II. Signal Processing Part 1

1. What determines Resolution?

1. Signal Variance >> Baseline Variance

SIGNAL $\oplus$ BASELINE NOISE $\Rightarrow$ SIGNAL + NOISE

$\Rightarrow$ Electronic (baseline) noise not important

Examples:
- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

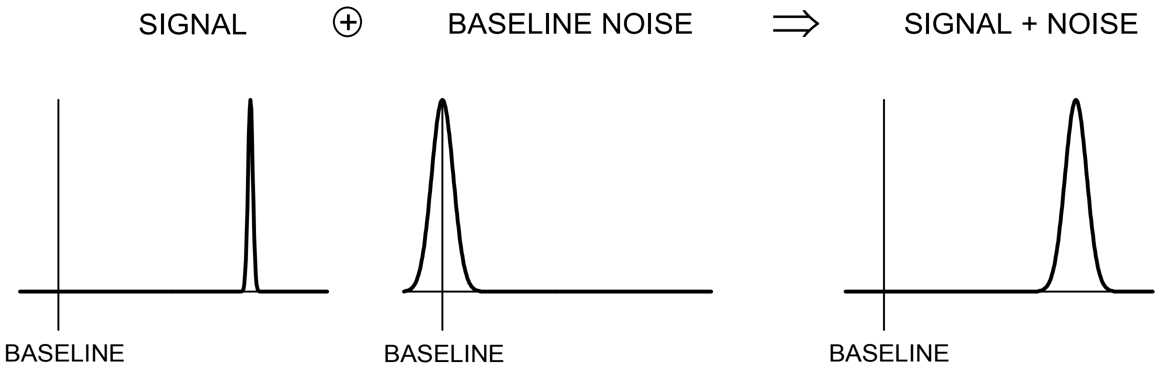
Number of photoelectrons: $N_{pe} \approx 8 \cdot 10^4$ [MeV^{-1}] $\times E_\gamma \times QE \approx 2.4 \cdot 10^4$

Variance typically: $\sigma_{pe} = N_{pe}^{1/2} \approx 160$ and $\sigma_{pe} / N_{pe} \approx 5$ - 8%

Signal at PMT anode (assume Gain= 10^4): $Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8$ el and
$\sigma_{sig} = G_{PMT}\, \sigma_{pe} \approx 1.2 \cdot 10^7$ el

whereas electronic noise easily < 10^4 el

2. Signal Variance << Baseline Variance



⇒ Electronic (baseline) noise critical for resolution

Examples:
- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si : Number of electron-hole pairs $N_{ep} = \dfrac{E_{dep}}{3.6\text{ eV}}$

Variance $\sigma_{ep} = \sqrt{F \cdot N_{ep}}$ (where F= Fano factor ≈ 0.1)

For 50 keV photons: $\sigma_{ep} \approx 40$ el $\Rightarrow$ $\sigma_{ep} / N_{ep} = 7.5 \cdot 10^{-4}$

Obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

2. Basic Noise Mechanisms

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\ e\ v}{l}\ .$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2 ,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.*	thermal noise
- number fluctuations, *e.g.*	shot noise excess or '1/f' noise

Thermal noise and shot noise are both “white” noise sources, i.e.

power per unit bandwidth (≡ spectral density) is constant: $\dfrac{dP_{noise}}{df} = const.$

Low Frequency Noise

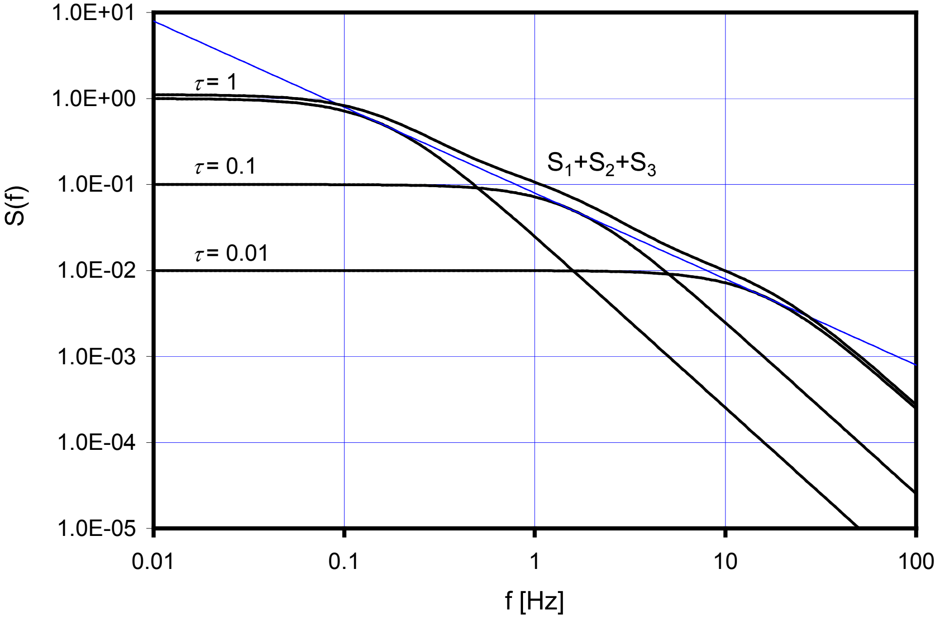
In a semiconductor, for example, charge can be trapped and then released after a characteristic lifetime τ.

The spectral density for a single lifetime $S(f) \propto \dfrac{\tau}{1+(2\pi f\tau)^2}$.

For $2\pi f\tau \gg 1$, $S(f) \propto \dfrac{1}{f^2}$.

However,
several traps with different time constants can yield a "1/f" distribution:

Traps with three time constants of 0.01, 0.1 and 1 s yield a 1/f distribution over two decades in frequency.



Low frequency noise is ubiquitous – must not have 1/f dependence, but commonly called 1/f noise.

Spectral power density: $\dfrac{dP_{noise}}{df} = \dfrac{1}{f^\alpha}$ (typically α= 0.5 – 2)

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f

$$\frac{dP_{noise}}{df} = 4kT$$

k = Boltzmann constant
T = absolute temperature

since $P = \frac{V^2}{R} = I^2 R$

R = DC resistance

the spectral noise voltage density

$$\frac{dV^2_{noise}}{df} \equiv e_n^2 = 4kTR$$

and the spectral noise current density

$$\frac{dI^2_{noise}}{df} \equiv i_n^2 = \frac{4kT}{R}$$

The total noise depends on the bandwidth of the system,
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \; A_v^2(f) \; df$$

Note: Since spectral noise components are non-correlated, one must integrate over the noise power.

2. Shot noise

A common example of noise due to number fluctuations is
"shot noise", which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode
(emission over a barrier)

Spectral noise current density: $i_n^2 = 2eI$

e = electronic charge
I = DC current

A more intuitive interpretation of this expression will be given later.

Note: Shot noise does not occur in "ohmic" conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

- For derivations of the thermal and shot noise spectral densities, see Appendix 1.

3. Noise Bandwidth vs. Signal Bandwidth

Consider an amplifier with the frequency response $A(f)$. This can be rewritten $A(f) \equiv A_0 G(f)$, where A_0 is the maximum gain and $G(f)$ describes the frequency response.

For example, for the simple amplifier described above

$$A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\ C_o \right)^{-1} = g_m R_L \ \frac{1}{1 + \mathbf{i}\omega R_L C_o}$$

and using the above convention $A_0 \equiv g_m R_L \quad \text{and} \quad G(f) \equiv \dfrac{1}{1 + \mathbf{i}\ (2\pi f\ R_L C_o)}$

If a “white” noise source with spectral density e_{ni} is present at the input, the total noise voltage at the output is

$$v_{no} = \sqrt{\int_0^\infty e_{ni}^2 \left| A_0 G(f) \right|^2 df} = e_{ni}\ A_0 \sqrt{\int_0^\infty G^2(f) df} \equiv e_{ni}\ A_0 \sqrt{\Delta f_n}$$

Δf_n is the “noise bandwidth”.
Note that, in general, the noise bandwidth and the signal bandwidth are not the same.
If the upper cutoff frequency is determined by a single RC time constant, as in the “simple amplifier”,

the signal bandwidth $\Delta f_s = f_u = \dfrac{1}{2\pi\ RC}$ and the noise bandwidth $\Delta f_n = \dfrac{1}{4RC} = \dfrac{\pi}{2} f_u$.

Independent noise contributions add in quadrature (additive in noise power)

$$v_{n,tot} = \sqrt{\sum_i v_{ni}^2}$$

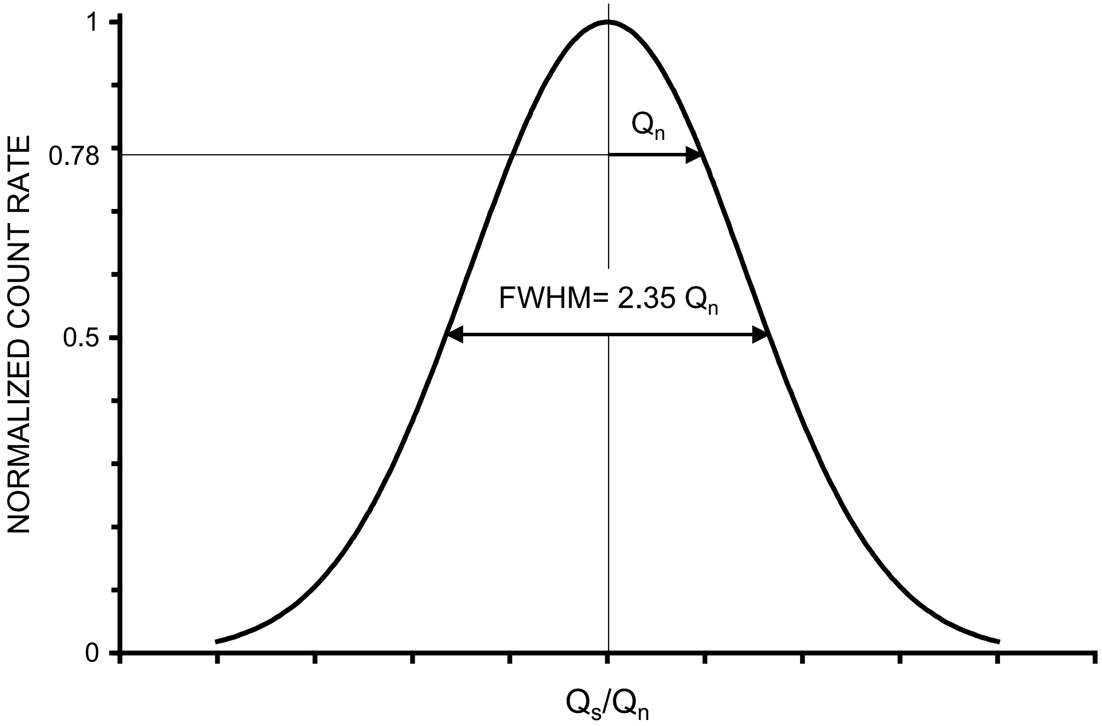
Both thermal and shot noise are purely random.

⇒ amplitude distribution is Gaussian

⇒ noise modulates baseline

⇒ baseline fluctuations superimposed on signal

⇒ output signal has Gaussian distribution



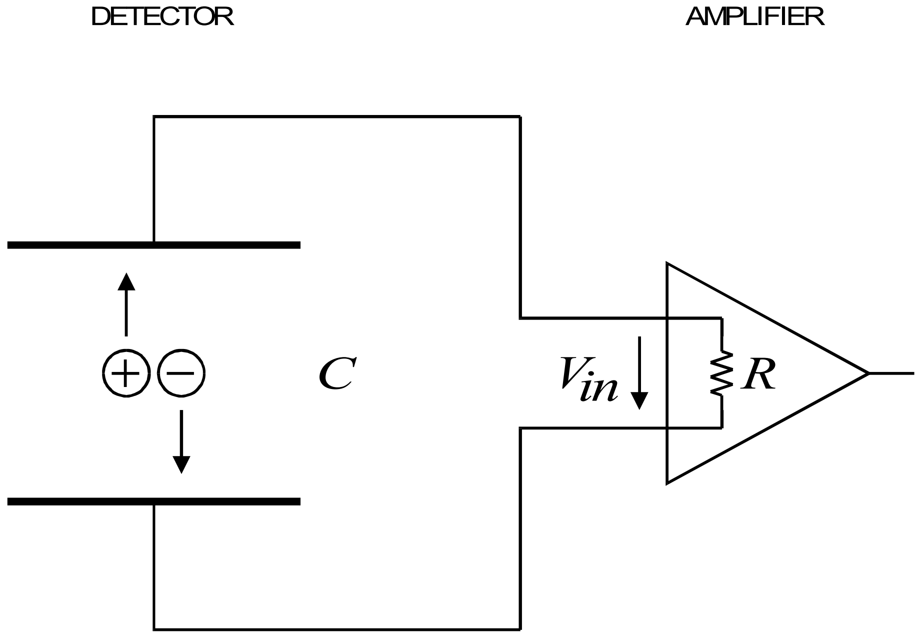
Measuring Resolution

Inject an input signal with known charge using a pulse generator set to approximate the detector signal (possible ballistic deficit).
Measure the pulse height spectrum.

peak centroid ⇒ signal magnitude

peak width ⇒ noise (FWHM= 2.35 rms)

4. Signal-to-Noise Ratio vs. Detector Capacitance



At long input time constants $\tau = RC$ the detector signal current is integrated on the detector capacitance.

