II. Signal Formation and Detection Thresholds

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1. Detector Models

We consider detectors that provide electrical signal outputs.

To extract the amplitude or timing information the electrical signal is coupled to an amplifier, sent through gain and filtering stages, and finally digitized to allow data storage and analysis.

Optimal signal processing depends on the primary signal.

The signal can be

- a continuously varying signal
- a sequence of pulses, occurring
    - periodically
    - at known times
    - randomly

All of these affect the choice of signal processing techniques.

First steps in signal processing:

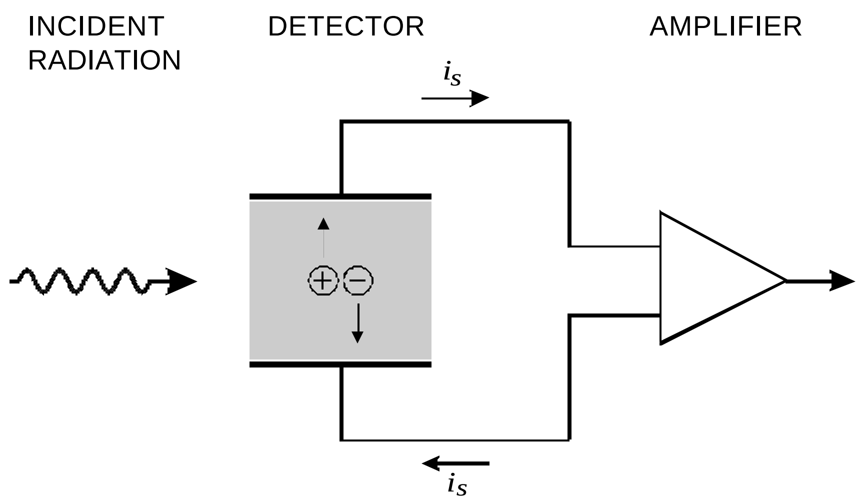
- Formation of the signal in the detector (sensor)
- Coupling the sensor to the amplifier

Detectors use either
- direct detection or
- indirect detection

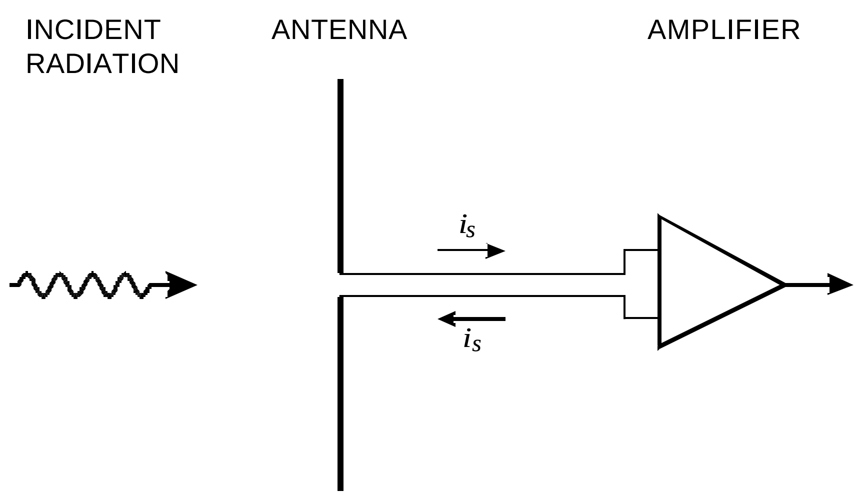
Examples:

1. Direct Detection

a) ionization chamber (>eV photons, charged particles)

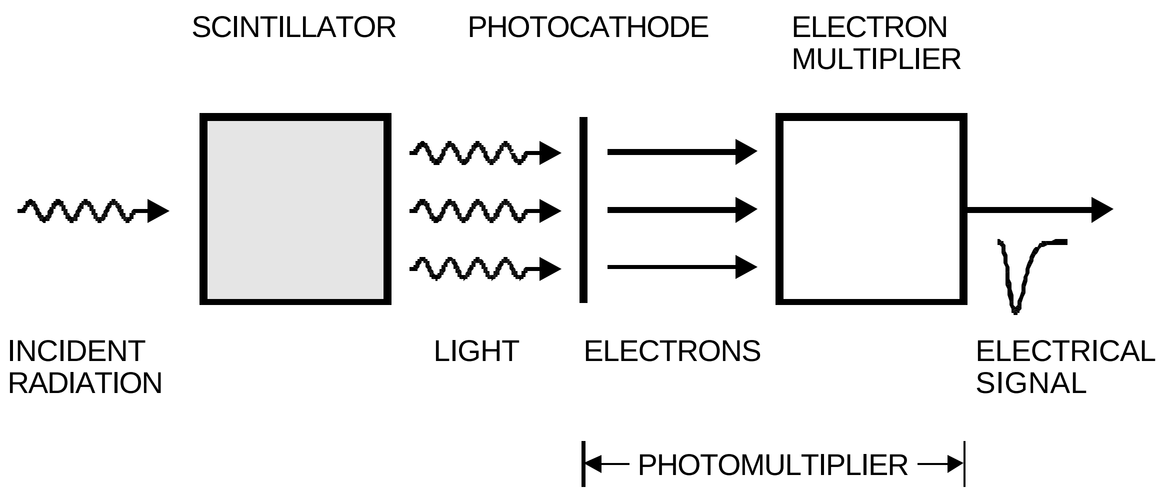


b) RF measurement (kHz ... THz), e.g. CMB



2. Indirect Detection

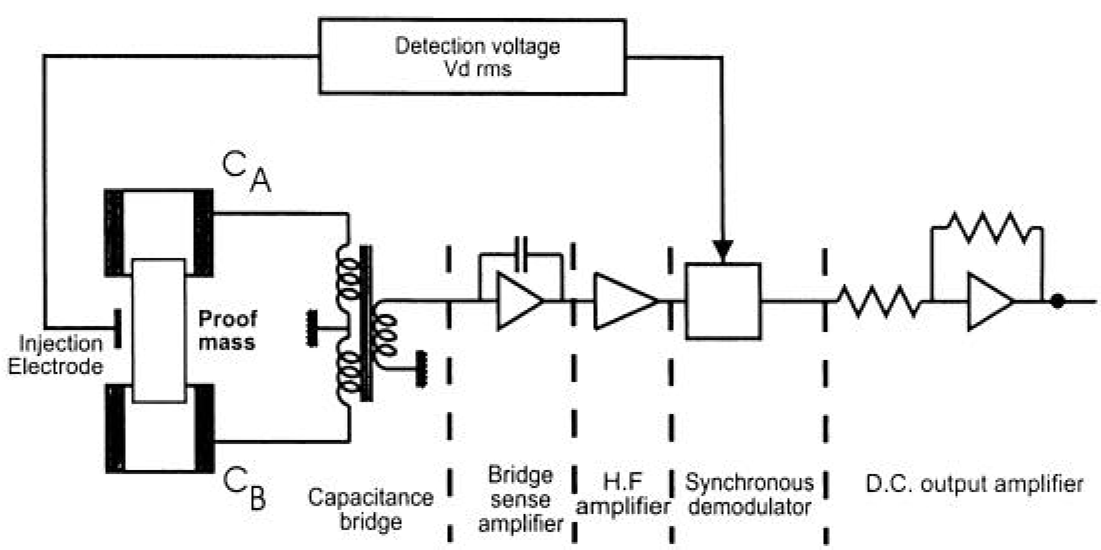
a) scintillation detector



b) gravity wave detector

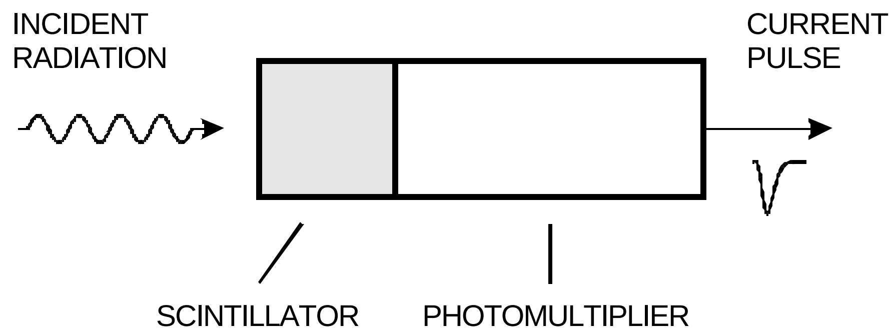
Motion of proof mass measured by capacitive sensor.

Schematic of LISA position sensor:

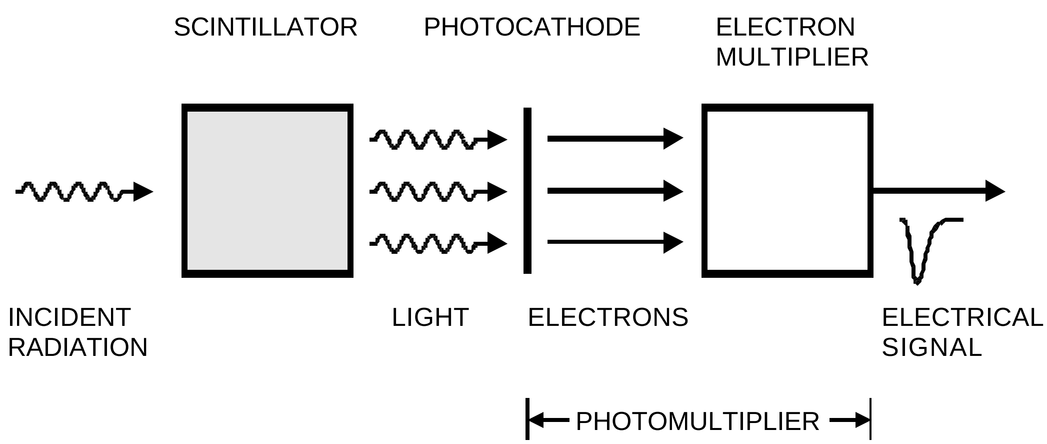


Capacitive sensors readily achieve sensitivities <100 pm.

Detector Functions



Processes in Scintillator – Photomultiplier

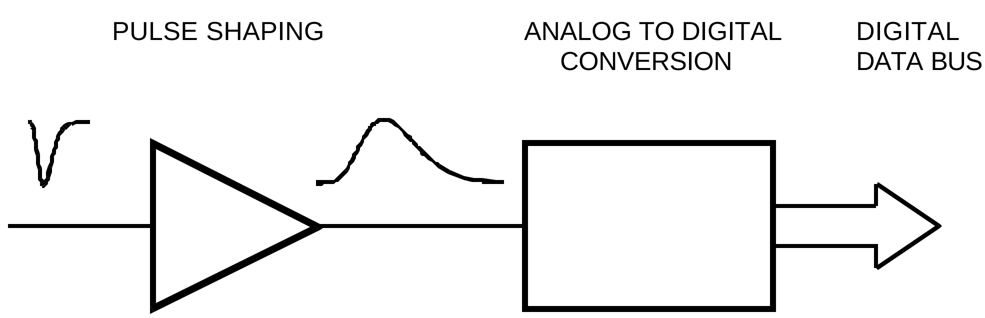


number of photons
$\propto$ absorbed energy

number of photoelectrons
$\propto$ absorbed energy

charge in pulse
$\propto$ abs. energy

Signal Processing



charge in pulse
$\propto$ abs. energy

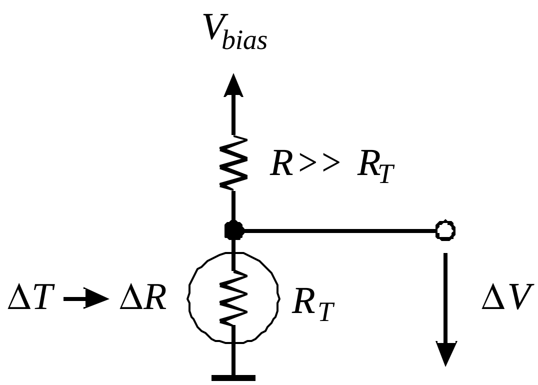
pulse height
$\propto$ absorbed energy

Example Detector Models

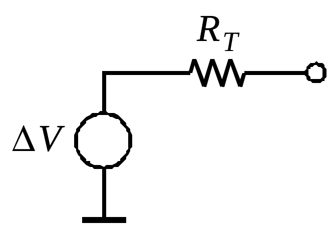
Although detectors take on many different forms, one can analyze the coupling to the amplifier with simple models.

1. Thermistor detecting IR radiation

Actual Circuit

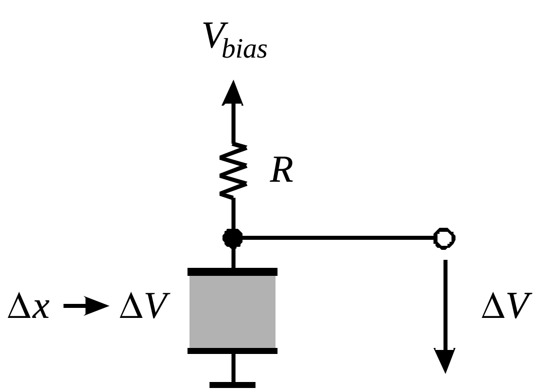


Model

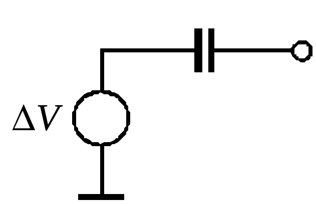


2. Piezoelectric Transducer

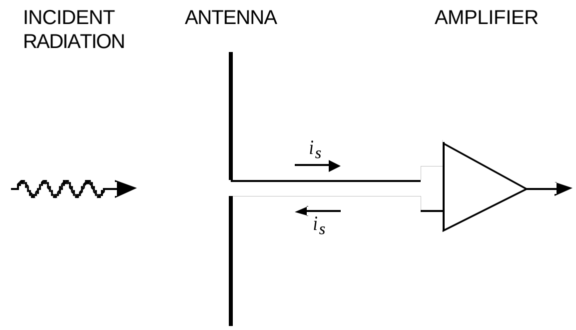
Actual Circuit



Model



3. Radio Antenna



To derive the equivalent circuit apply the reciprocity principle, i.e. analyze the antenna as a radiator driven by an RF generator.

