III. Electronic Noise

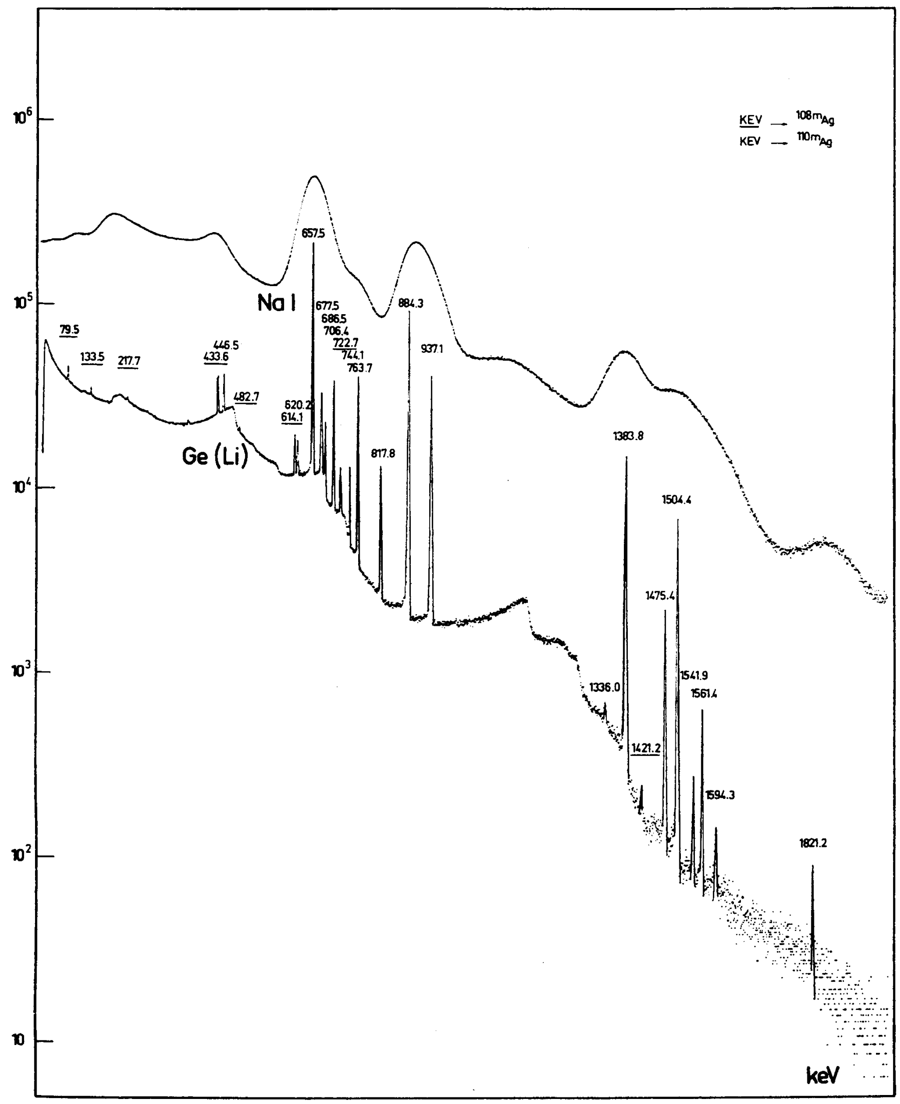
Resolution and Electronic Noise

Resolution: the ability to distinguish signal levels

1. Why?

a) Recognize structure in amplitude spectra

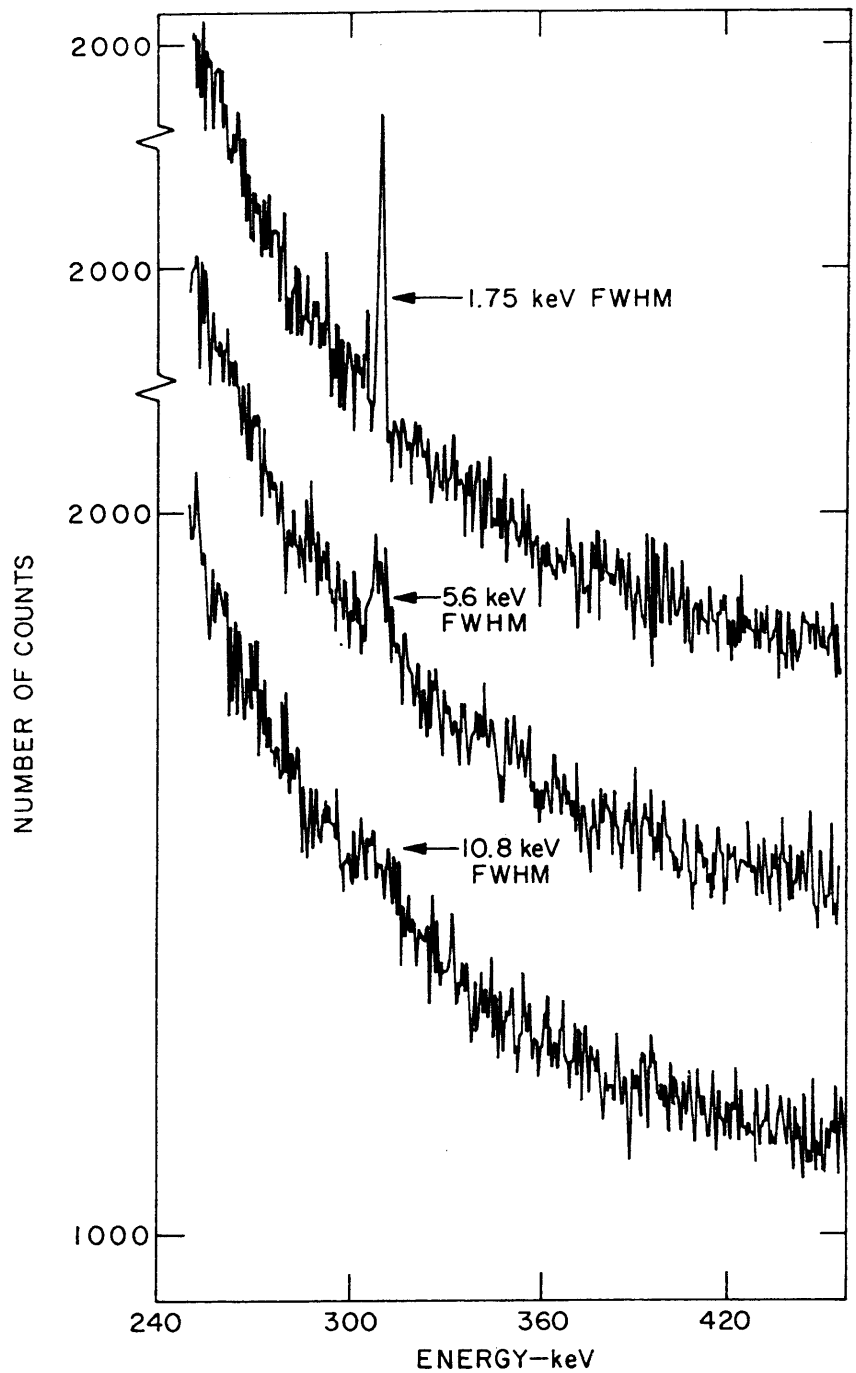
Comparison between NaI(Tl) and Ge detectors



(J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446)

