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Introduction to Analog and Digital Electronics for Detectors

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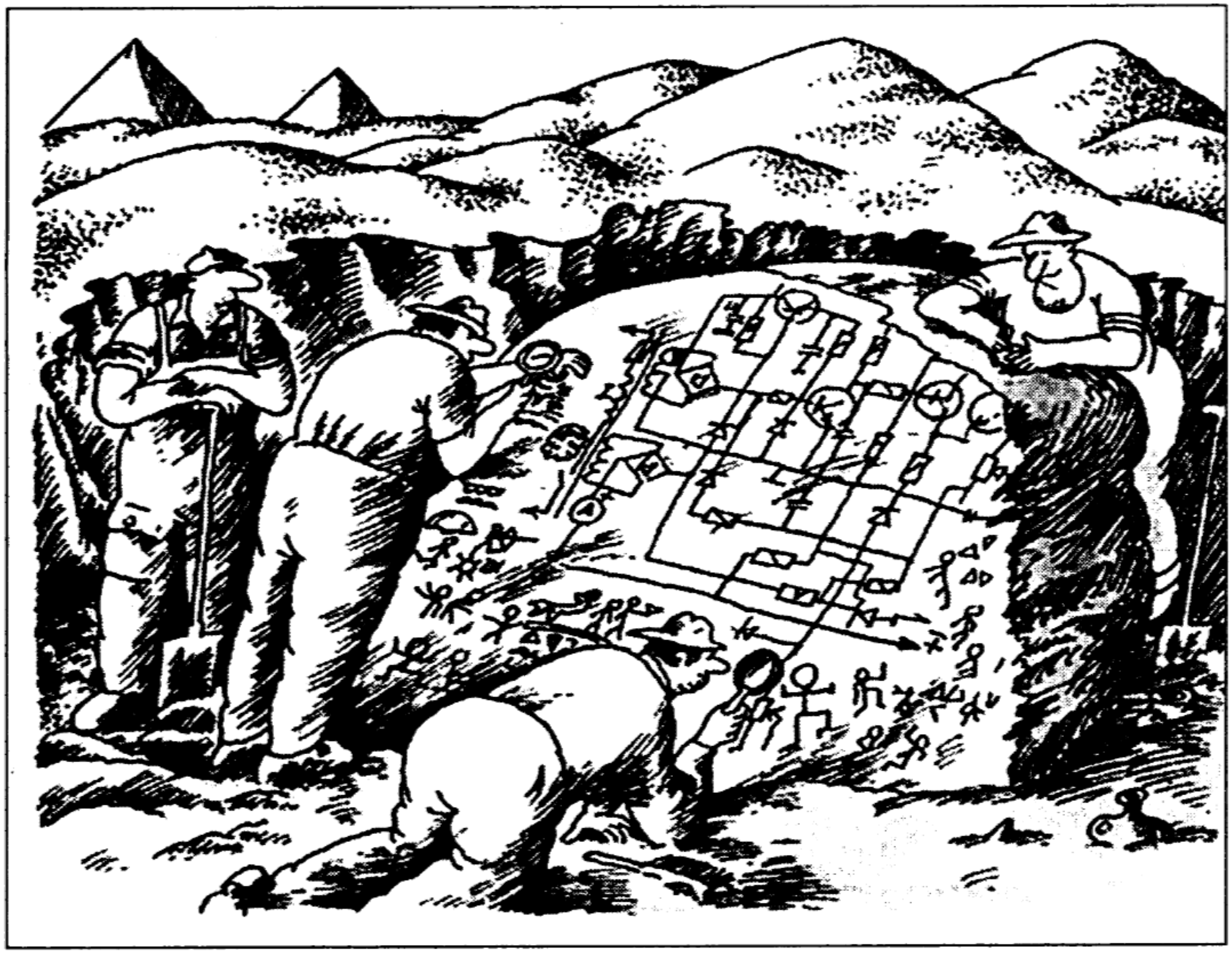

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I. Introduction

Purpose of pulse processing and analysis systems:

1. acquire electrical signal from detector

   typically a short current pulse

2. tailor the time response (i.e. “shape” the output pulse) of the system to optimize

   - minimum detectable signal (detect hit/no hit)
   - energy measurement (magnitude of signal)
   - event rate
   - time of arrival (timing measurement)
   - insensitivity to detector pulse shape
   - some combination of the above

   Generally, these cannot be optimized simultaneously

   ⇒ compromises

   Position-sensitive detectors use presence of hit, amplitude measurement or timing.

   ⇒ same problem

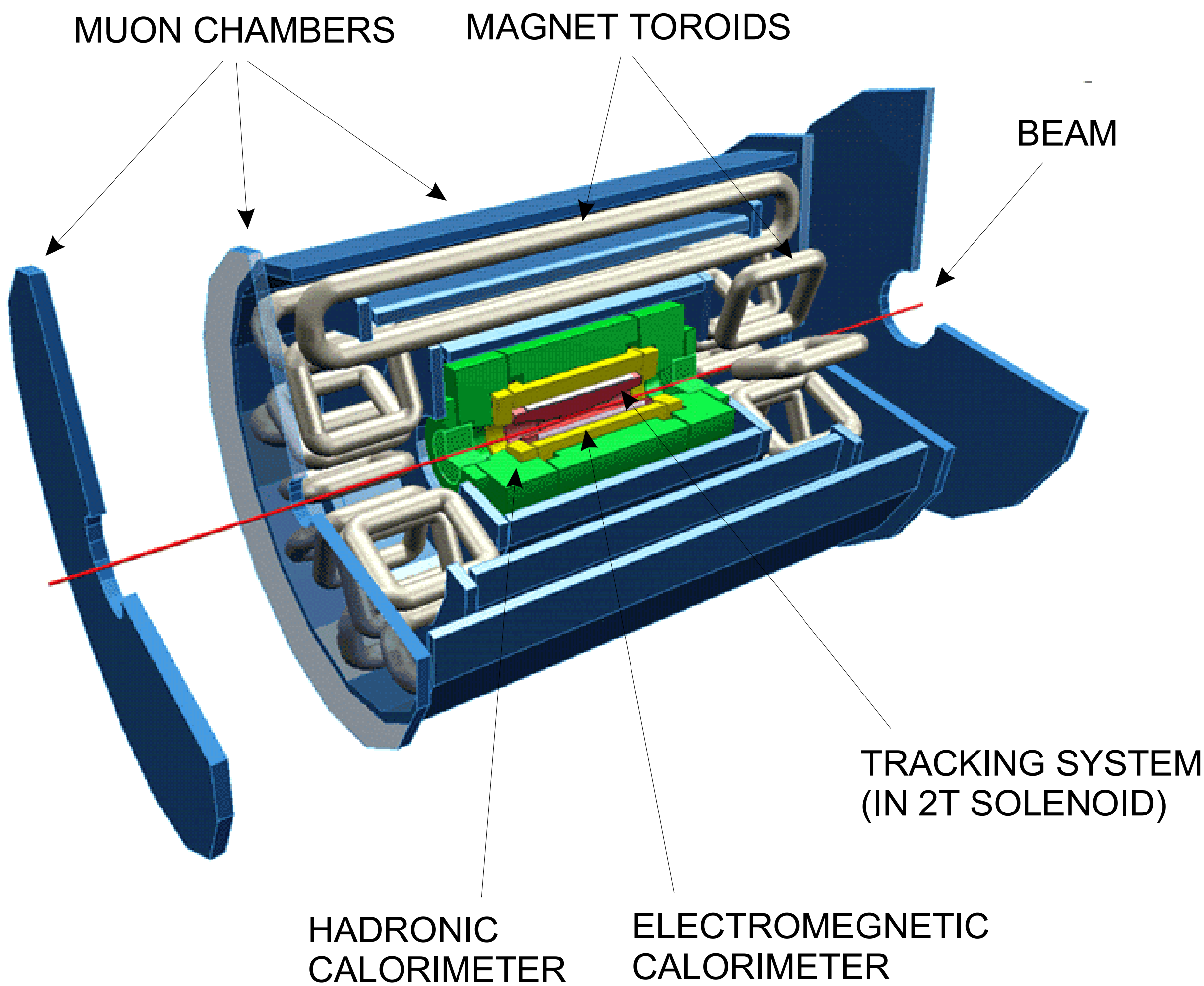
3. digitize the signal and store for subsequent analysis

Additional requirements, depending on specific application, e.g.

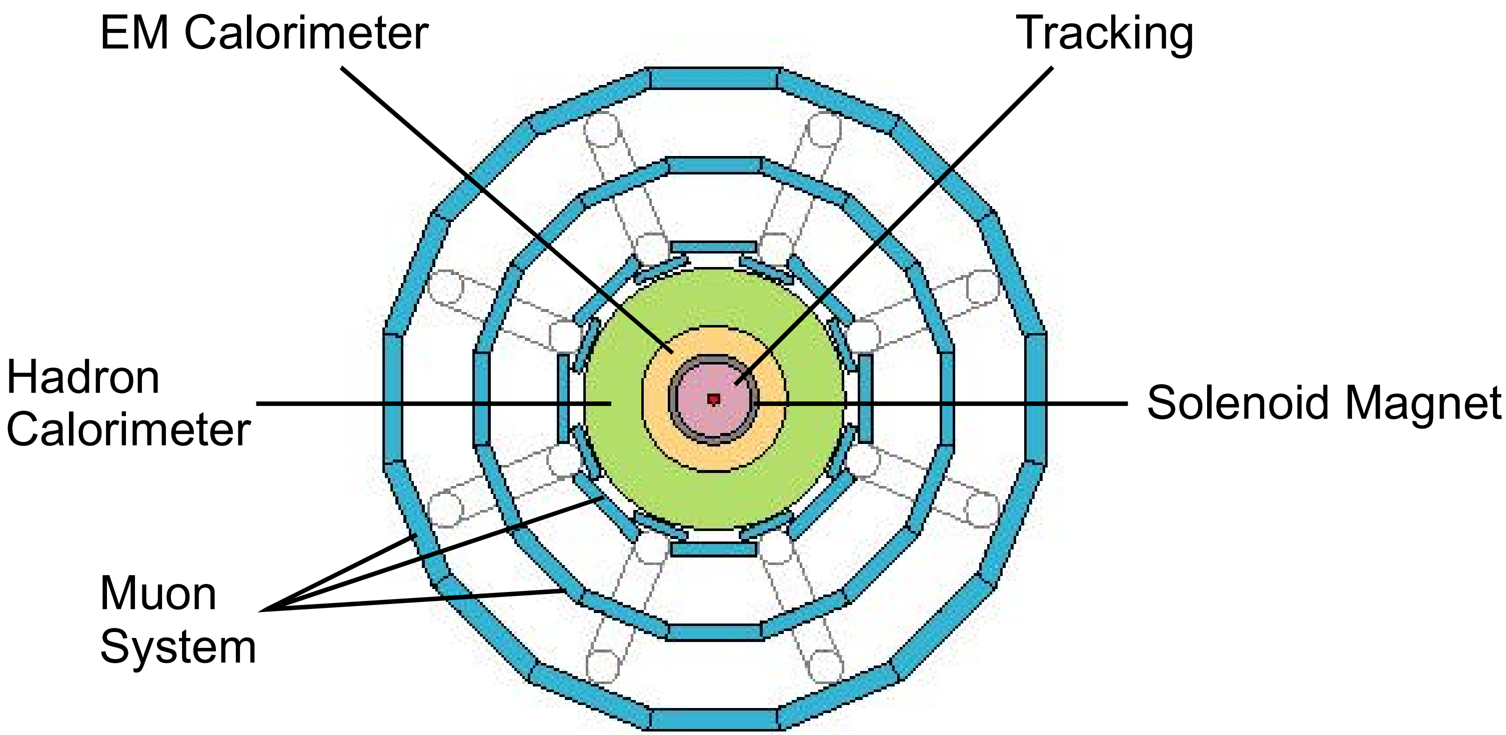
- radiation resistance
- low power
  - portable systems
  - large detector arrays, e.g. in HEP
- robustness
- cost

