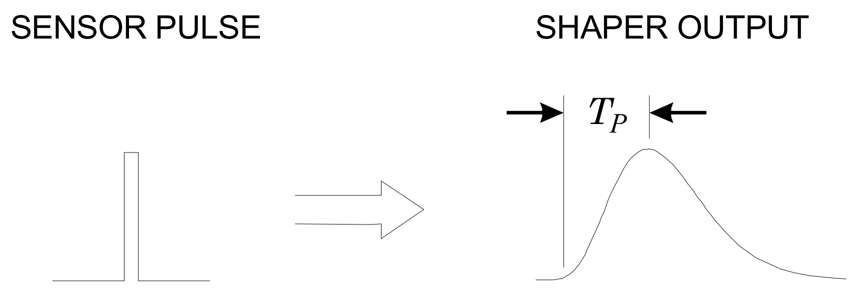
1. Improve Signal-to-Noise Ratio S/N

Restrict bandwidth to match measurement time $\Rightarrow$ Increase pulse width

Typically, the pulse shaper transforms a narrow detector current pulse to

a broader pulse (to reduce electronic noise),

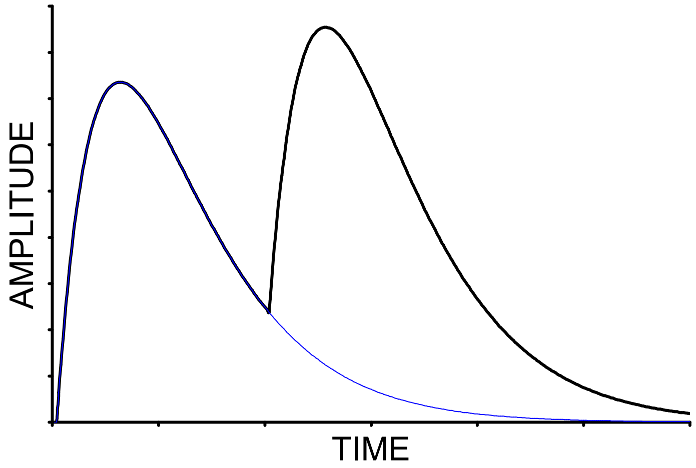
with a gradually rounded maximum at the peaking time T_P
(to facilitate measurement of the peak amplitude)



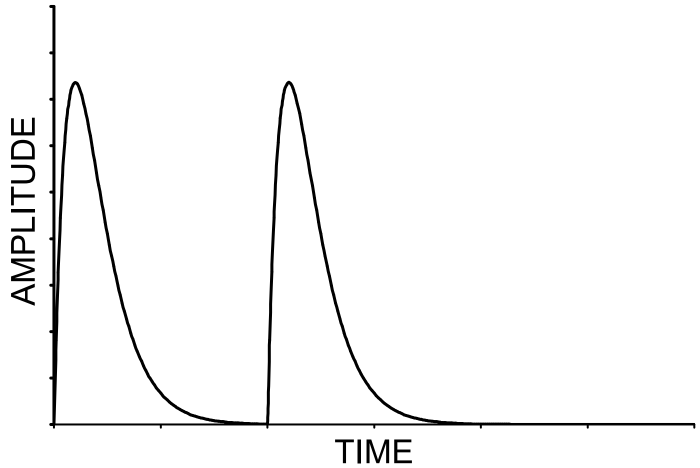
If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or pulse height analysis. The pulse height spectrum is the energy spectrum.

2. Improve Pulse Pair Resolution ⇒ Decrease pulse width

Pulse pile-up distorts amplitude measurement.



Reducing pulse shaping time to 1/3 eliminates pile-up.

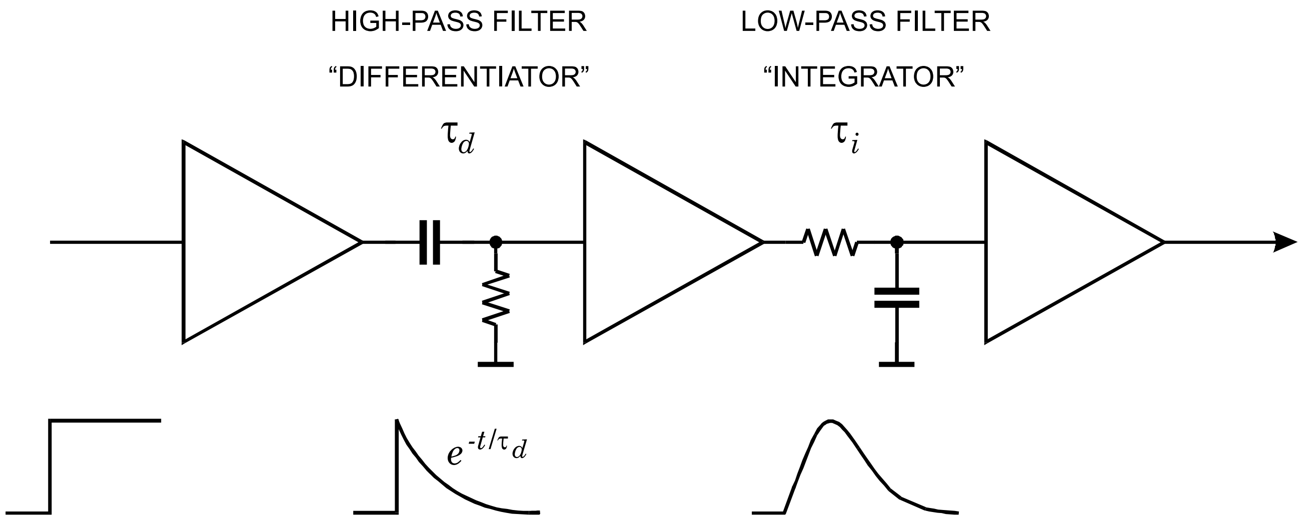


Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

Usually, many considerations combined lead to a “non-textbook” compromise.

- *“Optimum shaping” depends on the application!*
- Shapers need not be complicated – *Every amplifier is a pulse shaper!*

Simple Example: CR-RC Shaping



Simple arrangement: Noise only 36% worse than optimum filter with same time constants.

⇒ Useful for estimates, since simple to evaluate

Key elements:

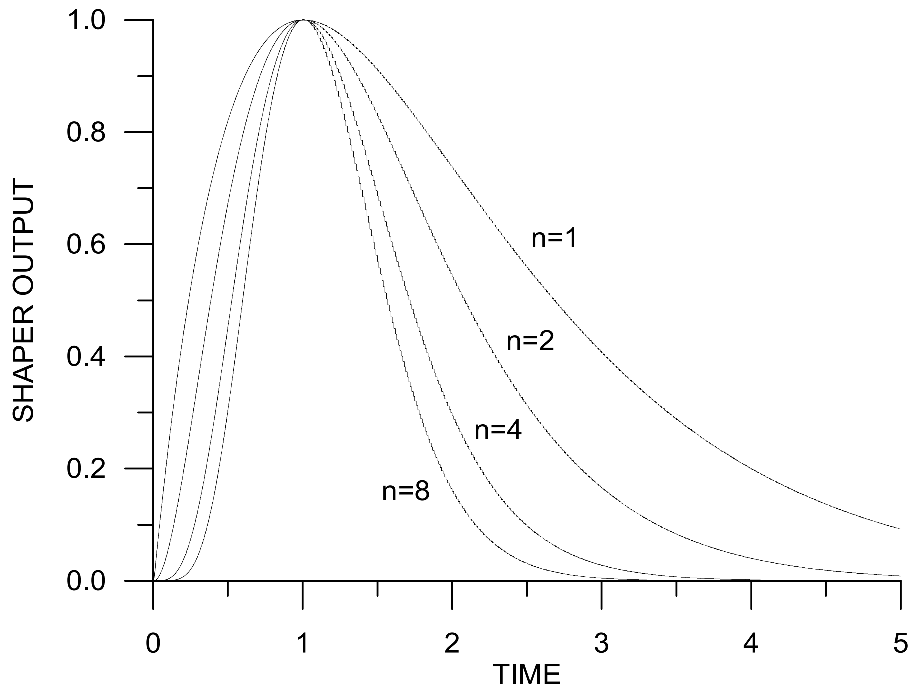
- lower frequency bound ($\triangleq$ pulse duration)
- upper frequency bound ($\triangleq$ rise time)

important in all shapers.

Analogous functions can be combined in digital signal processors.

Other Types of Shapers

Shapers with Multiple Integrators



Start with simple *CR-RC* shaper and add additional integrators ($n = 1$, $n = 2$, ... $n = 8$).

Change integrator time constants to preserve the peaking time $\tau_n = \tau_{n=1} / n$

Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

$\Rightarrow$ Improved rate capability at the same peaking time.

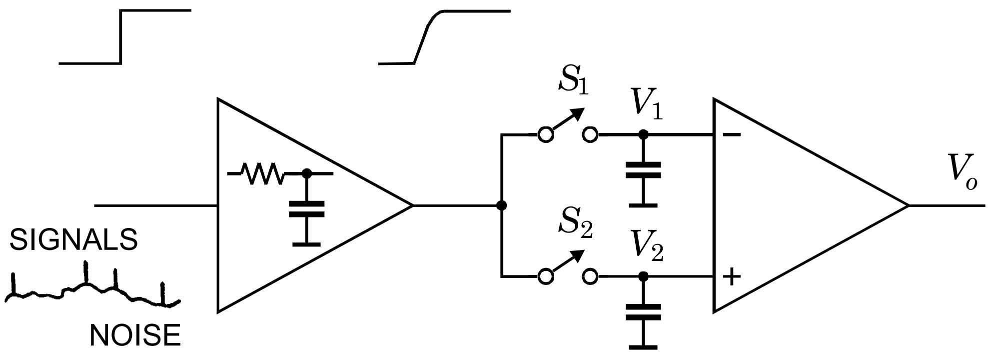
Shapers with the equivalent of 8 *RC* integrators are common.
Usually, this is achieved with active filters,

(i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

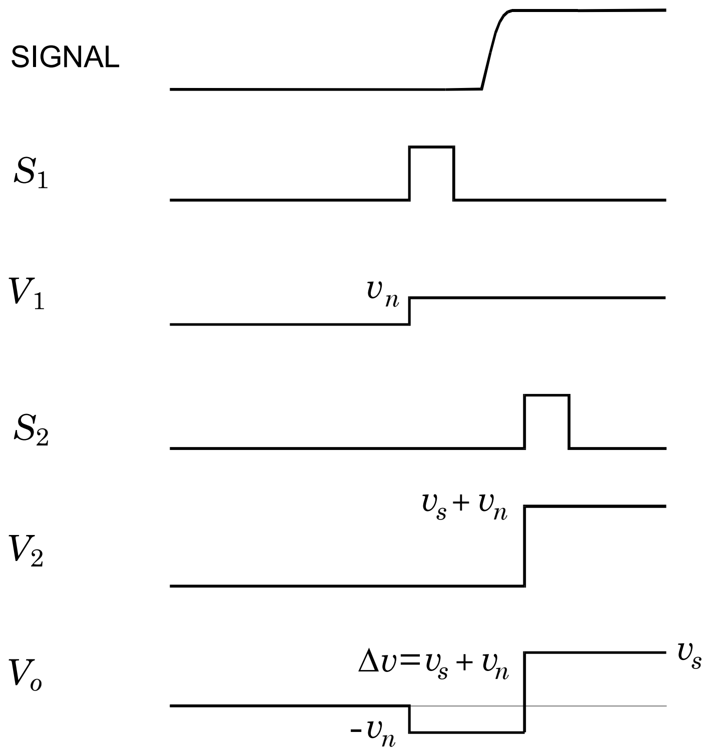
Time-Variant Shapers

Time variant shapers change the filter parameters during the processing of individual pulses.

A commonly used time-variant filter is the correlated double-sampler.



1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal



S/N depends on

1. time constant of prefilter
2. time difference between samples

See "Semiconductor Detector Systems" for a detailed noise analysis.

Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Pulse duration inversely proportional to bandwidth

$\Rightarrow$ fast shaping $\Rightarrow$ higher bandwidth $\Rightarrow$ increased thermal noise

Equivalent Noise Charge

- The primary signal quantity is a current.
- The basic quantities of electronic noise are current and voltage.
- However, the absorbed energy translates into signal charge.
- Signal processing integrates the signal current and noise spectrum to yield a measured signal proportional to charge (i.e. energy).
- Output noise can be characterized as an equivalent charge.
- Referred to the input, this is the **equivalent noise charge** (ENC).

Measurement: Inject known signal charge into preamp input (either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge $\equiv$ Input charge for which $S/N = 1$.