b) Improve sensitivity

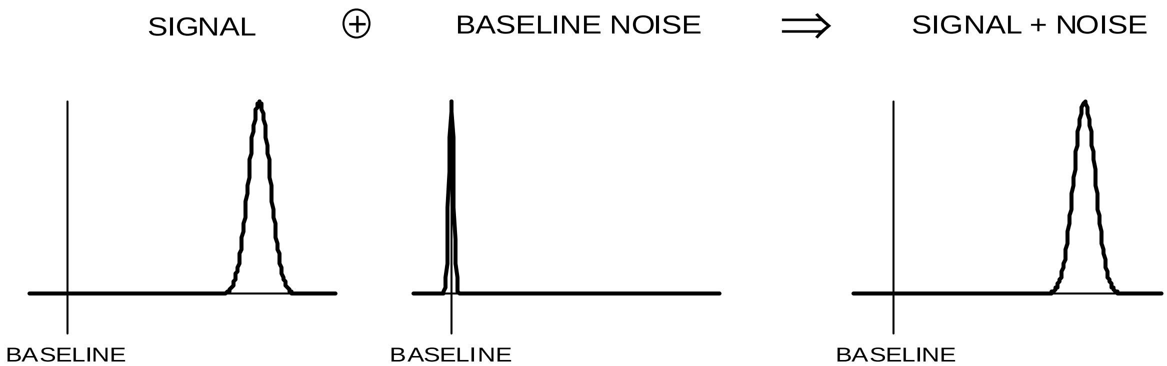
Signal to background ratio improves with better resolution
(signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

2. What determines Resolution?

1. Signal Variance >> Baseline Variance



Þ Electronic (baseline) noise not important

Examples:

- High-gain proportional chambers
- Scintillation Counters with High-Gain PMTs

e.g. 1 MeV γ-rays absorbed by NaI(Tl) crystal

Number of photoelectrons

$$N_{pe} \approx 8 \cdot 10^4 \text{ [MeV}^{-1}] \times E_g \times QE \approx 2.4 \cdot 10^4$$

Variance typically

$$s_{pe} = N_{pe}^{1/2} \approx 160 \text{ and } s_{pe} / N_{pe} \approx 5 - 8\%$$

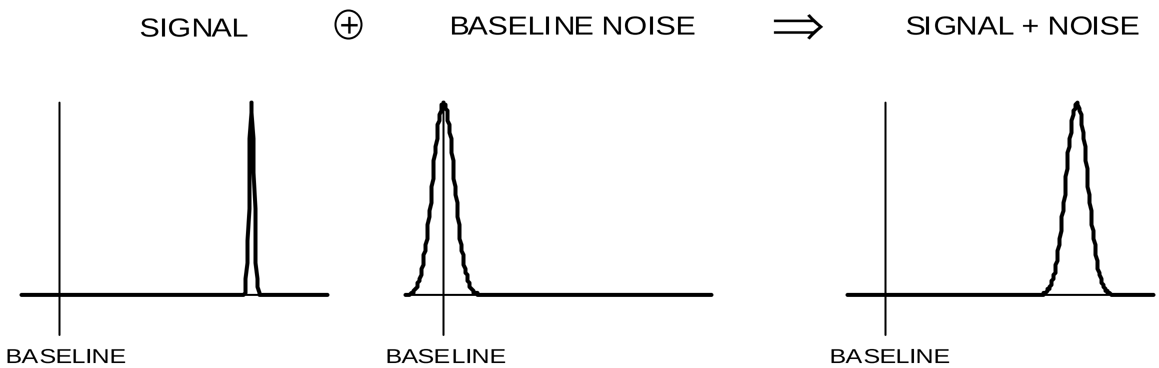
Signal at PMT anode (assume Gain= 10^4)

$$Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8 \text{ el and}$$

$$s_{sig} = G_{PMT}\, s_{pe} \approx 1.2 \cdot 10^7 \text{ el}$$

whereas electronic noise easily < 10^4 el

2. Signal Variance << Baseline Variance



Þ Electronic (baseline) noise critical for resolution

Examples

- Gaseous ionization chambers (no internal gain)
- Semiconductor detectors

e.g. in Si

Number of electron-hole pairs

$$N_{ep} = E_{dep} / (3.6 \text{ eV})$$

Variance

$$s_{ep} = \sqrt{F \cdot N_{ep}}$$

(where F= Fano factor ≈ 0.1)

For 50 keV photons

$$s_{ep} \approx 40 \text{ el} \Rightarrow \quad s_{ep} / N_{ep} = 7.5 \cdot 10^{-4}$$

obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

… etc.,

but the (practical) fundamental limit is electronic noise.

3. Basic Noise Mechanisms

Consider n carriers of charge e moving with a velocity v through a sample of length l. The induced current i at the ends of the sample is

$$i = \frac{n\ e\ v}{l}\ .$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, *e.g.* thermal noise
- number fluctuations, *e.g.* shot noise
  excess or '1/ f ' noise

Thermal noise and shot noise are both “white” noise sources, i.e.

power per unit bandwidth is constant:

$\equiv$ spectral density)

$$\frac{dP_{noise}}{df} = const.$$

or

$$\frac{dV_{noise}^2}{df} = const. \equiv e_n^2$$

whereas for “1/ f ” noise

$$\frac{dP_{noise}}{df} = \frac{1}{f^{\,a}}$$

(typically a = 0.5 – 2)

1. Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Spectral noise power density vs. frequency f

$$\frac{dP_{noise}}{df} = 4kT$$

k = Boltzmann constant
T = absolute temperature

Since

$$P = \frac{V^2}{R} = I^2 R$$

R = DC resistance

the spectral noise voltage density

$$\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$$

and the spectral noise current density

$$\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$$

The total noise depends on the bandwidth of the system.
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain $A_v(f)$ is

$$v_{on}^2 = \int_0^\infty e_n^2 \; |A_v(f)|^2 \; df$$

Note: Since spectral noise components are non-correlated, one must integrate over the noise power.

2. Shot noise

A common example of noise due to number fluctuations is "shot noise", which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode
(emission over a barrier)

Spectral noise current density:

$$i_n^2 = 2q_e I$$

q_e = electronic charge
I = DC current

A more intuitive interpretation of this expression will be given later.