1. Example Detector Systems

1.1. ATLAS Detector



Schematic End-View



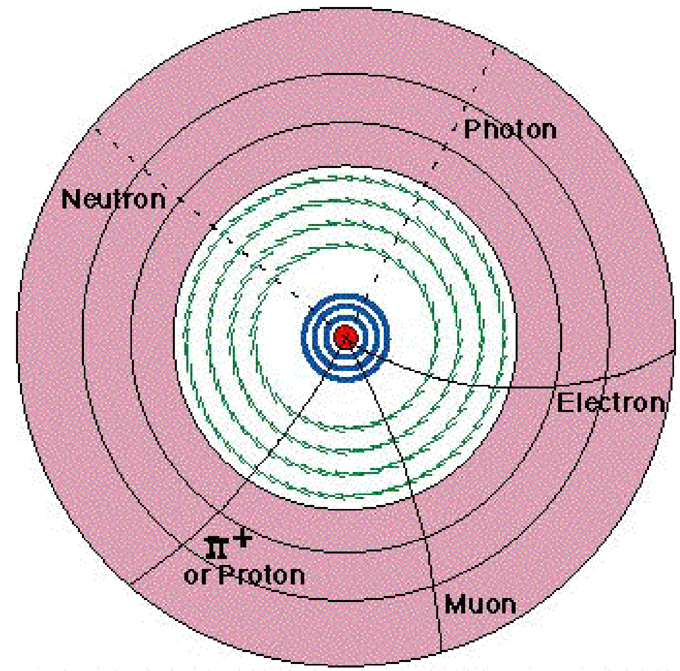
Tracking in 2T magnetic field

Separate particles by

sign of charge

magnetic rigidity q/m

$\Rightarrow$ position measurement layer by layer to reconstruct tracks



Inner layers: Silicon pixel and strip detectors

Measure presence of hit

Outer layers: “straw” drift chambers

timing provides position information (see muon system)

Calorimetry

Particles generate showers in calorimeters

Electromagnetic Calorimeter (yellow):

Absorbs and measures the energies of all electrons, photons

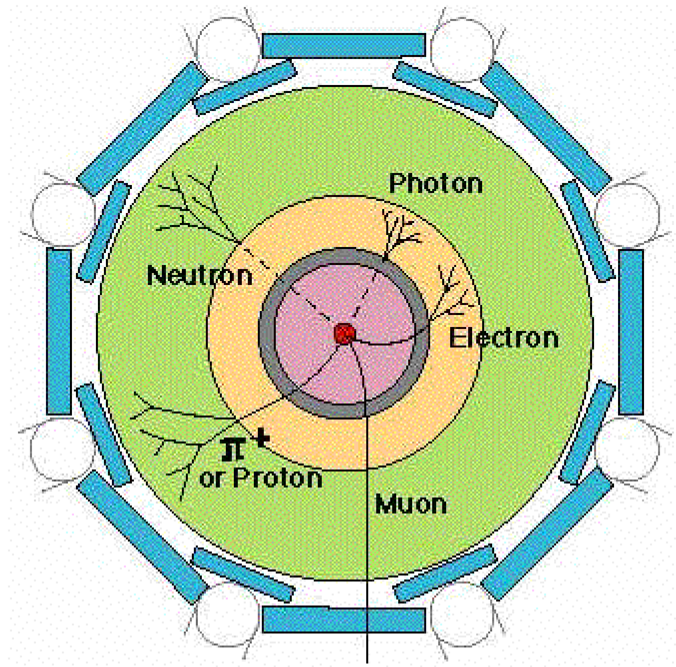
Hadronic Calorimeter (green)

Absorbs and measures the energies of hadrons, including protons and neutrons, pions and kaons

(electrons and photons have been absorbed in EM calorimeter)

⇒ amplitude measurement

position information provided by segmentation



Muon System

Muons are the only charged particle that can travel through all of the calorimeter material and reach the outer layer.

> muons with energy above, say, 5 GeV will penetrate about 5 meters of steel, whereas hadrons of almost any energy are completely absorbed in about 1.5 meters of steel.

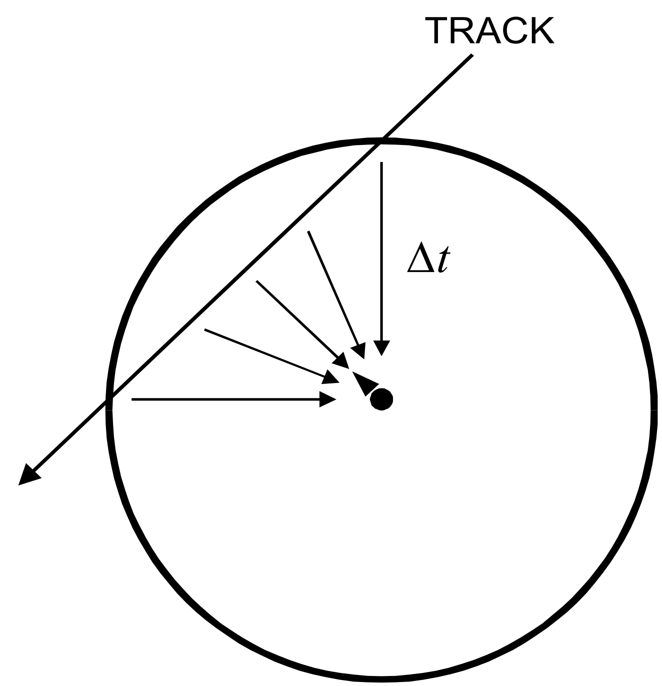
The muon sensors are gas proportional drift chambers: 3 cm in diameter, ~ 1 – 6 m long.

Electrons formed along the track drift towards the central wire.

The first electron to reach the high-field region initiates the avalanche, which is used to derive the timing pulse.

Since the initiation of the avalanche is delayed by the transit time of the charge from the track to the wire, the time of the avalanche can be used to determine the radial position.

Principle also used in straw tracker – need fast timing electronics



Summary of Measured Quantities

1. Si Tracking	position to ~10 µm accuracy in $r\varphi$ (through segmentation) timing to 25 ns accuracy to separate bunch crossings
2. Straw Tracker	position to 170 µm at r > 56 cm
3. EM calorimeter	energy via LAr ionization chambers position through segmentation
4. Hadron calorimeter	energy via plastic scintillator tiles position through segmentation
5. Muon System	signal via ionization chambers position through timing measurement

Although these various detector system look very different, they all follow the same principles.

Sensors must determine

1. presence of a particle
2. magnitude of signal
3. time of arrival

Some measurements depend on sensitivity, i.e. detection threshold.

example: silicon tracker, to detect presence of a particle in a given electrode

Others seek to determine a quantity very accurately, i.e. resolution

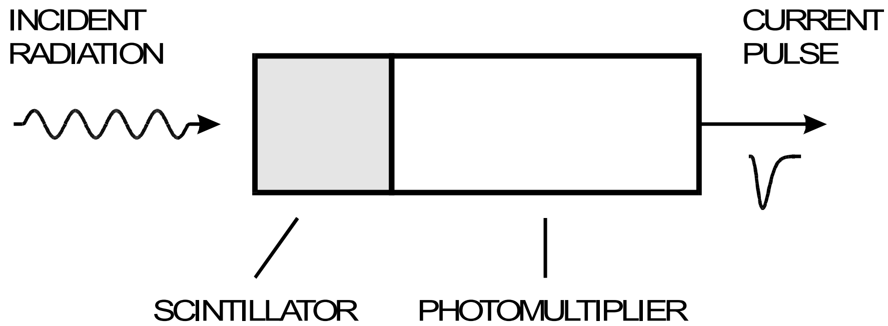
example: calorimeter – magnitude of absorbed energy

muon chambers – time measurement yields position

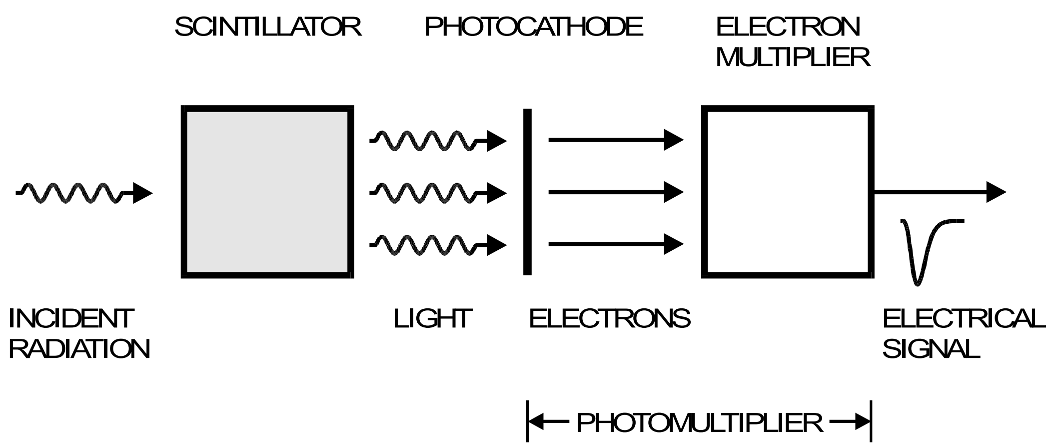
All have in common that they are sensitive to