Ballistic Deficit

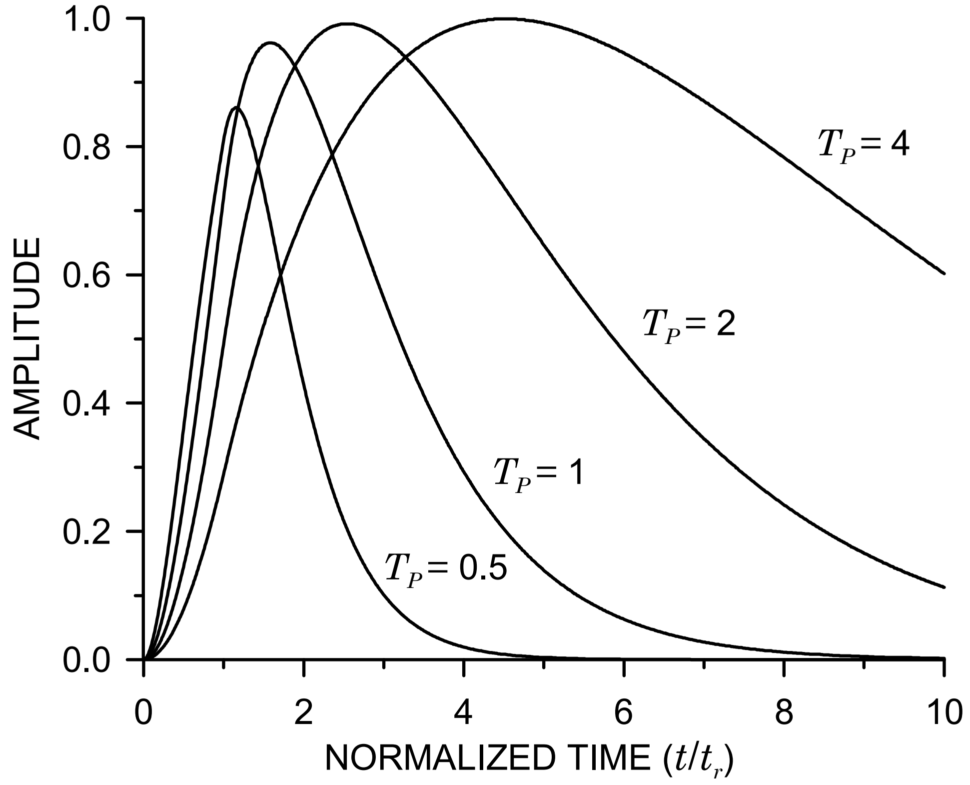
When the rise time of the input pulse to the shaper extends beyond the nominal peaking time, the shaper output is both stretched in time and the amplitude decreases

Shaper output for an input rise time $t_r = 1$ for various values of nominal peaking time.

Note that the shaper with $T_P = 0.5$ peaks at $t = 1.15 t_r$

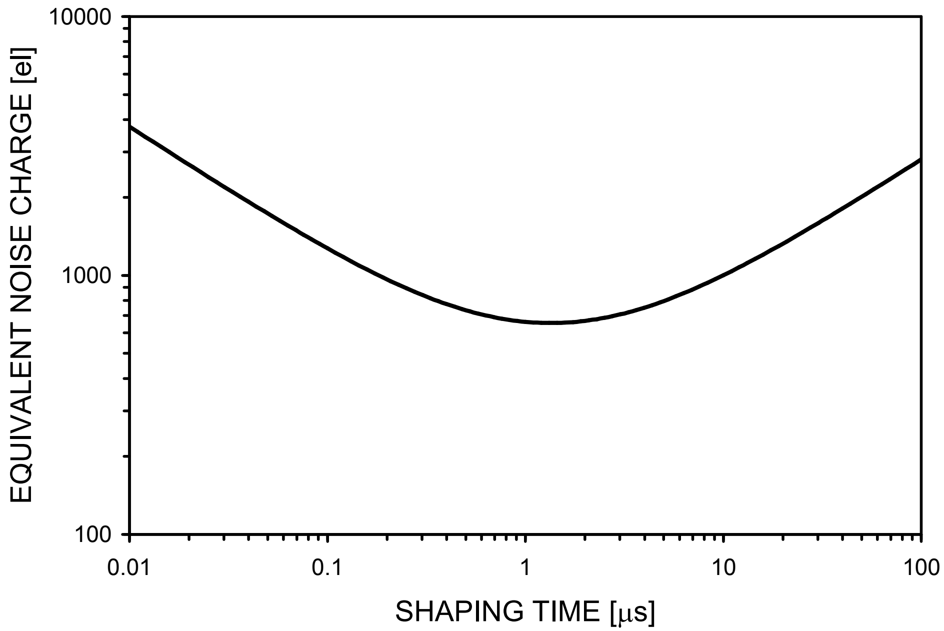
and

attains only 86% of the pulse height achieved at longer shaping times.



Ballistic deficit $\Rightarrow$ increased ENC

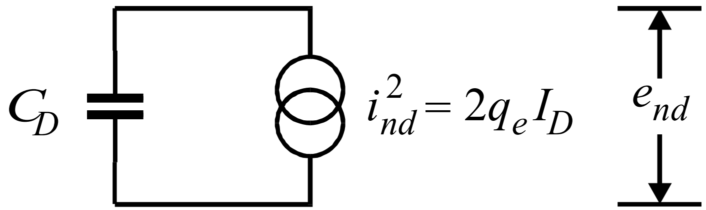
Dependence of Equivalent Noise Charge on Shaping Time



Noise sources (thermal and shot noise) have a flat (“white”) frequency distribution.

Why doesn’t the noise decrease monotonically with increasing shaping time (decreasing bandwidth)?

Shot noise current from detector flows through detector capacitance.



This yields the voltage

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega\, C_D)^2} = 2q_e I_D \frac{1}{(\omega\, C_D)^2}\,.$$

⇒ The noise contribution decreases with increasing frequency (shorter shaping time)

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of "counting electrons".

Assume an ideal integrator that records all charge uniformly within a time T. The number of electron charges measured is

$$N_e = \frac{I_D T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are "counted"?

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of "counting electrons" is actually the counting of zero crossings, which in a detailed analysis yields the same result.

Equivalent Noise Charge of a System

Two basic noise mechanisms: input noise current i_n
input noise voltage e_n

Equivalent Noise Charge: $$Q_n^2 = i_n^2\ T_S\ F_i + C^2\ e_n^2\ \frac{F_v}{T_S}$$

Where T_S Characteristic shaping time (*e.g.* peaking time)

$F_i,\ F_v$ "Shape Factors" that are determined by the shape of the pulse.

C Total capacitance at the input node (detector capacitance + input capacitance of preamplifier + stray capacitance + …)

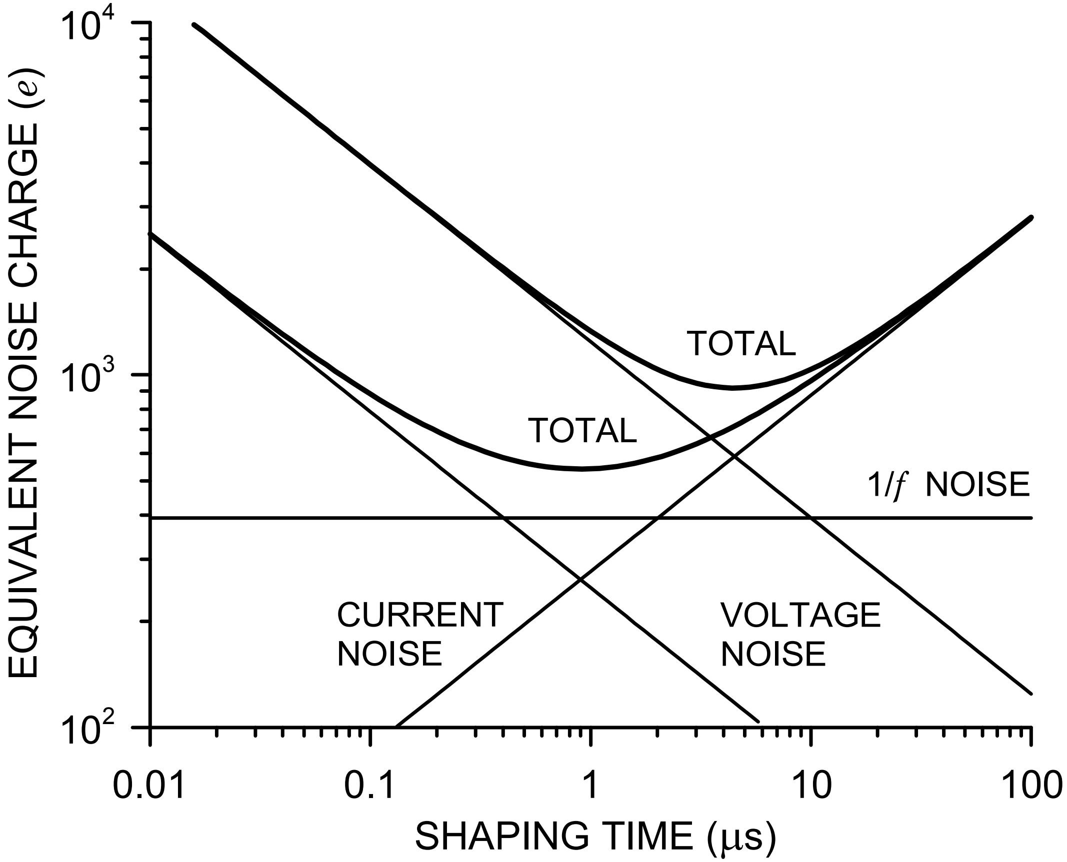
Typical values of F_i , F_v

CR-RC shaper	F_i = 0.924	F_v = 0.924
CR-(RC)^4 shaper	F_i = 0.45	F_v = 1.02
CR-(RC)^7 shaper	F_i = 0.34	F_v = 1.27
CAFE chip	F_i = 0.4	F_v = 1.2

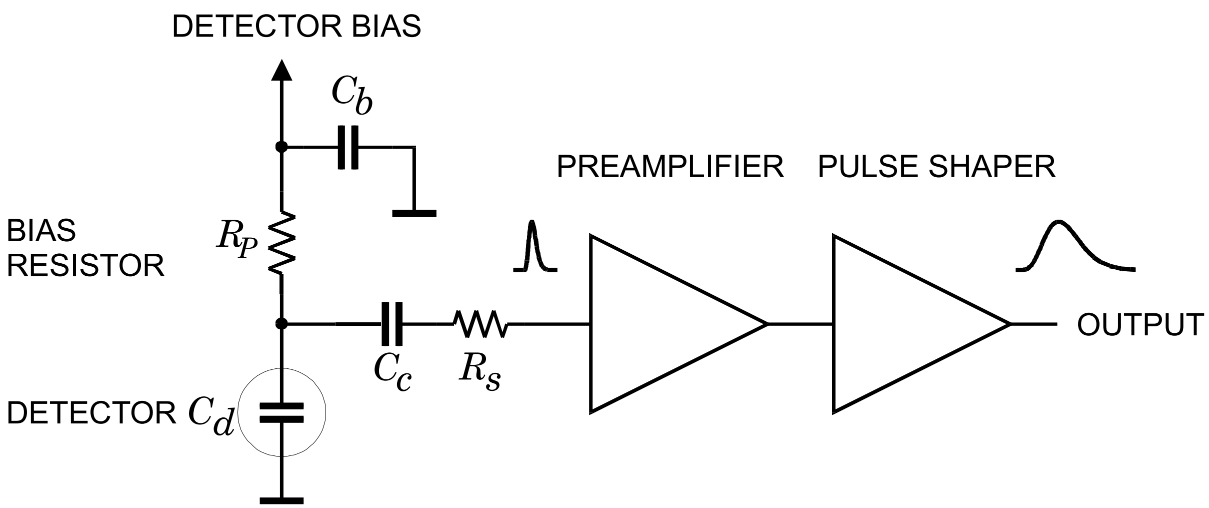
Note that $F_i < F_v$ for higher order shapers.

Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).
Minimum noise obtains when the current and voltage noise contributions are equal.

Equivalent Noise Charge vs. Shaping Time



For a practical circuit:



ENC using a simple CR-RC shaper with peaking time T:

$$Q_n^2 \approx \left[\left(2q_e I_d + \frac{4kT}{R_P} + i_{na}^2\right)\cdot T \;+\; \left(4kTR_S + e_{na}^2\right)\cdot \frac{C_d^2}{T} \;+\; 4A_f C_d^2\right]$$

current noise	voltage noise	$1/f$ noise
$\propto \tau$	$\propto 1/T$	independent of T
independent of C_d	$\propto C_d^2$	$\propto C_d^2$

$$Q_n^2 \approx \left[\left(2q_e I_d + \frac{4kT}{R_P} + i_{na}^2 \right) \cdot T \;+\; \left(4kTR_S + e_{na}^2 \right) \cdot \frac{C_d^2}{T} \;+\; 4A_f C_d^2 \right]$$

↑ current noise ↑ voltage noise ↑ $1/f$ noise

- Current noise depends on detector bias current I_d and resistance R_P shunting the input.

  Also includes noise current amplifier (spectral density i_{na}).

  Increases with detector size, strongly temperature dependent.

  Is independent of detector capacitance, consistent with the notion of “counting electrons”.

- Voltage noise increases with detector capacitance (reduced signal voltage).

  Nose sources: Amplifier voltage noise (spectral density e_{na}) and series resistance R_S

- $1/f$ noise is independent of shaping time.

  In general, the total noise of a $1/f$ source depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth.

  For a given shaper topology this ratio remains constant.

- detector leakage current and FET noise decrease with temperature

  ⇒ Si and Ge detectors for x-rays and gammas operate at cryogenic temperatures.