Radiated field

For simplicity assume the dipole length is a half wavelength λ/2. The angular distribution of the radiated power (see J.D. Jackson, Classical Electrodynamics)

$$\frac{dP}{d\Omega} = \frac{I^2}{2pc} \frac{\cos^2\left(\frac{p}{2}\cos\Theta\right)}{\sin^2\Theta}$$

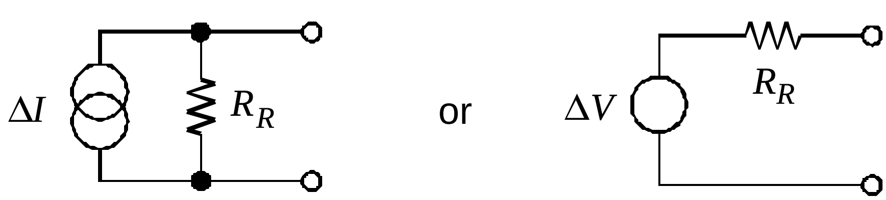
At the feed point the power fed to the dipole

$$P = I^2 R_R$$

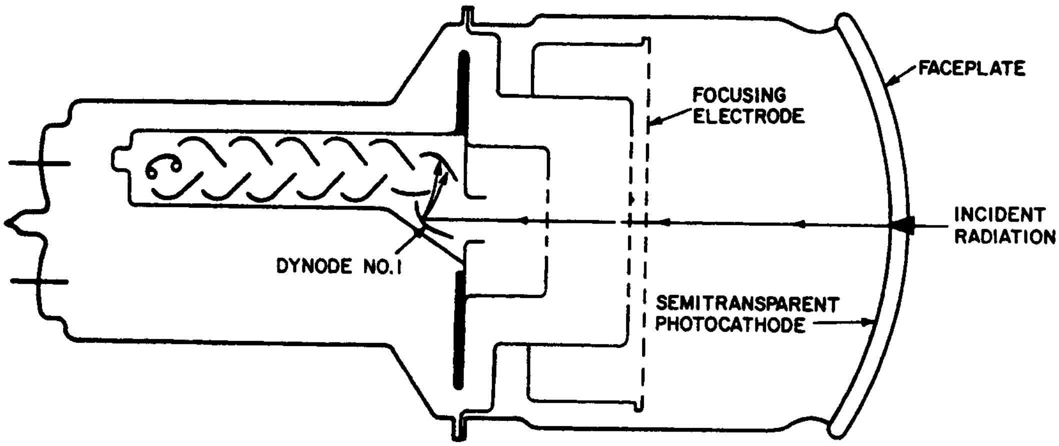
which is equal to the radiated power

$$P = \int_0^{2p} \frac{dP}{d\Omega} d\Omega$$

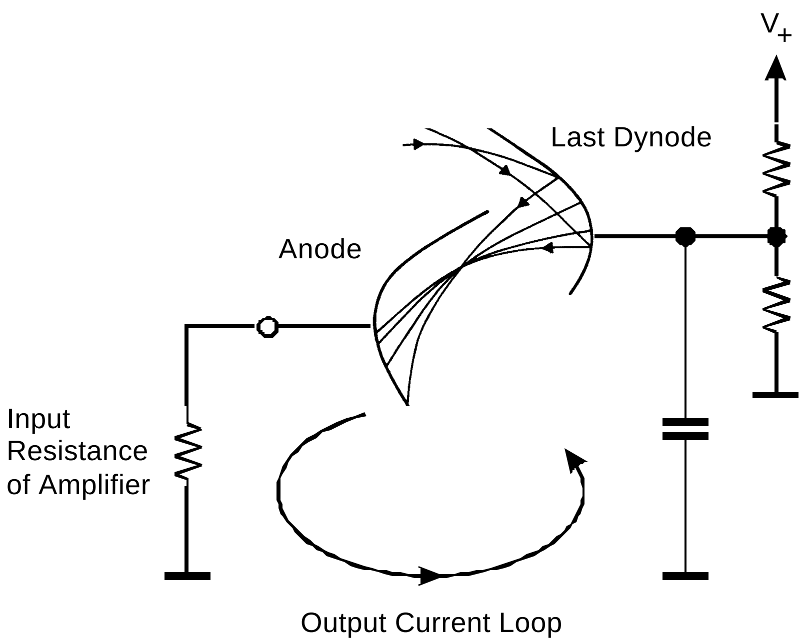
i.e. the dipole appears as a resistance, so the equivalent circuit



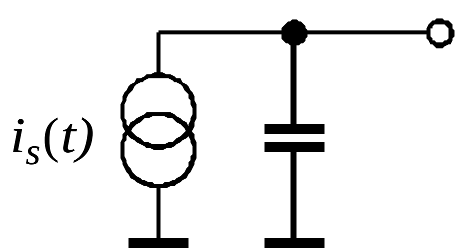
4. Photomultiplier Tube



Detail of output circuit



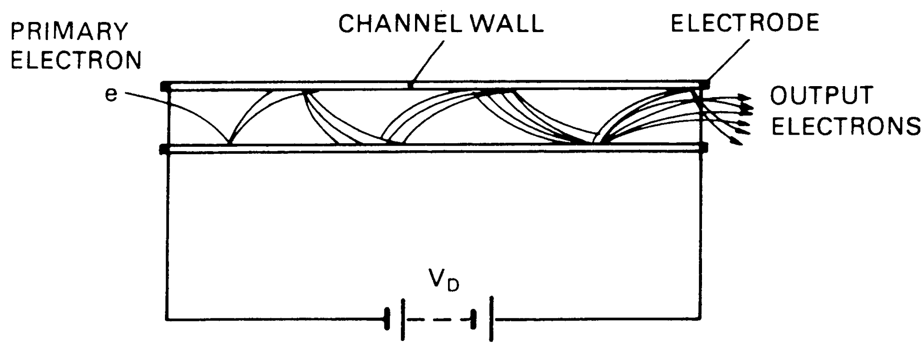
Model



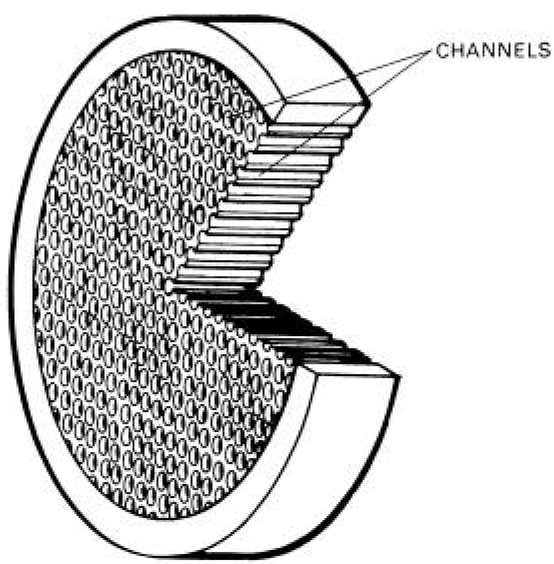
5. Channeltrons and Microchannel Plates

Channel electron multiplier

The inside of a glass capillary is coated with a secondary electron emitter that also forms a distributed resistance. Application of a voltage between the the two ends sets up a field, so that electrons in the structure are accelerated, strike the wall, and form secondaries.



Channel electron multipliers are used individually (“channeltrons”), with tube diameters of ~1 mm, and in arrays called “micro-channel plates”, which combine many small channels of order 10 µm diameter in the form of a plate.

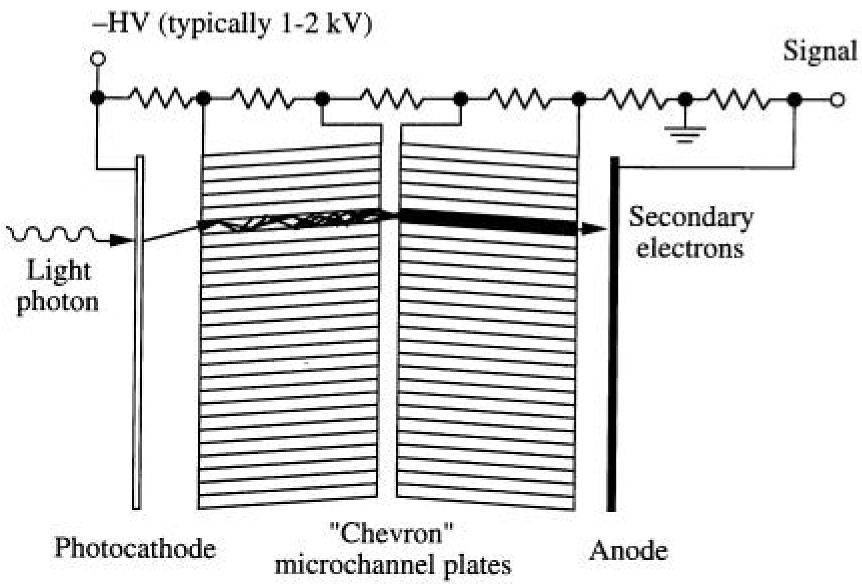


(Hamamatsu Photonics)

Microchannel plates are fabricated by stretching bundles of glass capillaries and then slicing the bundle to form 2 … 5 cm diameter plates of several hundred microns thickness.

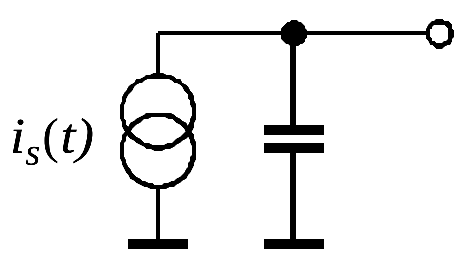
Microchannel plates are compact and fast. Transit time dispersion is < 1 ns due to the small dimensions of an individual channel. Pairs of microchannel plates can be combined to provide higher gain.

Connection scheme of a photon detector using microchannel plates



(from Derenzo)

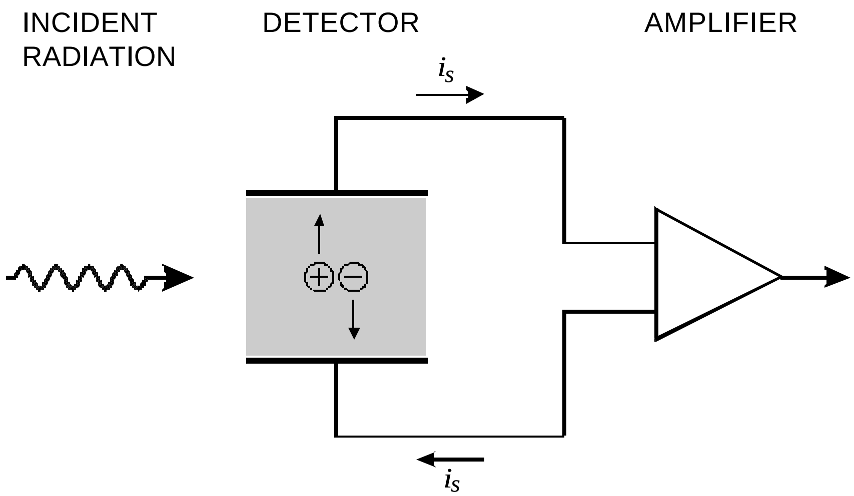
Model:



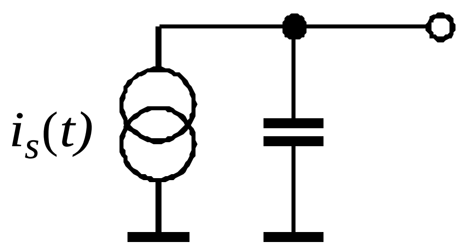
The shunt capacitor represents the capacitance between the exit face of the MCP and the anode.

6. Ionization Chamber

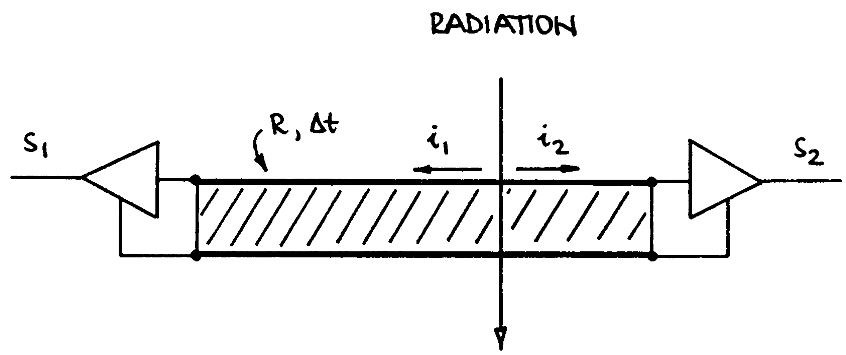
semiconductor detectors (pad, strip, pixel electrodes)
gas-filled ionization or proportional chambers, ...



Model:



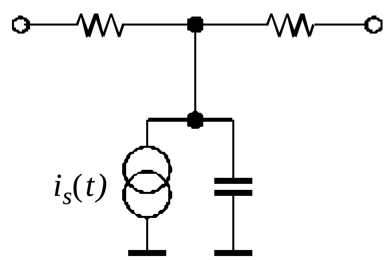
7. Position-Sensitive Detector with Resistive Charge Division



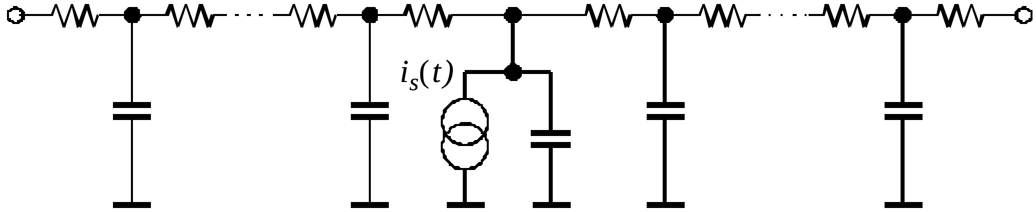
Electrode is made resistive with low-impedance amplifiers at each end. The signal current divides according to the ratio of resistances presented to current flow in the respective direction

$$\frac{i_1(x)}{i_2(x)} = \frac{R_2(x)}{R_1(x)}$$

Simplest Model:



Depending on the speed of the amplifier, a more accurate model of the electrode includes the distributed capacitance:



2. The Signal

Any form of elementary excitation can be used to detect the radiation signal.

$$\text{Magnitude of signal} = \frac{\text{absorbed energy}}{\text{excitation energy}}$$

An electrical signal can be formed directly by ionization.

Incident radiation quanta impart sufficient energy to individual atomic electrons to form electron-ion pairs (in gases) or electron-hole pairs (in semiconductors and metals).

Other detection mechanisms are

Excitation of optical states (scintillators)

Excitation of lattice vibrations (phonons)

Breakup of Cooper pairs in superconductors

Formation of superheated droplets in superfluid He

Typical excitation energies

Ionization in gases	~30 eV
Ionization in semiconductors	1 – 5 eV
Scintillation	~10 eV
Phonons	meV
Breakup of Cooper Pairs	meV

Fluctuations in the Signal Charge: the Fano Factor

The mean ionization energy exceeds the bandgap for two reasons

1. Conservation of momentum requires excitation of lattice vibrations

2. Many modes are available for the energy transfer with an excitation energy less than the bandgap.

Two types of collisions are possible:

a) Lattice excitation, i.e. phonon production (with no formation of mobile charge).

b) Ionization, i.e. formation of a mobile charge pair.