1. signal magnitude
2. fluctuations

1.2. A Typical Detector System – Scintillation Detector



Processes in Scintillator – Photomultiplier



number of photons
$\propto$ absorbed energy

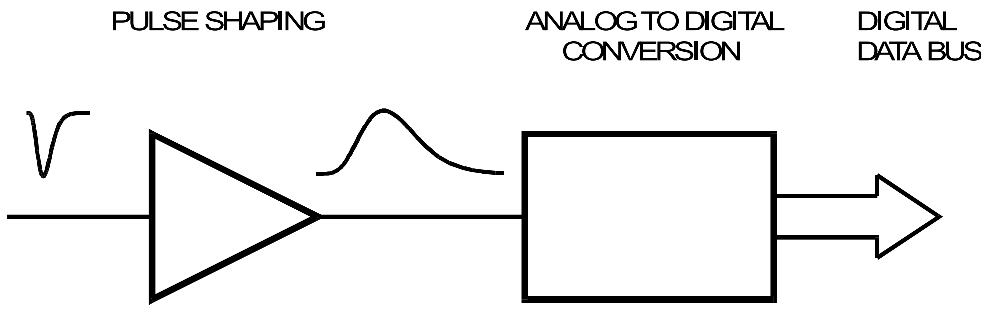
number of photoelectrons
$\propto$ absorbed energy

charge in pulse
$\propto$ abs. energy

Signal Processing

charge in pulse
∝ abs. energy

pulse height
∝ absorbed energy



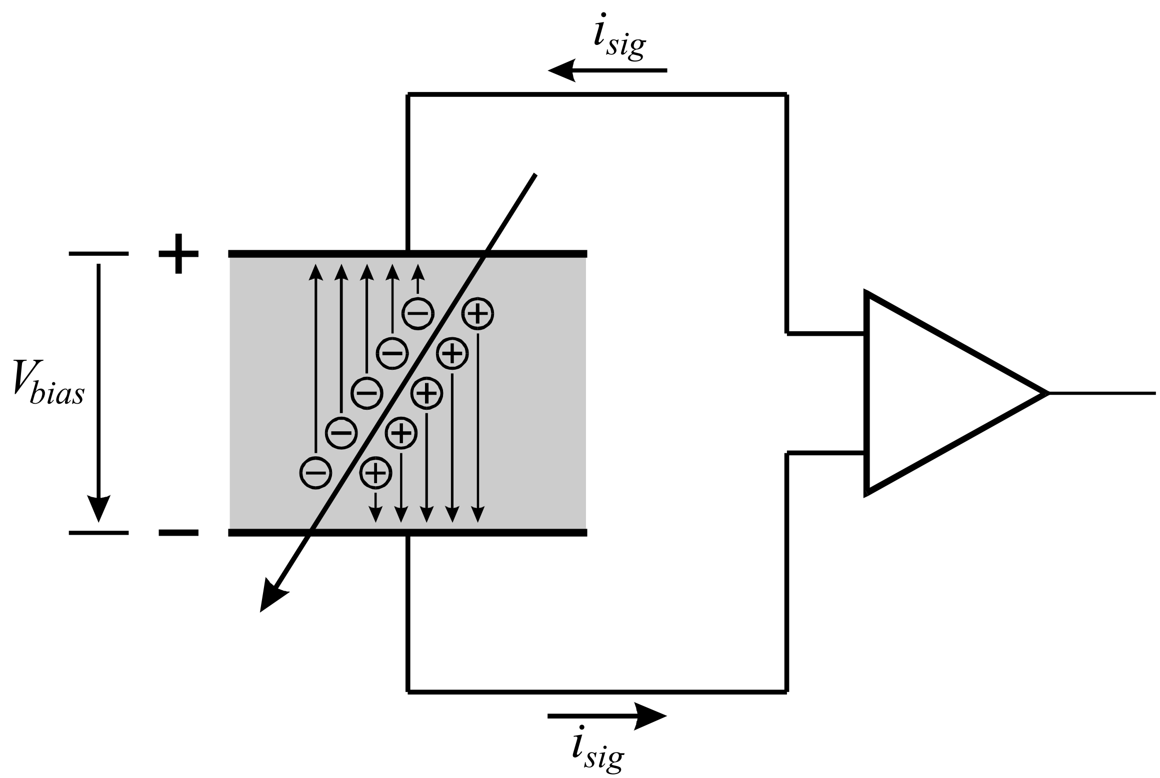
1.3. Ionization Chamber

All ionization chambers utilize the same principle:

1. Particles deposit energy in an absorber and create mobile charge carriers (positive and negative charge pairs).

   in solids, liquids: electrons and holes
   in gases: electrons and ions

2. Electric field applied to detector volume sweeps charge carriers towards electrodes and induces a signal current



2. The Signal

Any form of elementary excitation can be used to detect the radiation signal.

$$\text{Magnitude of signal} = \frac{\text{absorbed energy}}{\text{excitation energy}}$$

An electrical signal can be formed directly by ionization.

Incident radiation quanta impart sufficient energy to individual atomic electrons to form electron-ion pairs (in gases) or electron-hole pairs (in semiconductors and metals).

Other detection mechanisms are

Excitation of optical states (scintillators) $\rightarrow$ light intensity

Excitation of lattice vibrations (phonons) $\rightarrow$ temperature

Breakup of Cooper pairs in superconductors

Formation of superheated droplets in superfluid He

Typical excitation energies:		
	Ionization in gases	~30 eV
	Ionization in semiconductors	1 – 10 eV
	Scintillation	20 - 500 eV
	Phonons	meV
	Breakup of Cooper Pairs	meV

Precision of signal magnitude is limited by fluctuations

Two types of fluctuations

1. Fluctuations in signal charge for a given energy absorption in detector

   Signal formed by many elementary excitations

$$\text{Average number of signal quanta} = \frac{\text{absorbed energy}}{\text{excitation energy}} \quad \Rightarrow \quad N = \frac{E}{E_i}$$

   Number of signal quanta fluctuates statistically. $\Delta N = \sqrt{FN}$

   where F is the Fano factor (0.1 in Si, for example), so the energy resolution

$$\Delta E = E_i \Delta N = \sqrt{FEE_i} \quad \text{r.m.s.}$$

$$\Delta E_{FWHM} = 2.35 \times \Delta E_{rms}$$

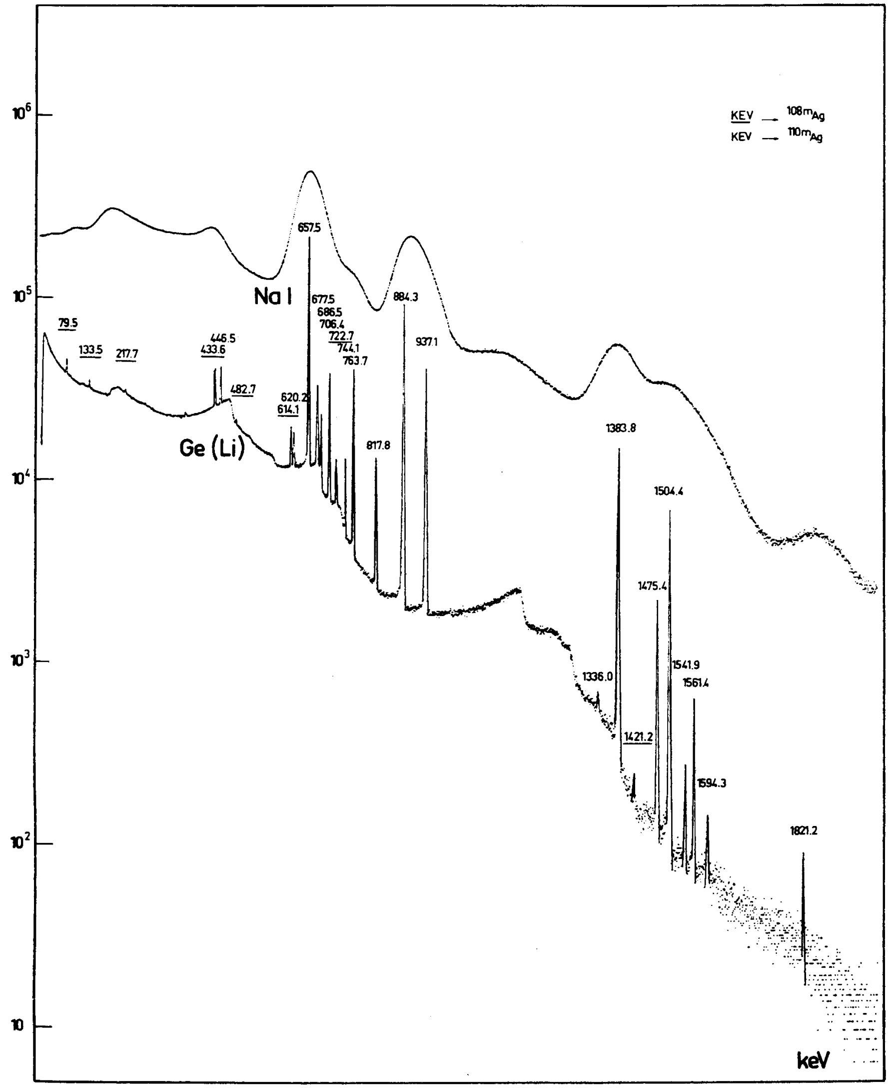
2. Baseline fluctuations in the electronics: "electronic noise"

The overall resolution is often the result of several contributions. Individual resolutions add in quadrature, for example

$$\Delta E = \sqrt{\Delta E_{fluc}^2 + \Delta E_{elec}^2}$$

If one contribution is 20% of the other, the overall resolution is increased by 10%.

Resolution of NaI(Tl) and Ge detectors



NaI(Tl) scintillation detector:

signal fluctuations

Ge detector:

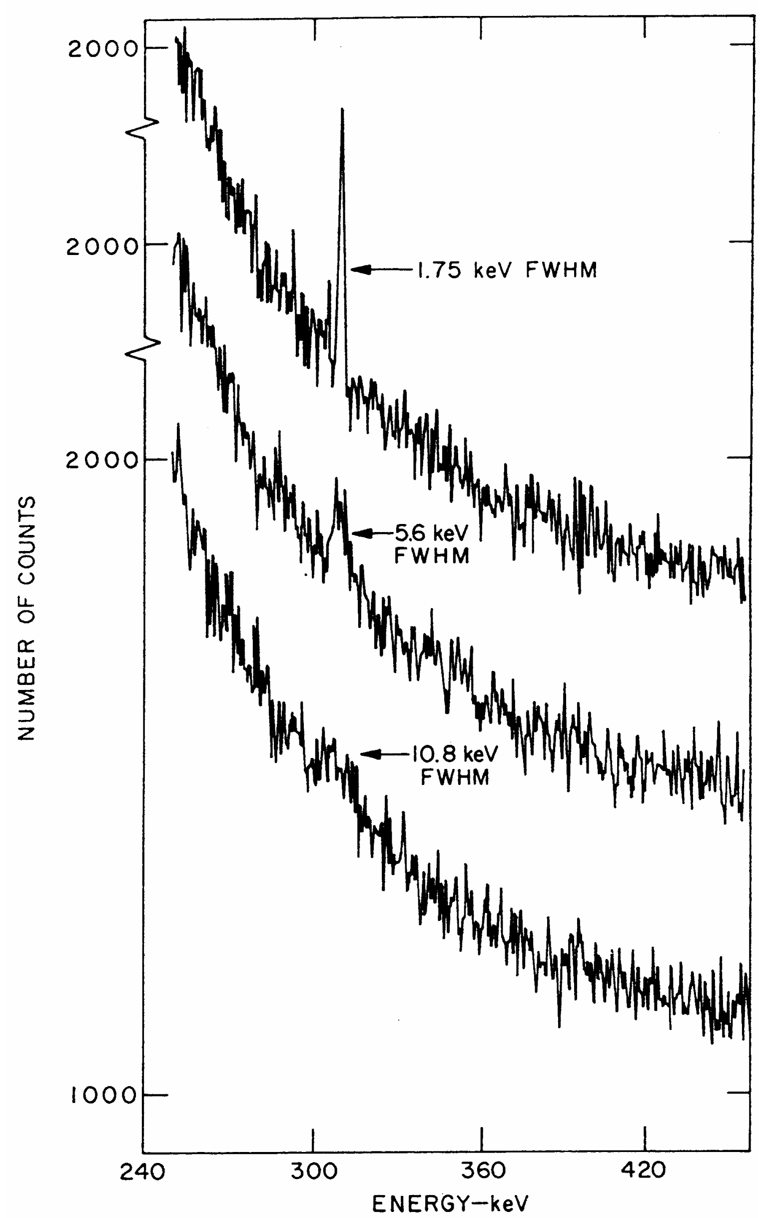
predominantly electronic noise

(J.Cl. Philippot,
IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446)

Resolution increases sensitivity

Signal to background ratio improves with better resolution

(narrow peak competes with fewer background counts)



G.A. Armantrout, *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972)

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Signal Fluctuations in a Scintillation Detector

Example:

a typical NaI(Tl) system (from Derenzo)

511 keV gamma ray

$\Downarrow$

25000 photons in scintillator

$\Downarrow$

15000 photons at photocathode

$\Downarrow$

3000 photoelectrons at first dynode

$\Downarrow$

$3 \cdot 10^9$ electrons at anode

2 mA peak current

Resolution of energy measurement determined by statistical variance of produced signal quanta.

$$\frac{\Delta E}{E} = \frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Resolution determined by smallest number of quanta in chain, i.e. number of photoelectrons arriving at first dynode.