Equivalent Noise Charge vs. Detector Capacitance ($C = C_d + C_a$)

$$Q_n = \sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}$$

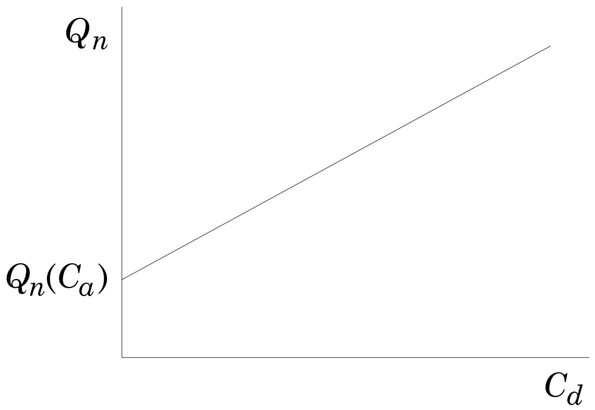
$$\frac{dQ_n}{dC_d} = \frac{2C_d e_n^2 F_v \frac{1}{T}}{\sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}}$$

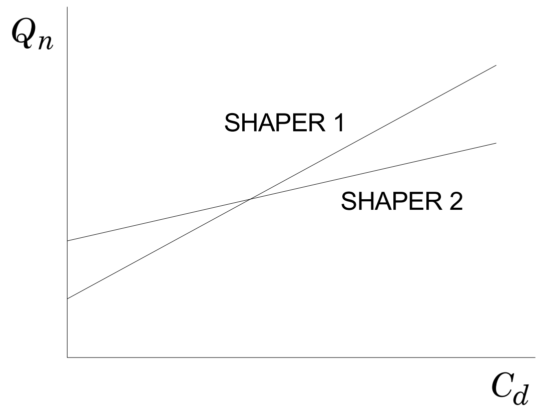
If current noise $i_n^2 F_i T$ is negligible, i.e. **voltage noise dominates:**

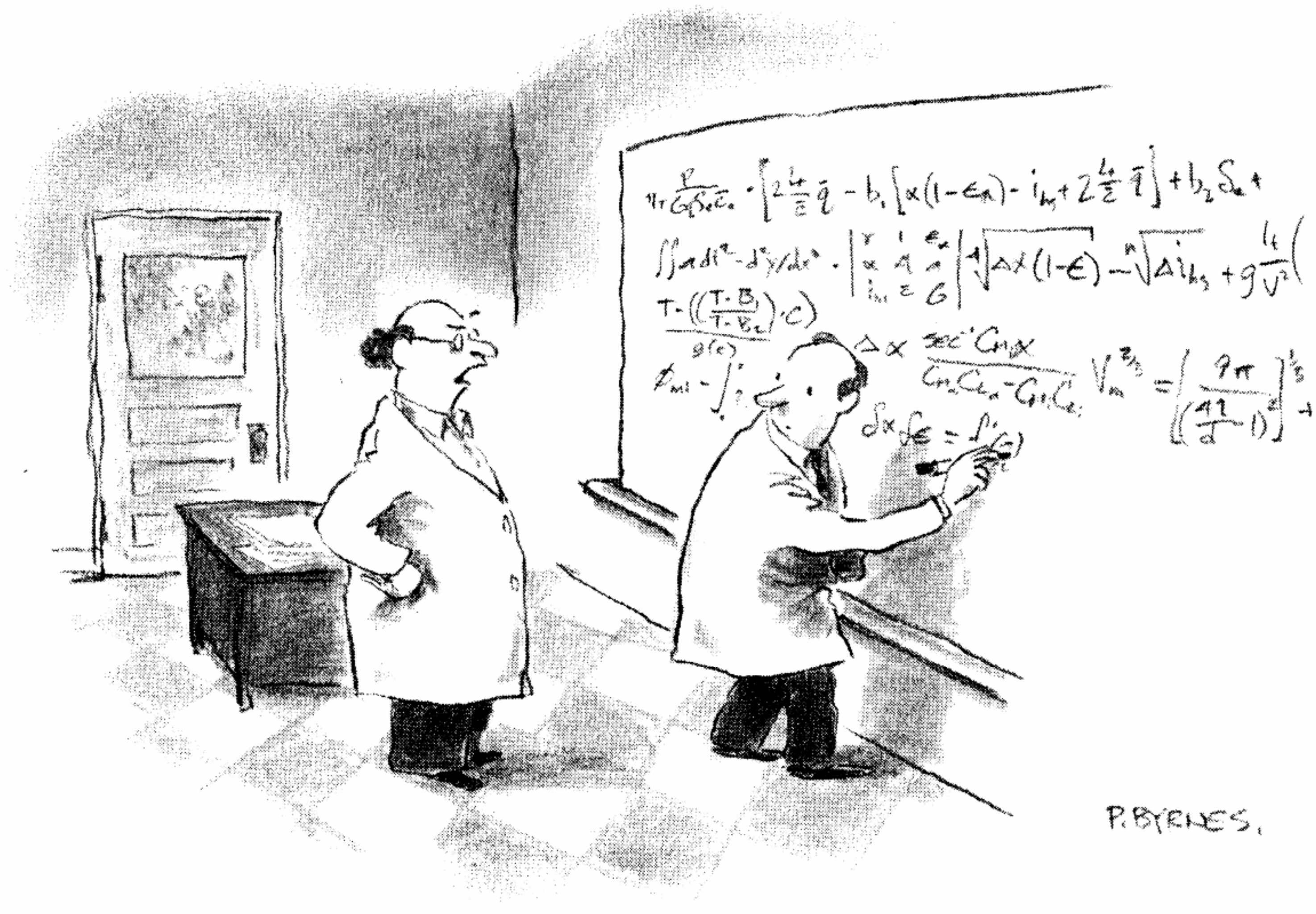
$$\frac{dQ_n}{dC_d} \approx 2e_n \cdot \sqrt{\frac{F_v}{T}}$$

↑ input stage ↑ shaper

Zero intercept: $Q_n\big|_{C_d=0} = C_a e_n \sqrt{F_v / T}$





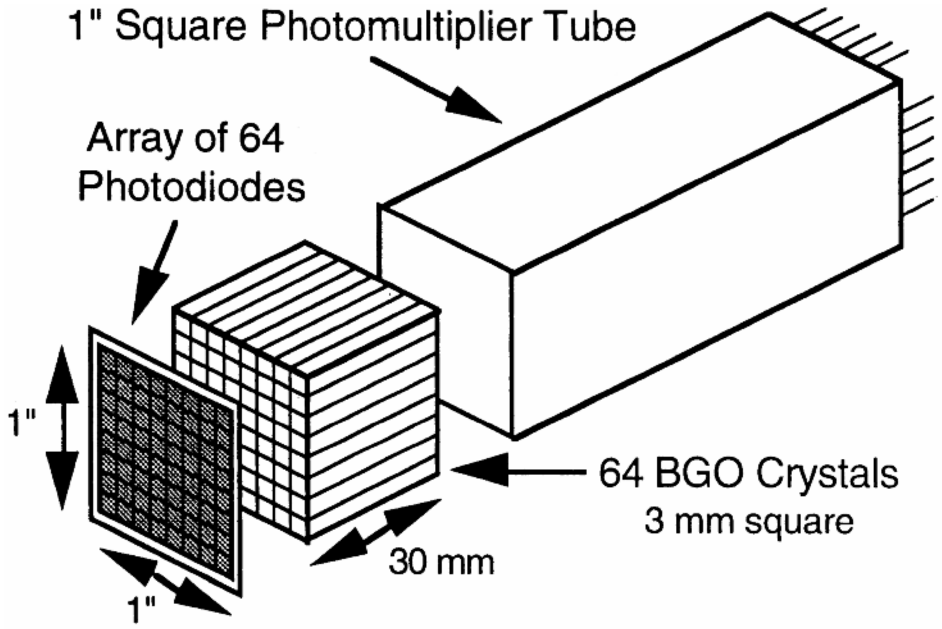


"Duh."

9. Examples: Photodiode Readout

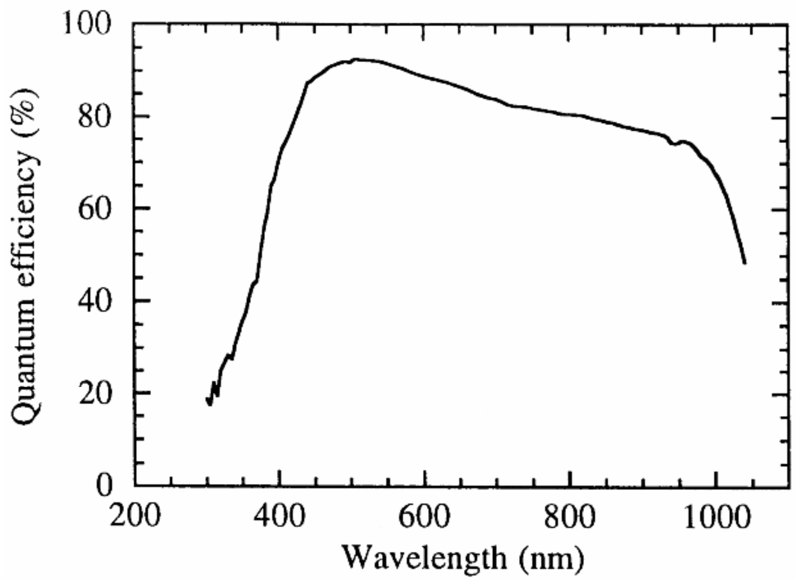
(S. Holland, N. Wang, I. Kipnis, B. Krieger, W. Moses, LBNL)

Medical Imaging (Positron Emission Tomography)



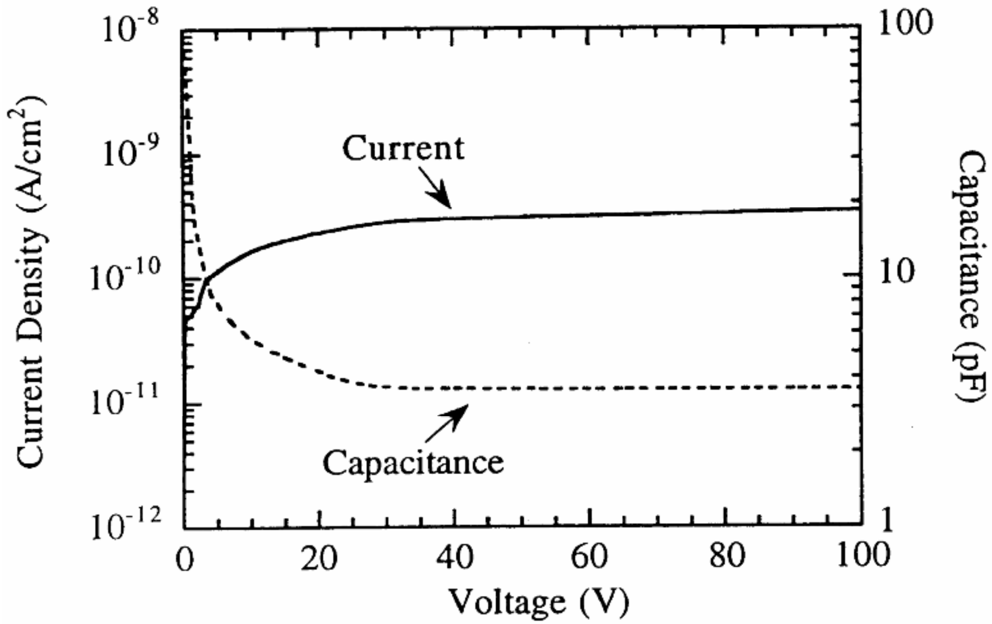
Read out 64 BGO crystals with one PMT (timing, energy) and tag crystal by segmented photodiode array.

Requires thin dead layer on photodiode to maximize quantum efficiency.



Thin electrode must be implemented with low resistance to avoid significant degradation of electronic noise.

Furthermore, low reverse bias current critical to reduce noise.

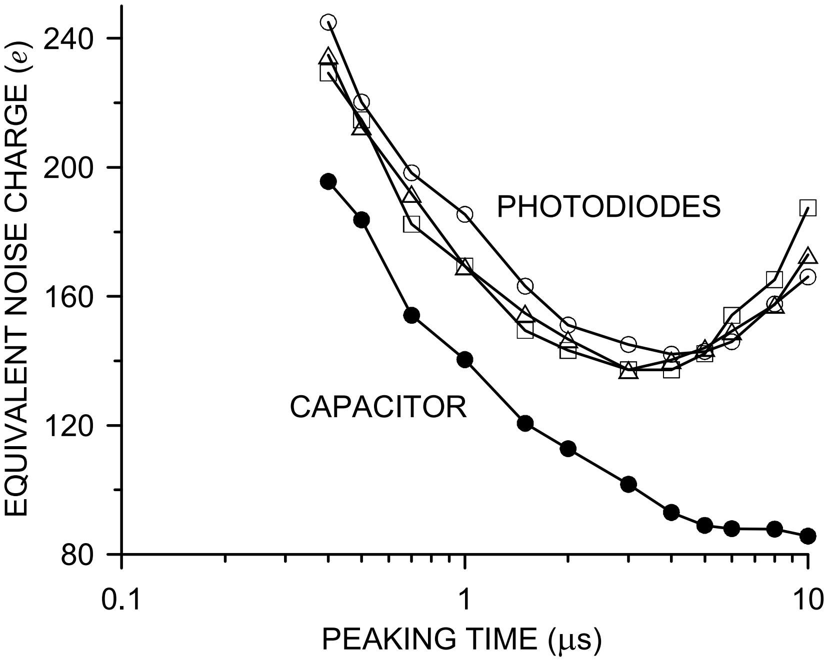


Photodiodes designed and fabricated in LBNL Microsystems Lab.

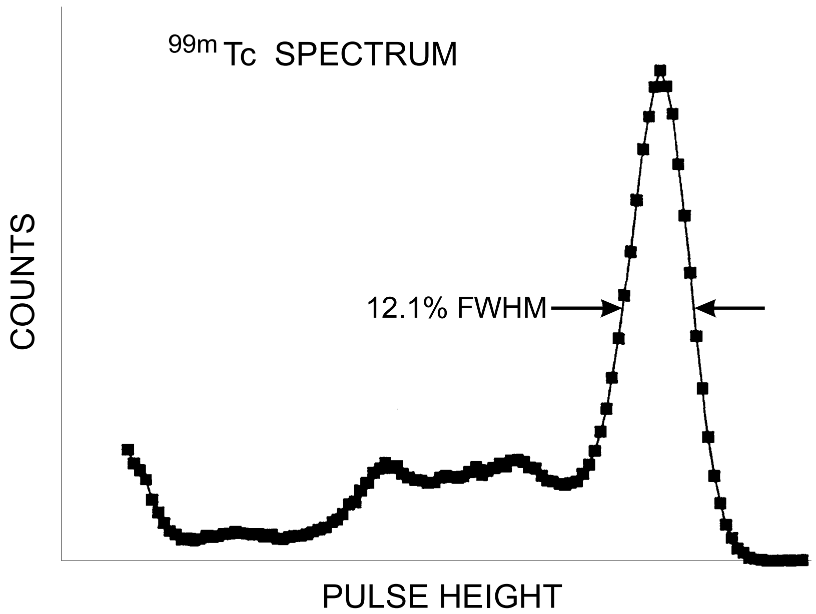
Front-end chip (preamplifier + shaper): 16 channels per chip, die size: 2 x 2 mm^2, 1.2 μm CMOS

continuously adjustable shaping time (0.5 to 50 μs)

Noise vs. shaping time



Energy spectrum with BGO scintillator



Note increase in noise at long shaping times when photodiode is connected - shot noise contribution.