- Shot noise does not occur in "ohmic" conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

3. 1/*f* Noise

The noise spectrum becomes non-uniform whenever the fluctuations are not purely random in time, for example when carriers are trapped and then released with a time constant τ. With an infinite number of uniformly distributed time constants the spectral power density assumes a pure 1/*f* distribution. However, with as few as 3 time constants spread over one or two decades, the spectrum is approximately 1/*f*, so this form of noise is very common.

- For a 1/*f* spectrum the total noise depends on the ratio of the upper to lower cutoff frequencies, rather than the absolute bandwidth.

Spectral Density of Thermal Noise

Two approaches can be used to derive the spectral distribution of thermal noise.

1. The thermal velocity distribution of the charge carriers is used to calculate the time dependence of the induced current, which is then transformed into the frequency domain.

2. Application of Planck's theory of black body radiation.

The first approach clearly shows the underlying physics, whereas the second "hides" the physics by applying a general result of statistical mechnics. However, the first requires some advanced concepts that go well beyond the standard curriculum, so the "black body" approach will be used.

In Planck's theory of black body radiation the energy per mode

$$\overline{E} = \frac{hn}{e^{hn/kT} - 1}$$

and the spectral density of the radiated power

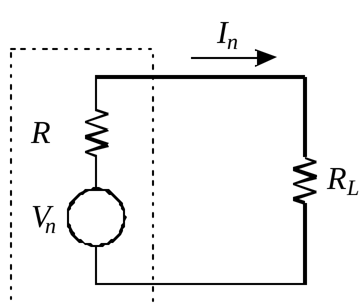
$$\frac{dP}{dn} = \frac{hn}{e^{hn/kT} - 1}$$

i.e. this is the power that can be extracted in equilibrium. At low frequencies $hn \ll kT$

$$\frac{dP}{dn} \approx \frac{hn}{\left(1 + \frac{hn}{kT}\right) - 1} = kT \;,$$

so at low frequencies the spectral density is independent of frequency and for a total bandwidth B the noise power that can be transferred to an external device $P_n = kTB$.

To apply this result to the noise of a resistor, consider a resistor R whose thermal noise gives rise to a noise voltage V_n. To determine the power transferred to an external device consider the circuit



The power dissipated in the load resistor R_L

$$\frac{V_{nL}^2}{R_L} = I_n^2 R_L = \frac{V_n^2 R_L}{(R+R_L)^2}$$

The maximum power transfer occurs when the load resistance equals the source resistance $R_T = R$, so

$$V_{nL}^2 = \frac{V_n^2}{4}$$

Since the power transferred to R_L is kTB

$$\frac{V_{nL}^2}{R} = \frac{V_n^2}{4R} = kTB$$

$$P_n = \frac{V_n^2}{R} = 4kTB$$

and the spectral density of the noise power

$$\frac{dP_n}{dn} = 4kT$$

Spectral Density of Shot Noise

If an excess electron is injected into a device, it forms a current pulse of duration t. In a thermionic diode t is the transit time from cathode to anode (see IX.2), for example. In a semiconductor diode t is the recombination time (see IX-2). If these times are short with respect to the periods of interest $t \ll 1/f$, the current pulse can be represented by a d pulse. The Fourier transform of a delta pulse yields a "white" spectrum, i.e. the amplitude distribution in frequency is uniform

$$\frac{dI_{n,pk}}{df} = 2q_e$$

Within an infinitesimally narrow frequency band the individual spectral components are pure sinusoids, so their rms value

$$i_n \equiv \frac{dI_n}{df} = \frac{2q_e}{\sqrt{2}} = \sqrt{2}q_e$$

If N electrons are emitted at the same average rate, but at different times, they will have the same spectral distribution, but the coefficients will differ in phase. For example, for two currents i_p and i_q with a relative phase j the total rms current

$$\left\langle i^2 \right\rangle = \left(i_p + i_q e^{ij}\right)\left(i_p + i_q e^{-ij}\right) = i_p^2 + i_q^2 + 2i_p i_q \cos j$$

For a random phase the third term averages to zero

$$\left\langle i^2 \right\rangle = i_p^2 + i_q^2 \;,$$

so if N electrons are randomly emitted per unit time, the individual spectral components simply add in quadrature

$$i_n^2 = 2Nq_e^2$$

The average current

$$I = Nq_e \;,$$

so the spectral noise density

$$i_n^2 \equiv \frac{dI_n^2}{df} = 2q_e I$$

“Noiseless” Resistances

a) Dynamic Resistance

In many instances a resistance is formed by the slope of a device’s current-voltage characteristic, rather than by a static ensemble of electrons agitated by thermal energy.