The resulting voltage sensed by the amplifier

$$V_{in} = \frac{Q_{\det}}{C} = \frac{\int i_s dt}{C}$$

Then the peak amplifier signal is inversely proportional to the **total capacitance at the input**, i.e. the sum of

detector capacitance,
input capacitance of the amplifier, and
stray capacitances.

For a constant noise voltage v_n , the signal-to-noise ratio

$$\frac{S}{N} = \frac{V_S}{v_n} \propto \frac{1}{C}$$

- However, S/N does not become infinite as $C \to 0$ (see Appendix 2)

Charge-Sensitive Preamplifier – Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,

  i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.
- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i >> C_{det}$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_{v0}$.
- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

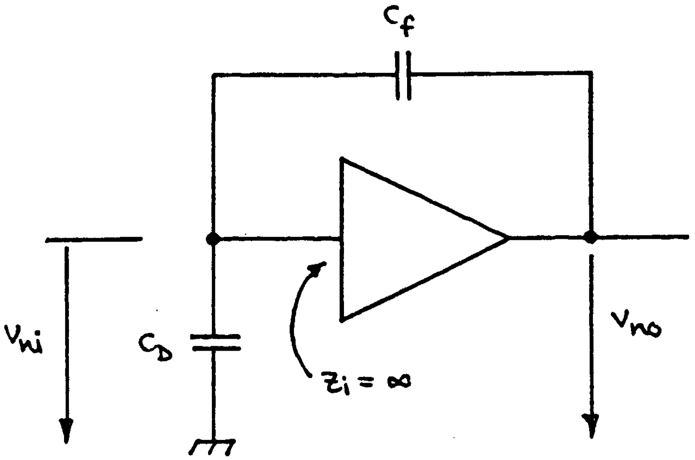
Note, that although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.

⇒ S/N **at the amplifier output depends on the amount of feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.

$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_D}}{X_{C_D}} = v_{ni} \frac{\frac{1}{\omega C_f} + \frac{1}{\omega C_D}}{\frac{1}{\omega C_D}}$$



$$v_{no} = v_{ni} \left(1 + \frac{C_D}{C_f} \right)$$

Equivalent input noise charge $Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$

Signal-to-noise ratio $\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_D + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$ $\Rightarrow$ $Q_{ni} = v_{ni} \left(C_D + C_f \right)$

Same result as for voltage-sensitive amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback. In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

Conclusion

In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.
- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

5. Pulse Shaping

Two conflicting objectives:

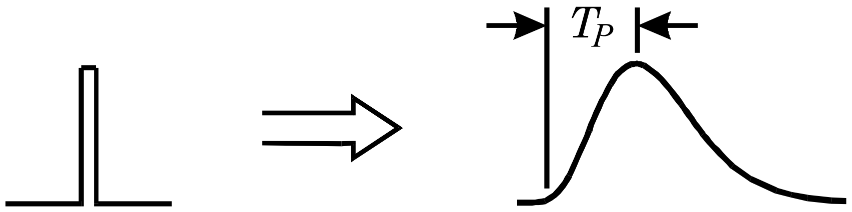
1. Improve Signal-to-Noise Ratio S/N

Restrict bandwidth to match measurement time ⇒ Increase pulse width

Typically, the pulse shaper transforms a narrow detector current pulse to a broader pulse (to reduce electronic noise),

with a gradually rounded maximum at the peaking time T_P (to facilitate measurement of the amplitude)

SENSOR PULSE SHAPER OUTPUT

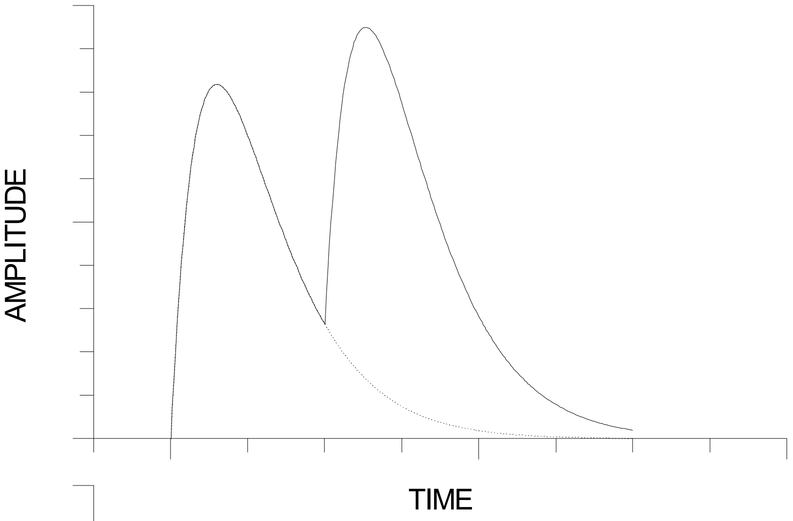


If the shape of the pulse does not change with signal level, the peak amplitude is also a measure of the energy, so one often speaks of pulse-height measurements or pulse height analysis.

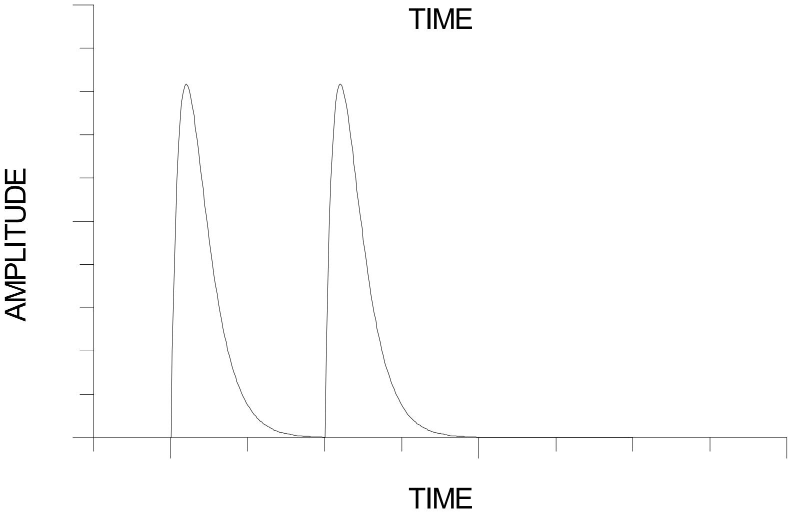
The pulse height spectrum is the energy spectrum.

2. Improve Pulse Pair Resolution

⇒ Decrease pulse width



Pulse pile-up distorts amplitude measurement



Reducing pulse shaping time to 1/3 eliminates pile-up.

Necessary to find balance between these conflicting requirements.

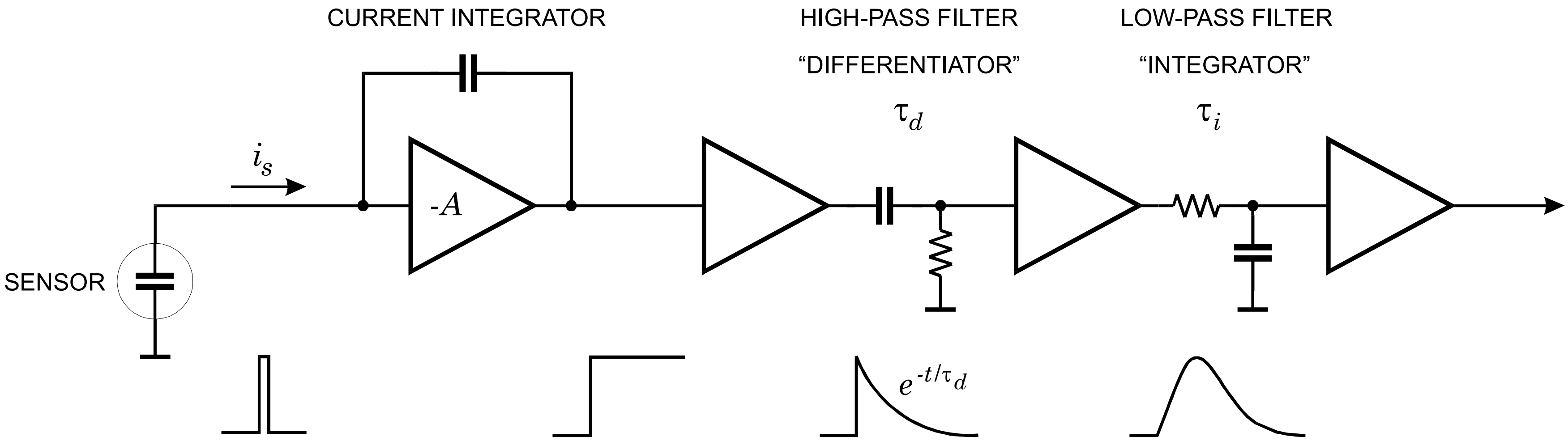
Sometimes minimum noise is crucial,

sometimes rate capability is paramount.

Usually, many considerations combined lead to a “non-textbook” compromise.

- *“Optimum shaping” depends on the application!*
- Shapers need not be complicated – *every amplifier is a pulse shaper!*

Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

⇒ Useful for estimates, since simple to evaluate

Key elements:
- lower frequency bound
- upper frequency bound

common to all shapers.

6. Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Equivalent Noise Charge

Inject known signal charge into preamp input
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

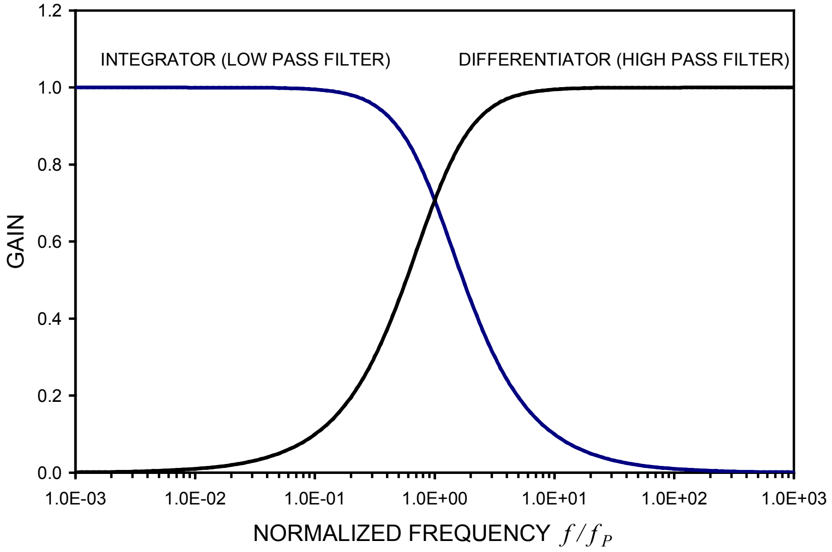
Equivalent Noise Charge $\equiv$ Input charge for which $S/N = 1$

Dependence of Equivalent Noise Charge on Shaping Time

Assume that differentiator and integrator time constants are equal $\tau_i = \tau_d \equiv \tau$.

⇒ Both cutoff frequencies equal $f_U = f_L \equiv f_P = 1/2\pi\tau$.