Example: Short-Strip Si X-Ray Detector

(B. Ludewigt, C. Rossington, I. Kipnis, B. Krieger, LBNL)

Use detector with multiple strip electrodes not for position resolution,

but for segmentation ⇒ distribute rate over many channels

⇒ reduced capacitance

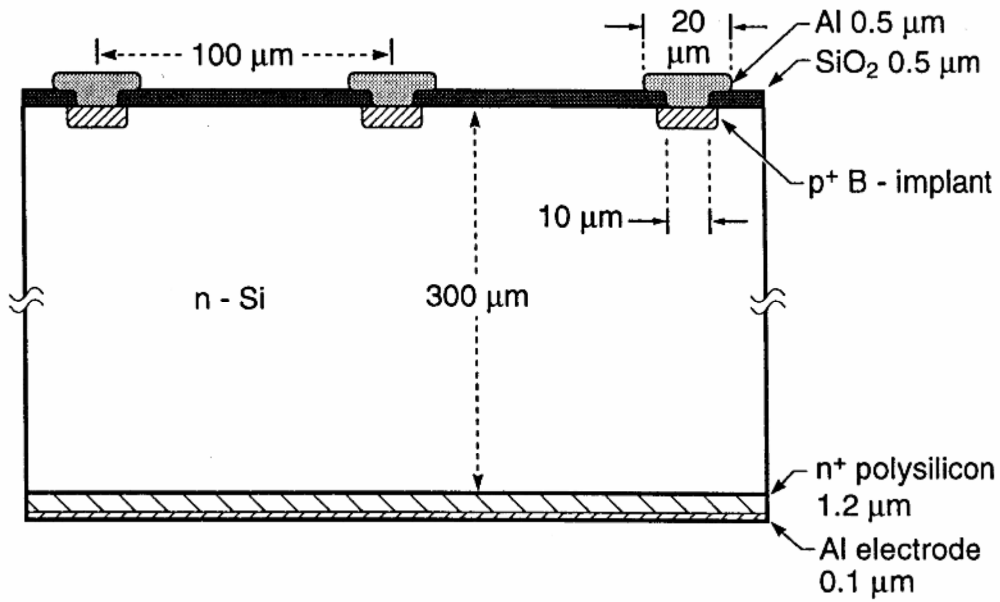
⇒ low noise at short shaping time

⇒ higher rate per detector element

For x-ray energies 5 – 25 keV ⇒ photoelectric absorption dominates (signal on 1 or 2 strips)

Strip pitch: 100 μm

Strip Length:
2 mm (matched to ALS)

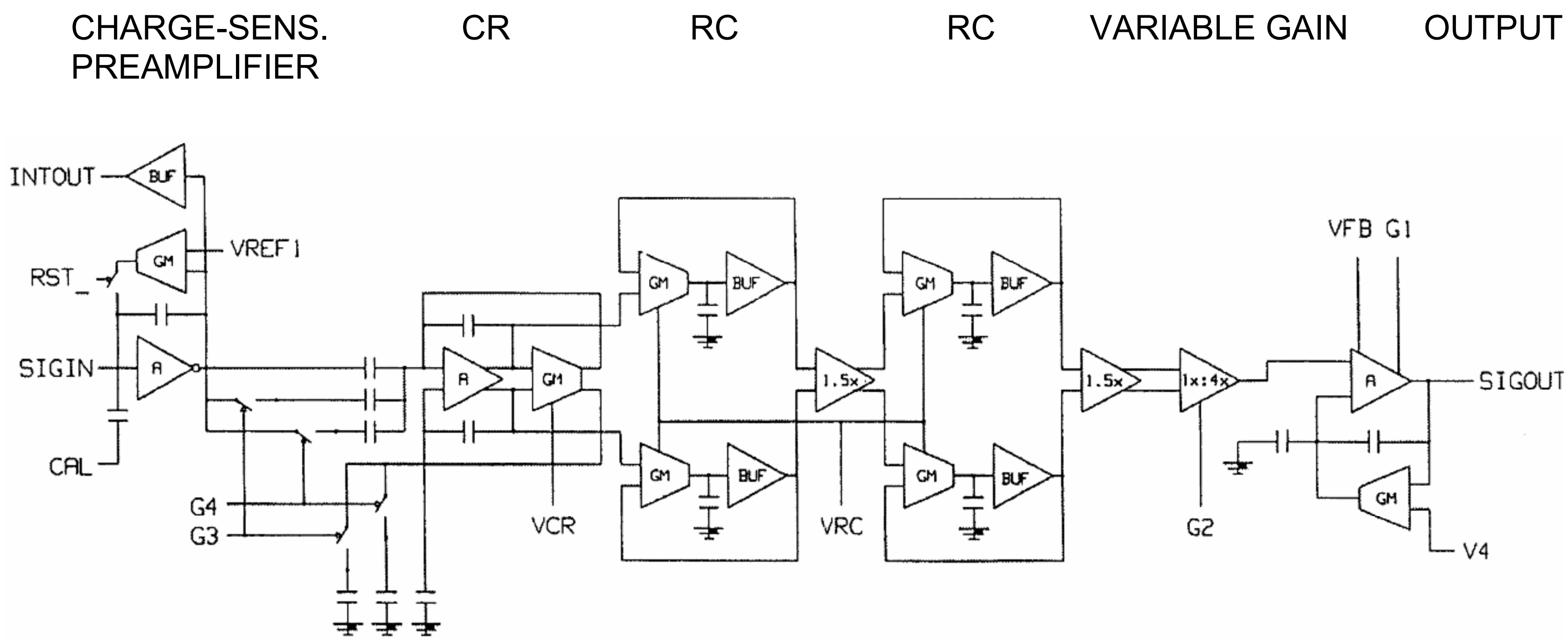


Readout IC tailored to detector

Preamplifier + CR-RC^2 shaper + cable driver to bank of parallel ADCs
(M. Maier + H. Yaver)

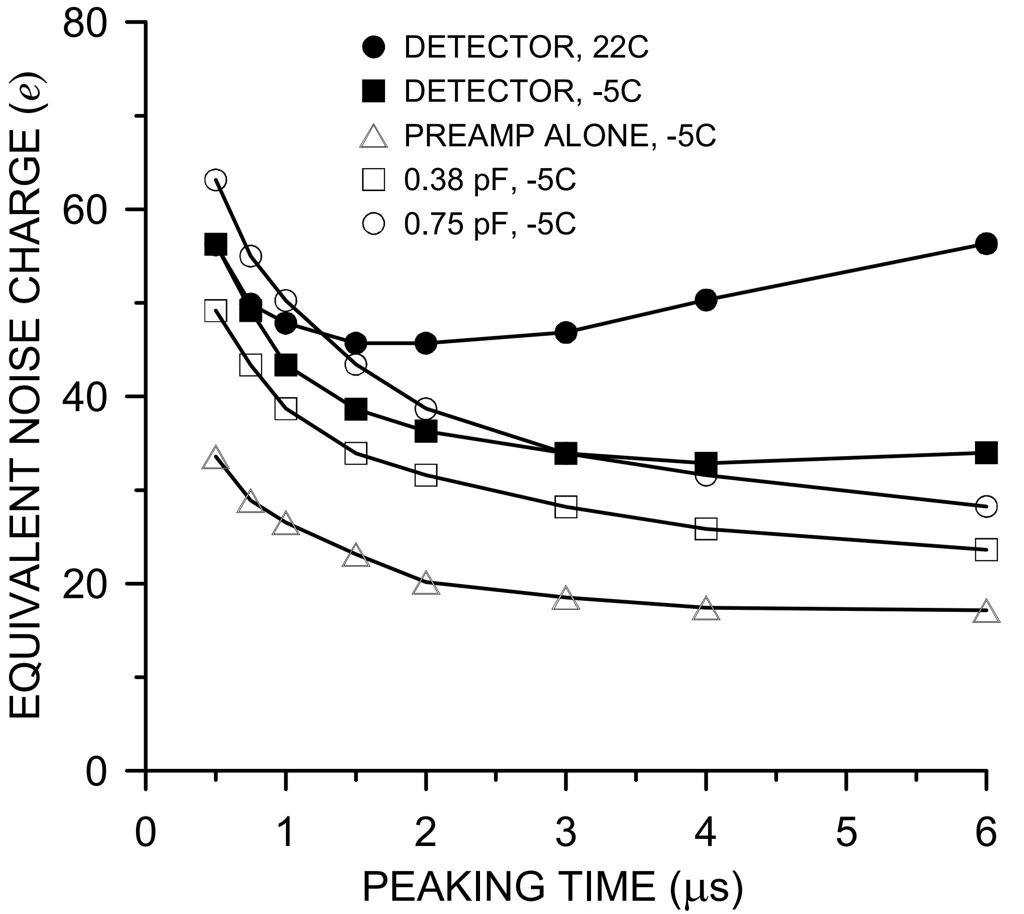
Preamplifier with pulsed reset.

Shaping time continuously variable 0.5 to 20 μs.



Noise Charge vs. Peaking Time

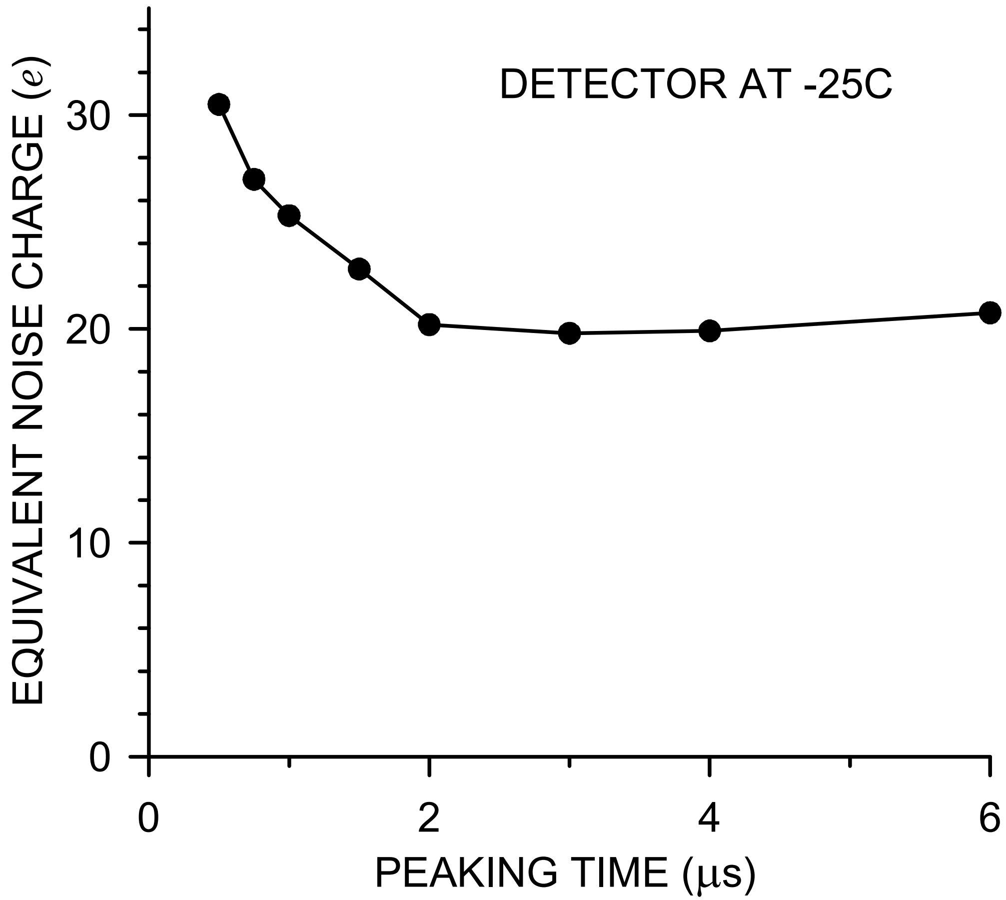
- Open symbols: preamplifier alone and with capacitors connected instead of a detector.
- Connecting the detector increases noise because of added capacitance and detector current (as indicated by increase of noise with peaking time).
- Cooling the detector reduces the current and noise improves.



Second prototype

Current noise negligible because of cooling.

“Flat” noise vs. shaping time indicates that $1/f$ noise dominates.



Numerical expression for the noise of a CR-RC shaper
(FET input, amplifier current noise negligible)

(note that some units are "hidden" in the numerical factors)

$$Q_n^2 = 12\, I_d\, \tau \;+\; 6\cdot 10^5 \frac{T}{R_P} \;+\; 3.6\cdot 10^4\, e_n^2 \frac{C^2}{T} \quad [\text{rms electrons}^2]$$

where

T shaping time constant [ns]

I_d detector bias current + amplifier input current [nA]

R_P input shunt resistance [kΩ]

e_n equivalent input noise voltage spectral density $[\mathrm{nV}/\sqrt{\mathrm{Hz}}]$

C total input capacitance [pF]

$Q_n = 1\, el$ corresponds to 3.6 eV in Si
2.9 eV in Ge

“Series” and “Parallel” Noise

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

The rest of the world uses equivalent noise voltage and current.

Since they are physically meaningful, use of these widely understood terms is preferable.