Assume that in the course of energy deposition

N_x excitations produce N_P phonons and

N_i ionization interactions form N_Q charge pairs.

On the average, the sum of the energies going into excitation and ionization is equal to the energy deposited by the incident radiation

$$E_0 = E_i N_i + E_x N_x$$

where E_i and E_x are the energies required for a single excitation or ionization.

Assuming gaussian statistics, the variance in the number of excitations

$$s_x = \sqrt{N_x}$$

and the variance in the number of ionizations

$$s_i = \sqrt{N_i}$$

For a single event, the energy E_0 deposited in the detector is fixed (although this may vary from one event to the next).

If the energy required for excitation E_x is much smaller than required for ionization E_i, sufficient degrees of freedom will exist for some combination of ionization and excitation processes to dissipate precisely the total energy. Hence, for a given energy deposited in the sample a fluctuation in excitation must be balanced by an equivalent fluctuation in ionization.

$$E_x \Delta N_x + E_i \Delta N_i = 0$$

If for a given event more energy goes into charge formation, less energy will be available for excitation. Averaging over many events this means that the variances in the energy allocated to the two types of processes must be equal

$$E_i s_i = E_x s_x$$

$$s_i = \frac{E_x}{E_i} \sqrt{N_x}$$

From the total energy $E_i N_i + E_x N_x = E_0$

$$N_x = \frac{E_0 - E_i N_i}{E_x}$$

yielding

$$s_i = \frac{E_x}{E_i} \sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x} N_i}$$

Since each ionization leads to a charge pair that contributes to the signal

$$N_i = N_Q = \frac{E_0}{e_i}$$

where e_i is the average energy loss required to produce a charge pair,

$$s_i = \frac{E_x}{E_i}\sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x}\frac{E_0}{e_i}}$$

$$s_i = \sqrt{\frac{E_0}{e_i}} \cdot \sqrt{\frac{E_x}{E_i}\left(\frac{e_i}{E_i} - 1\right)}$$

The second factor on the right hand side is called the Fano factor F.

Since s_i is the variance in signal charge Q and the number of charge pairs is $N_Q = E_0 / e_i$

$$s_Q = \sqrt{FN_Q}$$

In Silicon E_x= 0.037 eV
$E_i = E_g$= 1.1 eV
e_i = 3.6 eV

for which the above expression yields F= 0.08, in reasonable agreement with the measured value F= 0.1.

Þ The variance of the signal charge is smaller than naively expected

$$s_Q \approx 0.3\sqrt{N_Q}$$

A similar treatment can be applied if the degrees of freedom are much more limited and Poisson statistics are necessary.

However, when applying Poisson statistics to the situation of a fixed energy deposition, which imposes an upper bound on the variance, one can not use the usual expression for the variance

$$\mathrm{var}\, N = \overline{N}$$

Instead, the variance is

$$\overline{(N - \overline{N})^2} = F\,\overline{N}$$

as shown by Fano [1] in the original paper.

An accurate calculation of the Fano factor requires a detailed accounting of the energy dependent cross sections and the density of states of the phonon modes. This is discussed by Alkhazov [2] and van Roosbroeck [3].

References:

1. U. Fano, Phys. Rev. **72** (1947) 26

2. G.D. Alkhazov et al., NIM **48** (1967) 1

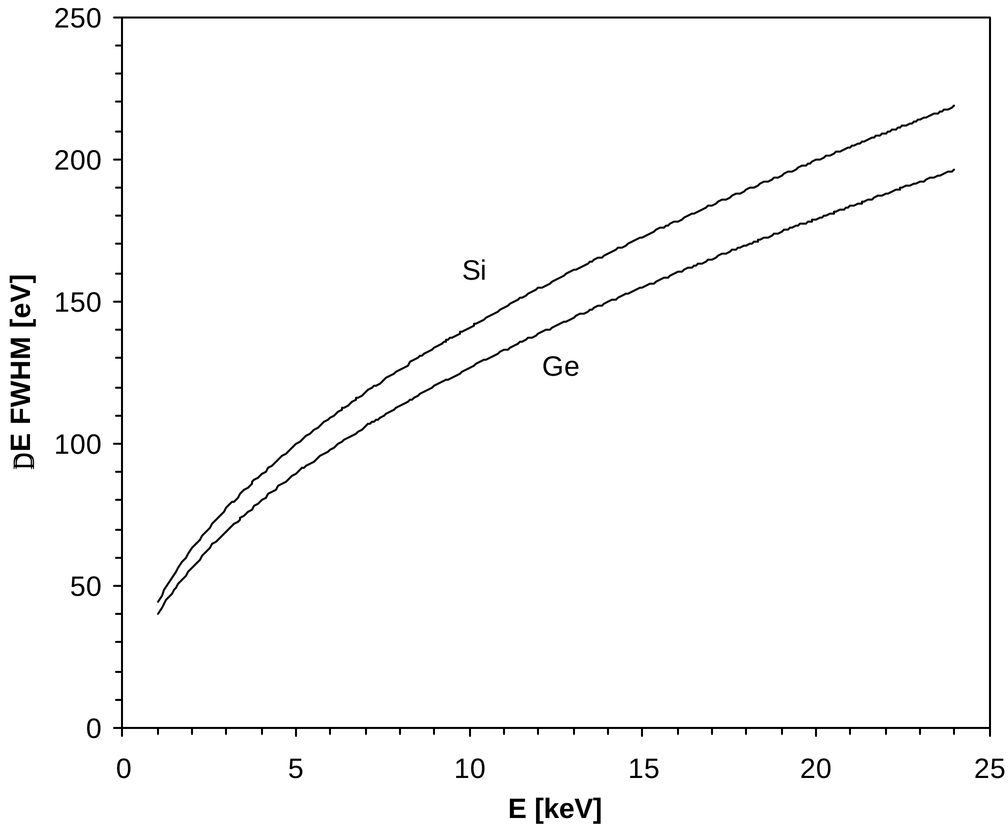
3. W. van Roosbroeck, Phys. Rev. **139** (1963) A1702

Intrinsic Resolution of Semiconductor Detectors

$$\Delta E = 2.35 \cdot e_i \sqrt{FN_Q} = 2.35 \cdot e_i \sqrt{F \frac{E}{w}} = 2.35 \cdot \sqrt{FEe_i}$$

Si:	e_i= 3.6 eV	F= 0.1
Ge:	e_i= 2.9 eV	F= 0.1

Intrinsic Resolution of Si and Ge Detectors



Detectors with good efficiency for this energy range have sufficiently small capacitance to allow electronic noise of ~100 eV FWHM, so the variance of the detector signal is a significant contribution.

At energies >100 keV the detector sizes required tend to increase the electronic noise to dominant levels.

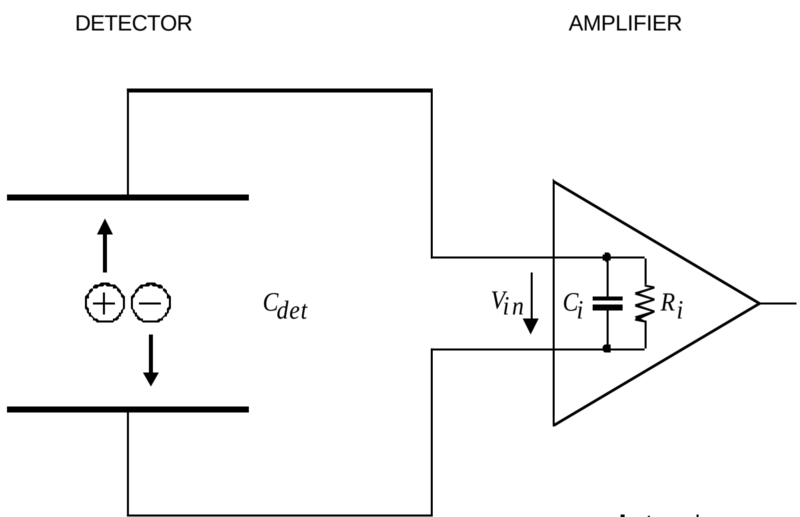
3. Signal Formation

Ionization Chambers:

Detection volume with electric field

Energy deposited ® positive and negative charge pairs

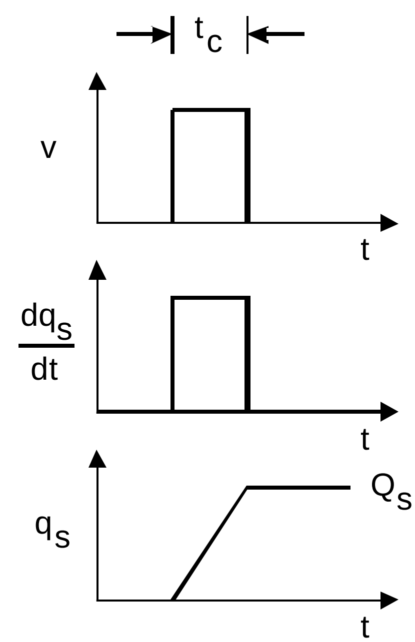
Charges move in field ® external electrical signal



velocity of charge carriers

rate of induced charge on
detector electrodes

signal charge



if R_i x $(C_{det} + C_i)$ >> collection time t_c:

peak voltage at amplifier input $$V_s = \frac{Q_s}{C_{\text{det}} + C_i}$$

Ionization chambers can be made with any medium that allows charge collection to a pair of electrodes.

Medium can be gas
liquid
solid

Crude comparison of relevant properties

	gas	**liquid**	**solid**
density	low	moderate	high
atomic number Z	low	moderate	moderate
ionization energy e_i	moderate	moderate	low
signal speed	moderate	moderate	fast

Desirable properties:

low ionization energy Þ 1. increased charge yield dq/dE

2. superior resolution

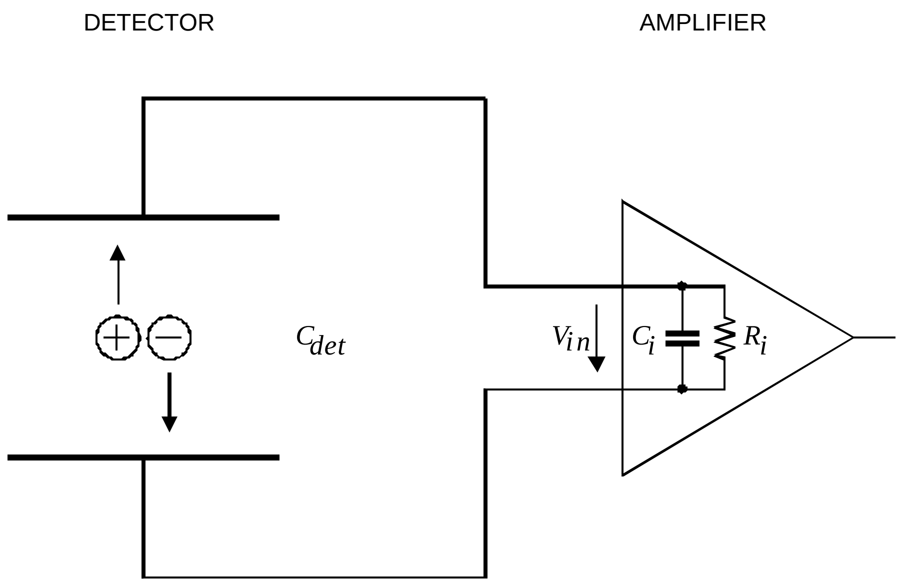
$$\frac{\Delta E}{E} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E/e_i}} \propto \sqrt{e_i}$$

high field in detection volume

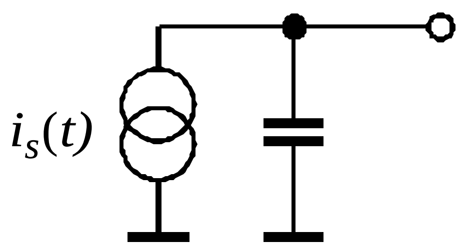
Þ 1. fast response

2. improved charge collection efficiency (reduced recombination or trapping)

Time Dependence of the Signal Current



Model:



When does the signal current begin?

a) when the charge reaches the electrode?

or

b) when the charge begins to move?

Although the first answer is quite popular (encouraged by the phrase "charge collection"), the second is correct.

When a charge pair is created, both the positive and negative charges couple to the electrodes and induce mirror charges of equal magnitude.

As the positive charge moves toward the negative electrode, it couples more strongly to it and less to the positive electrode.

Conversely, the negative charge couples more to the positive electrode and less to the negative electrode.

The net effect is a negative current at the positive electrode and a positive current at the negative electrode, due to both the positive and negative charges.

The following discussion applies to ALL types of structures that register the effect of charges moving in an ensemble of electrodes, i.e. not just semiconductor or gas-filled ionization chambers, but also resistors, capacitors, photoconductors, vacuum tubes, etc.

Induced Charge – Ramo's Theorem

W. Shockley, J. Appl. Phys. **9** (1938) 635
S. Ramo, Proc. IRE **27** (1939) 584

Consider a mobile charge in the presence of any number of grounded electrodes.

Surround the charge q with a small equipotential sphere. Then, if V is the potential of the electrostatic field, in the region between conductors

$$\nabla^2 V = 0$$

Call V_q the potential of the small sphere and note that $V = 0$ on the conductors. Applying Gauss' law yields

$$\int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V}{\partial n}\, ds = 4p\, q$$

Next, consider the charge removed and one conductor A raised to unit potential.

Call the potential V_1, so that

$$\nabla^2 V_1 = 0$$

in the space between the conductors, including the site where the charge was situated. Call the new potential at this point V_{q1}.

Green's theorem states that

$$\int_{\substack{\text{volume between}\\ \text{boundaries}}} (V_1 \nabla^2 V - V \ \nabla^2 V_1)\ dv = -\int_{\substack{\text{boundary}\\ \text{surfaces}}} \left[V_1 \frac{\partial V}{\partial n} - V \frac{\partial V_1}{\partial n} \right] ds$$

Choose the volume to be bounded by the conductors and the tiny sphere.

Then the left hand side is 0 and the right hand side may be divided into three integrals:

1. Over the surfaces of all conductors except A. This integral is 0 since on these surfaces $V = V_1 = 0$.

2. Over the surface of A. As $V_1 = 1$ and $V = 0$ this reduces to

$$-\int_{\text{surface A}} \frac{\partial V}{\partial n}\, ds$$

3. Over the surface of the sphere.

$$-V_{q1} \int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V}{\partial n}\, ds \quad + \quad V_q \int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V_1}{\partial n}\, ds$$

The second integral is 0 by Gauss' law, since in this case the charge is removed.