Frequency response of individual pulse shaping stages

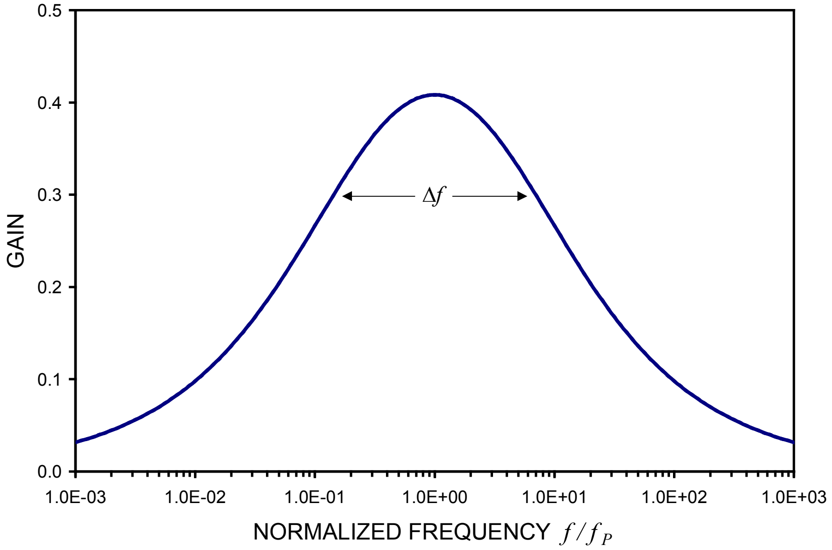


Combined frequency response

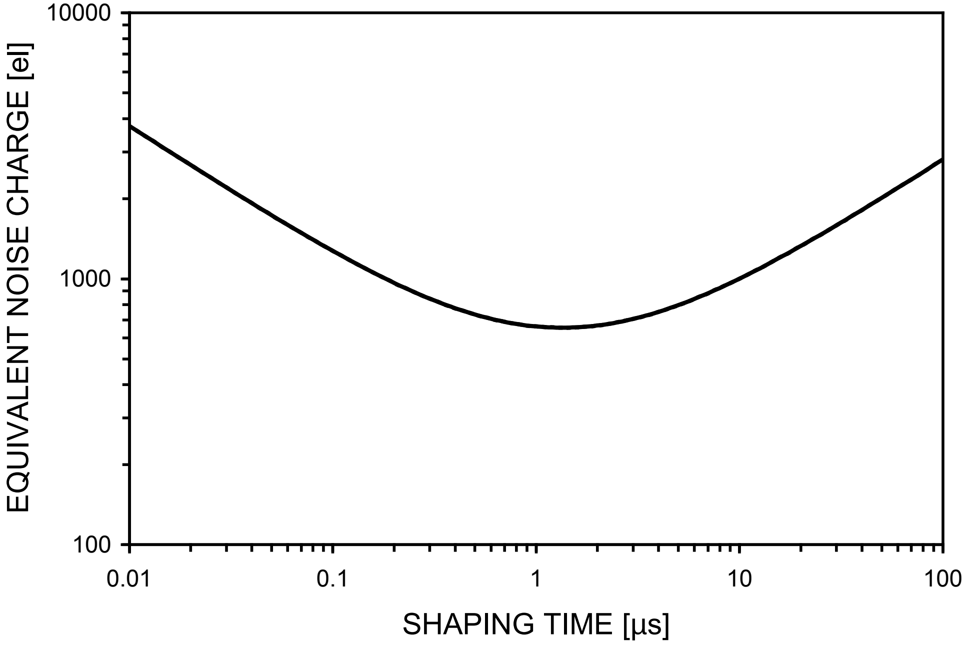
Logarithmic frequency scale
⇒ shape of response independent of τ.

However, bandwidth Δf decreases with increasing time constant τ.

⇒ for white noise sources expect noise to decrease with bandwidth, i.e. decrease with increasing time constant.



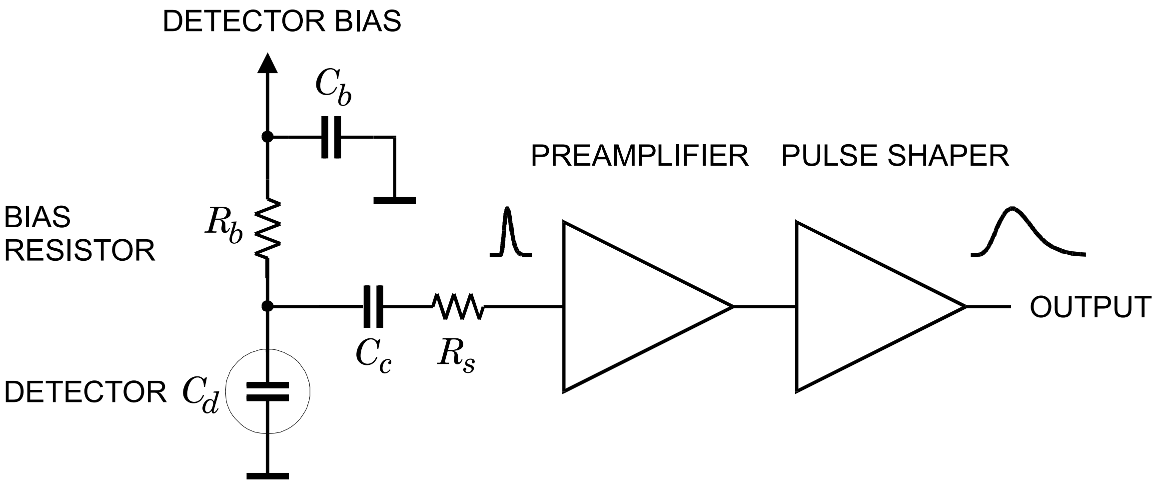
Result of typical noise measurement vs. shaping time



Noise sources (thermal and shot noise) have a flat (“white”) frequency distribution.

Why doesn’t the noise decrease monotonically with increasing shaping time (decreasing bandwidth)?

7. Analytical Analysis of a Detector Front-End



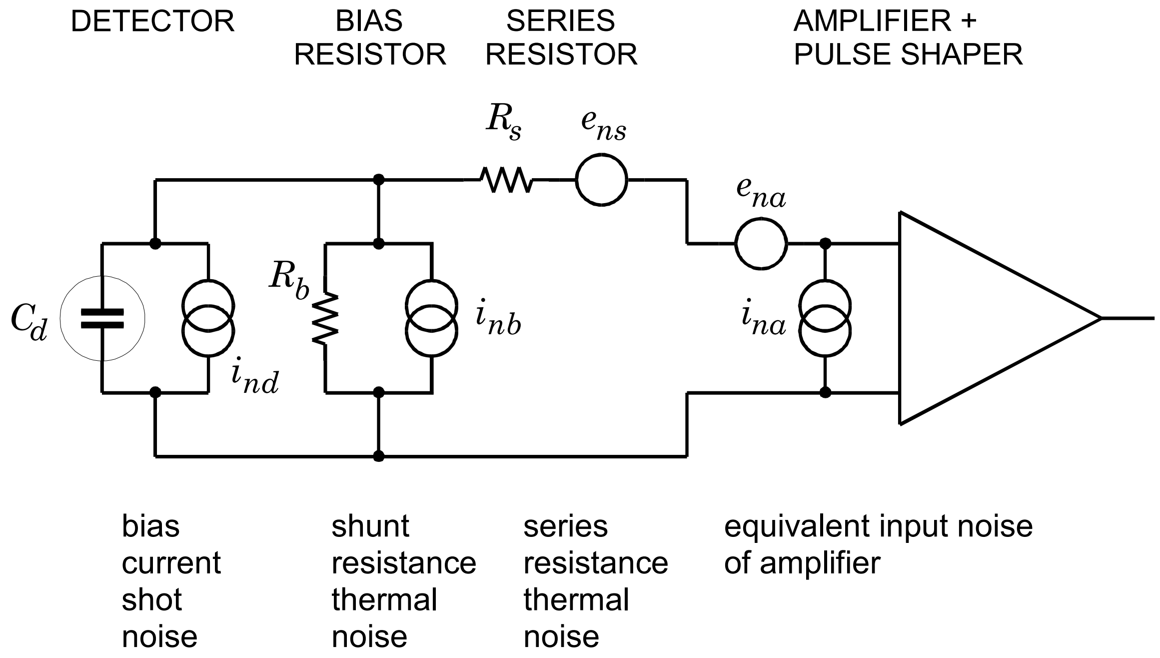
Detector bias voltage is applied through the resistor R_B. The bypass capacitor C_B serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the “far end” of the bias resistor to ground, so that R_B appears to be in parallel with the detector.

The coupling capacitor C_C in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why a capacitor serving this role is also called a “blocking capacitor”).

The series resistor R_S represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients (“input protection”)

Equivalent circuit for noise analysis



In this example a voltage-sensitive amplifier is used, so all noise contributions will be calculated in terms of the noise voltage appearing at the amplifier input.

Resistors can be modeled either as voltage or current generators.

- Resistors in parallel with the input act as current sources
- Resistors in series with the input act as voltage sources.

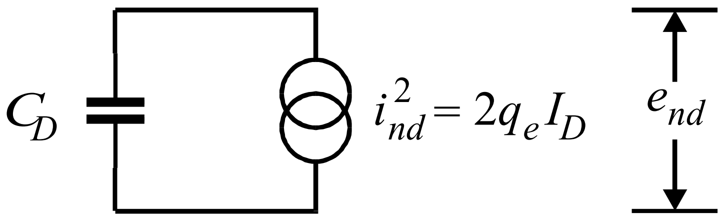
Steps in the analysis:

1. Determine the frequency distribution of the noise voltage presented to the amplifier input from all individual noise sources
2. Integrate over the frequency response of a CR-RC shaper to determine the total noise output.
3. Determine the output signal for a known signal charge and calculate equivalent noise charge (signal charge for S/N= 1)

First, assume a simple CR-RC shaper with equal differentiation and integration time constants $\tau_d = \tau_i = \tau$, which in this special case is equal to the peaking time.

Noise Contributions

1. Detector bias current



This model results from two assumptions:

1. The input impedance of the amplifier is infinite
2. The shunt resistance R_P is much larger than the capacitive reactance of the detector in the frequency range of the pulse shaper.

Does this assumption make sense?

If R_P is too small, the signal charge on the detector capacitance will discharge before the shaper output peaks. To avoid this

$$R_P C_D >> t_P \approx \frac{1}{\omega_P}$$

where ω_P is the midband frequency of the shaper. Therefore, $R_P >> \frac{1}{\omega_P C_D}$ as postulated.

Under these conditions the noise current will flow through the detector capacitance, yielding the voltage

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega\, C_D)^2} = 2q_e I_D \frac{1}{(\omega\, C_D)^2}$$

⇒ **the noise contribution decreases with increasing frequency (shorter shaping time)**

Note: Although shot noise is “white”, the resulting noise spectrum is strongly frequency dependent.

In the time domain this result is more intuitive. Since every shaper also acts as an integrator, one can view the total shot noise as the result of “counting electrons”.

Assume an ideal integrator that records all charge uniformly within a time T. The number of electron charges measured is

$$N_e = \frac{I_D T}{q_e}$$

The associated noise is the fluctuation in the number of electron charges recorded

$$\sigma_n = \sqrt{N_e} \propto \sqrt{T}$$

Does this also apply to an AC-coupled system, where no DC current flows, so no electrons are “counted”?

Since shot noise is a fluctuation, the current undergoes both positive and negative excursions. Although the DC component is not passed through an AC coupled system, the excursions are. Since, on the average, each fluctuation requires a positive and a negative zero crossing, the process of “counting electrons” is actually the counting of zero crossings, which in a detailed analysis yields the same result.

2. Parallel Resistance

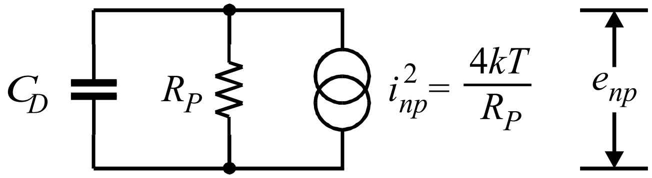
Any shunt resistance R_P acts as a noise current source. In the specific example shown above, the only shunt resistance is the bias resistor R_b.

Additional shunt components in the circuit:

1. bias noise current source (infinite resistance by definition)
2. detector capacitance

The noise current flows through both the resistance R_P and the detector capacitance C_D.

⇒ equivalent circuit



The noise voltage applied to the amplifier input is

$$e_{np}^2 = \frac{4kT}{R_P}\left(\frac{R_P \cdot \frac{-\mathbf{i}}{\omega C_D}}{R_P - \frac{\mathbf{i}}{\omega C_D}}\right)^2$$

$$e_{np}^2 = 4kTR_P \frac{1}{1+(\omega R_P C_D)^2}$$

Comment:

Integrating this result over all frequencies yields

$$\int_0^\infty e_{np}^2(\omega)d\omega = \int_0^\infty \frac{4kTR_P}{1+(\omega\, R_P C_D)^2}\; d\omega = \frac{kT}{C_D}\ ,$$

which is independent of R_P. Commonly referred to as "kTC" noise, this contribution is often erroneously interpreted as the "noise of the detector capacitance".

An ideal capacitor has no thermal noise; all noise originates in the resistor.

So, why is the result independent of R_P?

R_P determines the primary noise, but also the noise bandwidth of this subcircuit. As R_P increases, its thermal noise increases, but the noise bandwidth decreases, making the total noise independent of R_P.

However,
If one integrates e_{np} over a bandwidth-limited system (such as our shaper),

$$\upsilon_n^2 = \int_0^\infty 4kTR_P \left| \frac{G(i\omega\ )}{1-i\omega\, R_P C_D} \right|^2 d\omega$$

the total noise decreases with increasing R_P.

3. Series Resistance

The noise voltage generator associated with the series resistance R_S is in series with the other noise sources, so it simply contributes

$$e_{nr}^2 = 4kTR_S$$

4. Amplifier input noise

The amplifier noise voltage sources usually are not physically present at the amplifier input. Instead the amplifier noise originates within the amplifier, appears at the output, and is referred to the input by dividing the output noise by the amplifier gain, where it appears as a noise voltage generator.

$$e_{na}^2 = \underset{\substack{\uparrow \\ \text{“white noise”}}}{e_{nw}^2} + \underset{\substack{\uparrow \\ 1/f \text{ noise (can also originate in external components)}}}{\frac{A_f}{f}}$$

This noise voltage generator also adds in series with the other sources.