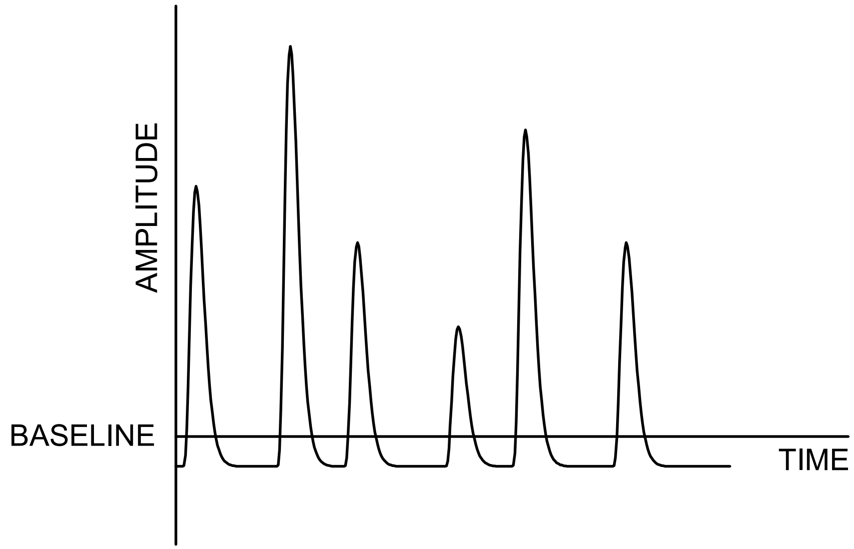
4. Some Other Aspects of Pulse Shaping

Baseline Restoration

Any series capacitor in a system prevents transmission of a DC component.

A sequence of unipolar pulses has a DC component that depends on the duty factor, i.e. the event rate.

⇒ The baseline shifts to make the overall transmitted charge equal zero.



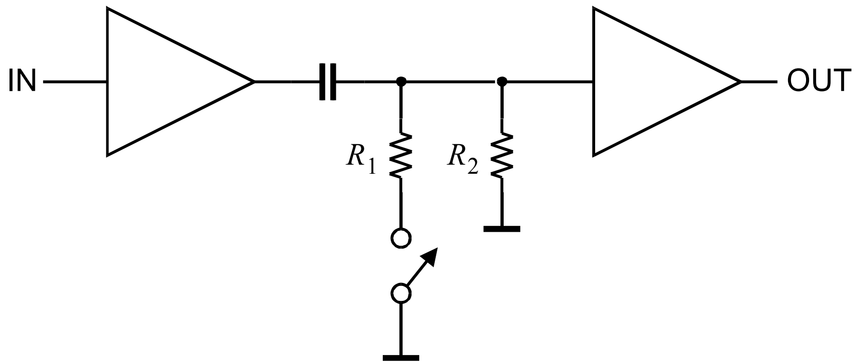
Random rates lead to random fluctuations of the baseline shift ⇒ spectral broadening

- These shifts occur whenever the DC gain is not equal to the midband gain

  The baseline shift can be mitigated by a baseline restorer (BLR).

Principle of a baseline restorer:

Connect signal line to ground during the absence of a signal to establish the baseline just prior to the arrival of a pulse.



R_1 and R_2 determine the charge and discharge time constants.
The discharge time constant (switch opened) must be much larger than the pulse width.

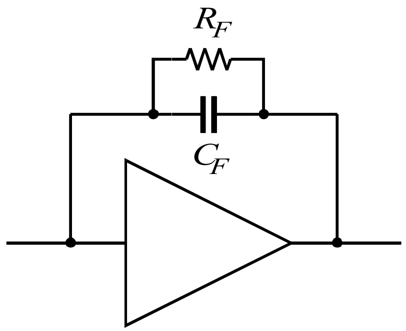
Originally performed with diodes (passive restorer), baseline restoration circuits now tend to include active loops with adjustable thresholds to sense the presence of a signal (gated restorer). Asymmetric charge and discharge time constants improve performance at high count rates.

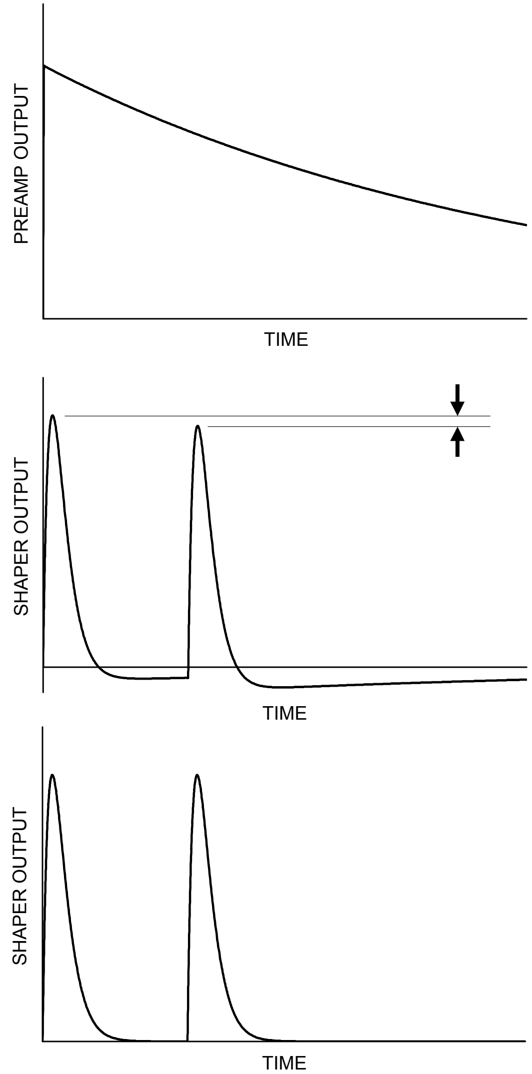
- This is a form of time-variant filtering. Care must be exercised to reduce noise and switching artifacts introduced by the BLR.

- Good pole-zero cancellation (next topic) is crucial for proper baseline restoration.

Tail (Pole Zero) Cancellation

Feedback capacitor in charge sensitive preamplifier must be discharged. Commonly done with resistor.





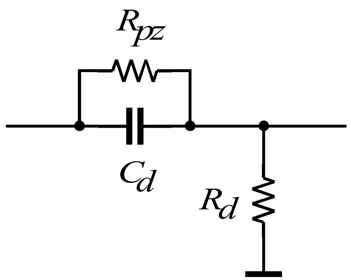
Output no longer a step, but decays exponentially

Exponential decay superimposed on shaper output.

⇒ undershoot

⇒ loss of resolution
due to baseline
variations

Add R_{pz} to differentiator:



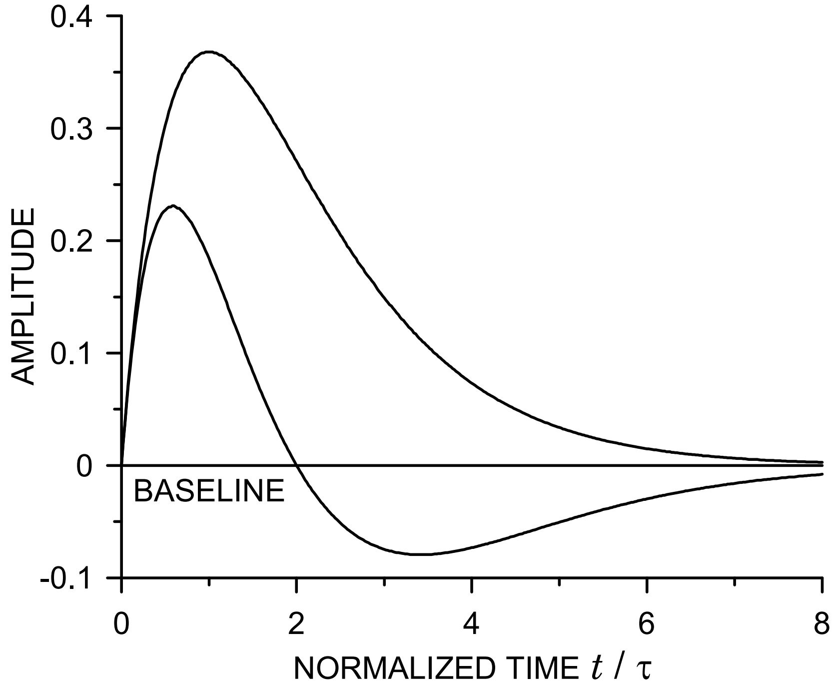
“zero” cancels “pole” of preamp when $R_F C_F = R_{pz} C_d$

Technique also used to compensate for “tails” of detector pulses: “tail cancellation”

Bipolar vs. Unipolar Shaping

Unipolar pulse + 2nd differentiator

→ Bipolar pulse



Electronic resolution with bipolar shaping typ. 25 – 50% worse than for corresponding unipolar shaper.

However …

- Bipolar shaping eliminates baseline shift (as the DC component is zero).
- Pole-zero adjustment less critical
- Added suppression of low-frequency noise (see Part 7).
- Not all measurements require optimum noise performance.
  Bipolar shaping is much more convenient for the user
  (important in large systems!) – often the method of choice.

2. Timing Measurements

Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

- Timing measurements optimize determination of time of occurrence.
- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

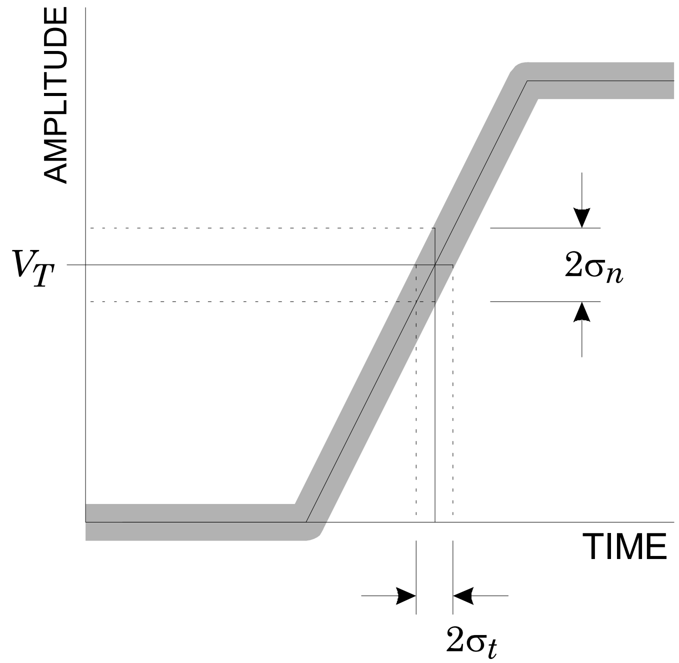
The instantaneous signal level is modulated by noise.

by

⇒ time of threshold crossing fluctuates

$$\sigma_t = \frac{\sigma_n}{\left.\frac{dV}{dt}\right|_{V_T}} \approx \frac{t_r}{S/N}$$

t_r = rise time



Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.

Pulse Shaping

Consider a system whose bandwidth is determined by a single RC integrator.

The time constant of the RC low-pass filter determines the

- rise time (and hence dV/dt)
- amplifier bandwidth (and hence the noise)

Time dependence: $V_o(t) = V_0(1 - e^{-t/\tau})$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2\ \tau$$

In terms of bandwidth

$$t_r = 2.2\ \tau = \frac{2.2}{2\pi\ f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

For a cascade of amplifiers: $t_r \approx \sqrt{t_{r1}^2 + t_{r2}^2 + \ ...\ + t_{rn}^2}$

Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude V_0 and a rise time t_c passing through an amplifier chain with a rise time t_{ra}.

1. amplifier rise time $\gg$ signal rise time:

$$\text{Noise} \propto \sqrt{f_u} \propto \sqrt{\frac{1}{t_{ra}}}$$

$$\frac{dV}{dt} \propto \frac{1}{t_{ra}} \propto f_u$$

increase in bandwidth $\Rightarrow$ improvement in dV/dt outweighs increase in noise.

2. amplifier rise time $\ll$ signal rise time

increase in noise without increase in dV/dt

Optimum: The amplifier rise time should be chosen to match the signal rise time.

Differentiation time constant: choose greater than rise time constant

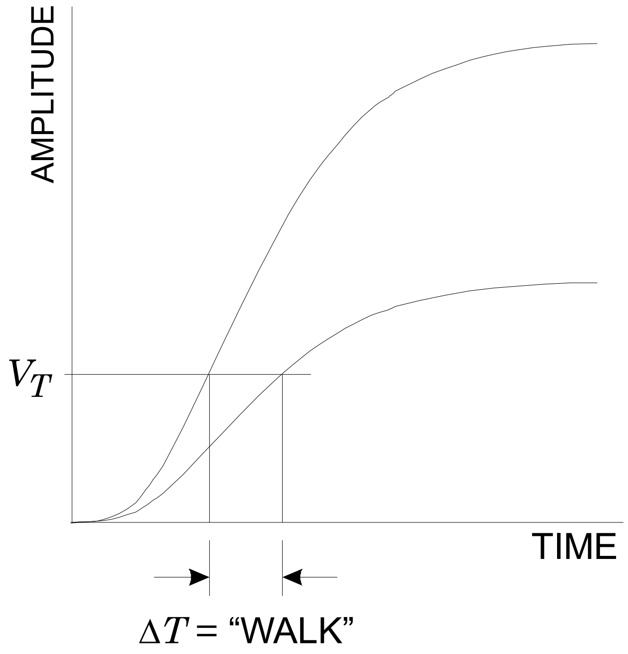
($\tau_{diff} = 10\tau_{\text{int}}$ incurs 20% loss in pulse height)

Time Walk

For a fixed trigger level the time of threshold crossing
depends on pulse amplitude.

⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)



If the rise time is known, "time walk" can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

Lowest Practical Threshold

Single RC integrator has maximum slope at t= 0: $$\frac{d}{dt}(1-e^{-t/\tau}) = \frac{1}{\tau}\ e^{-t/\tau}$$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small t the slope at the output of a single RC integrator is linear, so initially the pulse can be approximated by a ramp $\alpha\, t$.

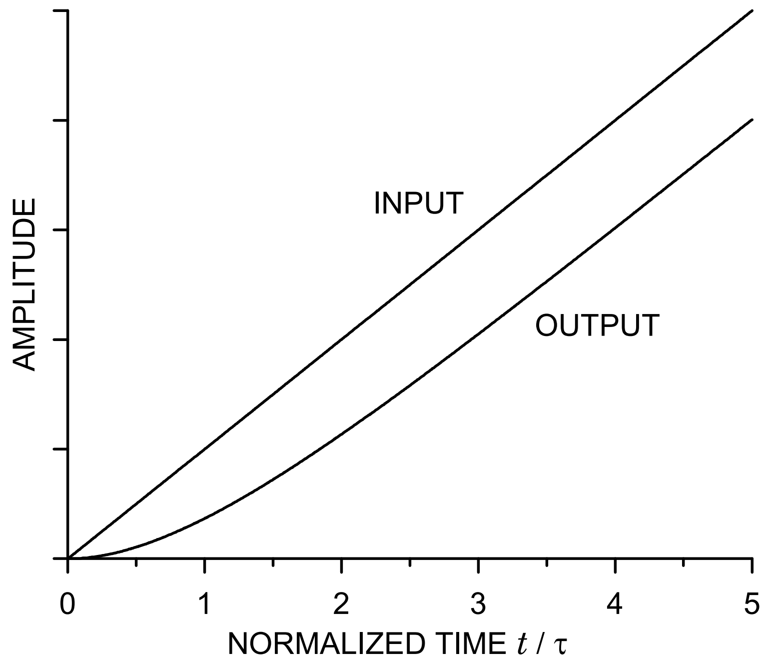
Response of the following integrator

$$V_i = \alpha\ t \quad \rightarrow \quad V_o = \alpha(t-\tau) - \alpha\ \tau\ e^{-t/\tau}$$

$\Rightarrow$ The output is delayed by τ
and curvature is introduced at small t.

Output attains 90% of input slope after $t = 2.3\tau$.

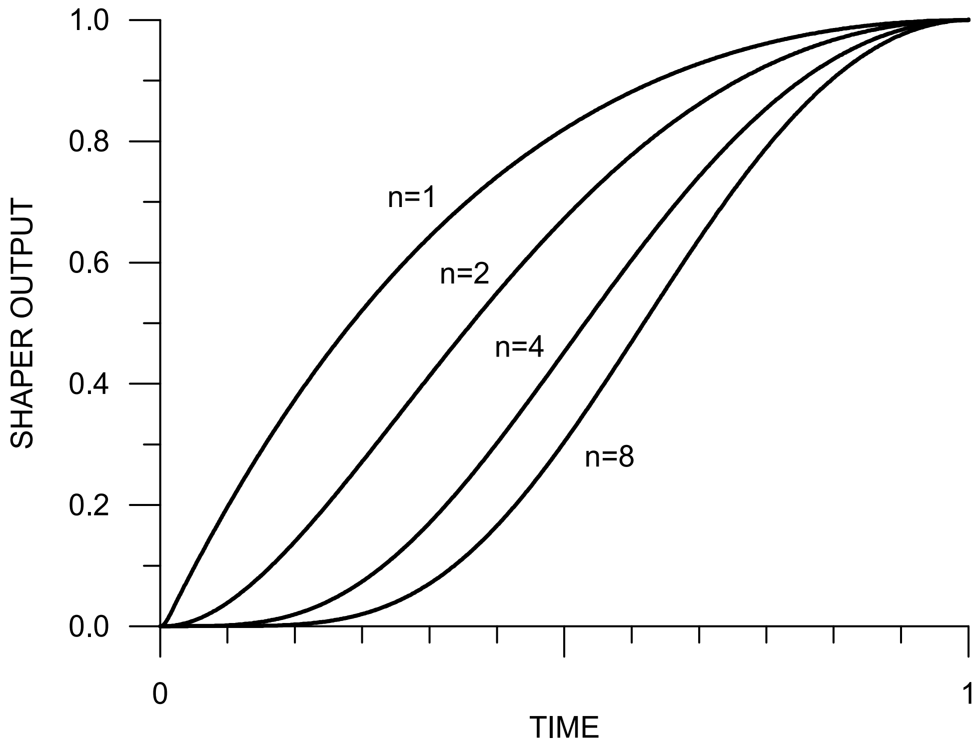
Delay for n integrators= $n\tau$



Additional RC integrators introduce more curvature at the beginning of the pulse.

Output pulse shape for multiple RC integrators

(normalized to preserve the peaking time, $\tau_n = \tau_{n-1} / n$)



Increased curvature at beginning of pulse limits the minimum threshold for good timing.

⇒ One dominant time constant best for timing measurements

Unlike amplitude measurements, where multiple integrators are desirable to improve pulse symmetry and count rate performance.

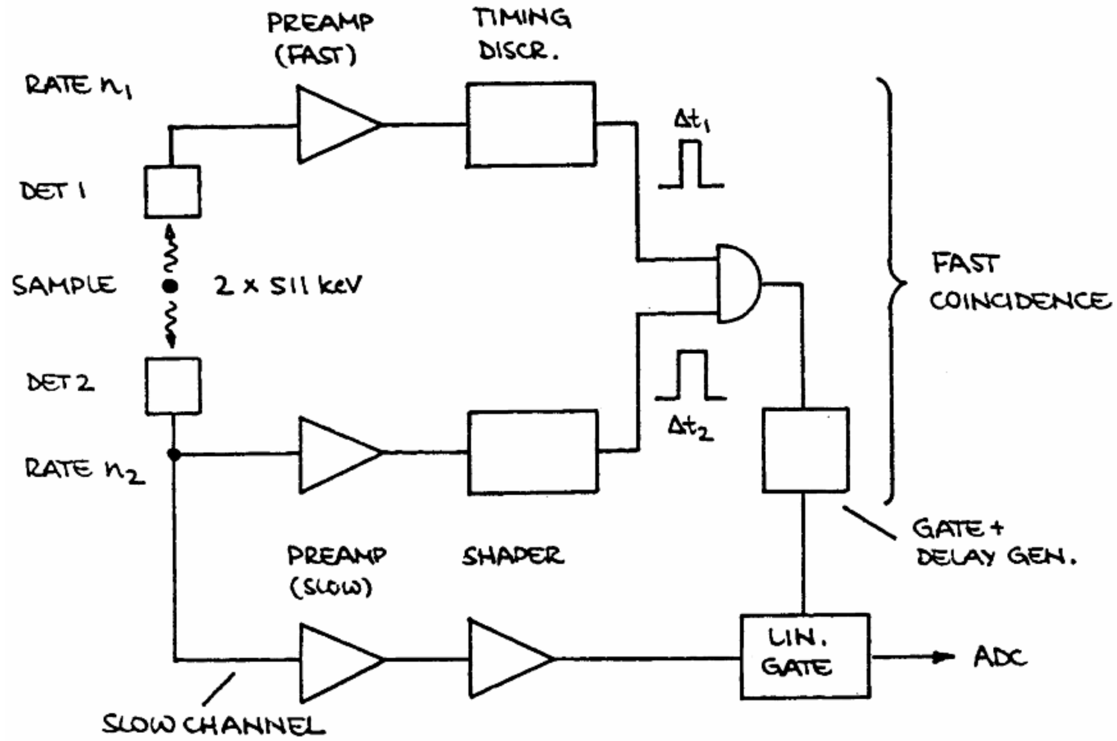
Example

γ-γ coincidence (as used in positron emission tomography)

Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.



- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output when two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.

This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are n_1 and n_2, and the timing pulse widths are Δt_1 and Δt_2, the probability of a pulse from the first source occurring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate n_2 , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$

$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

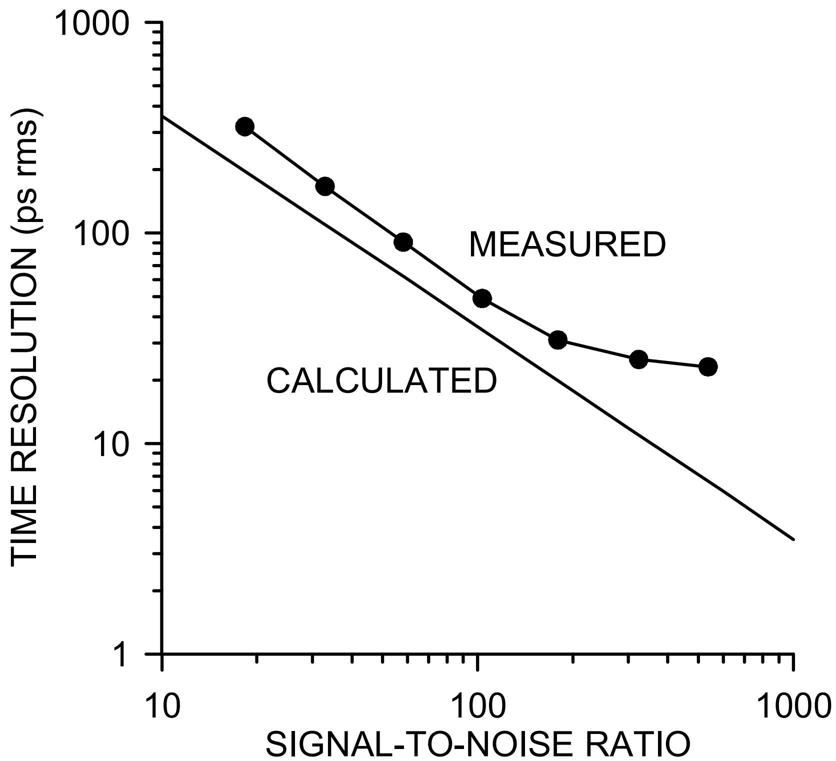
i.e. in the arrangement shown above, the chance coincidence rate increases with the *square* of the source strength.

Example: $n_1 = n_2 = 10^6\ \mathrm{s}^{-1}$

$\Delta t_1 = \Delta t_2 = 5\ \mathrm{ns} \quad \Rightarrow \quad n_c = 10^4\ \mathrm{s}^{-1}$

Fast Timing: Comparison between theory and experiment

Time resolution $\propto 1/(S/N)$



At $S/N<100$ the measured curve lies above the calculation because the timing discriminator limited the rise time.

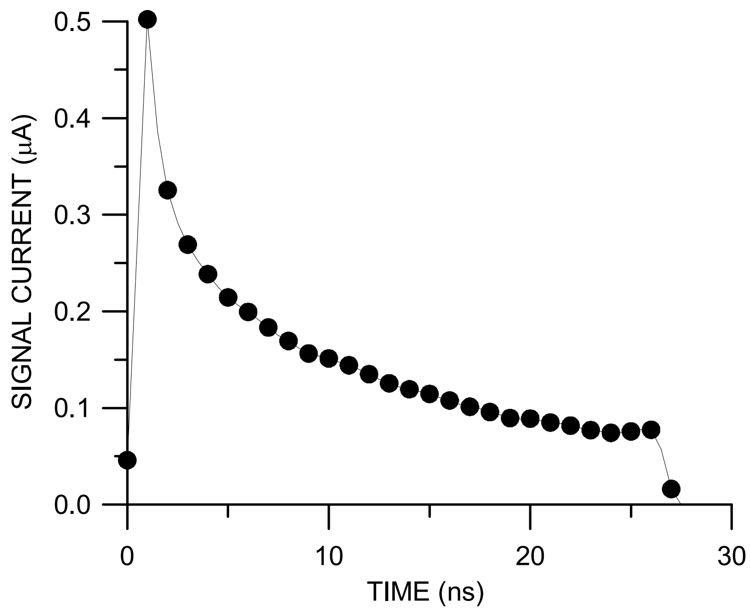
At high S/N the residual jitter of the time digitizer limits the resolution.

For more details on fast timing with semiconductor detectors, see

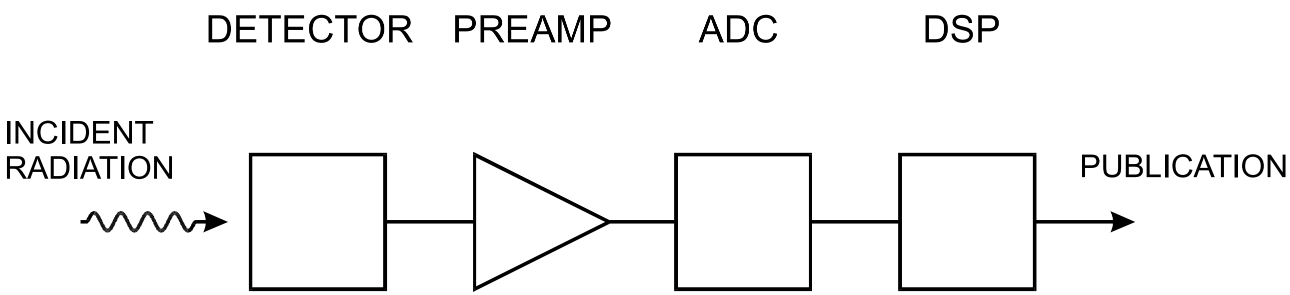
H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.

Digital Signal Processing

Sample detector signal with fast digitizer to reconstruct pulse:



Then use digital signal processor to perform mathematical operations for desired pulse shaping.