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{3000}} = 2\% \text{ r.m.s.} = 5\% \text{ FWHM}$$

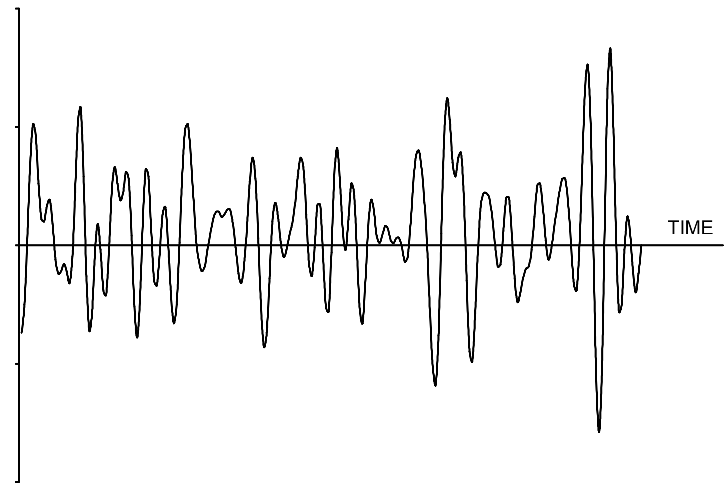
In this example

Typically 7 – 8% obtained, due to non-uniformity of light collection and gain.

Baseline Fluctuations (Electronic Noise)

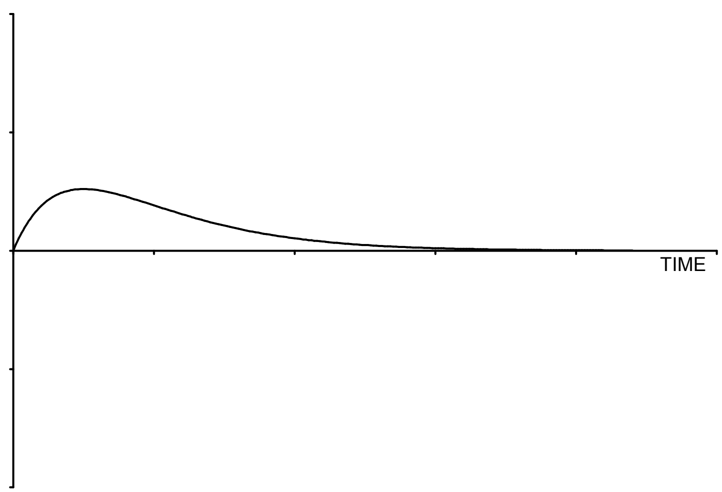
Choose a time when no signal is present.

Amplifier's quiescent output level (baseline):

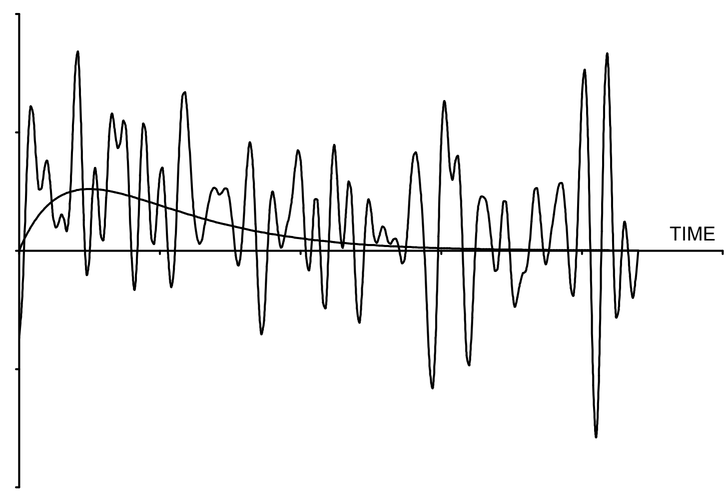


In the presence of a signal, noise + signal add.

Signal:



Signal+Noise (S/N = 1)



Measurement of peak amplitude yields signal amplitude + noise fluctuation

The preceding example could imply that the fluctuations tend to increase the measured amplitude, since the noise fluctuations vary more rapidly than the signal.

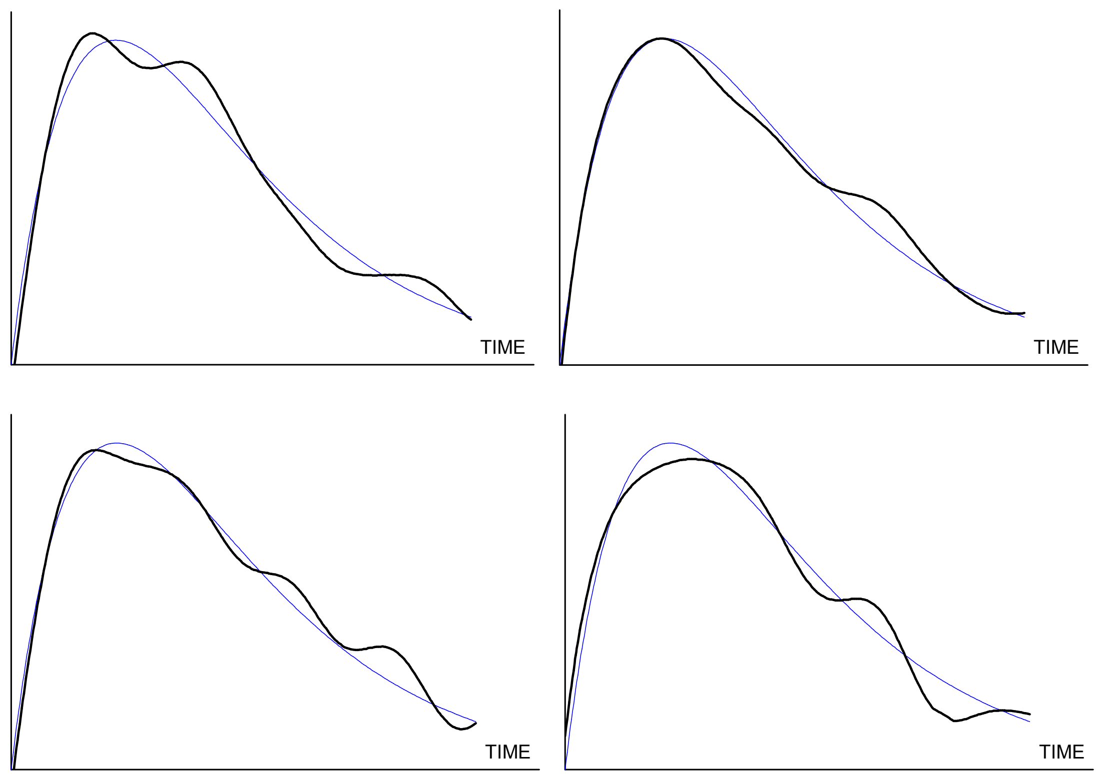
In an optimized system, the time scale of the fluctuation is comparable to the signal peaking time.

Then the measured amplitude fluctuates positive and negative relative to the ideal signal.

Measurements taken at 4 different times:

noiseless signal superimposed for comparison

S/N = 20



Noise affects

Peak signal

Time distribution

3. The Problem

Radiation impinges on a sensor and creates an electrical signal.

The signal level is low and must be amplified to allow digitization and storage.

Both the sensor and amplifiers introduce signal fluctuations – noise.

1. Fluctuations in signal introduced by sensor
2. Noise from electronics superimposed on signal

The detection limit and measurement accuracy are determined by the signal-to-noise ratio.

Electronic noise affects all measurements:

1. Detect presence of hit:
   Noise level determines minimum threshold.
   If threshold too low, output dominated by noise hits.
2. Energy measurement:
   noise "smears" signal amplitude
3. Time measurement
   noise alters time dependence of signal pulse

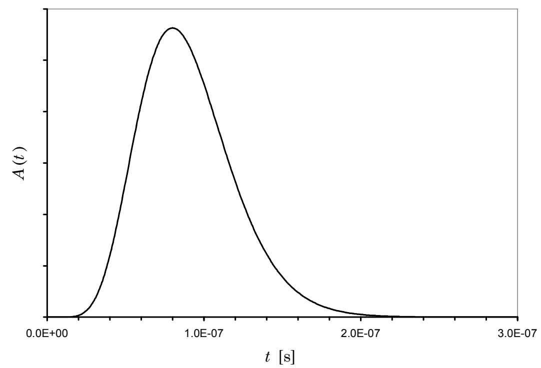
How to optimize the signal-to-noise ratio?

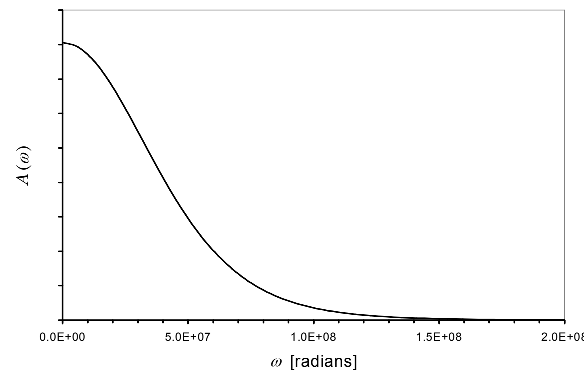
1. Increase signal and reduce noise
2. For a given sensor and signal: reduce electronic noise

Assume that the signal is a pulse.

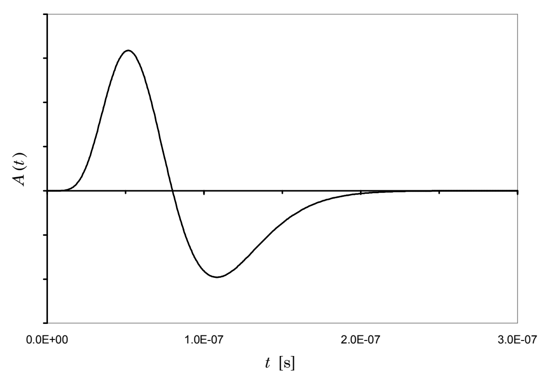
The time distribution of the signal corresponds to a frequency spectrum (Fourier transform).

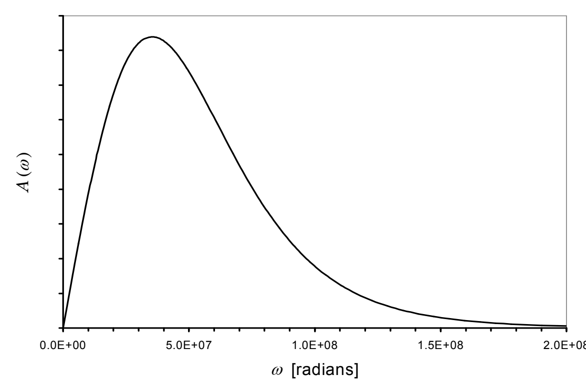
Examples: Time Domain Frequency Domain





The pulse is unipolar, so it has a DC component and the frequency spectrum extends down to 0.