- Amplifiers generally also exhibit input current noise, which is physically present at the input. Its effect is the same as for the detector bias current, so the analysis given in 1. can be applied.
- In a well-designed amplifier the noise is dominated by the input transistor (fast, high-gain transistors generally best). Noise parameters of transistors are discussed in the Appendix.

  Transistor input noise decreases with transconductance

  $\Rightarrow$ increased power
- Minimum device noise limited both by technology and fundamental physics.

Equivalent Noise Charge

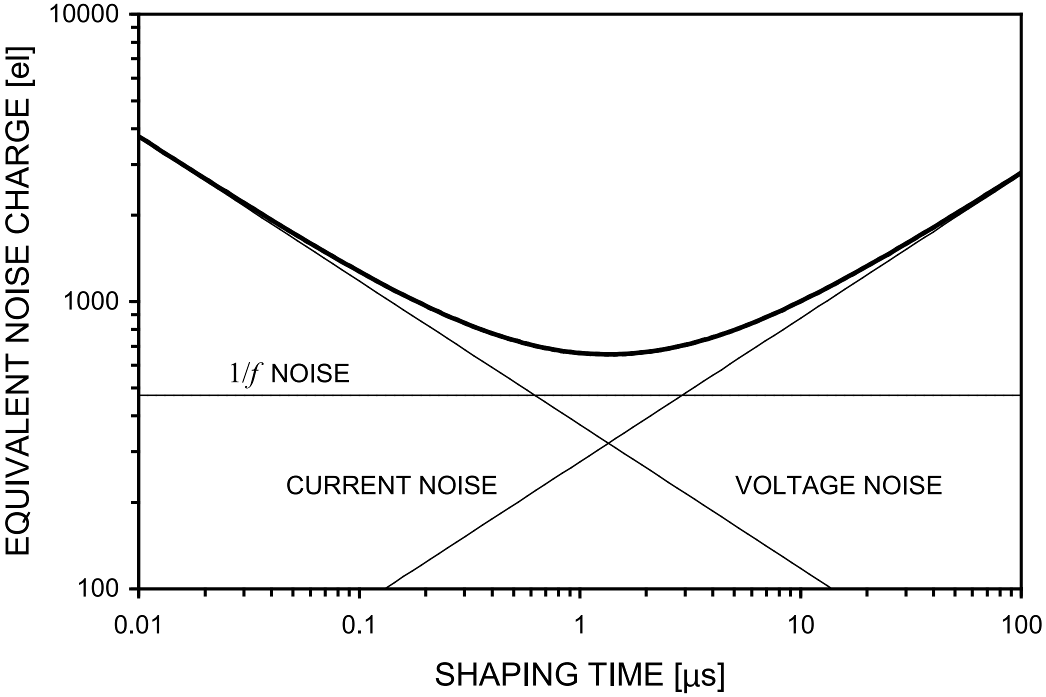
$$Q_n^2 = \left(\frac{\mathbf{e}^2}{8}\right)\left[\left(2q_e I_D + \frac{4kT}{R_P} + i_{na}^2\right)\cdot\tau + \left(4kTR_S + e_{na}^2\right)\cdot\frac{C_D^2}{\tau} + 4A_f C_D^2\right]$$

	↑	↑	↑
$\mathbf{e} = \exp(1)$	current noise	voltage noise	$1/f$ noise
	$\propto \tau$	$\propto 1/\tau$	independent of τ
	independent of C_D	$\propto C_D^2$	$\propto C_D^2$

- Current noise is independent of detector capacitance, consistent with the notion of "counting electrons".
- Voltage noise increases with detector capacitance
  (reduced signal voltage)
- $1/f$ noise is independent of shaping time.
  In general, the total noise of a $1/f$ source depends on the ratio of the upper to lower cutoff frequencies, not on the absolute noise bandwidth. If τ_d and τ_i are scaled by the same factor, this ratio remains constant.
- detector leakage current and FET noise decrease with temperature

  ⇒ high resolution Si and Ge detectors for x-rays and gamma rays operate at cryogenic temperatures.

The equivalent noise charge Q_n assumes a minimum when the current and voltage noise contributions are equal. Typical result:



↑ ↑

dominated by voltage noise current noise

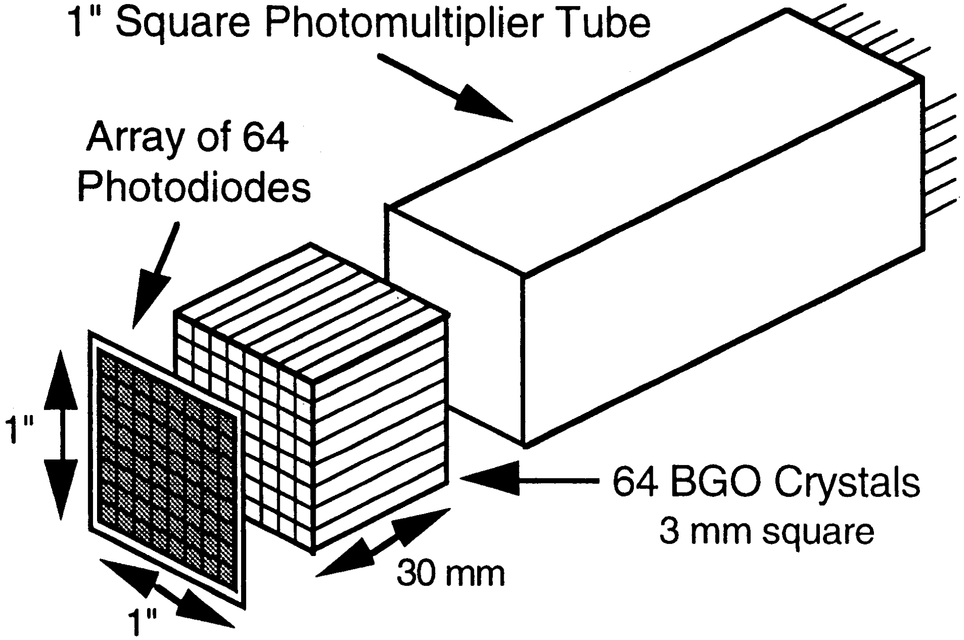
For a CR-RC shaper the noise minimum obtains for $\tau_d = \tau_i = \tau$.

This criterion does not hold for more sophisticated shapers.

8. Example: Photodiode Readout

(S. Holland, N. Wang, I. Kipnis, B. Krieger, W. Moses, LBNL)

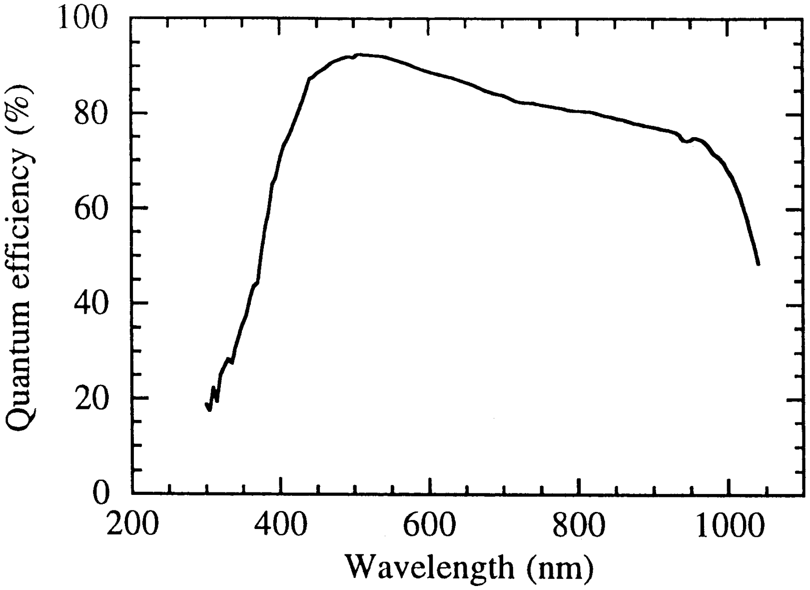
Medical Imaging (Positron Emission Tomography)



Read out 64 BGO crystals with one PMT (timing, energy) and tag crystal by segmented photodiode array.

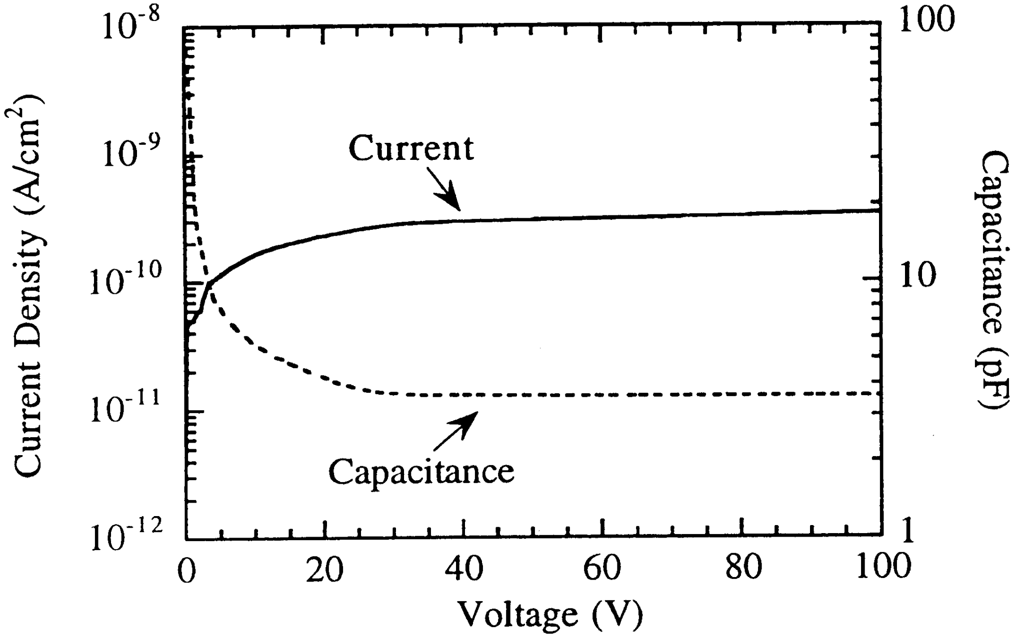
Requires thin dead layer on photodiode to maximize quantum efficiency.

Thin electrode must be implemented with low resistance to avoid significant degradation of electronic noise.



Furthermore, low reverse bias current critical to reduce noise.

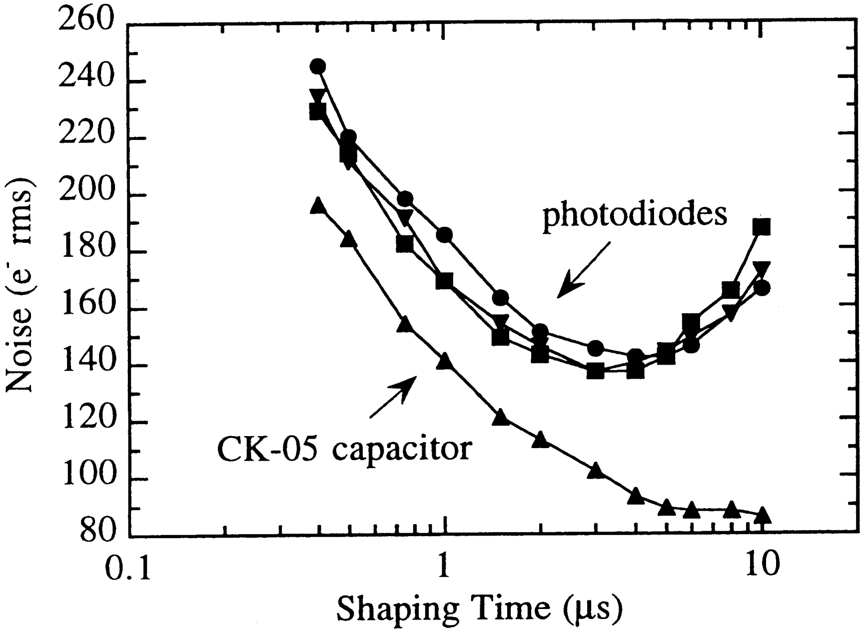
Photodiodes designed and fabricated in LBNL Microsystems Lab.