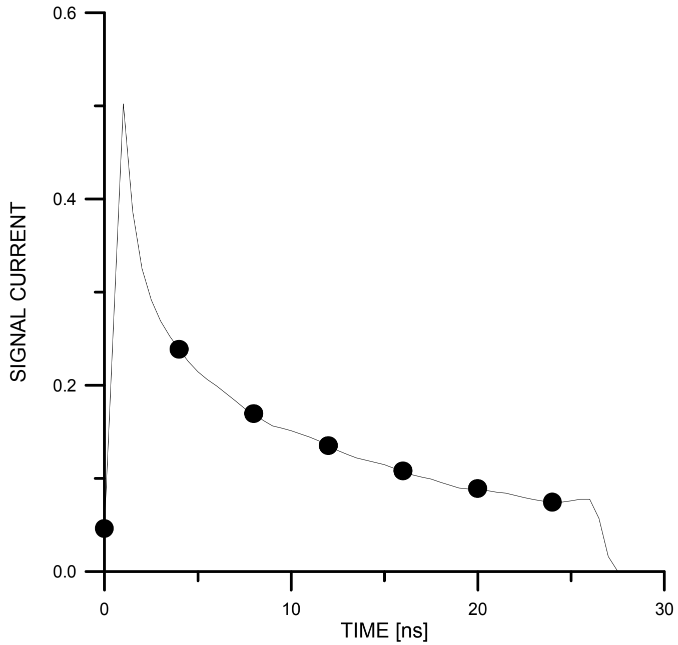


DSP allows great flexibility in implementing filtering functions.

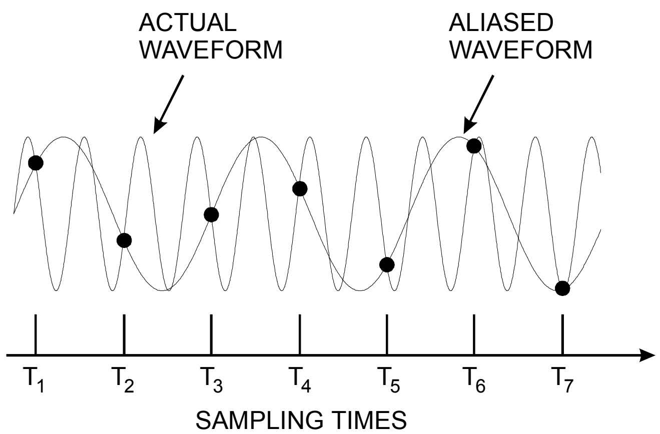
However: increased circuit complexity
increased demands on ADC, compared to traditional shaping.

Important to choose sample interval sufficiently small
to capture pulse structure.

Sampling interval of 4 ns misses initial peak.



With too low a sampling rate high frequency components will be "aliased" to lower frequencies:



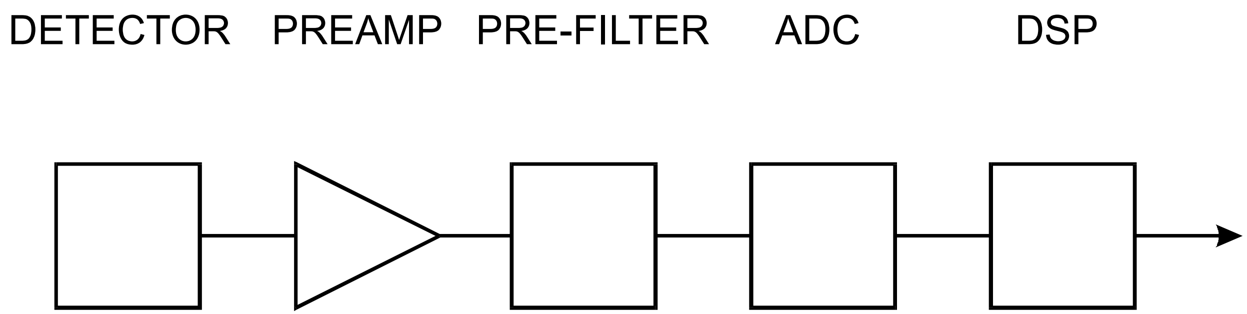
Applies to any form of sampling
(time waveform, image, ...)

Nyquist condition:

Sampling frequency > 2x highest signal frequency

⇒ Requires

- Fast ADC +
- Pre-Filter to limit signal bandwidth



- Dynamic range requirements for ADC may be more severe than in analog filtered system (depending on pulse shape and pre-filter).

- Digitization introduces additional noise ("quantization noise")

  If one bit corresponds to an amplitude interval Δ, the quantization noise

$$\sigma_v^2 = \int_{-\Delta/2}^{\Delta/2} \frac{v^2}{\Delta}\, dv = \frac{\Delta^2}{12}\,.$$

  (differential non-linearity introduces quasi-random noise)

- Electronics preceding ADC and front-end of ADC must exhibit same precision as analog system:

  Baseline and other pulse-to-pulse amplitude fluctuations less than order Q_n/10, i.e. typically 10^{-4} in high-resolution systems.

  For 10 V FS at the ADC input this corresponds to < 1 mV.

⇒ ADC must provide high performance at short conversion times.

Today this is technically feasible for some applications, e.g. detectors with moderate to long collection times (γ and x-ray detectors).

Digital Filtering

Filtering is performed by convolution:

$$S_o(n) = \sum_{k=0}^{N-1} W(k) \cdot S_i(n-k)$$

$W(k)$ is a set of coefficients that describes the weighting function yielding the desired pulse shape.

A filter performing this function is called a Finite Impulse Response (FIR) filter.

This is analogous to filtering in the frequency domain.

In the frequency domain the result of filtering is determined by multiplying the responses of the individual stages:

$$G(f) = G_1(f) \cdot G_2(f)$$

where $G_1(f)$ and $G_1(f)$ are complex numbers.

The theory of Fourier transforms states that the equivalent result in the time domain is formed by convolution of the individual time responses:

$$g(t) = g_1(t) * g_2(t) \equiv \int_{-\infty}^{+\infty} g_1(\tau) \cdot g_2(t-\tau) d\tau \, ,$$

analogously to the discrete sum shown above.

Benefits of digital signal processing:

- Flexibility in implementing filter functions
- Filters possible that are impractical in hardware
- Simple to change filter parameters
- Tail cancellation and pile-up rejection easily incorporated
- Adaptive filtering can be used to compensate for pulse shape variations.

Where is digital signal processing appropriate?

- Systems highly optimized for
  Resolution
  High counting rates
- Variable detector pulse shapes

Where is analog signal processing best (most efficient)?

- Fast shaping (e.g. LHC)
- Systems not sensitive to pulse shape (fixed shaper constants)
- High density systems that require small circuit area or low power

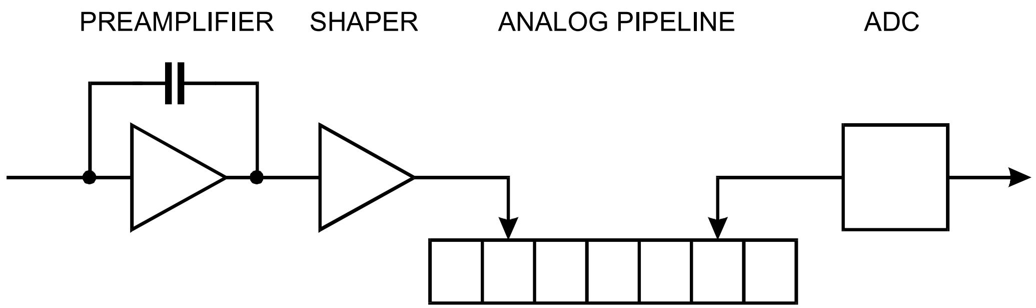
Both types of systems require careful analog design.

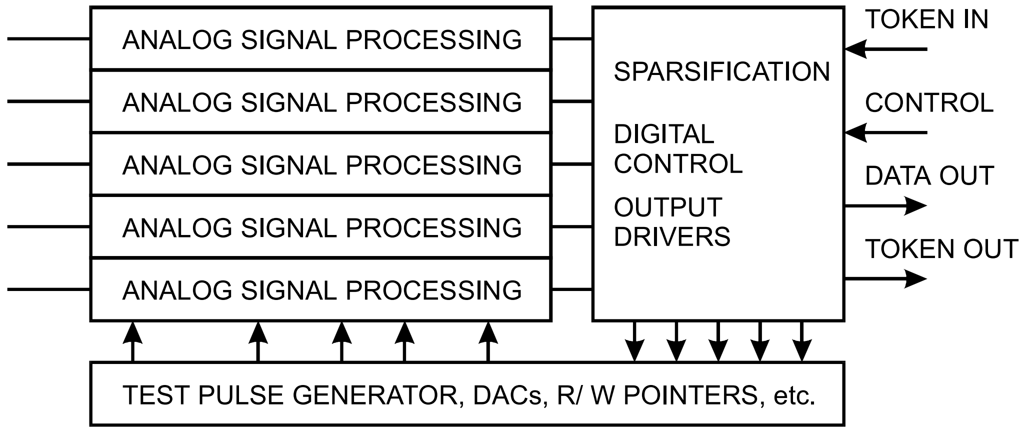
Progress in fast ADCs (precision, reduced power) will expand range of DSP applications.

8. Readout Systems

Example: Si strip detector

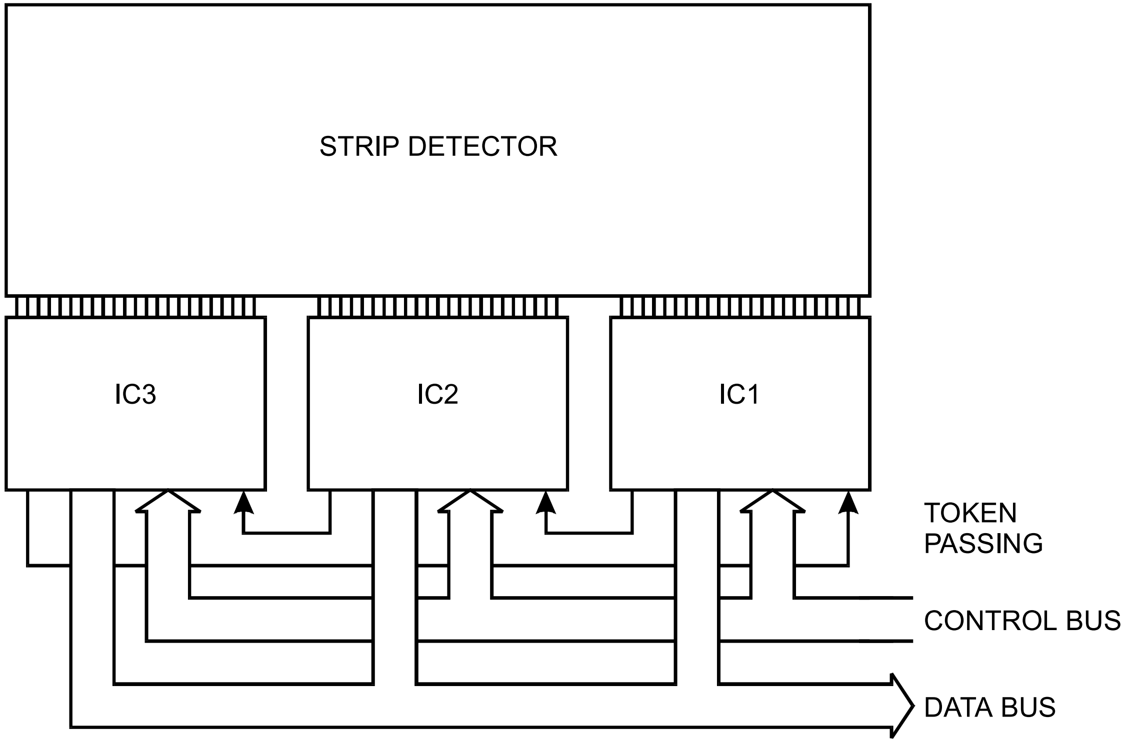
On-chip Circuits





Inside a typical readout IC: 128 parallel channels of analog front-end electronics
Logic circuitry to decode control signals, load DACs, etc.
Digital circuitry for zero-suppression, readout

Readout of Multiple ICs



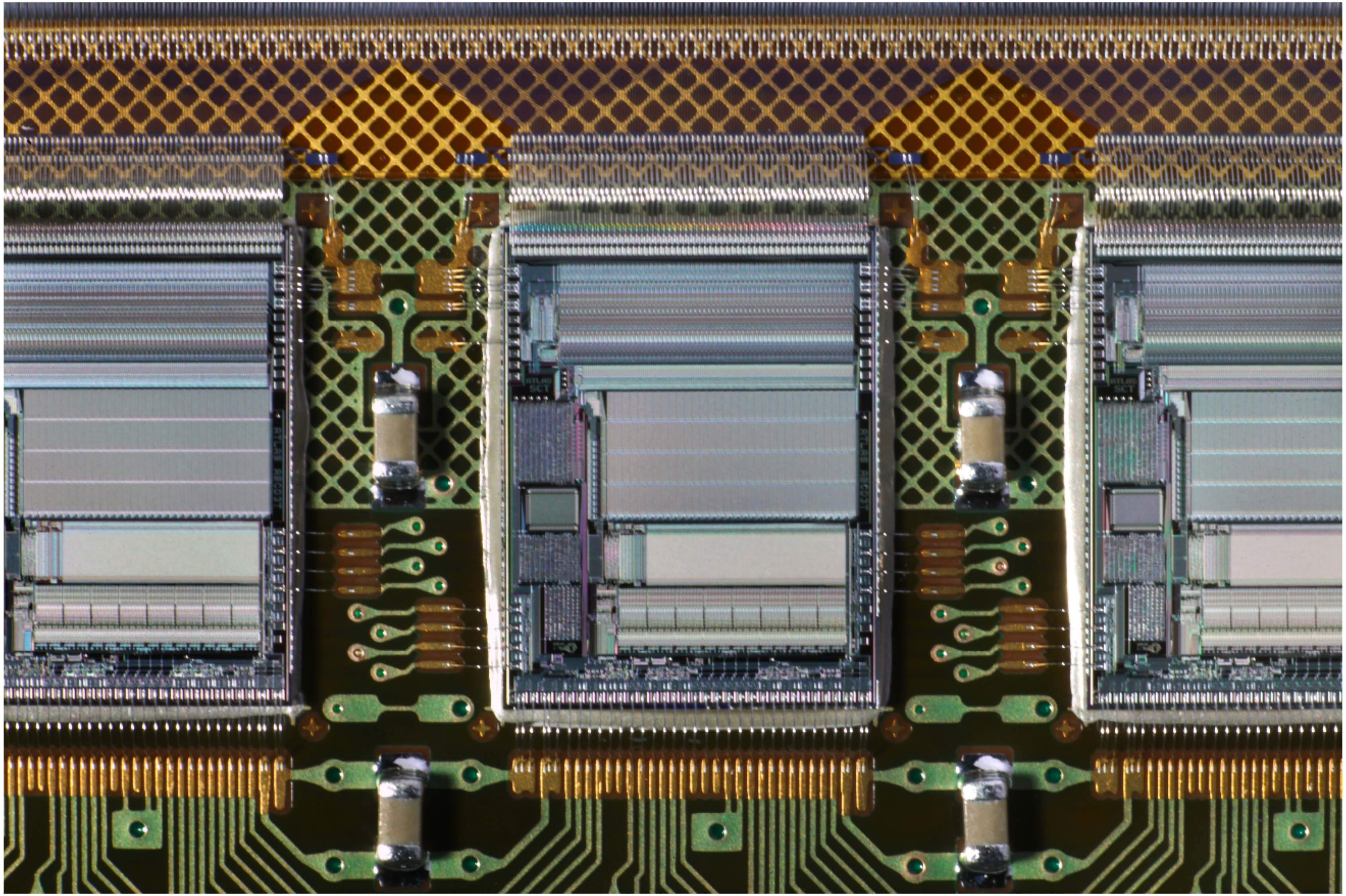
IC1 is designated as master.