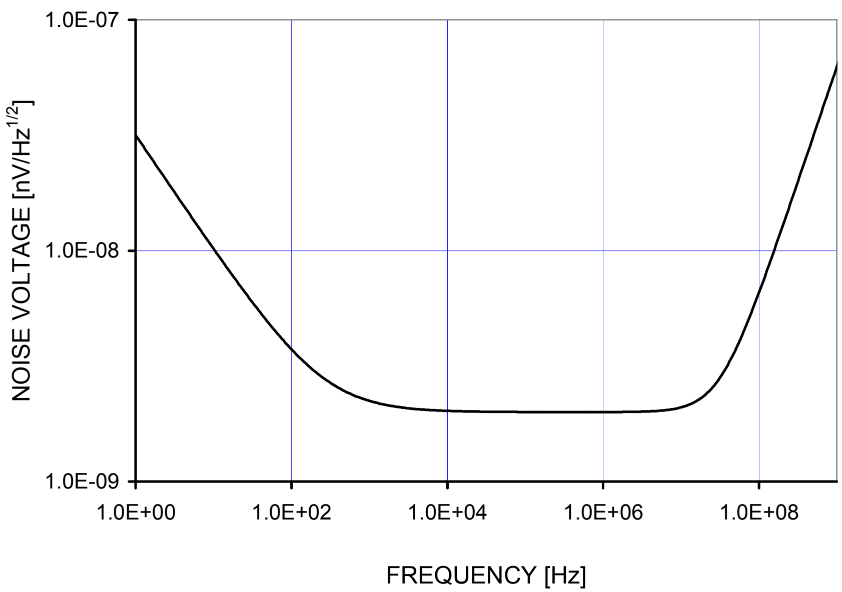




This bipolar pulse carries no net charge, so the frequency spectrum falls to zero at low frequencies.

The noise spectrum generally not the same as the signal spectrum.

Typical Noise Spectrum:



$\Rightarrow$ tailor frequency response of measurement system to optimize signal-to-noise ratio.

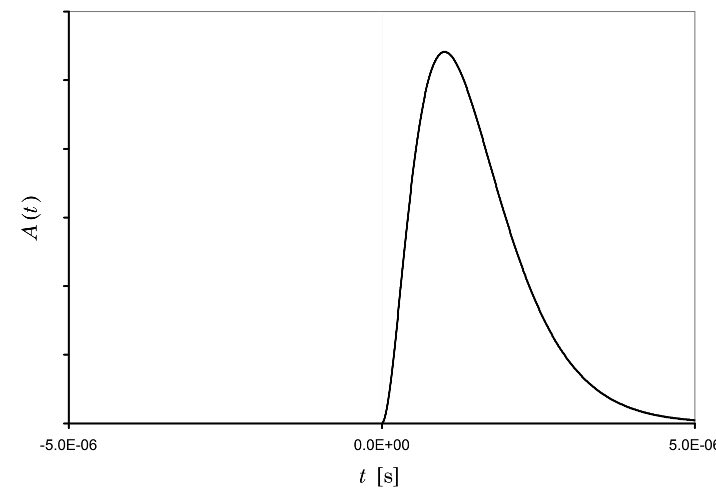
Frequency response of measurement system affects both

- signal amplitude and
- noise.

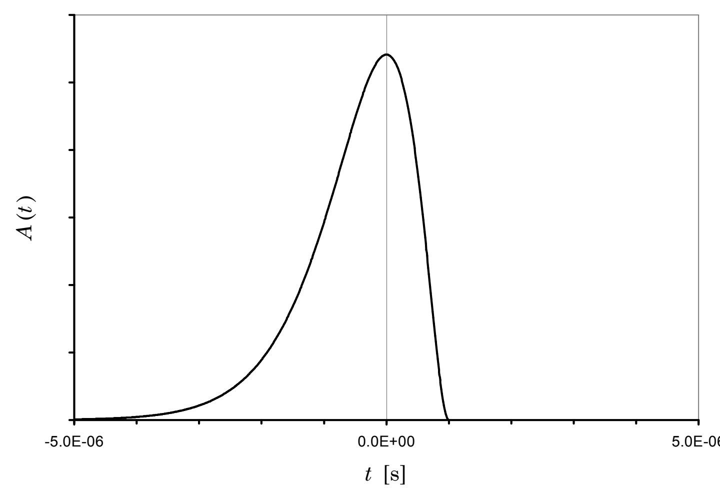
There is a general solution to this problem:

Apply a filter to make the noise spectrum white (constant over frequency). Then the optimum filter has an impulse response that is the signal pulse *mirrored in time* and shifted by the measurement time.

For example, if the signal pulse shape is:



The response of the optimum filter:



This is an “acausal” filter, i.e. it must act before the signal appears.

⇒ only useful if the time of arrival is known in advance.

Not good for random events
need time delay buffer memory

⇒ complexity!

Does that mean our problem is solved (and the lecture can end)?

1. The “optimum filter” preserves all information in signal, i.e. magnitude, timing, structure.

   Usually, we need only subset of the information content, i.e.
   area (charge) or time-of-arrival.

   Then the raw detector signal is not of the optimum form for the information that is required.

   For example, a short detector pulse would imply a fast filter function. This retains both amplitude and timing information.
   If only charge information is required, a slower filter is better, as will be shown later.

2. The optimum filter is often difficult or impractical to implement

   Digital signal processing would seem to remove this restriction, but this approach is not practical for very fast signals or systems that require low power.

4. Simpler filters often will do nearly as well

5. Even a digital system requires continuous (“analog”)
   pre-processing.

6. It’s often useful to understand what you’re doing, so we’ll spend some more time to bring out the physical background of signal formation and processing.

4. Signal processing systems

Large detector systems may consist of several subsystems especially designed to perform specific functions, for example

- position sensing (tracking)
- energy measurement (spectroscopy, calorimeters)
- timing
- particle identification

Functions

Although these subsystems may look very different and use radically differing technologies, they all tend to comprise the same basic functions:

1. Radiation deposits energy in a detecting medium.

   The medium may be gas, solid or liquid.

   In a tracking detector one wishes to detect the presence of a particle without affecting its trajectory, so the medium will be chosen to minimize energy loss and particle scattering.

   Conversely, if one wishes to measure the total energy (energy spectrometry or calorimetry), the absorber will be chosen to optimize energy loss (high density, high Z).

2. Energy is converted into an electrical signal, either directly or indirectly. Each detected particle will appear as a pulse of electric charge.

   Direct conversion:

   incident radiation ionizes atoms/molecules in absorber, creating mobile charges that are detected. (ionization chambers)

   Indirect conversion:

   incident radiation excites atomic/molecular states that decay by emission of light, which in a second step is converted into charge. (scintillation detectors)

- The primary signal charge is proportional to the energy absorbed.

  Some typical values of energy required to form a signal charge of 1 electron:

gases	30 eV
semiconductors	1 to 10 eV
scintillators	20 to 500 eV

In neither of these schemes is the signal charge available instantaneously.

Scintillation detector: The pulse duration is determined by the decay time of the optical transitions.

Ionization chamber: The charges must move to the electrodes to obtain the full signal.

Typical pulse durations: 1 ns – 10 µs

3. The electrical signal is amplified.

a) electronic circuitry

b) gain by secondary multiplication

Primary charge is accelerated to sufficient energy for it to liberate additional charge carriers by impact ionization.

Examples: proportional chambers
avalanche photodiodes
photomultiplier

Both techniques may introduce significant random fluctuations (electronic noise, avalanche noise).

Ideally, a gain stage would increase only the magnitude of the detector pulse, without affecting its time dependence.

This ideal behavior is never strictly realized in practice, as it would require amplifiers with infinite bandwidth.

However, this is not a severe limitation, as in many applications it is quite acceptable and even desirable to change the pulse shape.

4. Pulse shaping

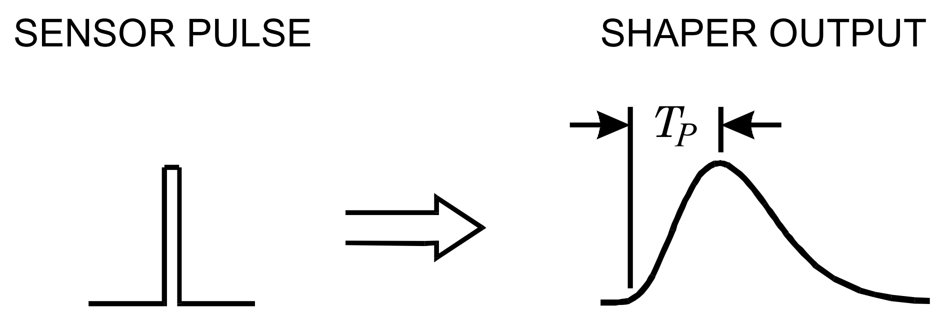
(not always necessary, but always present in some form)

The time response of the system is tailored to optimize the measurement of signal magnitude or time and the rate of signal detection.

The output of the signal chain is a pulse (current or voltage) whose area is proportional to the original signal charge, i.e. the energy deposited in the detector.

Typically, the pulse shaper transforms a narrow detector current pulse to

- a broader pulse (to reduce electronic noise),
- with a gradually rounded maximum at the peaking time T_P
  (to facilitate measurement of the amplitude)



However, to measure pulses in rapid succession, the duration of the pulse must be limited to avoid overlapping signals.

If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or analysis.

The pulse height spectrum is the energy spectrum.

5. Digitization

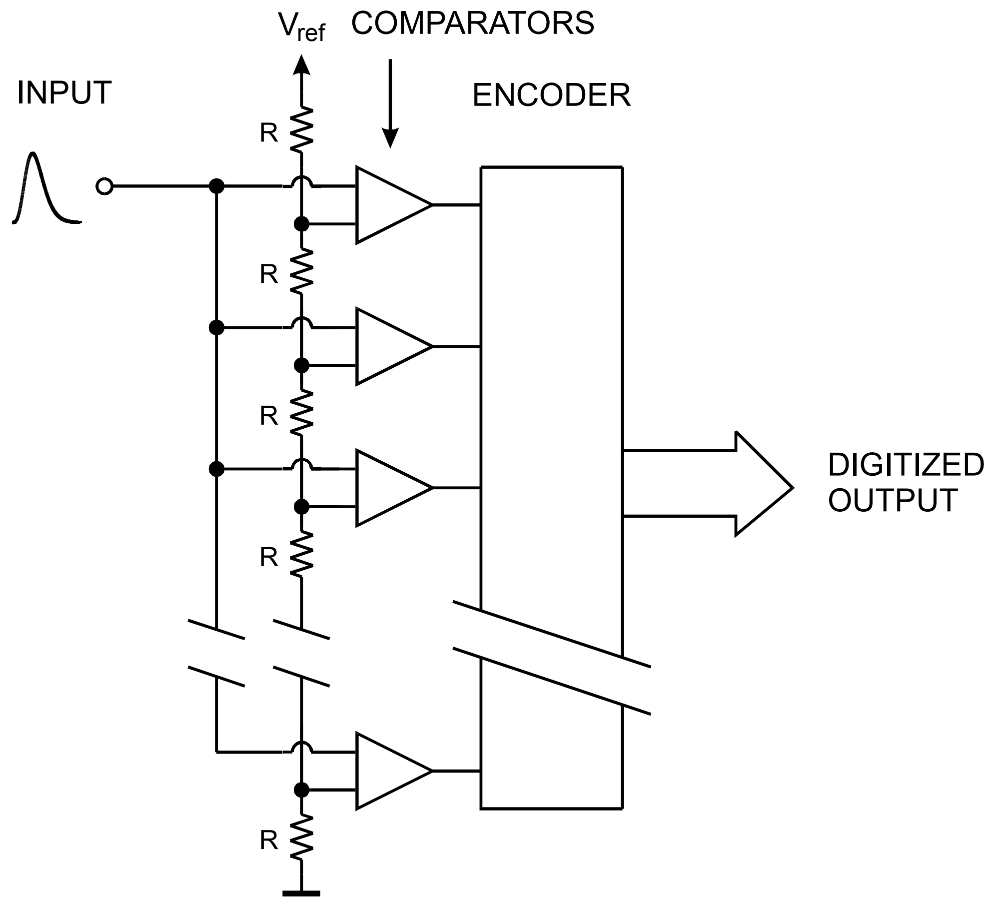
5.1 Signal Magnitude
(analog-to-digital converter,
viz. ADC or A/D)

Example: Flash ADC

The input signal is applied to n comparators in parallel. The switching thresholds are set by a resistor chain, such that the voltage difference between individual taps is equal to the desired measurement resolution.