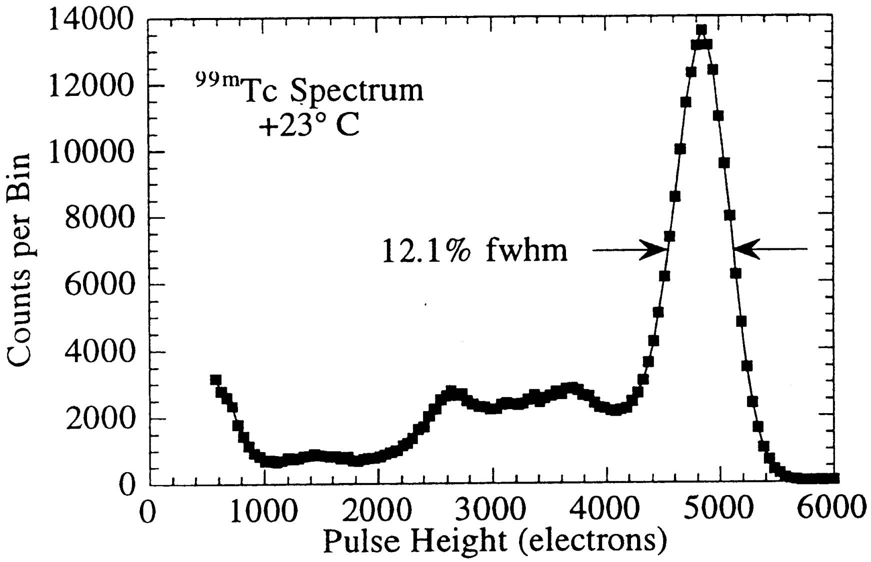
Front-end chip (preamplifier + shaper): 16 channels per chip, die size: 2 x 2 mm^2, 1.2 µm CMOS

continuously adjustable shaping time (0.5 to 50 µs)

Noise vs. shaping time



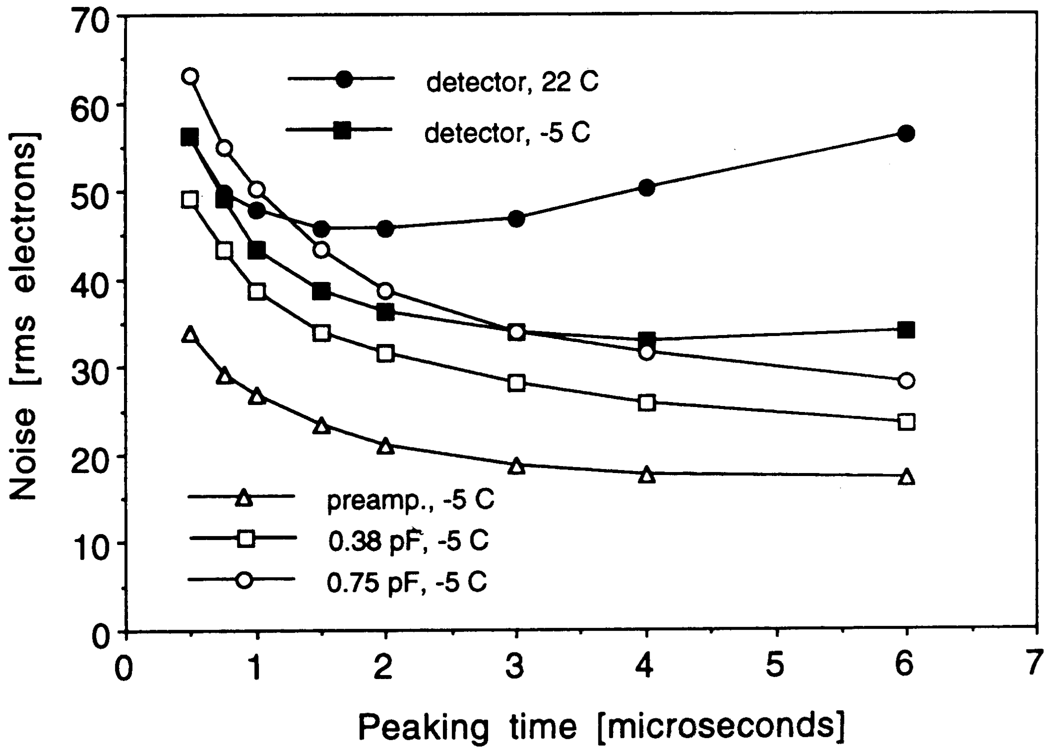
Energy spectrum with BGO scintillator



Note increase in noise at long shaping times when photodiode is connected - shot noise contribution.

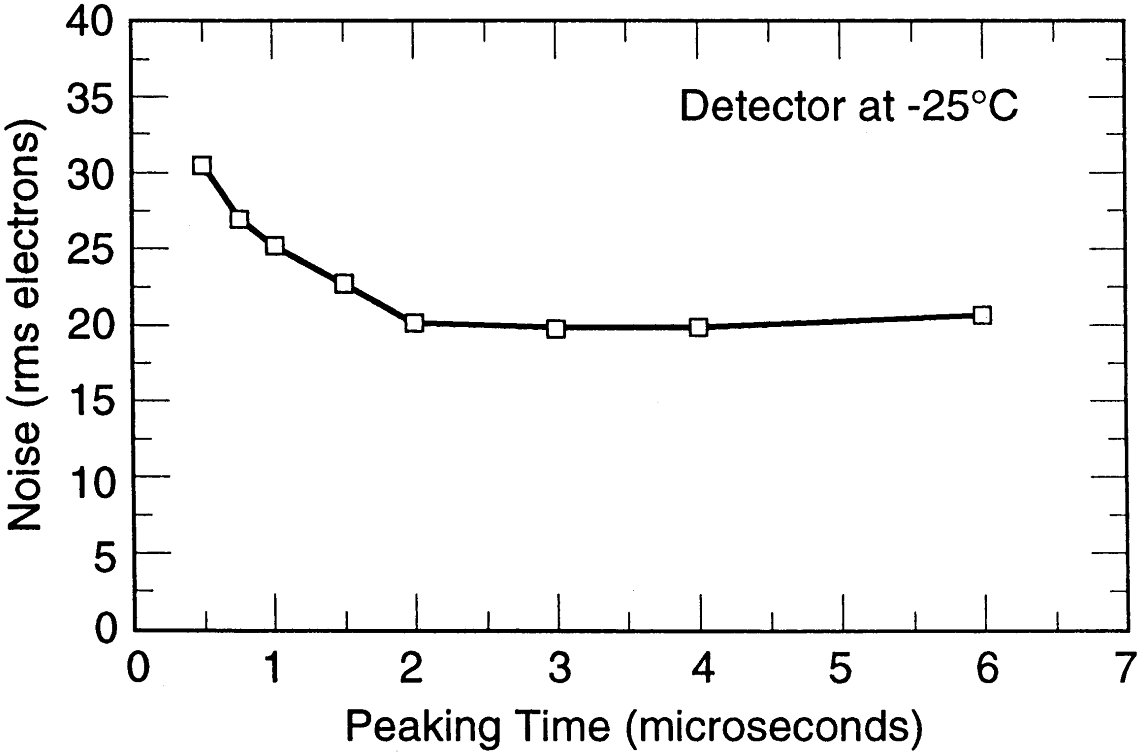
Another Example: short-strip Si x-ray detector

(B. Ludewigt, C. Rossington, I. Kipnis, B. Krieger, LBNL)



- Connecting the detector increases noise because of added capacitance and detector current (as indicated by increase of noise with peaking time).
- Cooling the detector reduces the current and noise improves.

Second prototype



Current noise negligible because of cooling –

“flat” noise vs. shaping time indicates that $1/f$ noise dominates.

Numerical expression for the noise of a CR-RC shaper
(amplifier current noise negligible)

(note that some units are "hidden" in the numerical factors)

$$Q_n^2 = 12\, I_B\, \tau \;+\; 6\cdot 10^5\, \frac{\tau}{R_P} \;+\; 3.6\cdot 10^4\, e_n^2\, \frac{C^2}{\tau} \quad \text{[rms electrons}^2\text{]}$$

where

τ	shaping time constant [ns]
I_B	detector bias current + amplifier input current [nA]
R_P	input shunt resistance [kΩ]
e_n	equivalent input noise voltage spectral density [nV/√Hz]
C	total input capacitance [pF]

Q_n= 1 el corresponds to 3.6 eV in Si
2.9 eV in Ge

Note:

For sources connected in parallel, currents are additive.

For sources connected in series, voltages are additive.

⇒ In the detector community voltage and current noise are often called “series” and “parallel” noise.

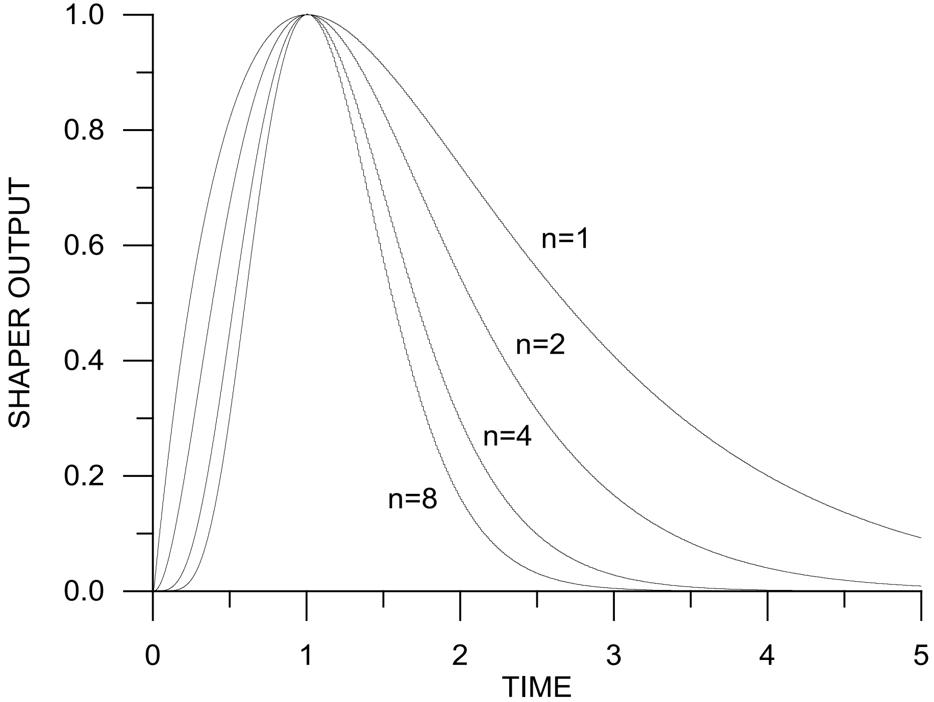
The rest of the world uses equivalent noise voltage and current.

Since they are physically meaningful, use of these widely understood terms is preferable.

9. Shapers with Multiple Integrators

Start with simple *CR-RC* shaper and add additional integrators ($n=$ 1 to $n=$ 2, ... $n=$ 8).

Change integrator time constants to preserve the peaking time $\tau_n = \tau_{n=1} / n$



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

⇒ improved rate capability at the same peaking time

Shapers with the equivalent of 8 *RC* integrators are common.
Usually, this is achieved with active filters
(i.e. circuitry that synthesizes the bandpass with amplifiers and feedback networks).

10. Noise Analysis in the Time Domain

The noise analysis of shapers is rather straightforward if the frequency response is known.

On the other hand, since we are primarily interested in the pulse response, shapers are often designed directly in the time domain, so it seems more appropriate to analyze the noise performance in the time domain also.

Clearly, one can take the time response and Fourier transform it to the frequency domain, but this approach becomes problematic for time-variant shapers.

The CR-RC shapers discussed up to now utilize filters whose time constants remain constant during the duration of the pulse, i.e. they are time-invariant.

Many popular types of shapers utilize signal sampling or change the filter constants during the pulse to improve pulse characteristics, i.e. faster return to baseline or greater insensitivity to variations in detector pulse shape.

These time-variant shapers cannot be analyzed in the manner described above. Various techniques are available, but some shapers can be analyzed only in the time domain.

References:

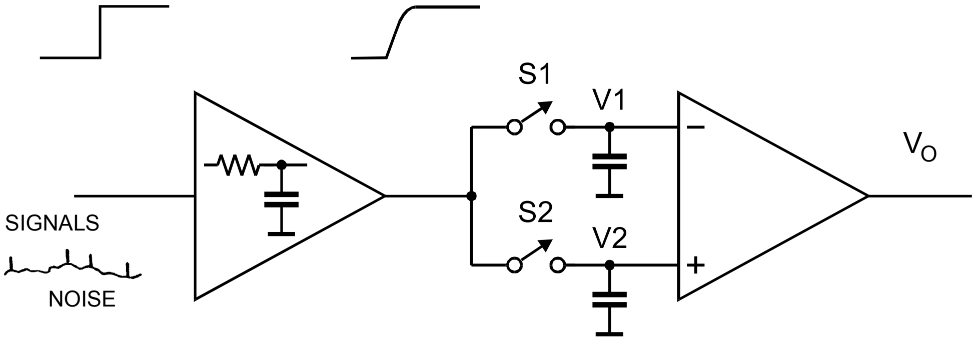
V. Radeka, Nucl. Instr. and Meth. **99** (1972) 525
V. Radeka, IEEE Trans. Nucl. Sci. **NS-21** (1974) 51
F.S. Goulding, Nucl. Instr. and Meth. **100** (1972) 493
F.S. Goulding, IEEE Trans. Nucl. Sci. **NS-29** (1982) 1125

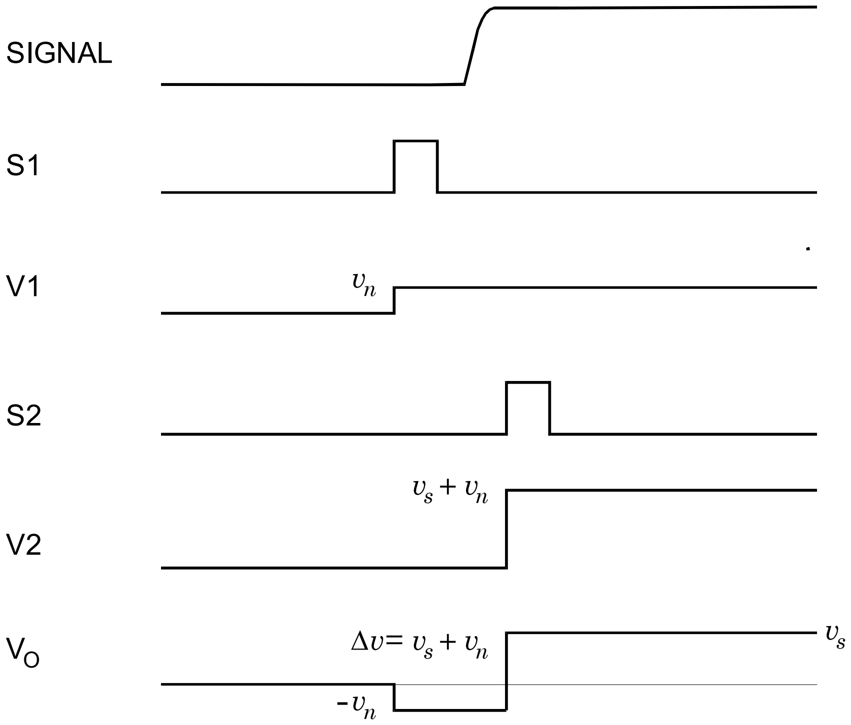
Example:

A commonly used time-variant filter is the correlated double-sampler.