Readout is initiated by a trigger signal selecting appropriate time stamp to IC1.

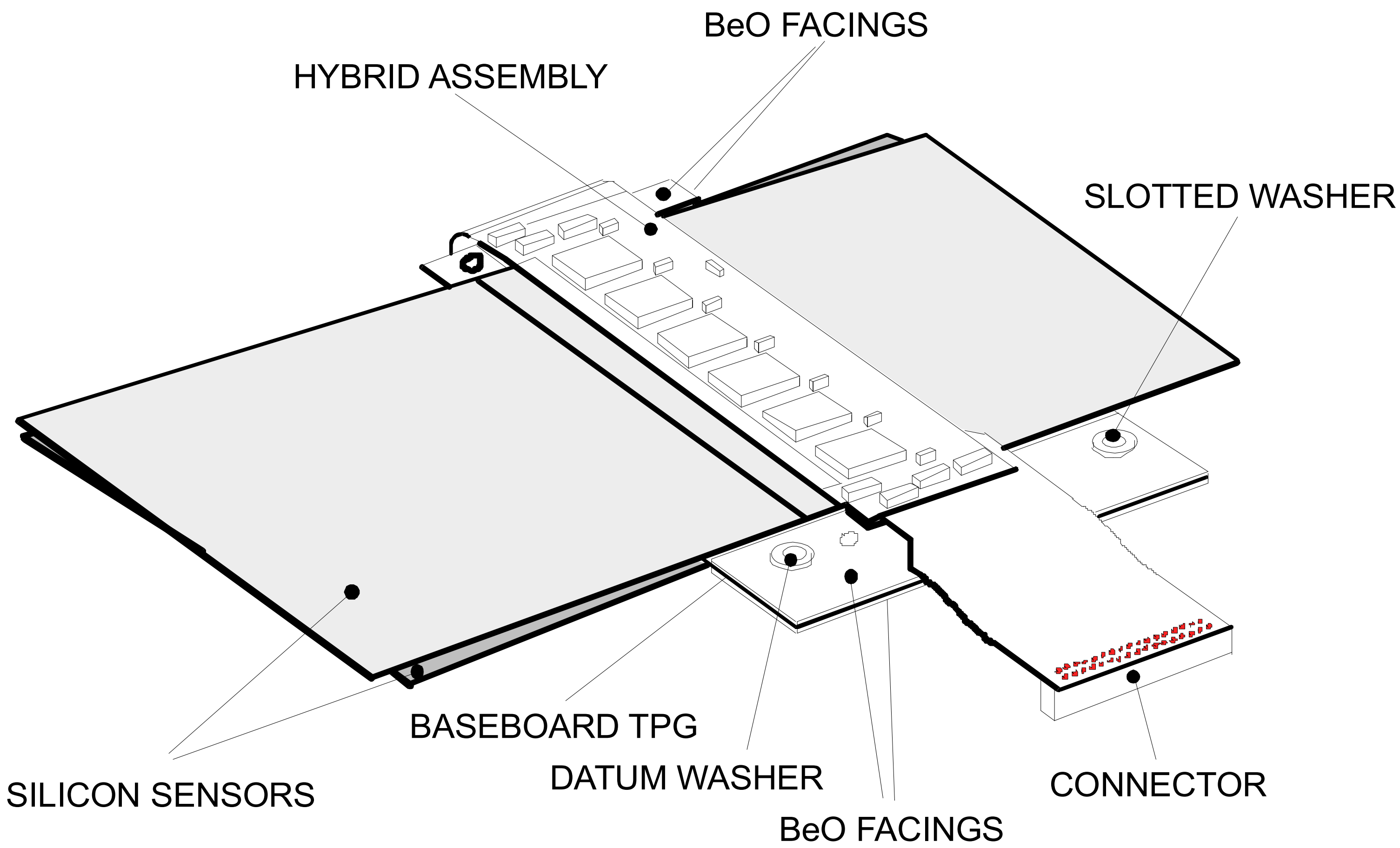
When all data from IC1 have been transferred, a token is passed to IC2.

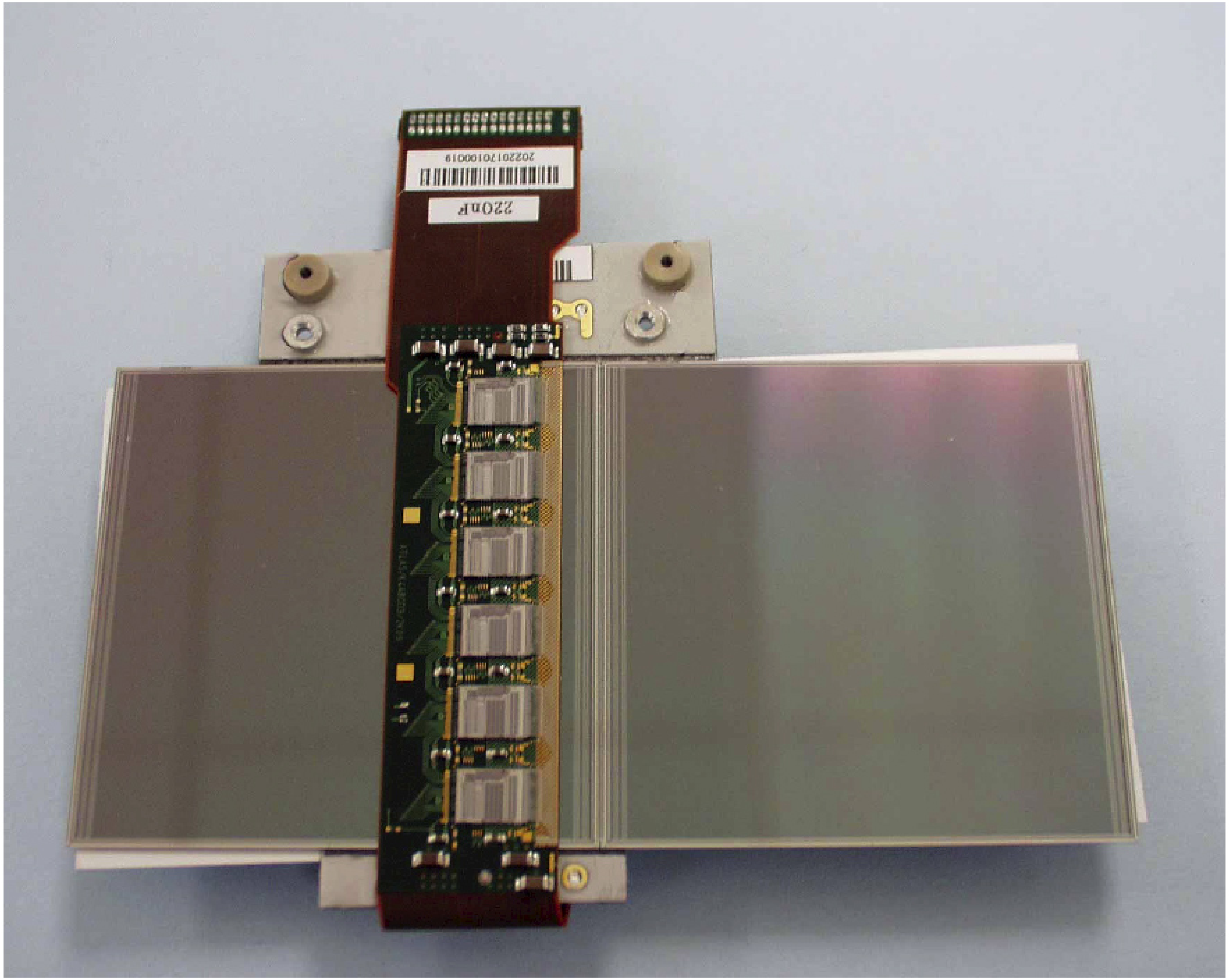
When IC3 has finished, the token is passed back to IC1, which can begin a new cycle.

ATLAS Silicon Strip system (SCT): ABCD chips mounted on hybrid

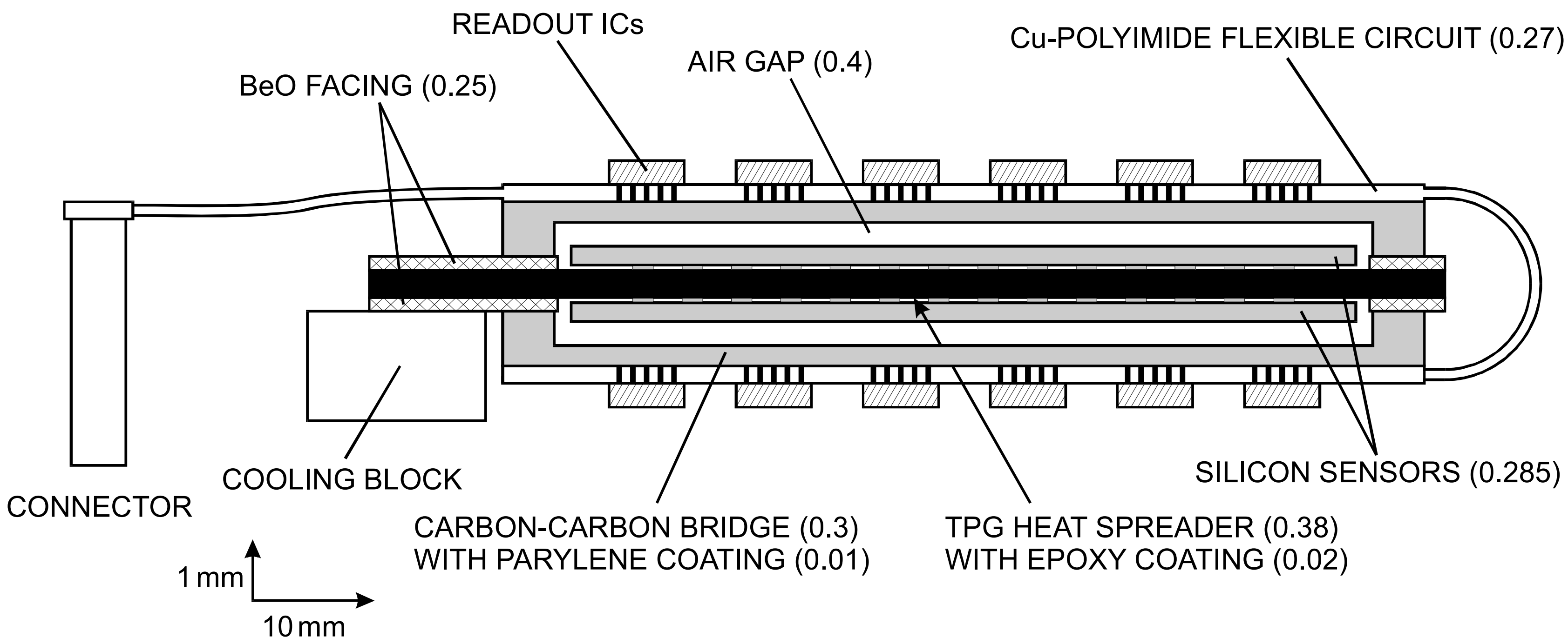


ATLAS SCT Detector Module





Cross Section of Module



9. Summary

- Front-end electronics critical to obtaining high energy and time resolution.
- Equivalent noise charge increases with detector capacitance.
- Pulse shaping is compromise between low noise and pulse rate.

  Fast shaping $\Rightarrow$ Higher noise

- Pulse shapers can be implemented with both analog and digital circuitry

  Digital processing well-suited for small scale systems

  In high-density systems where size and power are crucial, analog processing is more efficient.

- Detector systems utilizing high-density integrated circuits are widely used in high-energy physics (10^6 – 10^8 channel systems) and now in astrophysics (e.g. GLAST).

  Technology making its way into medical imaging and light source applications.

- Successful systems rely on many details that go well beyond “headline specs”.
- Crucial technology for many applications, well into the future!