Combining these three integrals yields

$$0 = -\int_{\text{surface A}} \frac{\partial V}{\partial n} ds \quad - \quad V_{q1} \int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V}{\partial n} ds = 4p\ Q_A - 4p\ q V_{q1}$$

or

$$Q_A = q V_{q1}$$

If the charge q moves in direction x, the current on electrode A is

$$i_A = \frac{dQ_A}{dt} = q\frac{dV_{q1}}{dt} = q\left(\frac{\partial V_{q1}}{\partial x}\;\frac{dx}{dt}\right)$$

Since the velocity of motion

$$\frac{dx}{dt} = v_x$$

the induced current on electrode A is

$$i_A = q\; v_x \frac{\partial V_{q1}}{dx}$$

where V_{q1} is the "weighting potential" that describes the coupling of a charge at any position to electrode A.

The weighting potential is for a specific electrode is obtained by setting the potential of the electrode to 1 and setting all other electrodes to potential 0.

- If a charge q moves along any path s from position 1 to position 2, the net induced charge on electrode k is

$$\Delta Q_k = q(V_{q1}(2) - V_{q1}(1)) \equiv q\left(\Phi_k(2) - \Phi_k(1)\right)$$

- The instantaneous current can be expressed in terms of a weighting field

$$i_k = -q\;\vec{v}\cdot\;\overrightarrow{F_k}$$

The weighting field is determined by applying unit potential to the measurement electrode and 0 to all others.

Note that the electric field and the weighting field are distinctly different.

- The electric field determines the charge trajectory and velocity

- The weighting field depends only on geometry and determines how charge motion couples to a specific electrode.

- Only in 2-electrode configurations are the electric field and the weighting field of the same form.

Example 1: Parallel plate geometry with uniform field
(semiconductor detector with very large overbias)

Assume a voltage V_b applied to the detector. The distance between the two parallel electrodes is d.

The electric field that determines the motion of charge in the detector is

$$E = \frac{V_b}{d}$$

Assume that the velocity of the charge carriers is collision limited, so the velocity of the charge

$$v = \mu E = \mu \frac{V_b}{d}$$

The weighting field is obtained by applying unit potential to the collection electrode and grounding the other.

$$E_Q = \frac{1}{d}$$

so the induced current

$$i = qvE_Q = q\mu \frac{V_b}{d}\frac{1}{d} = q\mu \frac{V_b}{d^2}$$

since both the electric field and the weighting field are uniform throughout the detector, the current is constant until the charge reaches its terminal electrode.

Assume that the charge is created at the opposite electrode and traverses the detector thickness d.

The required collection time, i.e. the time required to traverse the detector thickness d

$$t_c = \frac{d}{v} = \frac{d}{m \dfrac{V_b}{d}} = \frac{d^2}{m\ V_b}$$

The induced charge

$$Q = i t_c = qm \frac{V_b}{d^2} \frac{d^2}{m\ V_b} = q$$

Next, assume an electron-hole pair formed at coordinate x from the positive electrode.

The collection time for the electron

$$t_{ce} = \frac{x}{v_e} = \frac{xd}{m_e V_b}$$

and the collection time for the hole

$$t_{ch} = \frac{d-x}{v_h} = \frac{(d-x)d}{m_h V_b}$$

Since electrons and holes move in opposite directions, they induce current of the same sign at a given electrode, despite their opposite charge.

The induced charge due to the motion of the electron

$$Q_e = q_e m_e \frac{V_b}{d^2} \frac{xd}{m_e V_b} = q_e \frac{x}{d}$$

whereas the hole contributes

$$Q_h = q_e m_h \frac{V_b}{d^2} \frac{(d-x)d}{m_h V_b} = q_e \left(1 - \frac{x}{d}\right)$$

Assume that $x = d/2$. After the collection time for the electron

$$t_{ce} = \frac{d^2}{2 m_e V_b}$$

it has induced a charge $q_e/2$.

At this time the hole, due to its lower mobility m_h» $m_e/3$, has induced $q_e/6$, yielding a cumulative induced charge of $2q_e/3$.

After the additional time for the hole collection, the remaining charge $q_e/3$ is induced, yielding the total charge q_e.

If one carrier species is much slower than the other, as in gases, the induced current will be predominantly due to one carrier. Then the total induced charge will be position dependent and on the average, the signal charge will be only $q_e/2$.

In this configuration

- Electrons and holes contribute equally to the currents on both electrodes
- The instantaneous current at any time is the same (although of opposite sign) on both electrodes

  The continuity equation (Kirchhoff's law) must be satisfied

$$\sum_k i_k = 0$$

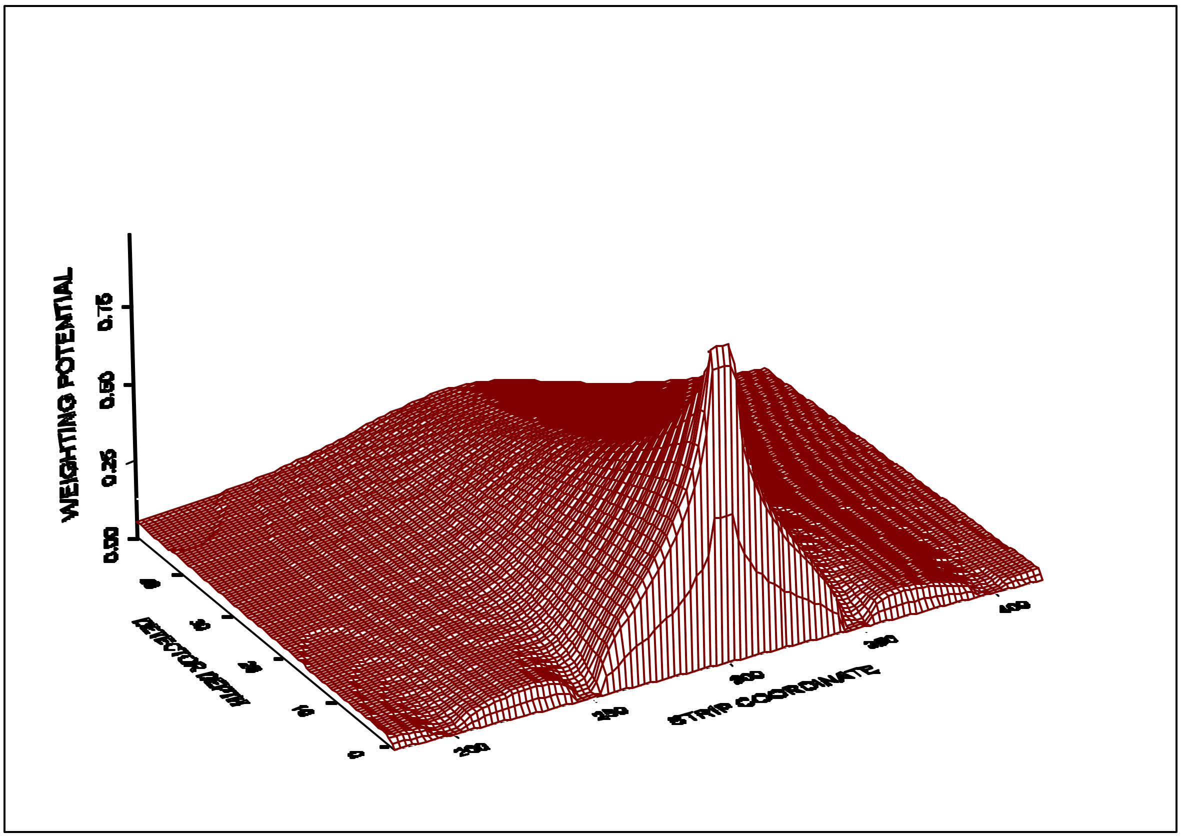
Since k=2: $i_1 = -i_2$

Example 2: Double-Sided Strip Detector

The strip pitch is assumed to be small compared to the thickness.

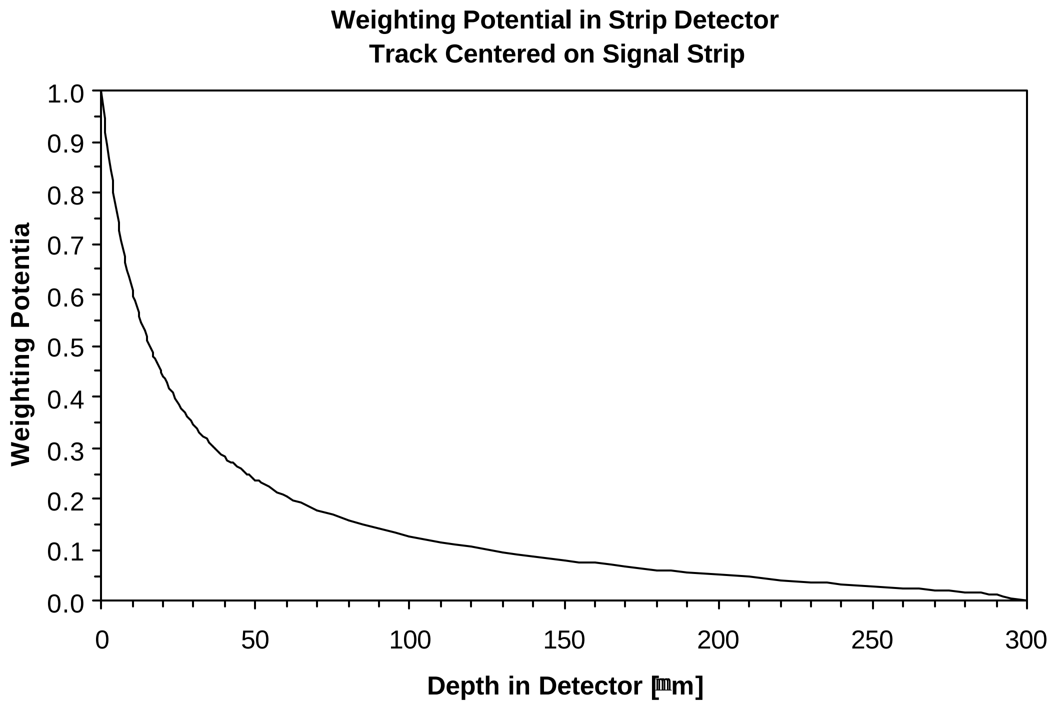
The electric field is similar to a parallel-plate geometry, except in the immediate vicinity of the strips.

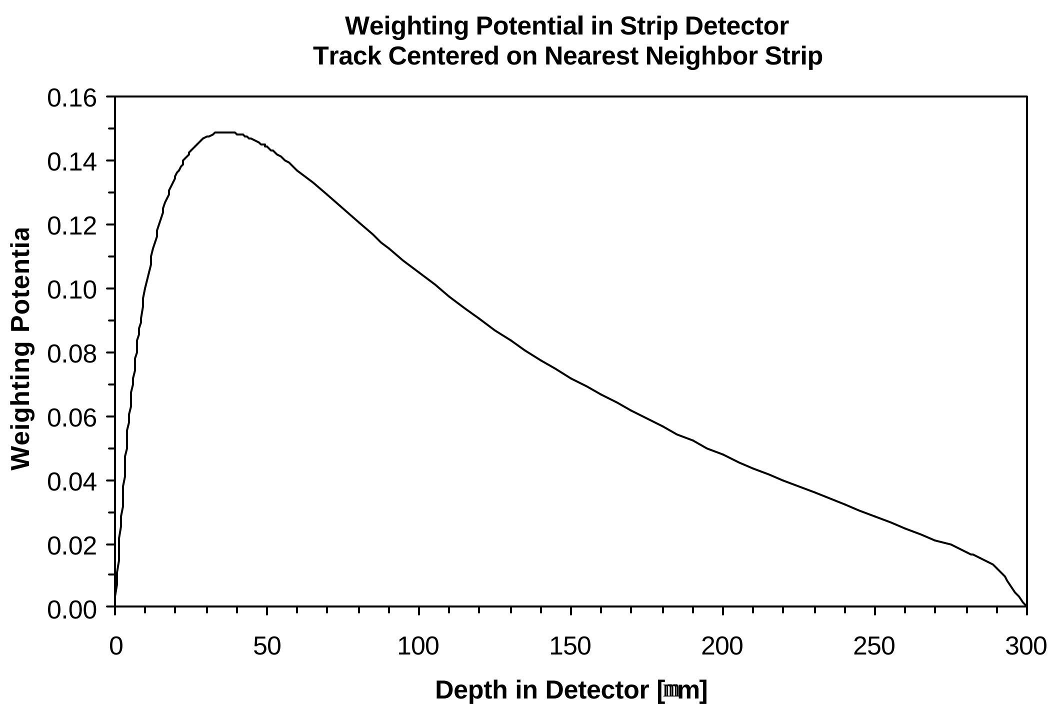
The signal weighting potential, however is very different.



Weighting potential for a 300 µm thick strip detector with strips on a pitch of 50 µm. Only 50 µm of depth are shown.

Cuts through the weighting potential





Consider an electron-hole pair q_n, q_p originating on a point x_0 on the center-line of two opposite strips of a double-sided strip detector. The motion of the electron towards the n-electrode x_n is equivalent to the motion of a hole in the opposite direction to the p-electrode x_p. The total induced charge on electrode k after the charges have traversed the detector is

$$Q_k = q_p[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] + q_n[\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$

since the hole charge $q_p = q_e$ and $q_n = -q_e$

$$Q_k = q_e[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] - q_e[\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$

$$Q_k = q_e[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_n)]$$

If the signal is measured on the p-electrode, collecting the holes,

$$\Phi_{Qk}(x_p) = 1,$$
$$\Phi_{Qk}(x_n) = 0$$

and $Q_k = q_e$.