This shaper can be analyzed exactly only in the time domain.

1. Signals are superimposed on a (slowly) fluctuating baseline
2. To remove baseline fluctuations the baseline is sampled prior to the arrival of a signal.
3. Next, the signal + baseline is sampled and the previous baseline sample subtracted to obtain the signal





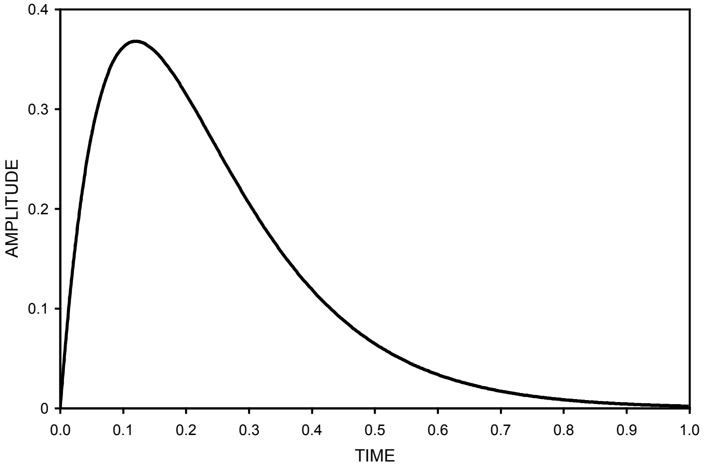
S/N depends on

1. time constant of prefilter
2. time difference between samples

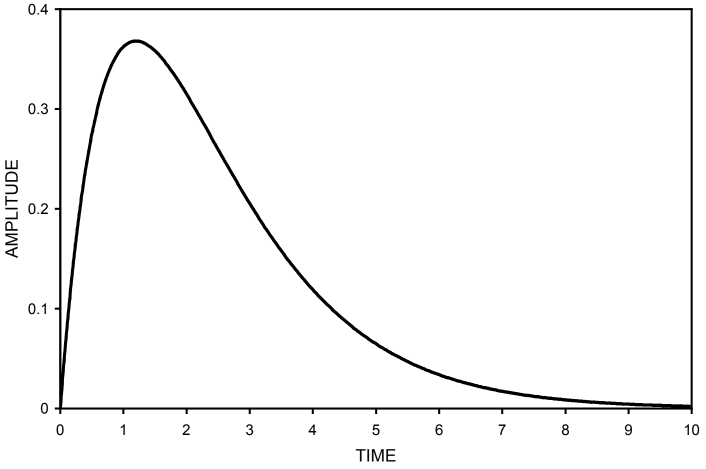
11. Scaling of Filter Parameters

Pulse shape is the same when shaping time is changed

shaping time = τ



shaping time = 10τ



Since the pulse width is directly related to the noise bandwidth (Parseval's Theorem),

$$\int_0^\infty |A(f)|^2 \, df = \int_{-\infty}^{\infty} [F(t)]^2 \, dt \ ,$$

the noise charge

$$Q_n^2 = \left(\frac{e^2}{8}\right) \left[\left(2q_e I_D + \frac{4kT}{R_P} + i_{na}^2 \right) \cdot \tau + \left(4kTR_S + e_{na}^2 \right) \cdot \frac{C_D^2}{\tau} + 4A_f C_D^2 \right]$$

can be written in a general form that applies to any shaper.

$$Q_n^2 = \ i_n^2 \ T \ F_i \ \ + \ \ C^2 \ e_n^2 \ \frac{1}{T} \ F_v \ \ + \ \ C^2 \ A_f \ F_{vf}$$

The individual current and voltage noise contributions are combined:

current noise $\quad i_n^2 = 2q_e I_b + \dfrac{4kT}{R_P} + i_{na}^2 \quad$ and voltage noise $\quad e_n^2 = 4kTR_S + e_{na}^2$

The shaper is characterized by noise coefficients F_i, F_v and F_{vf} , which depend only on the shape of the pulse.

The noise bandwidth scales with a characteristic time T.
In the specific case of a CR-RC shaper T is equal to the peaking time T_p , the time at which the pulse assumes its maximum value. For a correlated double sampler, the sampling time is an appropriate measure.

The first term describes the current noise contribution, whereas the second and third terms describe the voltage noise contributions due to white and $1/f$ noise sources.

- Generally, the noise indices or “shape factors” F_i , F_v and F_{vf} characterize the type of shaper, for example CR-RC or CR-$(RC)^4$.
- They depend only on the ratio of time constants τ_d / τ_i, rather than their absolute magnitude.
- The noise contribution then scales with the characteristic time T. The choice of characteristic time is somewhat arbitrary. so any convenient measure for a given shaper can be adopted in deriving the noise coefficients F.

The shape factors F_i, F_v are easily calculated

$$F_i = \frac{1}{2T_S} \int_{-\infty}^{\infty} \left[W(t) \right]^2 dt \ , \qquad F_v = \frac{T_S}{2} \int_{-\infty}^{\infty} \left[\frac{dW(t)}{dt} \right]^2 dt$$

For time invariant pulse shaping $W(t)$ is simply the system’s impulse response, with the peak output signal normalized to unity.

Recipe:

Inject a short current pulse injected into the preamplifier

For a charge-sensitive preamp this is generated by a voltage step applied to the test input.

$W(t)$ is the output signal as seen on an oscilloscope. With a digitizing oscilloscope the signal can be recorded and numerically normalized, squared, and integrated.

12. Summary

Two basic noise mechanisms:	input noise current i_n
	input noise voltage e_n

Equivalent Noise Charge:

$$Q_n^2 = i_n^2 \, T_s \, F_i \; + \; C^2 \, e_n^2 \, \frac{F_v}{T_s}$$

Where T_s	Characteristic shaping time (*e.g.* peaking time)
F_i , F_v	"Shape Factors" that are determined by the shape of the pulse.
C	Total capacitance at the input node (detector capacitance + input capacitance of preamplifier + stray capacitance + …)

Typical values of F_i , F_v

CR-RC shaper	F_i = 0.924	F_v = 0.924
CR-(RC)^4 shaper	F_i = 0.45	F_v = 1.02
CR-(RC)^7 shaper	F_i = 0.34	F_v = 1.27
CAFE chip	F_i = 0.4	F_v = 1.2

Note that $F_i < F_v$ for higher order shapers.

Shapers can be optimized to reduce current noise contribution relative to the voltage noise (mitigate radiation damage!).

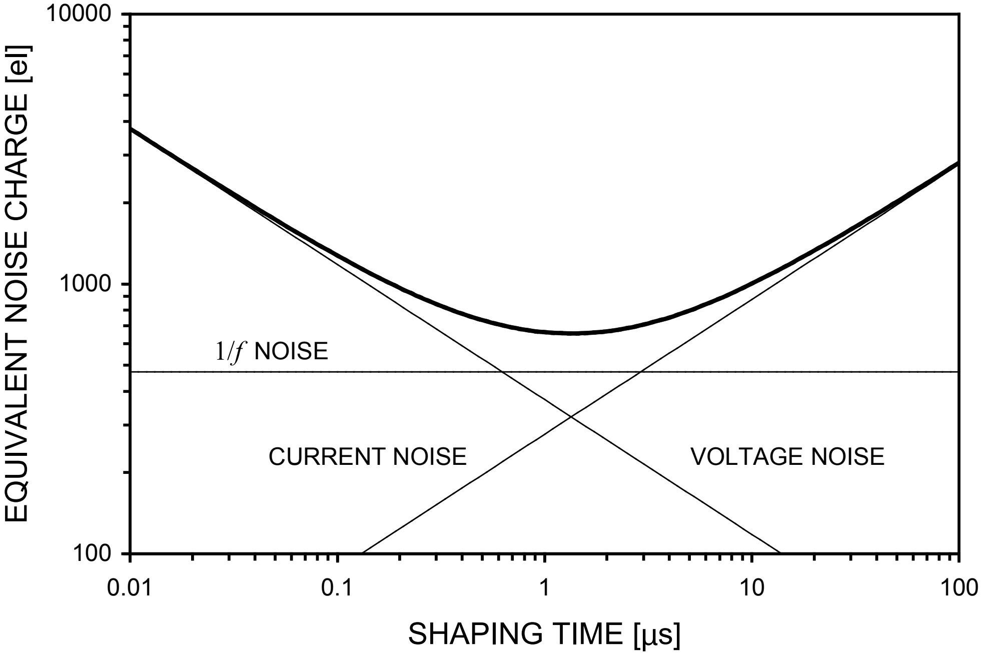
Minimum noise obtains when the current and voltage noise contributions are equal.

Current noise

- detector bias current
  increases with detector size, strongly temperature dependent
- resistors shunting the input
  increases as resistance is decreased
- input transistor – low for FET, higher for BJTs

Voltage noise

- input transistor (see Appendix)
- series resistance
  e.g. detector electrode, protection circuits



FETs commonly used as input devices – improved noise performance when cooled ($T_{opt} \approx 130$ K)

Bipolar transistors advantageous at short shaping times (<100 ns).
When collector current is optimized, bipolar transistor equivalent noise charge is independent of shaping time (see Appendix).

Equivalent Noise Charge vs. Detector Capacitance ($C = C_d + C_a$)

$$Q_n = \sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}$$

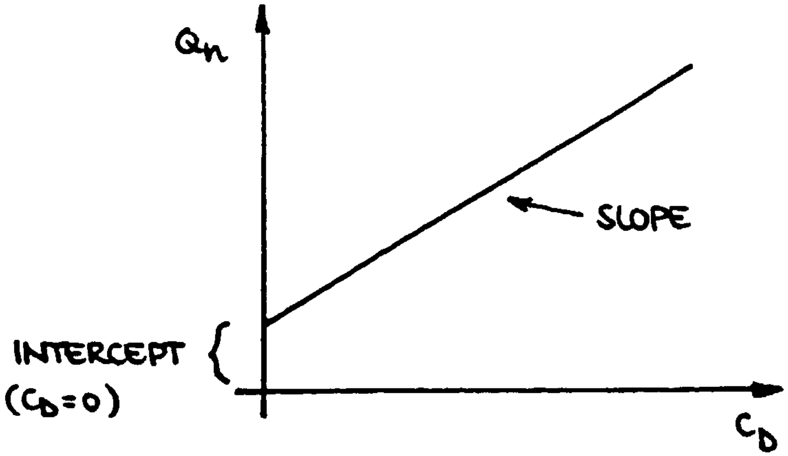
$$\frac{dQ_n}{dC_d} = \frac{2C_d e_n^2 F_v \frac{1}{T}}{\sqrt{i_n^2 F_i T + (C_d + C_a)^2 e_n^2 F_v \frac{1}{T}}}$$

If current noise $i_n{}^2 F_i T$ is negligible, i.e. **voltage noise dominates:**

$$\frac{dQ_n}{dC_d} \approx \underset{\substack{\uparrow \\ \text{input} \\ \text{stage}}}{2e_n} \cdot \underset{\substack{\uparrow \\ \text{shaper}}}{\sqrt{\frac{F_v}{T}}}$$