In the presence of a signal all comparators with threshold levels less than the signal amplitude will fire. A decoder converts the parallel bit pattern into a more efficient form, for example binary code.

This type of ADC is fast, but requires as many comparators as measurement bins. Other converter types provide higher resolution and simpler circuitry at the expense of speed.



5.2 Time difference between the detected signal and a reference signal (time-to-digital converter, TDC)

The reference signal can be derived from another detector or from a common system clock, the crossing time of colliding beams, for example.

Circuit implementations include schemes that count “clock ticks” in fully digital circuitry or combine time-to-amplitude and amplitude-to-digital conversion in mixed analog-digital arrangements.

In complex detector systems the individual digitized outputs may require rather complex circuitry to combine the signal associated with a specific event and “package” them for efficient transfer.

5. Acquiring the Detector Signal

- Determine energy deposited in detector

- Detector signal generally a short current pulse

  Typical durations

Thin silicon detector (10 ... 300 µm thick):	100 ps – 30 ns
Thick (~cm) Si or Ge detector:	1 – 10 µs
Proportional chamber (gas):	10 ns – 10 µs
Gas microstrip or microgap chamber:	10 – 50 ns
Scintillator + PMT/APD:	100 ps – 10 µs

  The total charge Q_s contained in the detector current pulse $i_s(t)$ is proportional to the energy deposited in the detector

$$E \propto Q_s = \int i_s(t)\, dt$$

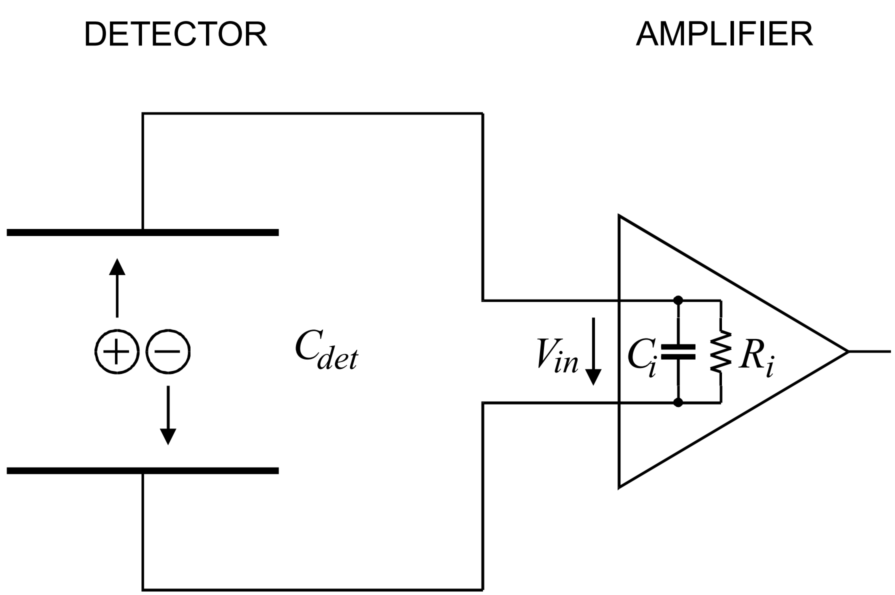
- Necessary to integrate the detector signal current.

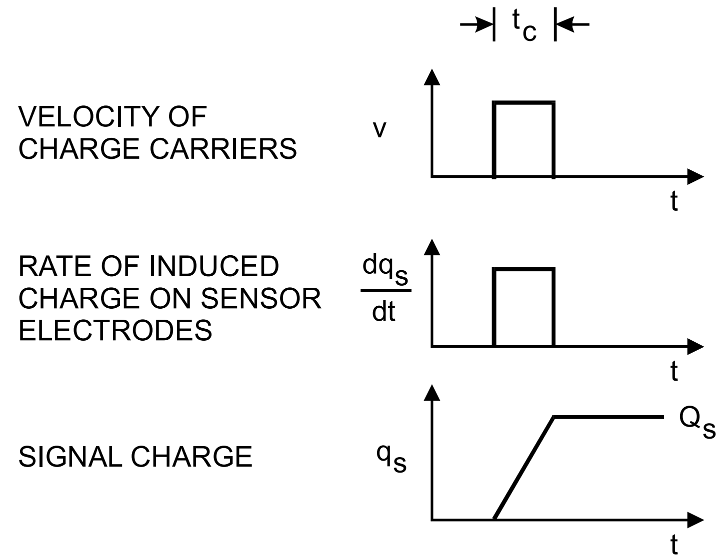
  Possibilities: 1. Integrate charge on input capacitance

  2. Use integrating ("charge sensitive") preamplifier

  3. Amplify current pulse and use integrating ("charge sensing") ADC

Integration on Input Capacitance





if R_i x $(C_{det} + C_i)$ >> collection time

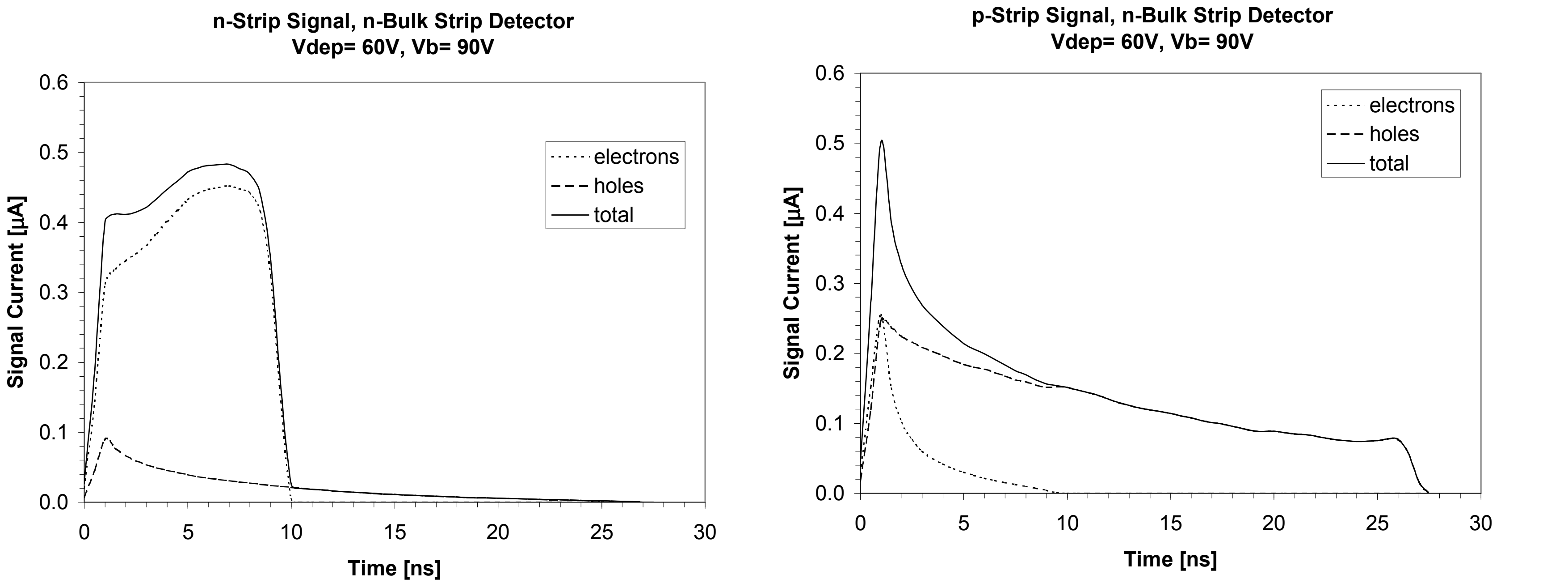
peak voltage at amplifier input $$V_{in} = \frac{Q_s}{C_{\det} + C_i}$$

↑

Magnitude of voltage depends on detector capacitance!

In reality the current pulses are more complex.

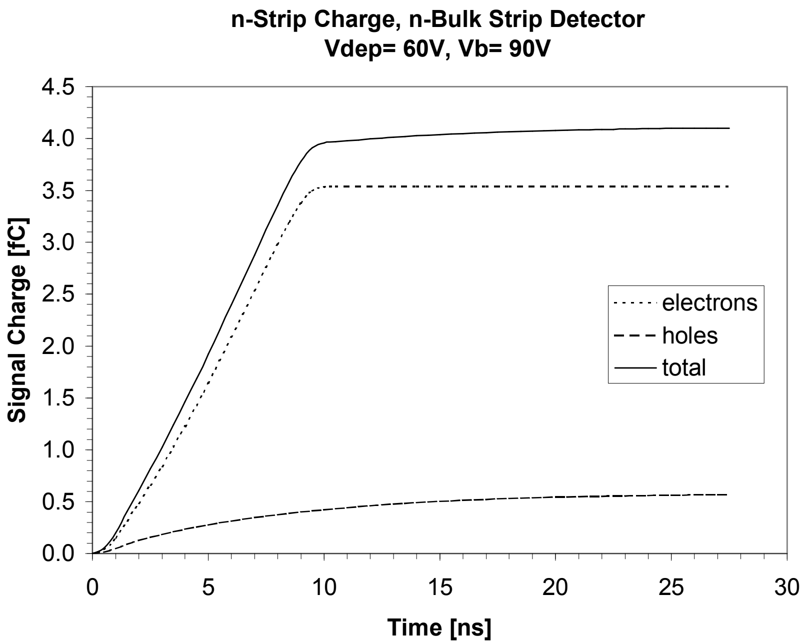
Current pulses on opposite sides (n-strip and p-strip) of a double-sided silicon strip detector (track traversing the detector) Although both pulses originate from the same particle track, the shapes are very different.