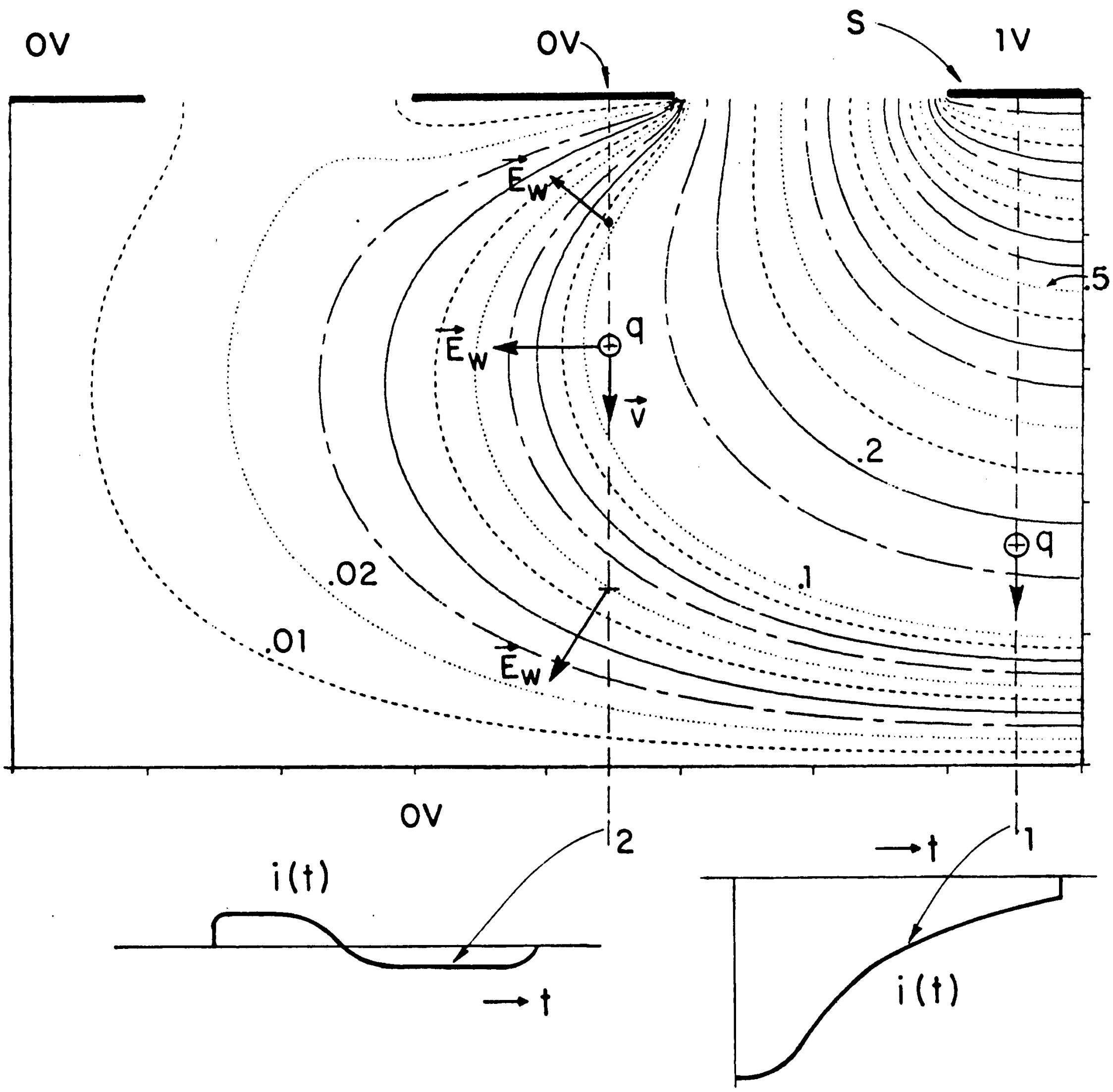
If, however, the charge is collected on the neighboring strip k+1, then

$$\Phi_{Qk+1}(x_p) = 0,$$
$$\Phi_{Qk+1}(x_n) = 0$$

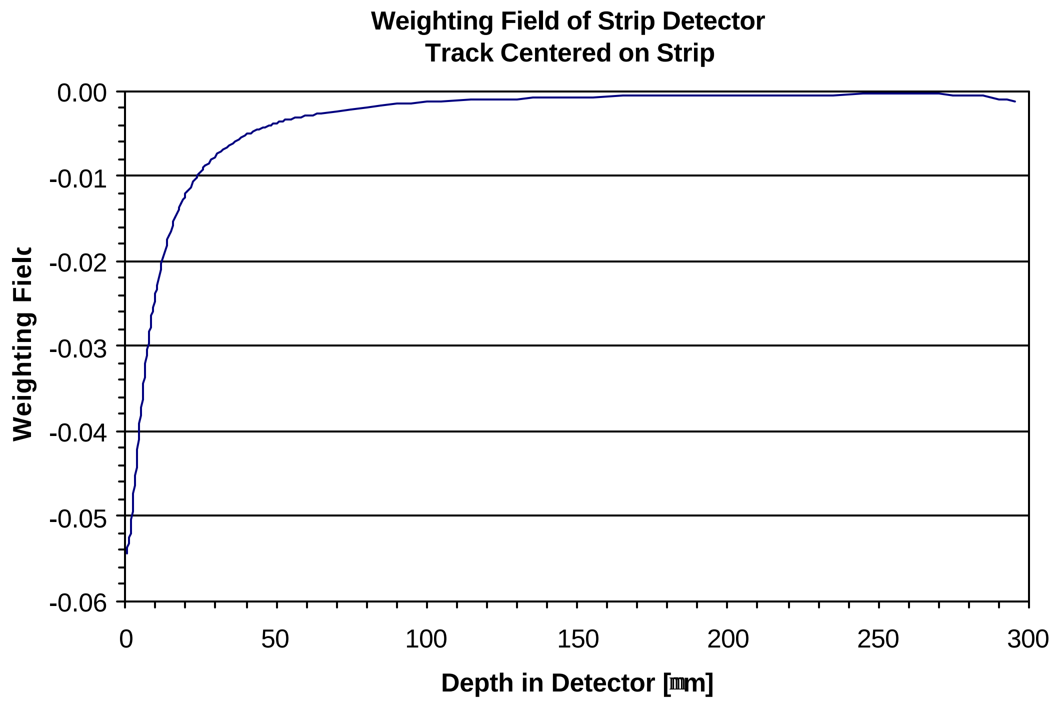
and $Q_{k+1} = 0$.

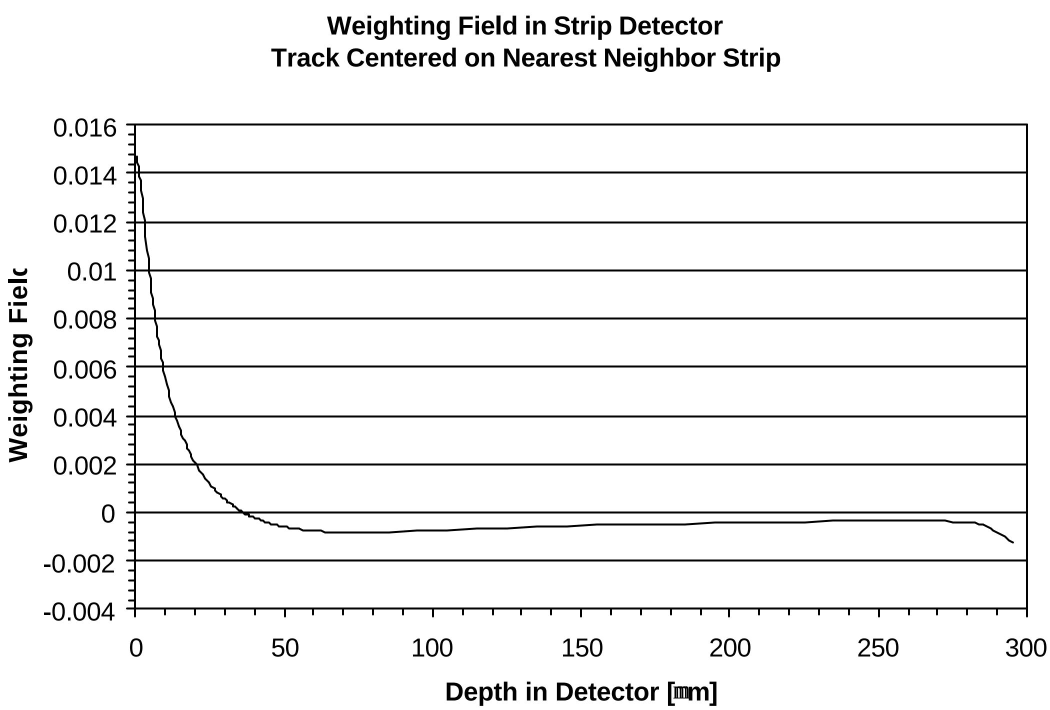
In general, if moving charge does not terminate on the measurement electrode, signal current will be induced, but the current changes sign and integrates to zero.

This is illustrated in the following schematic plot of the weighting field in a strip detector (from Radeka)



Cuts through the Weighting Field in a Strip Detector
(d= 300 µm, p= 50 µm)





Note, however that this charge cancellation on “non-collecting” electrodes relies on the motion of both electrons and holes.

Assume, for example, that the holes are stationary, so they don't induce a signal. Then the first term of the first equation above vanishes, which leaves a residual charge

$$Q_k = q_e[\Phi_{Qk}(x_0) - \Phi_{Qk}(x_n)]$$

since for any coordinate not on an electrode

$$Q_k(x_0) \neq 0,$$

although it may be very small.

An important consequence of this analysis is that one cannot simply derive pulse shapes by analogy with a detector with contiguous electrodes (i.e. a parallel plate detector of the same overall dimensions as a strip detector). Specifically,

1. the shape of the current pulses can be quite different,

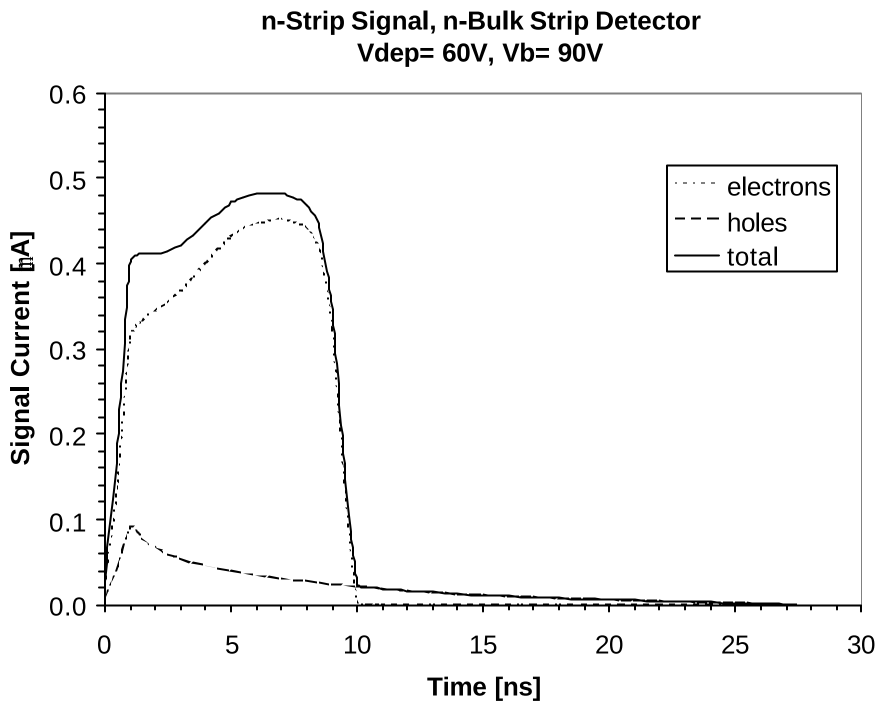
2. the signals seen on opposite strips of a double-sided detector are not the same (although opposite in sign), and

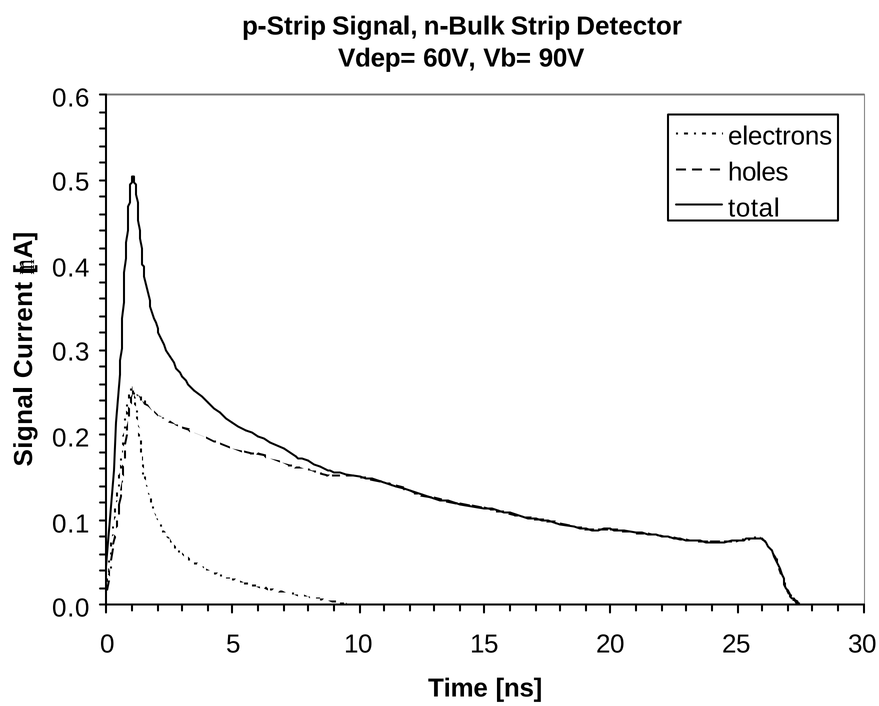
3. the net induced charge on the p- or n-side is not split evenly between electrons and holes.

- Because the weighting potential is strongly peaked near the signal electrode, most of the charge is induced when the moving charge is near the signal electrode.

- As a result, most of the signal charge is due to the charge terminating on the signal electrode.