Zero intercept

$$Q_n\big|_{C_d=0} = C_a e_n \sqrt{F_v / T}$$



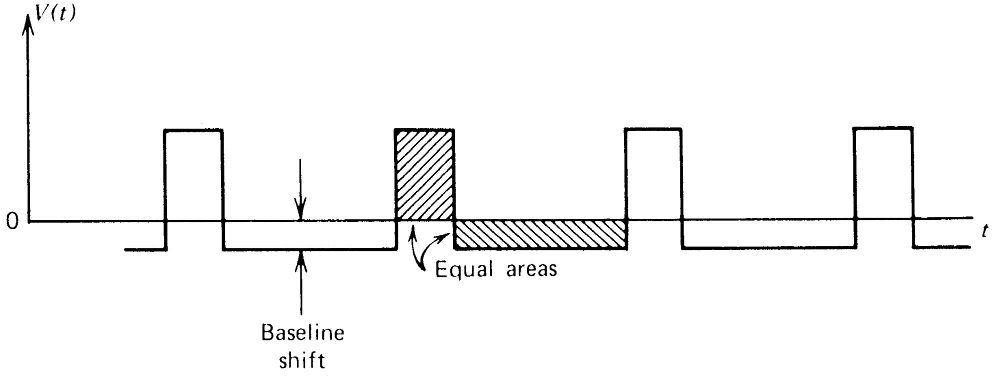
13. Some Other Aspects of Pulse Shaping

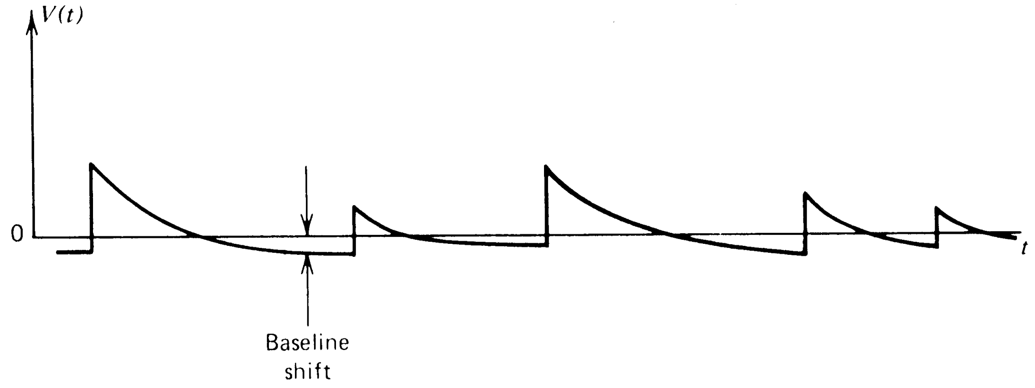
13.1 Baseline Restoration

Any series capacitor in a system prevents transmission of a DC component.

A sequence of unipolar pulses has a DC component that depends on the duty factor, i.e. the event rate.

⇒ The baseline shifts to make the overall transmitted charge equal zero.





Random rates lead to random fluctuations of the baseline shift

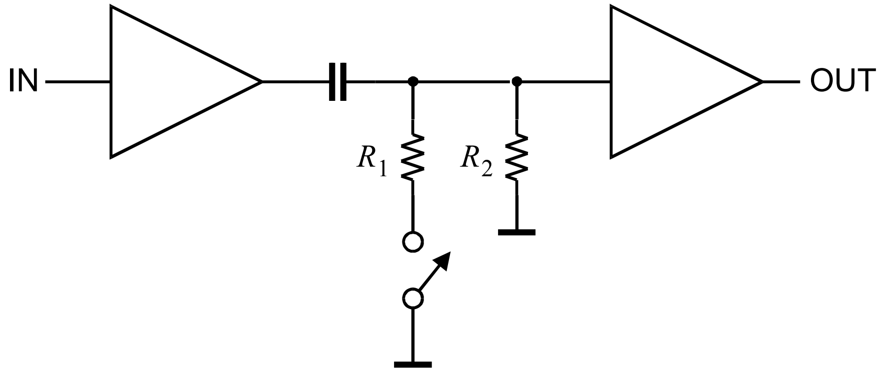
⇒ spectral broadening

- These shifts occur whenever the DC gain is not equal to the midband gain

The baseline shift can be mitigated by a baseline restorer (BLR).

Principle of a baseline restorer:

Connect signal line to ground during the absence of a signal to establish the baseline just prior to the arrival of a pulse.



R_1 and R_2 determine the charge and discharge time constants.
The discharge time constant (switch opened) must be much larger than the pulse width.

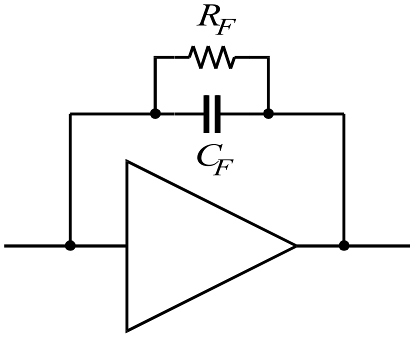
Originally performed with diodes (passive restorer), baseline restoration circuits now tend to include active loops with adjustable thresholds to sense the presence of a signal (gated restorer).
Asymmetric charge and discharge time constants improve performance at high count rates.

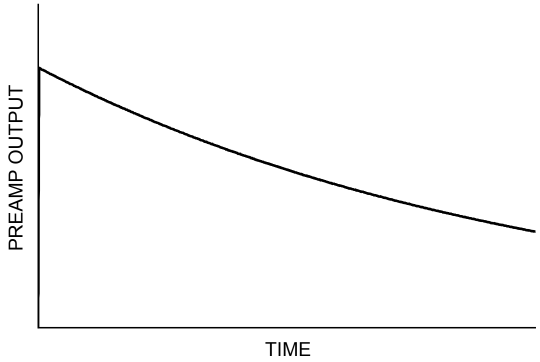
- This is a form of time-variant filtering. Care must be exercised to reduce noise and switching artifacts introduced by the BLR.

- Good pole-zero cancellation (next topic) is crucial for proper baseline restoration.

13.2 Tail (Pole Zero) Cancellation

Feedback capacitor in charge sensitive preamplifier must be discharged. Commonly done with resistor.



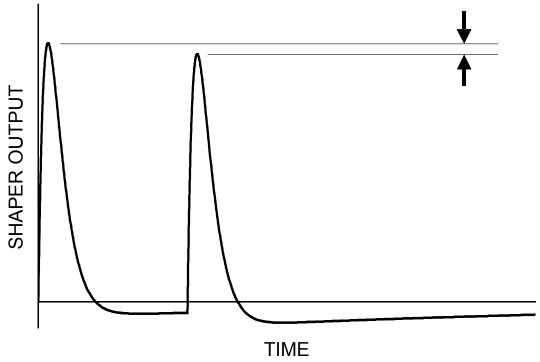


Output no longer a step, but decays exponentially

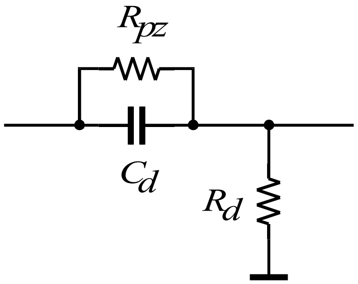
Exponential decay superimposed on shaper output.

⇒ undershoot

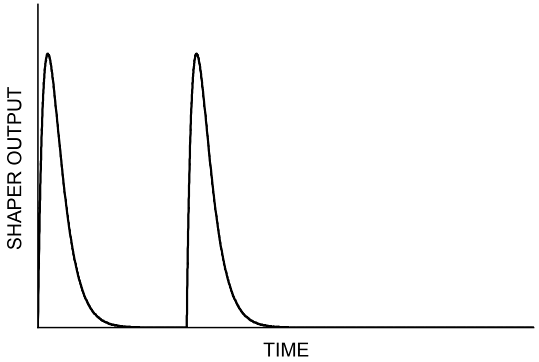
⇒ loss of resolution due to baseline variations



Add R_{pz} to differentiator:



“zero” cancels “pole” of preamp when $R_F C_F = R_{pz} C_d$

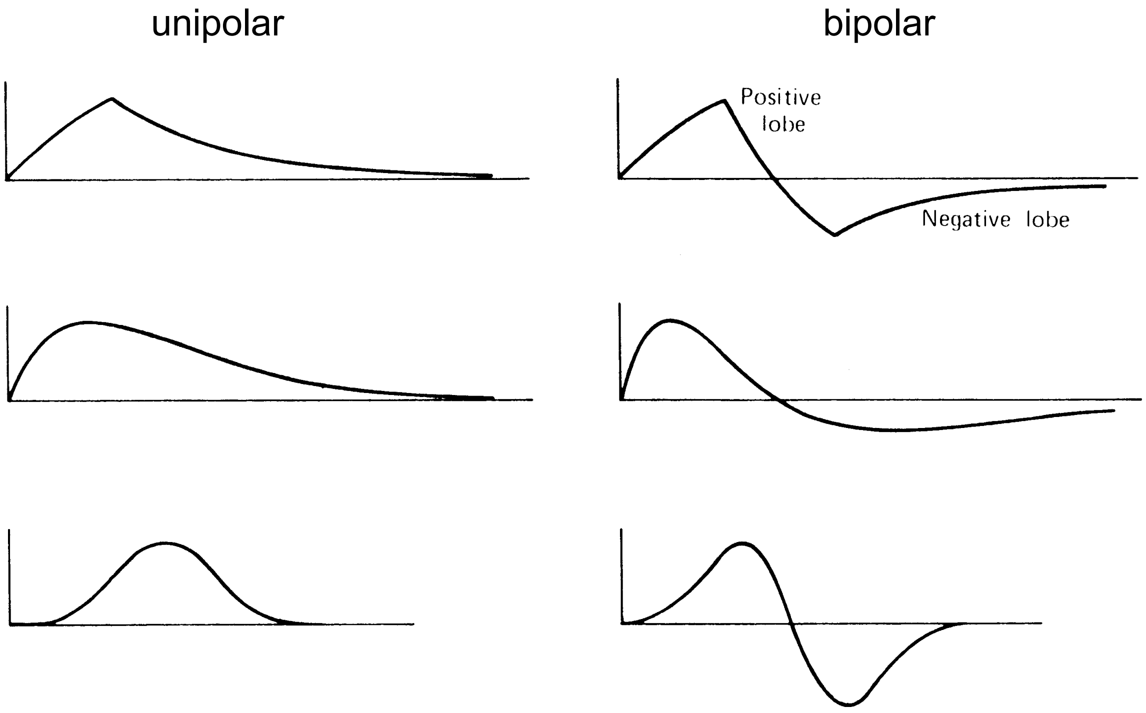


Technique also used to compensate for “tails” of detector pulses: “tail cancellation”

13.3 Bipolar vs. Unipolar Shaping

Unipolar pulse + 2nd differentiator

→ Bipolar pulse



Electronic resolution with bipolar shaping typ. 25 – 50% worse than for corresponding unipolar shaper.

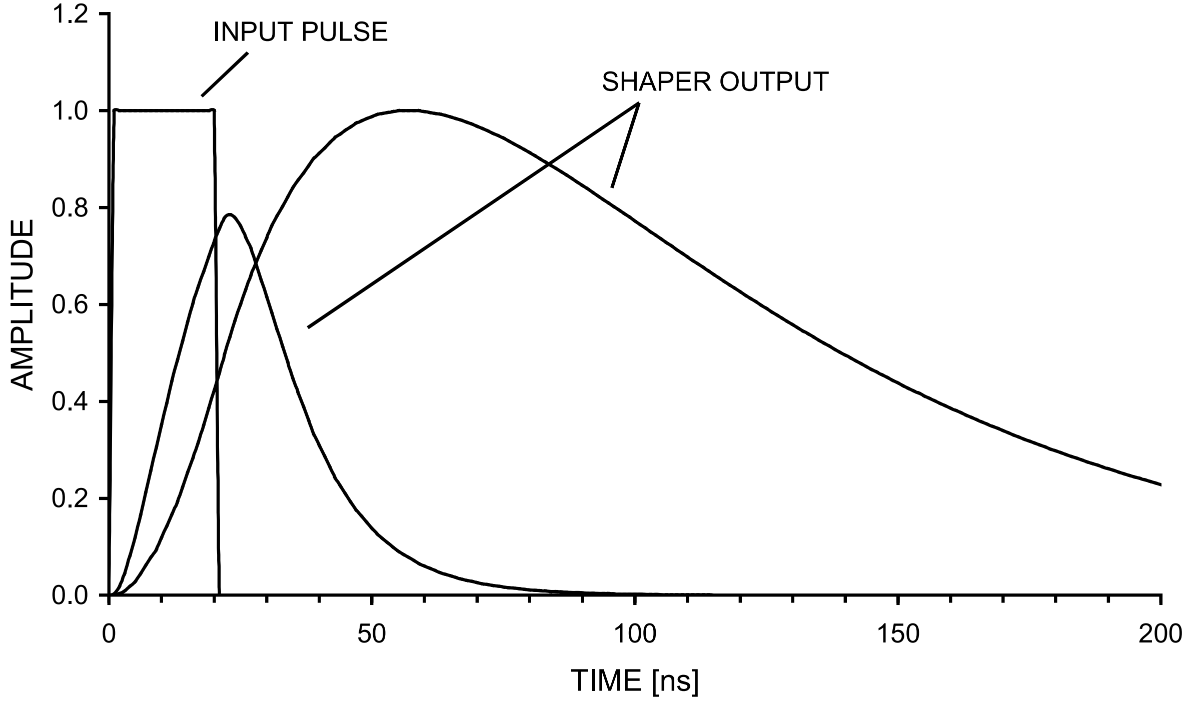
However …

- Bipolar shaping eliminates baseline shift (as the DC component is zero).
- Pole-zero adjustment less critical
- Added suppression of low-frequency noise (see Part 7).
- Not all measurements require optimum noise performance.
  Bipolar shaping is much more convenient for the user
  (important in large systems!) – often the method of choice.