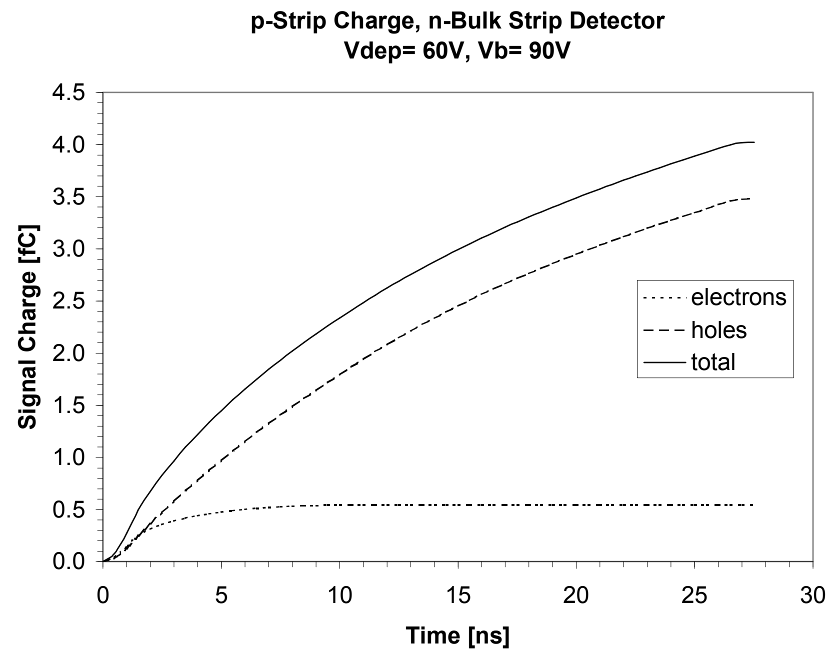


However, although the peak voltage or current signal measured by the amplifier may be quite different, the signal charge

$$Q_s = \int i_s dt$$

is the same.





$\Rightarrow$ **Desirable to measure signal charge**

- independent of detector pulse shape

When the input time constant RC is much greater than the signal duration, the peak voltage is a measure of the charge

$$V = \frac{1}{C}\int i_s dt = \frac{Q_s}{C}$$

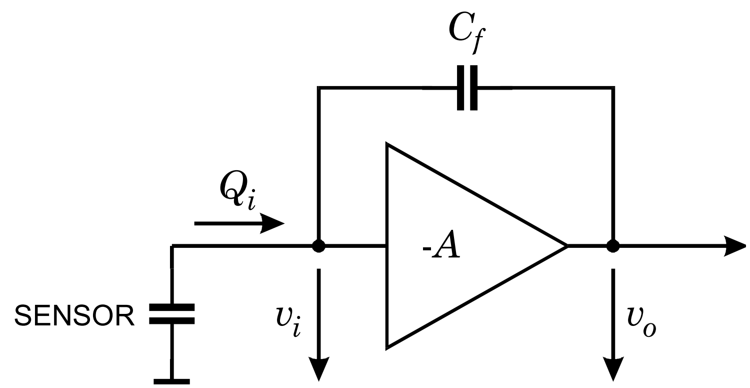
The measured signal depends on the total capacitance at the input.

Awkward in system where the detector capacitance varies, e.g.

- different detector geometries
  (e.g. strip detectors with different lengths)
- varying detector capacitance
  (e.g. partially depleted semiconductor detectors)

Use system whose response is independent of detector capacitance.

Active Integrator ("charge-sensitive amplifier")



Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow \quad v_o = -Av_i$

Input impedance = ∞ (i.e. no signal current flows into amplifier input)

Connect feedback capacitor C_f between output and input.

Voltage difference across C_f: $\quad v_f = (A+1)v_i$

⇒ Charge deposited on C_f: $\quad Q_f = C_f v_f = C_f (A+1) v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

⇒ Effective input capacitance $\quad C_i = \dfrac{Q_i}{v_i} = C_f (A+1)$ ("dynamic" input capacitance)

Gain

$$A_Q = \frac{dV_o}{dQ_i} = \frac{A \cdot v_i}{C_i \cdot v_i} = \frac{A}{C_i} = \frac{A}{A+1} \cdot \frac{1}{C_f} \approx \frac{1}{C_f} \quad (A >> 1)$$

Q_i is the charge flowing into the preamplifier .... but some charge remains on C_{det}.

What fraction of the signal charge is measured?

$$\frac{Q_i}{Q_s} = \frac{C_i v_i}{Q_{det} + Q_i} = \frac{C_i}{Q_s} \cdot \frac{Q_s}{C_i + C_{det}}$$

$$= \frac{1}{1 + \frac{C_{det}}{C_i}} \approx 1 \quad (\text{if } C_i >> C_{det})$$

Example: $A = 10^3$

$C_f = 1$ pF $\Rightarrow$ $C_i = 1$ nF

$C_{det} = 10$ pF: $Q_i / Q_s = 0.99$

$C_{det} = 500$ pF: $Q_i / Q_s = 0.67$

↑

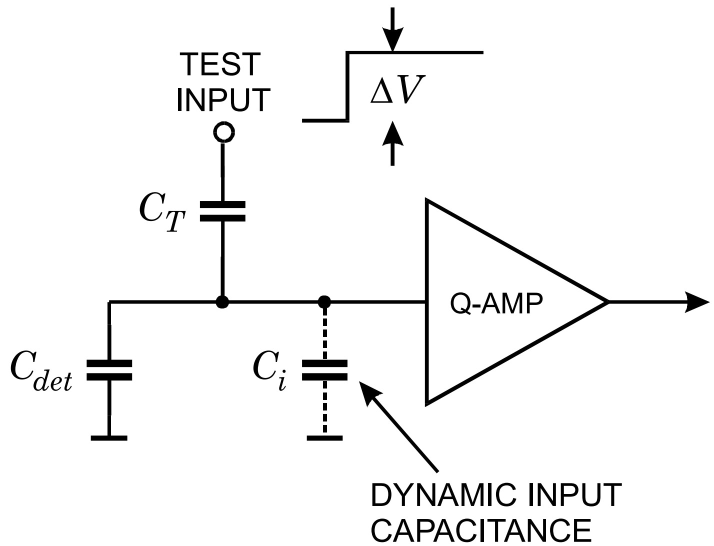
Si Det.: 50 µm thick
250 mm^2 area

Note: Input coupling capacitor must be $>> C_i$ for high charge transfer efficiency.

Calibration

Inject specific quantity of charge - measure system response

Use voltage pulse (can be measured conveniently with oscilloscope)



$C_i >> C_T \quad \Rightarrow$ Voltage step applied to test input develops over C_T.

$\Rightarrow \quad Q_T = \Delta V \cdot C_T$

Accurate expression:

$$Q_T = \frac{C_T}{1 + \frac{C_T}{C_i}} \cdot \Delta V \approx C_T \left(1 - \frac{C_T}{C_i}\right) \Delta V$$

Typically: $C_T / C_i = 10^{-3} - 10^{-4}$

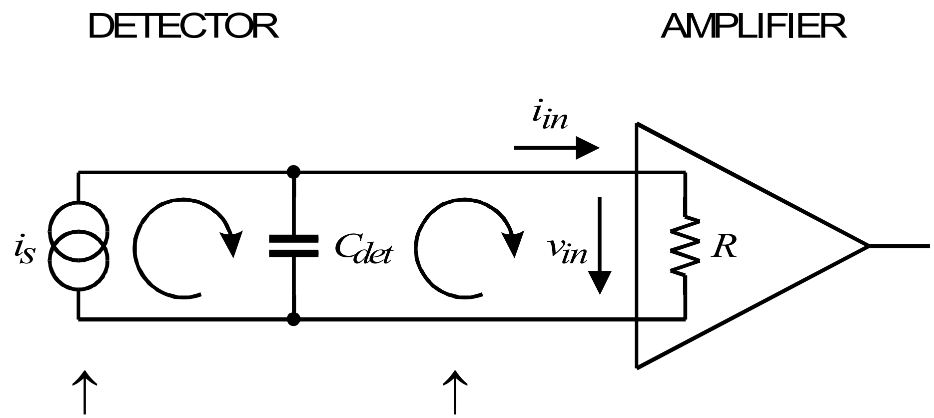
Realistic Charge-Sensitive Preamplifiers

The preceding discussion assumed idealized amplifiers with infinite speed.

In reality, amplifiers may be too slow to follow the instantaneous detector pulse.

Does this incur a loss of charge?

Equivalent Circuit:



charges moving in detector induce change of charge on detector electrodes

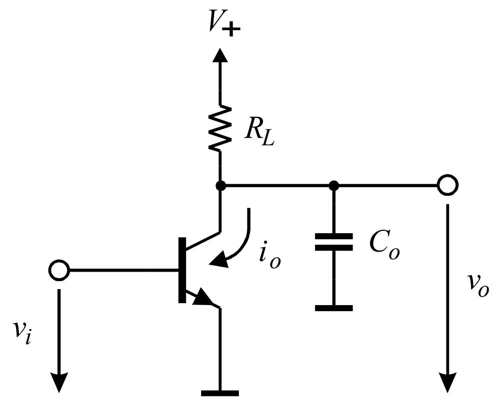
detector capacitance discharges into amplifier

Signal is preserved even if the amplifier responds much more slowly than the detector signal.

However, the response of the amplifier affects the measured pulse shape.

- How do “real” amplifiers affect the measured pulse shape?
- How does the detector affect amplifier response?

A Simple Amplifier



Voltage gain: $A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$

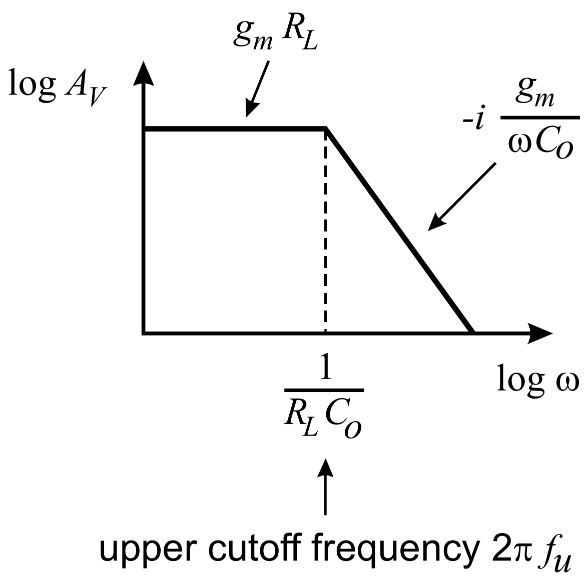
$g_m \equiv$ transconductance

$$Z_L = R_L \; // \; C_o$$

$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega C_o$$

$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega C_o \right)^{-1}$$

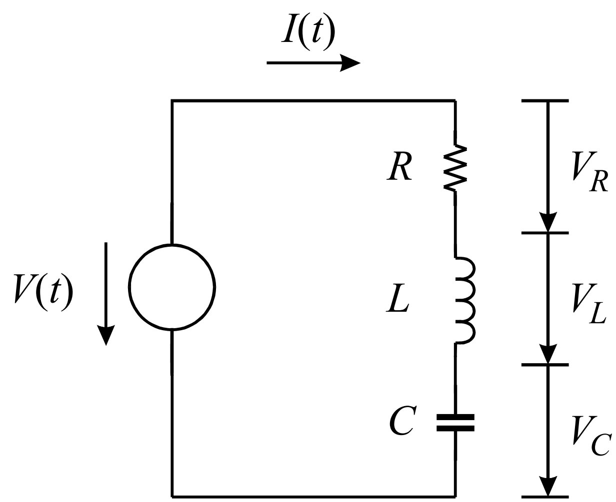
↑ low freq. ↑ high freq.