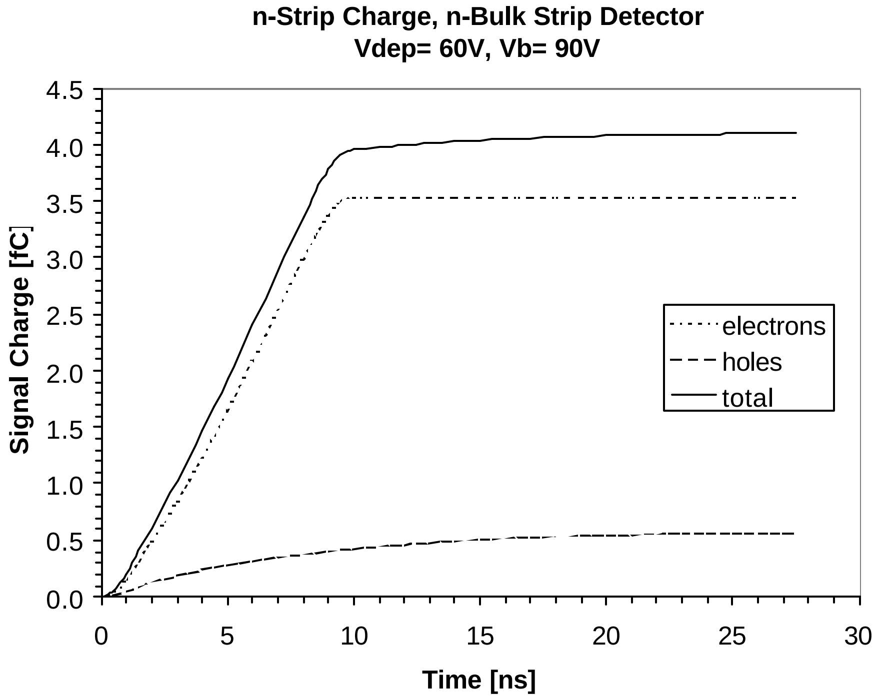
Current pulses in strip detectors (track traversing the detector)

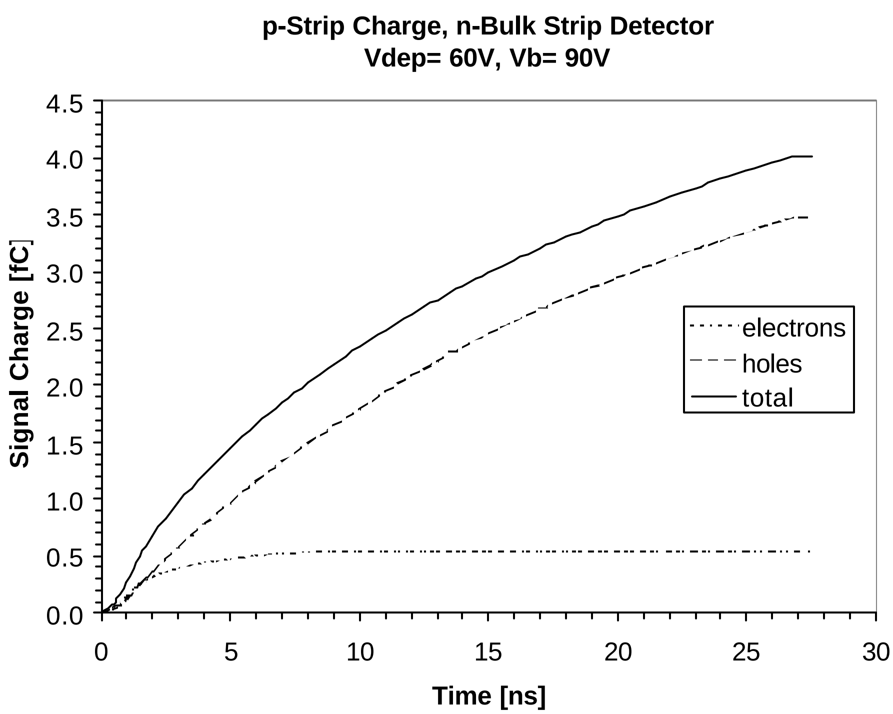




The duration of the electron and hole pulses is determined by the time required to traverse the detector as in a parallel-plate detector, but the shapes are very different.

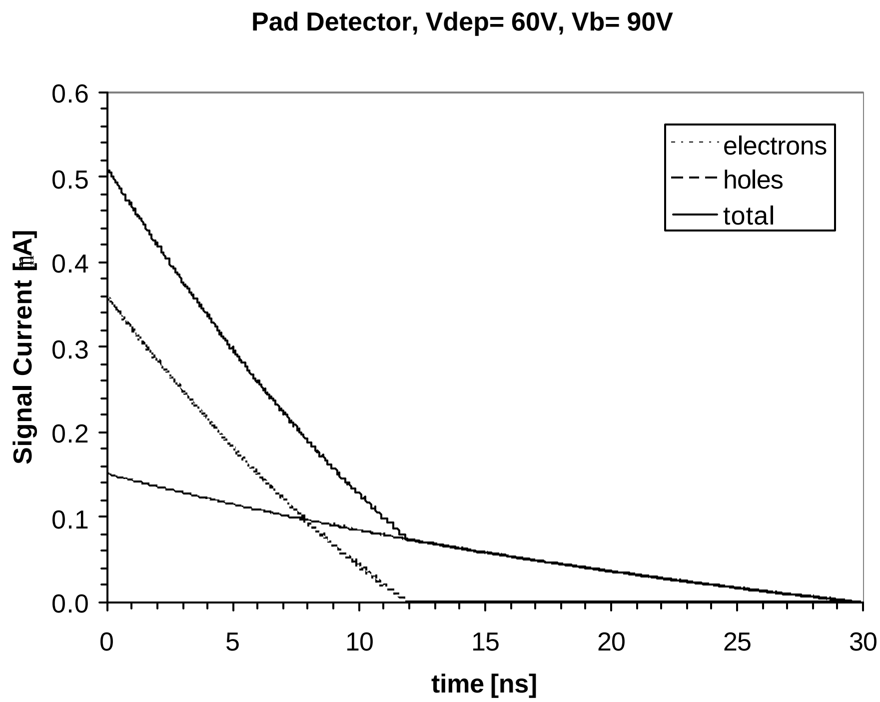
Strip Detector Signal Charge Pulses



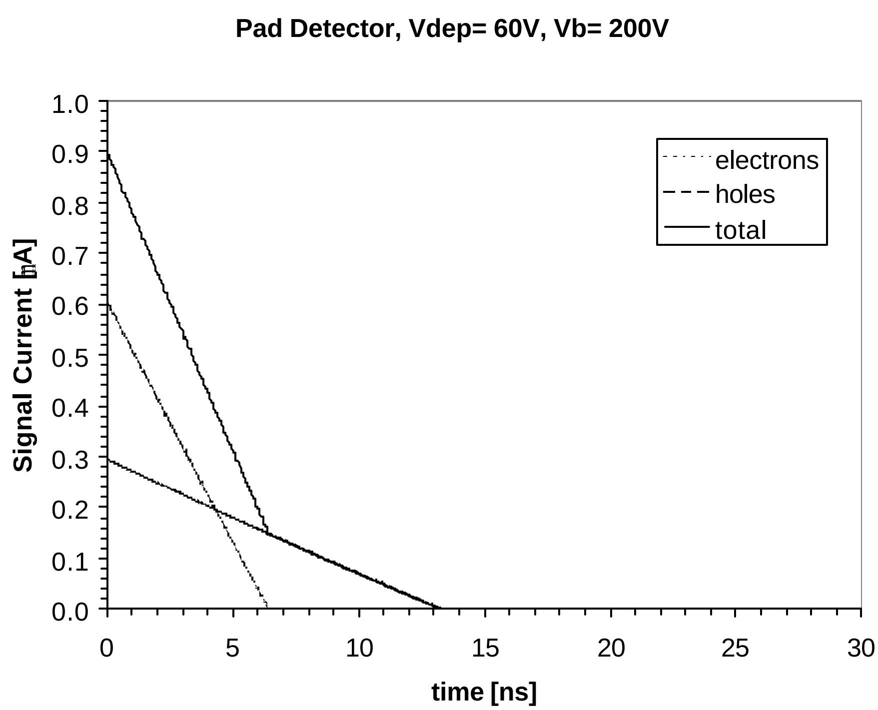


For comparison:

Current pulses in pad detectors (track traversing the detector)



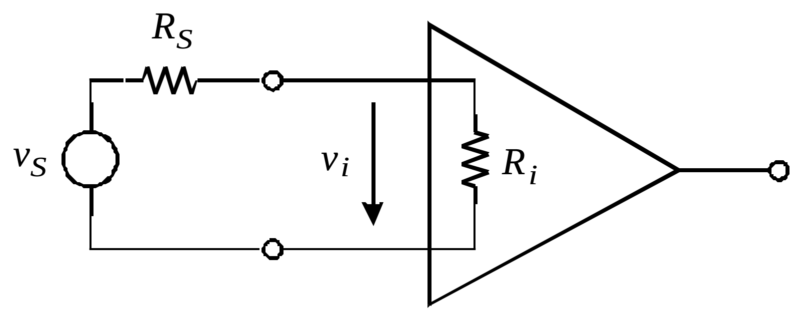
For the same depletion and bias voltages the pulse durations are the same as in strip detectors. Overbias decreases the collection time.



4. Signal Acquisition

Amplifier Types

a) Voltage-Sensitive Amplifier



The signal voltage at the amplifier input

$$v_i = \frac{R_i}{R_S + R_i} v_S$$

If the signal voltage at the amplifier input is to be approximately equal to the signal voltage

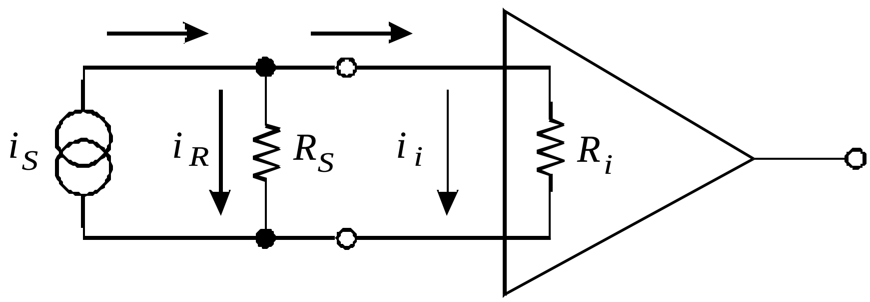
$$v_i \approx v_S \qquad \text{Þ} \qquad R_i \gg R_S$$

To operate in the voltage-sensitive mode, the amplifier's input resistance (or impedance) must be large compared to the source resistance (impedance).

In ideal voltage amplifiers one sets $R_i = \infty$, although this is never true in reality, although it can be fulfilled to a good approximation.

To provide a voltage output, the amplifier should have a low output resistance, i.e. its output resistance should be small compared to the input resistance of the following stage.

b) Current-Sensitive Amplifier



The signal current divides into the source resistance and the amplifier’s input resistance. The fraction of current flowing into the amplifier

$$i_i = \frac{R_s}{R_s + R_i} i_S$$

If the current flowing into the amplifier is to be approximately equal to the signal current

$$i_i \approx i_S \qquad Þ \qquad R_i \ll R_S$$

To operate in the current-sensitive mode, the amplifier’s input resistance (or impedance) must be small compared to the source resistance (impedance).

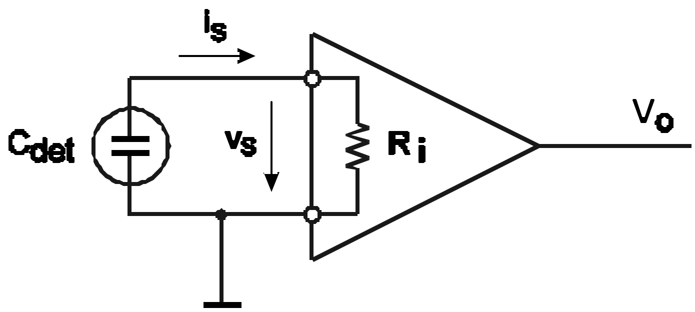
One can also model a current source as a voltage source with a series resistance. For the signal current to be unaffected by the amplifier input resistance, the input resistance must be small compared to the source resistance, as derived above.

- *Whether a specific amplifier operates in the current or voltage mode depends on the source resistance.*

At the output, to provide current drive the output resistance should be high, i.e. large compared to the input resistance of the next stage.

- Amplifiers can be configured as current mode input and voltage mode output or, conversely, as voltage mode input and current mode output. The gain is then expressed as V/A or A/V.

c) Voltage and Current Mode with Capacitive Sources



Output voltage V_o = voltage gain $A_v \times$ input voltage v_s.

Operating mode depends on charge collection time t_{coll} and the input time constant $R_i C_{det}$:

a) $R_i C_{det} << t_{coll}$ detector capacitance discharges rapidly

$$\Rightarrow \quad V_o \propto i_s(t)$$

current sensitive amplifier

b) $R_i C_{det} >> t_{coll}$ detector capacitance discharges slowly

$$\Rightarrow \quad V_o \propto \int i_s(t)dt$$

voltage sensitive amplifier

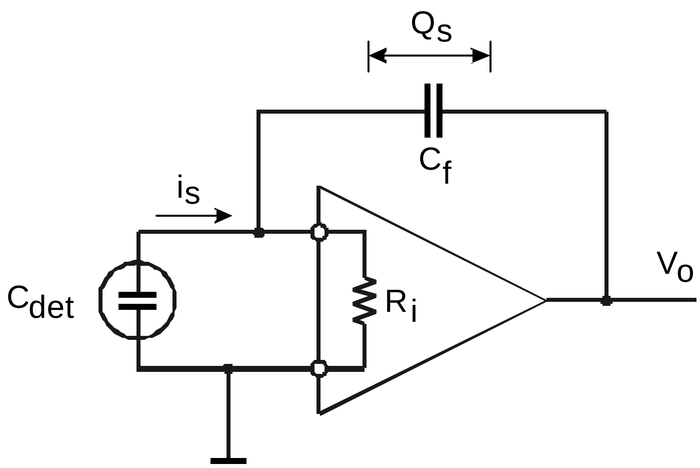
Note that in both cases the amplifier is providing voltage gain, so output signal voltage is determined directly by the input voltage. The difference is that the shape of the input voltage pulse is determined either by the instantaneous current or by the integrated current and the decay time constant.

d) Feedback Amplifiers

Basic amplifier as used above.

High input resistance: $R_i C_{det} >> t_{coll}$

Add feedback capacitance C_f



Signal current i_s is integrated on feedback capacitor C_f :

$$V_o \; \mu \; Q_s / C_f$$

Amplifier output directly determined by signal charge, insensitive to detector capacitance

$\Rightarrow$ charge-sensitive amplifier

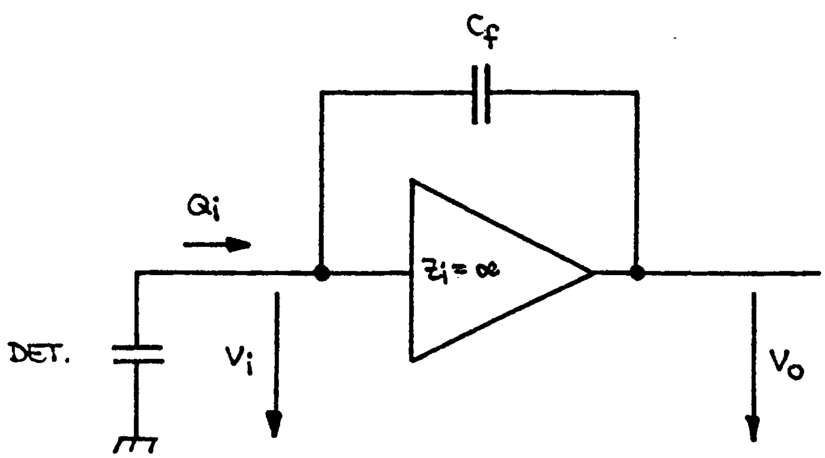
Active Integrator ("charge-sensitive amplifier")

Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow \quad v_o = -Av_i$

Input impedance = ∞ (i.e. no signal current flows into amplifier input)

Connect feedback capacitor C_f between output and input.