13.4 Ballistic Deficit

Peaking time must be longer than input pulse, else loss in pulse height.

Shaper output for short and long peaking times



Loss in signal translates directly into decreased signal-to-noise ratio,
i.e. the equivalent noise charge increases.

14. Timing Measurements

Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

- Timing measurements optimize determination of time of occurrence.
- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

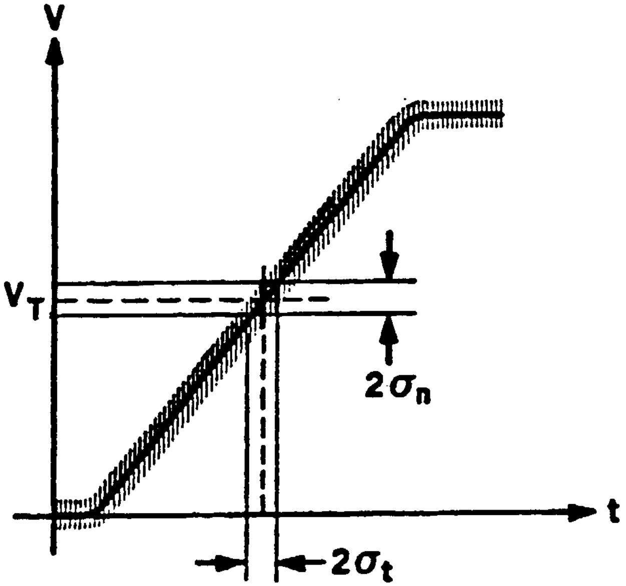
Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

The instantaneous signal level is modulated by noise.

⇒ time of threshold crossing fluctuates

$$\sigma_t = \frac{\sigma_n}{\left.\frac{dV}{dt}\right|_{V_T}} \approx \frac{t_r}{S/N}$$

t_r = rise time



Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.

Pulse Shaping

Consider a system whose bandwidth is determined by a single *RC* integrator.

The time constant of the *RC* low-pass filter determines the

- rise time (and hence *dV/dt*)
- amplifier bandwidth (and hence the noise)

Time dependence: $V_o(t) = V_0(1 - e^{-t/\tau})$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2\ \tau$$

In terms of bandwidth

$$t_r = 2.2\ \tau = \frac{2.2}{2\pi\ f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

For a cascade of amplifiers: $t_r \approx \sqrt{t_{r1}^2 + t_{r2}^2 + \ \ldots\ + t_{rn}^2}$

Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude V_0 and a rise time t_c passing through an amplifier chain with a rise time t_{ra}.

1. amplifier rise time $\gg$ signal rise time:

$$\text{Noise} \propto \sqrt{f_u} \propto \sqrt{\frac{1}{t_{ra}}}$$

$$\frac{dV}{dt} \propto \frac{1}{t_{ra}} \propto f_u$$

increase in bandwidth $\Rightarrow$ improvement in dV/dt outweighs increase in noise.

2. amplifier rise time $\ll$ signal rise time

increase in noise without increase in dV/dt

Optimum: The amplifier rise time should be chosen to match the signal rise time.

Differentiation time constant: choose greater than rise time constant

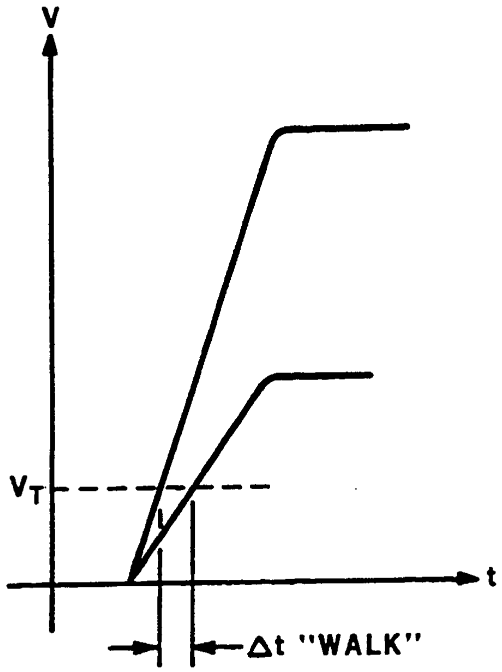
($\tau_{diff} = 10\tau_{\text{int}}$ incurs 20% loss in pulse height)

Time Walk

For a fixed trigger level the time of threshold crossing depends on pulse amplitude.

⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)



If the rise time is known, “time walk” can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

Lowest Practical Threshold

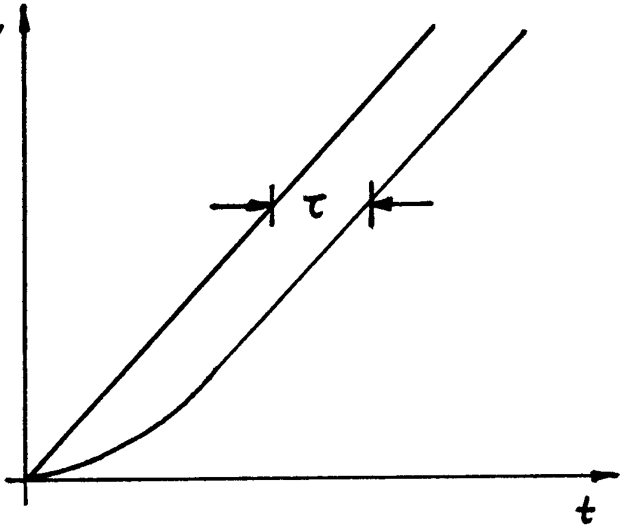
Single RC integrator has maximum slope at t= 0: $\frac{d}{dt}(1-e^{-t/\tau}) = \frac{1}{\tau}\; e^{-t/\tau}$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small t the slope at the output of a single RC integrator is linear, so initially the pulse can be approximated by a ramp $\alpha\, t$.

Response of the following integrator

$$V_i = \alpha\; t \quad \rightarrow \quad V_o = \alpha(t-\tau) - \alpha\;\tau\; e^{-t/\tau}$$



⇒ The output is delayed by τ
and curvature is introduced at small t.

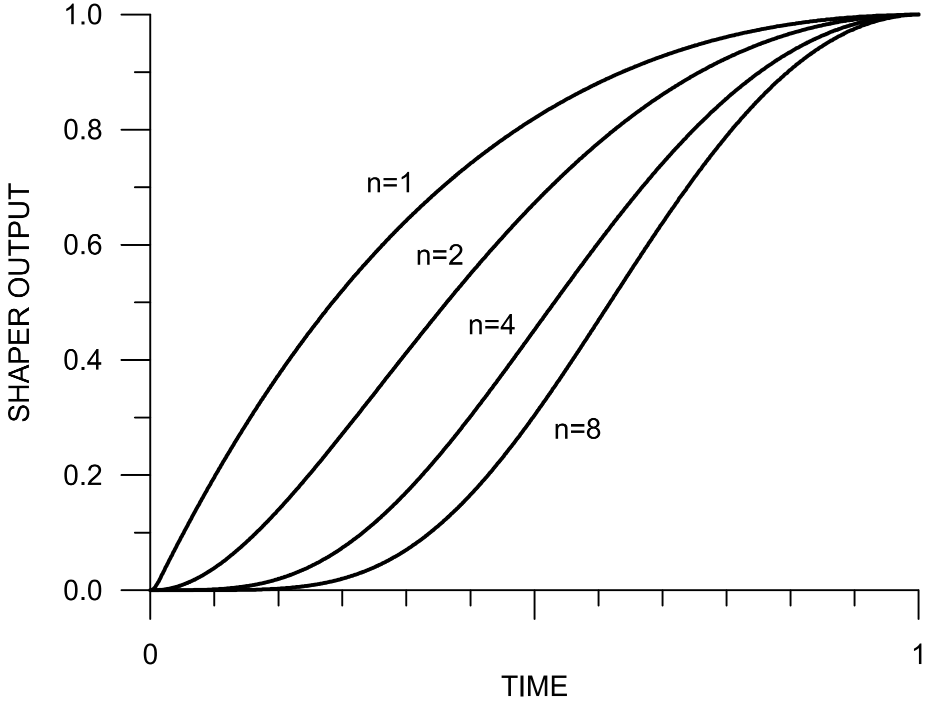
Output attains 90% of input slope after t= 2.3τ.

Delay for n integrators= $n\tau$

Additional RC integrators introduce more curvature at the beginning of the pulse.

Output pulse shape for multiple RC integrators

(normalized to preserve the peaking time, $\tau_n = \tau_{n-1} / n$)



Increased curvature at beginning of pulse limits the minimum threshold for good timing.

⇒ One dominant time constant best for timing measurements

Unlike amplitude measurements, where multiple integrators are desirable to improve pulse symmetry and count rate performance.

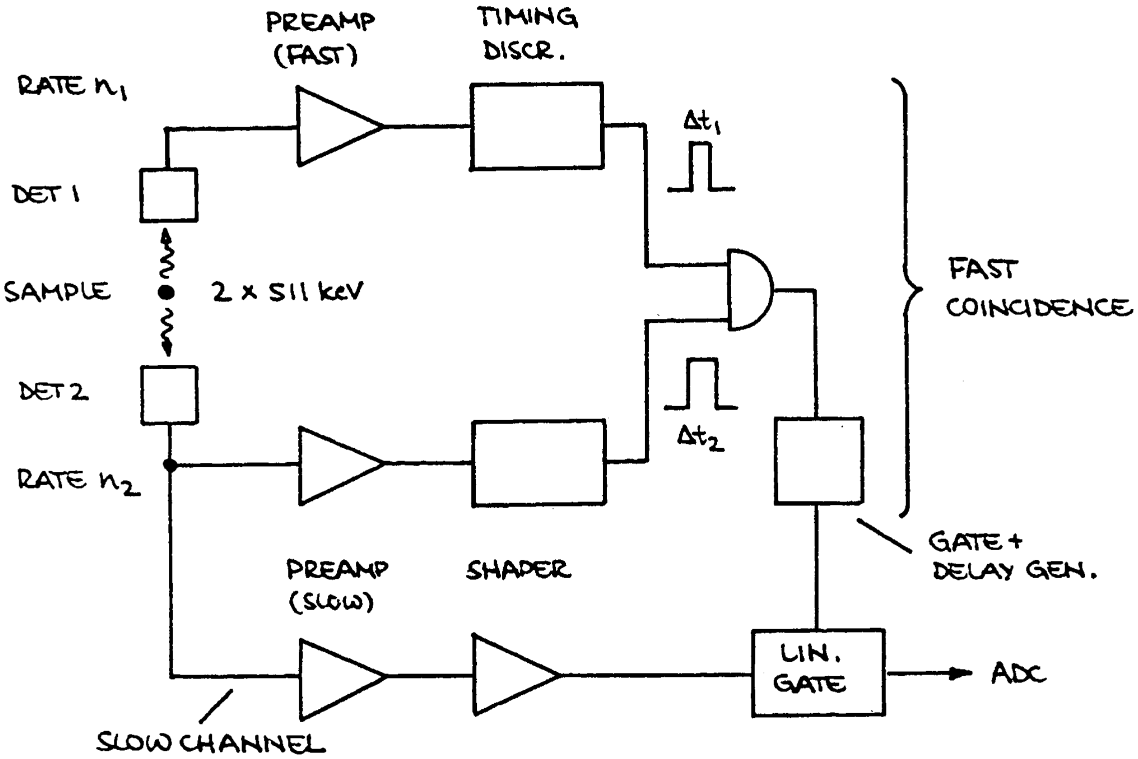
Example

γ-γ coincidence (as used in positron emission tomography)

Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.



- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output if the two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.

This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are n_1 and n_2, and the timing pulse widths are Δt_1 and Δt_2, the probability of a pulse from the first source ccurring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate n_2 , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$
$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

i.e. in the arrangement shown above, the chance coincidence rate increases with the square of the source strength.

Example: $n_1 = n_2 = 10^6$ s^{-1}

$\Delta t_1 = \Delta t_2 = 5$ ns $\Rightarrow$ $n_c = 10^4$ s^{-1}

Fast Timing: Comparison between theory and experiment

Time resolution $\propto 1/(S/N)$

At $S/N<100$ the measured curve lies above the calculation because the timing discriminator limited the rise time.

At high S/N the residual jitter of the time digitizer limits the resolution.

For more details on fast timing with semiconductor detectors, see

H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.