Appendix 1

Phasors and Complex Algebra in Electrical Circuits

Consider the *RLC* circuit



$$V = V_R + V_L + V_C$$

$$V = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{dI}{dt}R + L\frac{d^2 I}{dt^2} + \frac{I}{C}$$

Assume that $V(t) = V_0 e^{\mathbf{i}\omega t}$ and $I(t) = I_0 e^{\mathbf{i}(\omega t+\varphi)}$

$$i\omega V_0 e^{\mathbf{i}\omega t} = i\omega R I_0 e^{\mathbf{i}(\omega t-\varphi)} - \omega^2 L I_0 e^{\mathbf{i}(\omega t-\varphi)} + \frac{1}{C} I_0 e^{\mathbf{i}(\omega t-\varphi)}$$

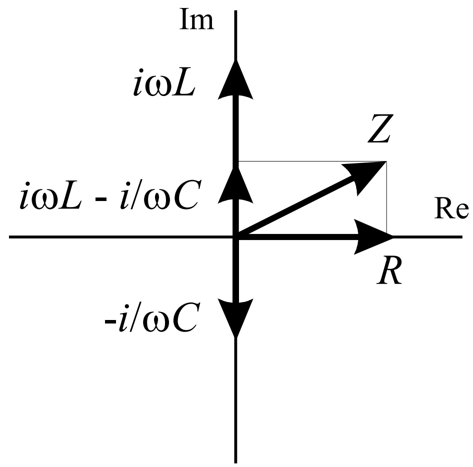
$$\frac{V_0}{I_0} e^{\mathbf{i}\varphi} = R + \mathbf{i}\omega L - \mathbf{i}\frac{1}{\omega C}$$

Thus, we can express the total impedance $Z \equiv (V_0 / I_0)\ e^{\mathbf{i}\varphi}$ of the circuit as a complex number with the magnitude $|Z| = V_0 / I_0$ and phase φ.

In this representation the equivalent resistances (reactances) of L and C are imaginary numbers

$$X_L = \mathbf{i}\omega L \quad \text{and} \quad X_C = -\frac{\mathbf{i}}{\omega C} \; .$$

Plotted in the complex plane:

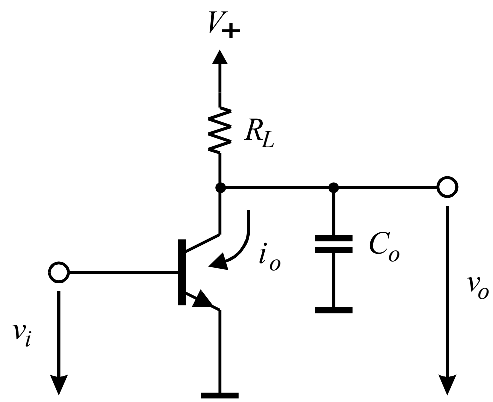


Relative to V_R, the voltage across the inductor V_L is shifted in phase by +90°.

The voltage across the capacitor V_C is shifted in phase by -90°.

Use to represent any element that introduces a phase shift, e.g. an amplifier. A phase shift of +90° appears as $+\mathbf{i}$, -90° as $-\mathbf{i}$.

A Simple Amplifier



Voltage gain: $$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

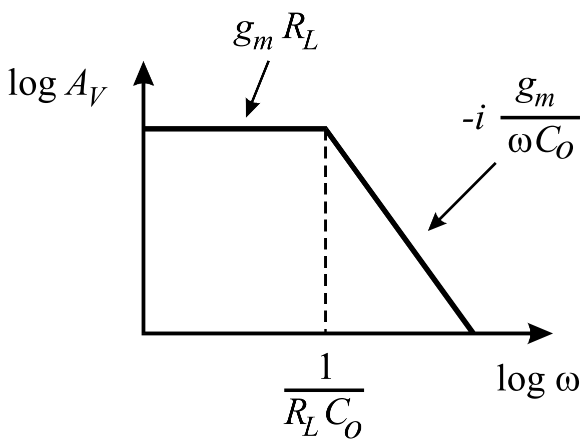
$g_m \equiv$ transconductance

$$Z_L = R_L \,//\, C_o$$

$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega C_o$$

$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega C_o \right)^{-1}$$

↑ low freq. ↑ high freq.



↑

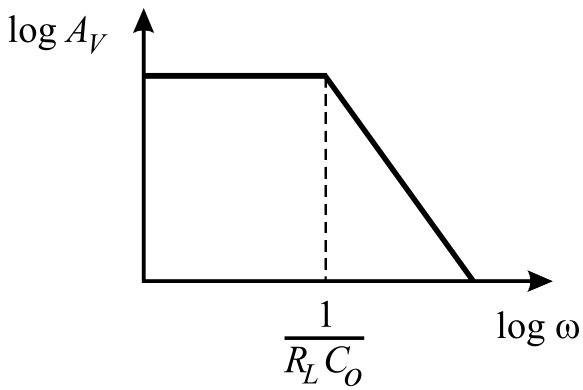
upper cutoff frequency $2\pi f_u$

Pulse Response of the Simple Amplifier

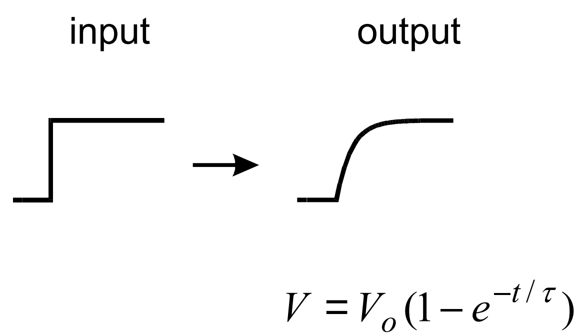
A voltage step $v_i(t)$ at the input causes a current step $i_o(t)$ at the output of the transistor.

For the output voltage to change, the output capacitance C_o must first charge up.

⇒ The output voltage changes with a time constant $\tau = R_L C_o$



Frequency Domain



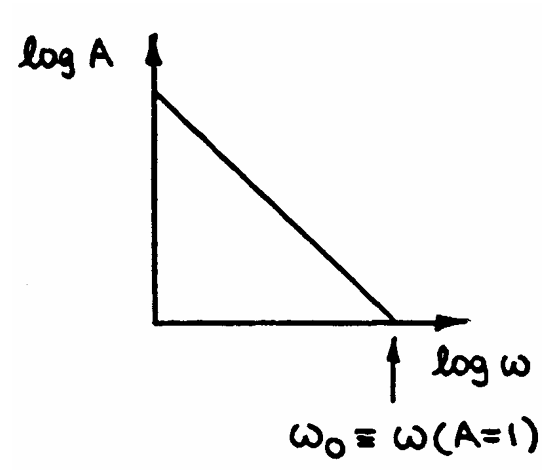
Time Domain

The time constant τ corresponds to the upper cutoff frequency

$$\tau = \frac{1}{2\pi f_u}$$

Input Impedance of a Charge-Sensitive Amplifier

Input impedance $$Z_i = \frac{Z_f}{A+1} \approx \frac{Z_f}{A} \quad (A >> 1)$$



Gain-Bandwidth Product

Amplifier gain vs. frequency beyond the upper cutoff frequency
Feedback Impedance

$$A = -\mathbf{i}\ \frac{\omega_0}{\omega}$$

Feedback impedance $$Z_f = -\mathbf{i}\ \frac{1}{\omega\ C_f}$$

$\Rightarrow$ Input Impedance $$Z_i = -\frac{\mathbf{i}}{\omega\ C_f} \cdot \frac{1}{-\mathbf{i}\ \dfrac{\omega_0}{\omega}}$$

$$Z_i = \frac{1}{\omega_0 C_f}$$

Imaginary component vanishes $\Rightarrow$ *Resistance:* $Z_i \rightarrow R_i$

$\Rightarrow$ low frequencies ($f < f_u$): capacitive input
high frequencies ($f > f_u$): resistive input

Time Response of a Charge-Sensitive Amplifier

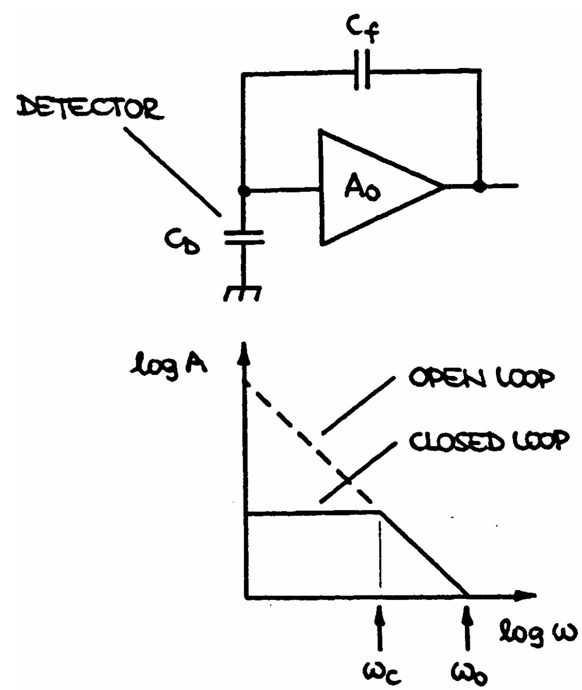
Input resistance and detector capacitance form RC time constant

$$\tau_i = R_i C_D$$

$$\tau_i = \frac{1}{\omega_0 C_f} \cdot C_D$$

⇒ **Rise time increases with detector capacitance.**

Or apply feedback theory:



Closed Loop Gain $$A_f = \frac{C_D + C_f}{C_f} \quad (A_f << A_0)$$

$$A_f \approx \frac{C_D}{C_f} \quad (C_D >> C_f)$$

Closed Loop Bandwidth $$\omega_C A_f = \omega_0$$

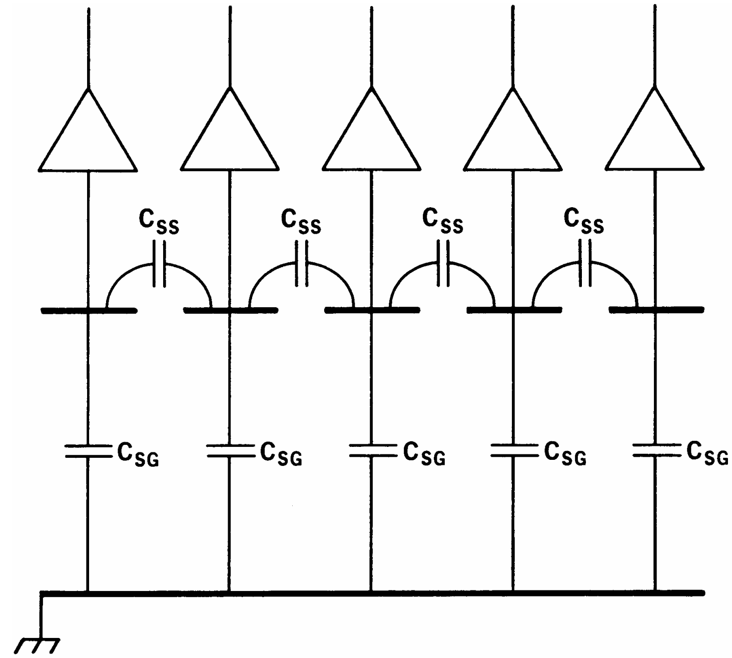
Response Time $$\tau_{amp} = \frac{1}{\omega_C} = C_D \frac{1}{\omega_0 C_f}$$

Same result as from input time constant.

Input impedance is critical in strip or pixel detectors:

Amplifiers must have a low input impedance to reduce transfer of charge through capacitance to neighboring strips

strip detector electrodes



For strip pitches that are smaller than the bulk thickness, the capacitance is dominated by the fringing capacitance to the neighboring strips C_{SS}.

Typically: 1 – 2 pF/cm for strip pitches of 25 – 100 µm on Si.

The backplane capacitance C_{SG} is typically 20% of the strip-to-strip capacitance.

Negligible cross-coupling at shaping times T_P > (2 … 3) x $R_i C_D$ and if $C_i \gg C_D$.