Voltage difference across C_f: $v_f = v_i - v_o = v_i - (-Av_i) = (A+1)v_i$

⇒ Charge deposited on C_f: $Q_f = C_f v_f = C_f (A+1) v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

⇒ Effective input capacitance

$$C_i = \frac{Q_i}{v_i} = C_f (A+1)$$

("dynamic" input capacitance)

Gain

$$A_Q = \frac{dV_o}{dQ_i} = \frac{A \cdot v_i}{C_i \cdot v_i} = \frac{A}{C_i} = \frac{A}{A+1} \cdot \frac{1}{C_f} \approx \frac{1}{C_f} \quad (A >> 1)$$

Q_i is the charge flowing into the preamplifier ....

but some charge remains on C_{det}.

What fraction of the signal charge is measured?

$$\frac{Q_i}{Q_s} = \frac{C_i v_i}{Q_{\text{det}} + Q_i} = \frac{C_i}{Q_s} \cdot \frac{Q_s}{C_i + C_{\text{det}}}$$

$$= \frac{1}{1 + \frac{C_{\text{det}}}{C_i}} \approx 1 \;\; (\text{if } C_i >> C_{\text{det}})$$

Example:

A = 10^3

C_f = 1 pF Þ C_i = 1 nF

C_{det} = 10 pF: Q_i / Q_s = 0.99

C_{det} = 500 pF: Q_i / Q_s = 0.67

↑

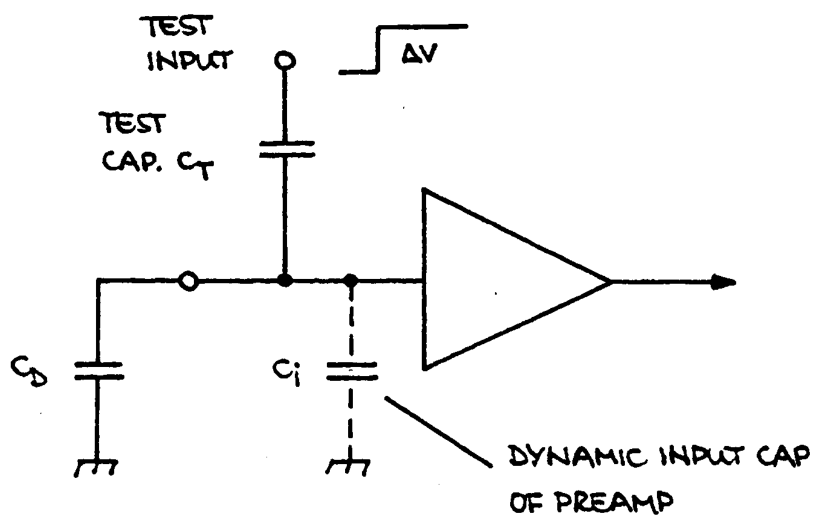
Si Det.: 50 µm thick
500 mm^2 area

Note: Input coupling capacitor must be >>C_i for high charge transfer efficiency.

Calibration

Inject specific quantity of charge - measure system response

Use voltage pulse (can be measured conveniently with oscilloscope)



$C_i >> C_T$ Þ Voltage step applied to test input develops over C_T .

Þ $Q_T = \Delta V \cdot C_T$

Accurate expression:

$$Q_T = \frac{C_T}{1+\frac{C_T}{C_i}} \cdot \Delta V \approx C_T \left(1 - \frac{C_T}{C_i}\right) \Delta V$$

Typically: $C_T / C_i = 10^{-3} - 10^{-4}$

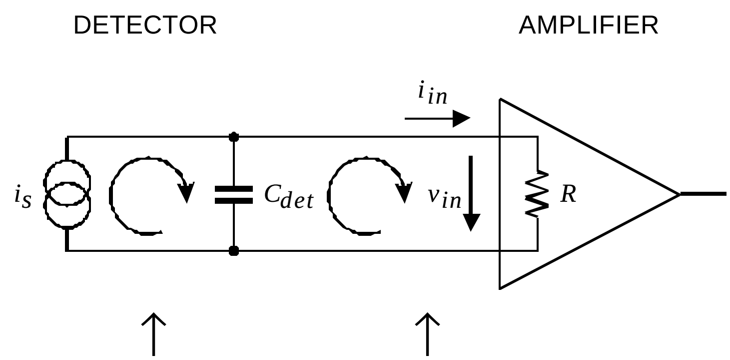
Realistic Charge-Sensitive Preamplifiers

The preceding discussion assumed idealized amplifiers with infinite speed.

In reality, amplifiers may be too slow to follow the instantaneous detector pulse.

Does this incur a loss of charge?

Equivalent Circuit:



charges moving in detector induce change of charge on detector electrodes

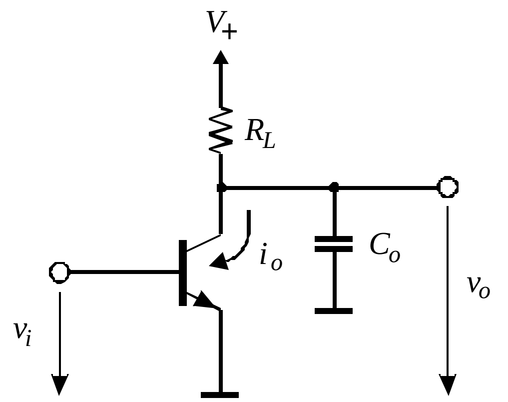
detector capacitance discharges into amplifier

Signal is preserved even if the amplifier responds much more slowly than the detector signal.

However, the response of the amplifier affects the measured pulse shape.

- How do “real” amplifiers affect the measured pulse shape?
- How does the detector affect amplifier response?

A Simple Amplifier



Voltage gain:

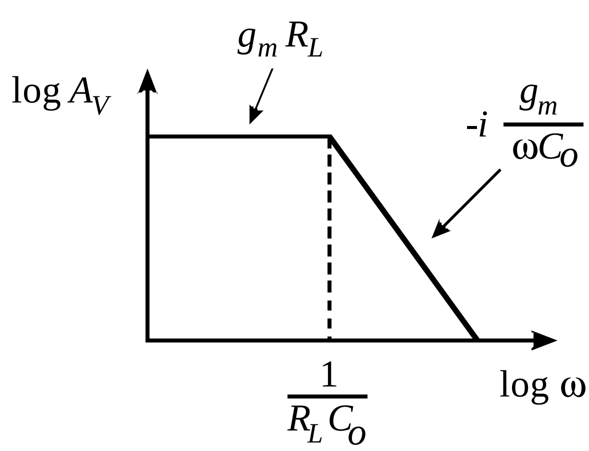
$$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

$g_m \equiv$ transconductance

$$Z_L = R_L \,//\, C_o$$

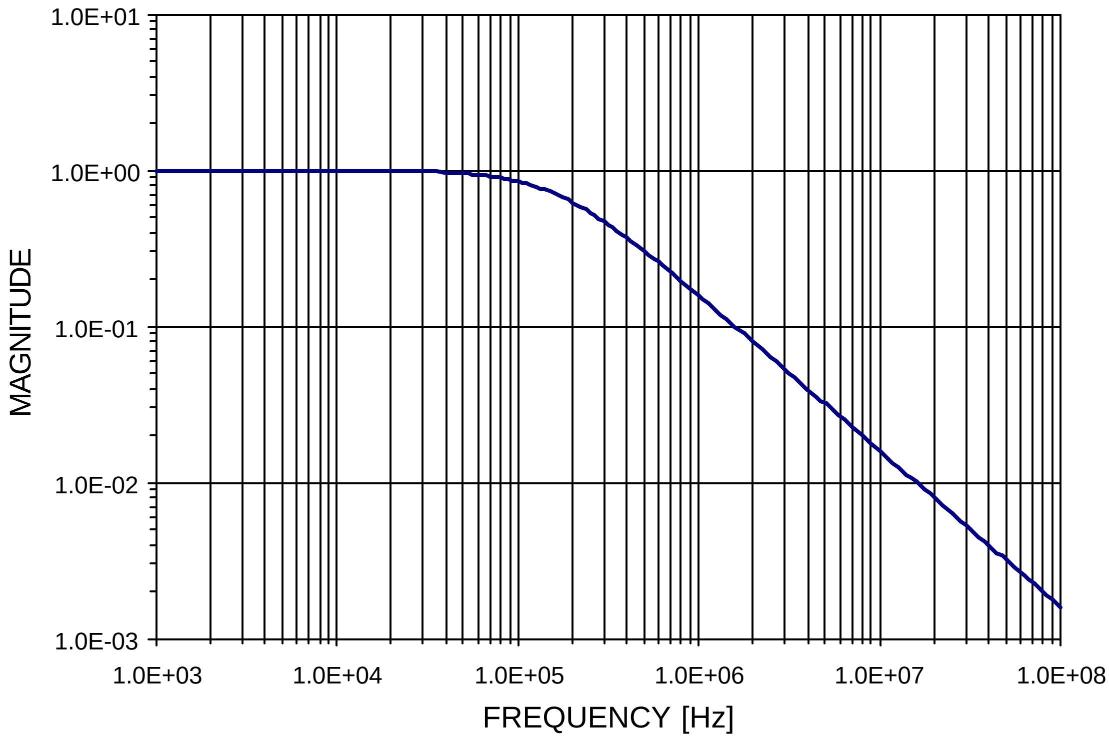
$$\frac{1}{Z_L} = \frac{1}{R_L} + iw\, C_o \qquad \Rightarrow \qquad A_V = g_m \left(\frac{1}{R_L} + iw\, C_o \right)^{-1}$$

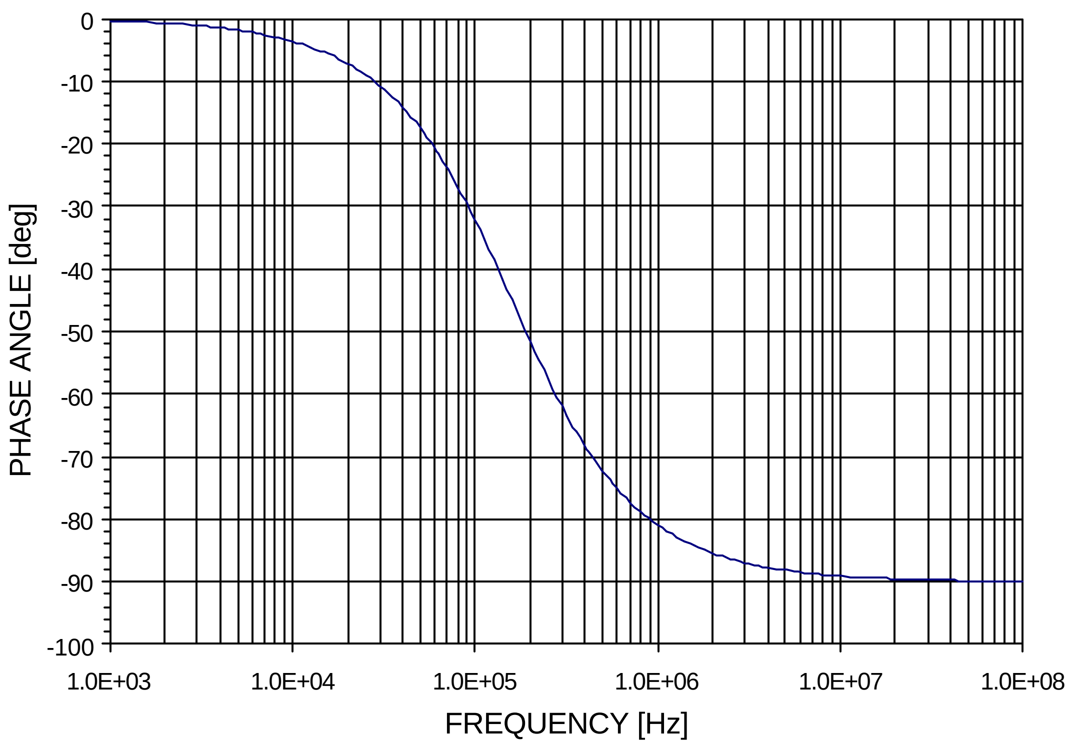
low freq. high freq.



upper cutoff frequency $2\pi\, f_u$

Exact amplitude and phase response:



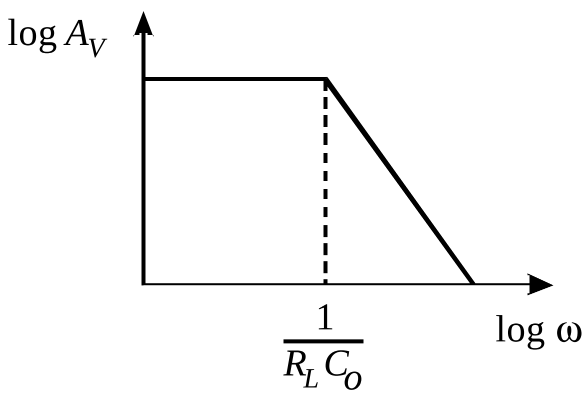


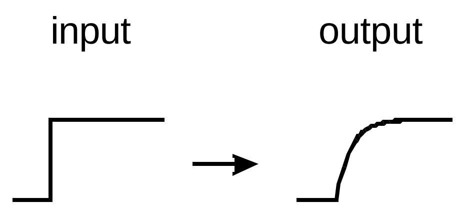
Pulse Response of the Simple Amplifier

A voltage step $v_i\,(t)$ at the input causes a current step $i_o\,(t)$ at the output of the transistor.

For the output voltage to change, the output capacitance C_o must first charge up.

Þ The output voltage changes with a time constant $t = R_L C_o$





$$V = V_o\,(1 - e^{-t/t})$$

Frequency Domain

Time Domain

The time constant t corresponds to the upper cutoff frequency

$$t = \frac{1}{2p\ f_u}$$

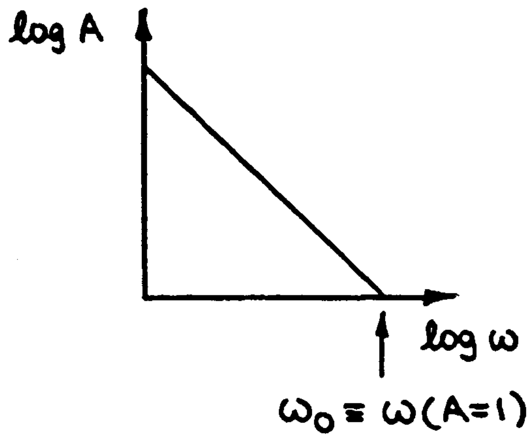
Input Impedance of a Charge-Sensitive Amplifier

Input impedance

$$Z_i = \frac{Z_f}{A+1} \approx \frac{Z_f}{A} \quad (A >> 1)$$

Amplifier gain vs. frequency beyond the upper cutoff frequency

$$A = -i \frac{\omega_0}{\omega}$$



Gain-Bandwidth Product

Feedback Impedance

$$Z_f = -i \frac{1}{\omega C_f}$$

⇒ Input Impedance

$$Z_i = -\frac{i}{\omega C_f} \cdot \frac{1}{-i \frac{\omega_0}{\omega}}$$

$$Z_i = \frac{1}{\omega_0 C_f}$$

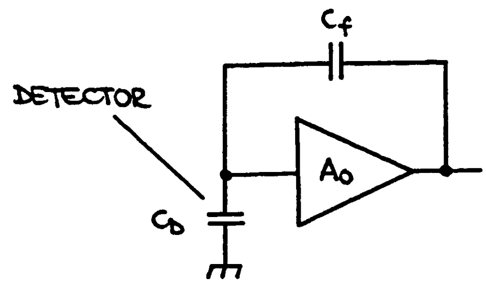
Imaginary component vanishes ⇒ *Resistance:* $Z_i \to R_i$

Note: can synthesize resistance with less than thermal noise

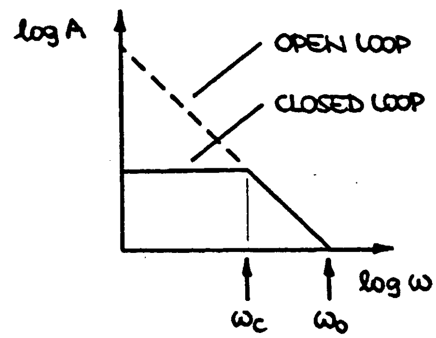
Time Response of a Charge-Sensitive Amplifier

Closed Loop Gain

$$A_f = \frac{C_D + C_f}{C_f} \quad (A_f << A_0)$$



$$A_f \approx \frac{C_D}{C_f} \quad (C_D >> C_f)$$



Closed Loop Bandwidth

$$w_C A_f = w_0$$

Response Time

$$t_{amp} = \frac{1}{w_C} = C_D \; \frac{1}{w_0 C_f}$$

Þ **Rise time increases with detector capacitance.**

Alternative Picture: Input Time Constant

$$t_i = R_i C_D$$

$$t_i = \frac{1}{w_0 C_f} \cdot C_D = t_{amp}$$

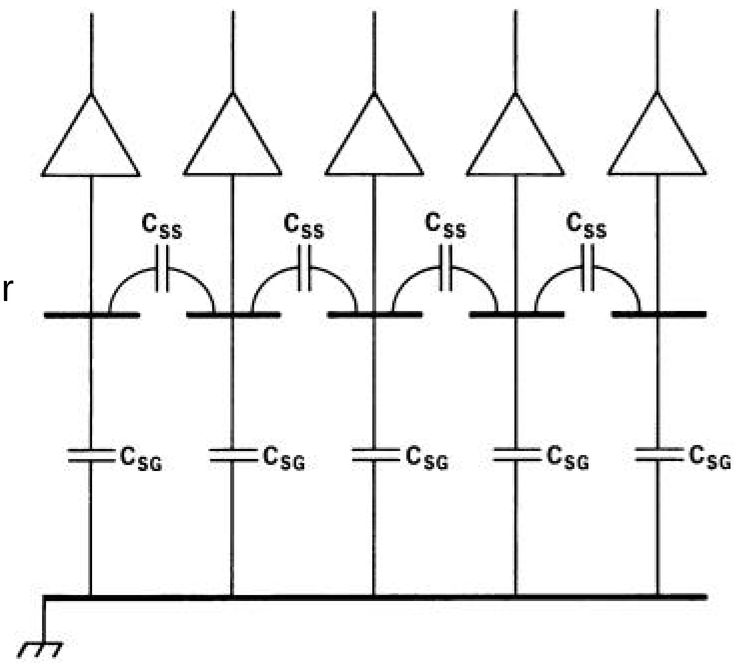
Same result as from conventional feedback theory.

Appication to Strip and Pixel Detectors

Input impedance is critical in strip or pixel detectors:

Amplifiers must have a low input impedance to reduce transfer of charge through capacitance to neighboring strips

strip detector electrodes



For strip pitches that are smaller than the bulk thickness the capacitance is dominated by the fringing capacitance to the neighboring strips C_{SS}.

Typically: 1 – 2 pF/cm for strip pitches of 25 – 100 μm on Si.

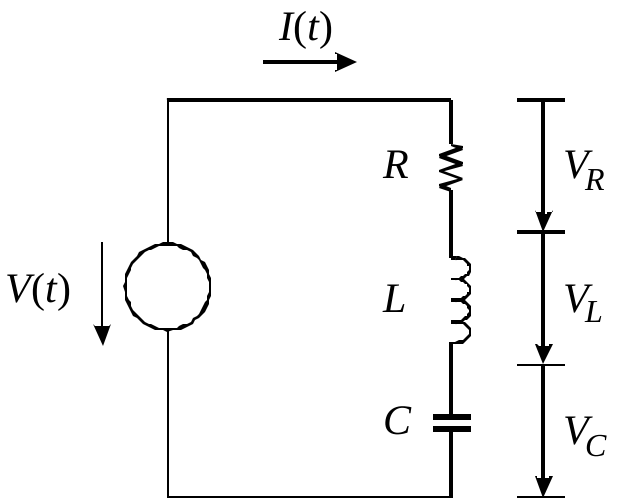
The backplane capacitance C_{SG} is typically 20% of the strip-to-strip capacitance.

Negligible cross-coupling at times t > (2 … 3) x R_iC_D and if $C_i >> C_D$.

Appendix 1

Phasors and Complex Algebra in Electrical Circuits

Consider the *RLC* circuit



$$V = V_R + V_L + V_C$$

$$V = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{dI}{dt}R + L\frac{d^2 I}{dt^2} + \frac{I}{C}$$

Assume that $V(t) = V_0 e^{iwt}$ and $I(t) = I_0 e^{i(wt+j)}$

$$iwV_0 e^{iwt} = iwRI_0 e^{i(wt-j)} - w^2 LI_0 e^{i(wt-j)} + \frac{1}{C} I_0 e^{i(wt-j)}$$

$$\frac{V_0}{I_0} e^{ij} = R + iwL - i\frac{1}{wC}$$

Thus, we can express the total impedance $Z \equiv (V_0 / I_0)\ e^{ij}$ of the circuit as a complex number with the magnitude $|Z| = V_0 / I_0$ and phase φ.

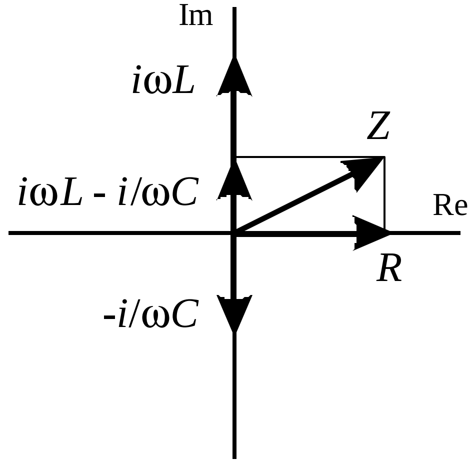
In this representation the equivalent resistances (reactances) of L and C are imaginary numbers

$$X_L = iwL \quad \text{and} \quad X_C = -\frac{i}{wC}$$

Plotted in the complex plane:

Relative to V_R, the voltage across the inductor V_L is shifted in phase by +90°.

The voltage across the capacitor V_C is shifted in phase by -90°.



The total impedance has the magnitude

$$|Z| = \sqrt{[\mathrm{Re}(Z)]^2 + [\mathrm{Im}(Z)]^2} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

and the phase

$$\tan j = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{wL - \frac{1}{wC}}{R}$$

From this one sees immediately that the impedance Z assumes a minimum at

$$w = \frac{1}{\sqrt{LC}} \; ,$$

the resonant frequency of the tuned circuit. The impedance vs. frequency yields the resonance curve. At resonance the phase φ becomes zero. At frequencies above resonance the inductive reactance dominates (as in the drawing above) and the asymptotic phase is +90°. Below resonance the capacitive reactance dominates and the asymptotic phase is -90°.

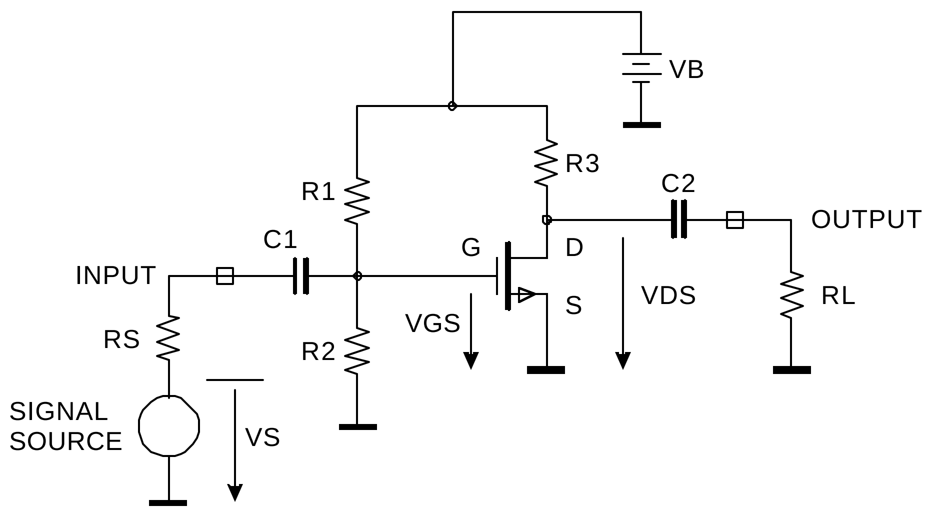
Use to represent any element that introduces a phase shift, e.g an amplifier. A phase shift of +90° appears as + i , -90° as $-i$.

Appendix 2

Equivalent Circuits

Take a simple amplifier as an example.

a) full circuit diagram



First, just consider the DC operating point of the circuitry between *C1* and *C2*:

1. The n-type MOSFET requires a positive voltage applied from the gate G to the source S.

$$V_{GS} = \frac{R2}{R1 + R2} V_B$$

2. The gate voltage V_{GS} sets the current flowing into the drain electrode D.
3. Assume the drain current is I_D. Then the DC voltage at the drain is

$$V_{DS} = V_B - I_D R3$$

Next, consider the AC signal V_S provided by the signal source.

Assume that the signal at the gate G is dV_G/dt.

1. The current flowing through *R2* is

$$\frac{dI}{dt}(R2) = \frac{dV_G}{dt} \cdot \frac{1}{R2}$$

2. The current flowing through *R1* is

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{d}{dt}(V_G + \mathrm{V_B})$$

Since the battery voltage $\mathrm{V_B}$ is constant,

$$\frac{d\mathrm{V_B}}{dt} = 0$$

so that

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{dV_G}{dt}$$

3. The total time-dependent input current is

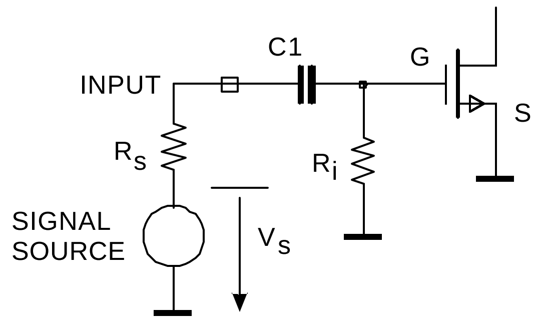
$$\frac{dI}{dt} = \frac{dI_{R1}}{dt} + \frac{dI_{R2}}{dt} = \left(\frac{1}{R1} + \frac{1}{R2}\right) \cdot \frac{dV_G}{dt} \equiv \frac{1}{R_i} \cdot \frac{dV_G}{dt}$$

where

$$R_i = \frac{R1 \cdot R2}{R1 + R2}$$

is the parallel connection of *R1* and *R2*.

Consequently, for the AC input signal the circuit is equivalent to



At the output, the voltage signal is formed by the current of the transistor flowing through the combined output load formed by R_L and $R3$.

For the moment, assume that $R_L >> R3$. Then the output load is dominated by $R3$.

The voltage at the drain D is

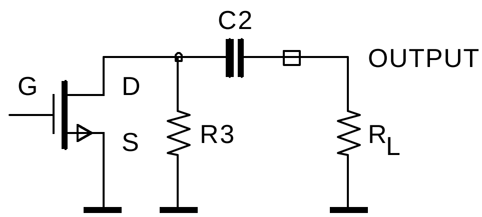
$$V_o = \mathrm{V_B} - i_D R3$$

If the gate voltage is varied, the transistor drain current changes, with a corresponding change in output voltage

$$\frac{dV_o}{di_D} = \frac{d}{dI_D}(\mathrm{V_B} - i_D R3) = R3$$

Þ The DC supply voltage does not directly affect the signal formation.

If we remove the restriction $R_L >> R3$, the total load impedance for time-variant signals is the parallel connection of $R3$ and $(X_{C2} + R_L)$, yielding the equivalent circuit at the output



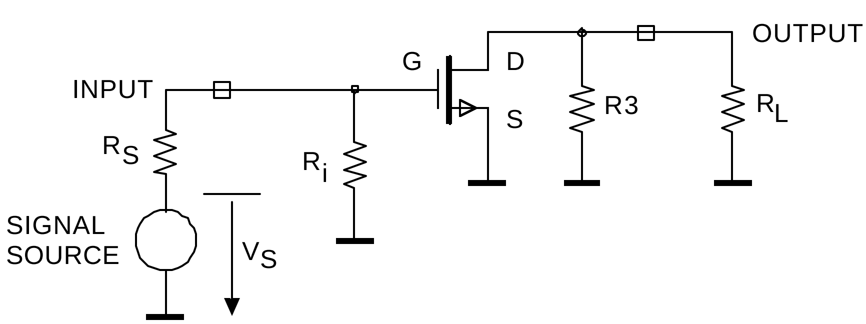
If the source resistance of the signal source $R_S << R_i$, the input coupling capacitor $C1$ and input resistance R_i form a high-pass filter. At frequencies where the capacitive reactance is $<<R_i$, i.e.

$$f >> \frac{1}{2\pi\ R_i C1}$$

the source signal v_s suffers negligible attenuation at the gate, so that

$$\frac{dV_G}{dt} = \frac{dV_s}{dt}$$

Correspondingly, at the output, if the impedance of the output coupling capacitor $C2<<R_L$, the signal across R_L is the same as across $R3$, yielding the simple equivalent circuit



Note that this circuit is only valid in the “high-pass” frequency regime.

Equivalent circuits are an invaluable tool in analyzing systems, as they remove extraneous components and show only the components and parameters essential for the problem at hand.

Often equivalent circuits are tailored to very specific questions and include simplifications that are not generally valid. Conversely, focussing on a specific question with a restricted model may be the only way to analyze a complicated situation.