Example: forward-biased semiconductor diode

Diode current vs. voltage

$$I = I_0 (e^{q_e V / kT} - 1)$$

The differential resistance

$$r_d = \frac{dV}{dI} = \frac{kT}{q_e I}$$

i.e. at a given current the diode presents a resistance, e.g. 26 Ω at I = 1 mA and T = 300 K.

Note that two diodes can have different charge carrier concentrations, but will still exhibit the same dynamic resistance at a given current, so the dynamic resistance is not uniquely determined by the number of carriers, as in a resistor.

There is no thermal noise associated with this “dynamic” resistance, although the current flow carries shot noise.

b) Radiation Resistance of an Antenna

Consider a receiving antenna with the normalized power pattern $P_n(q,f)$ pointing at a brightness distribution $B(q,f)$ in the sky. The power per unit bandwidth received by the antenna

$$w = \frac{A_e}{2} \iint B(q,f) P_n(q,f) d\Omega$$

where A_e is the effective aperture, i.e. the “capture area” of the antenna. For a given field strength E, the captured power $W \propto E A_e$.

If the brightness distribution is from a black body radiator and we’re measuring in the Rayleigh-Jeans regime,

$$B(q,f) = \frac{2kT}{l^2}$$

and the power received by the antenna

$$w = \frac{kT}{l^2} A_e \Omega_A \quad .$$

Ω_A is the beam solid angle of the antenna (measured in rad^2), i.e. the angle through which all the power would flow if the antenna pattern were uniform over its beamwidth.

Since $A_e \Omega_A = l^2$ (see antenna textbooks), the received power

$$w = kT$$

The received power is independent of the radiation resistance, as would be expected for thermal noise.

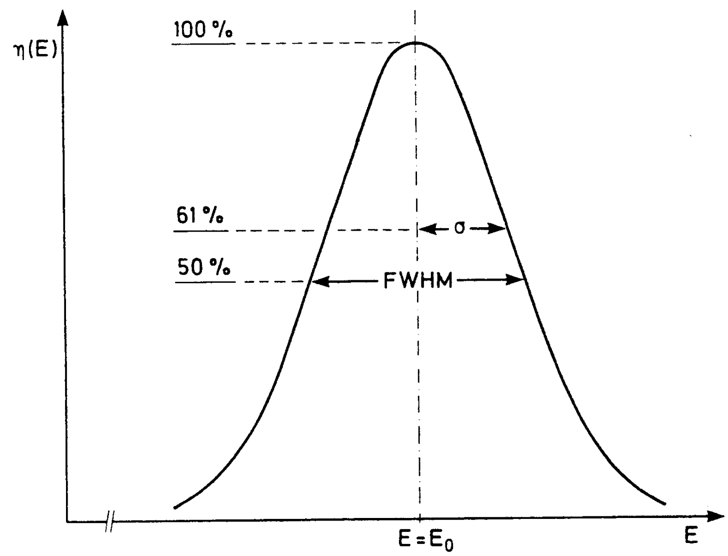
However, it is not determined by the temperature of the antenna, but by the temperature of the sky the antenna pattern is subtending.

For example, for a region dominated by the CMB, the measured power corresponds to a resistor at a temperature of ~3K, although the antenna may be at 300K.

Noise characteristics

Both thermal and shot noise are purely random.

Þ amplitude distribution is gaussian



Þ noise modulates baseline

Þ baseline fluctuations superimposed on signal

Þ output signal has gaussian distribution

Correlated Noise

Generally, noise power is additive.

$$P_{n,tot} = P_{n1} + P_{n1} + ...$$

However, in a coherent system (i.e. a system that preserves phase), the power often results from the sum of voltages or currents, which is sensitive to relative phase.

For two correlated noise sources N_1 and N_2 the total noise

$$N = N_1^2 + N_2^2 + 2CN_1N_2$$

where the correlation coefficient C can range from –1 (anti-correlated, i.e. identical, but 180° out of phase) to +1 (fully correlated).

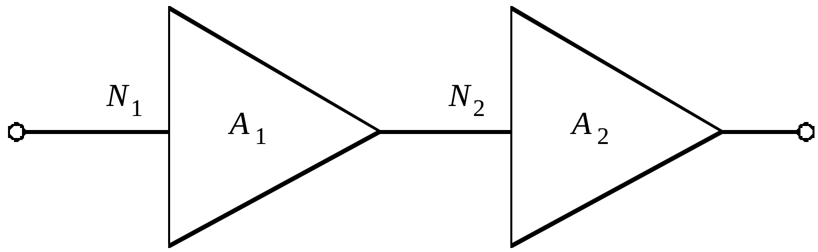
For uncorrelated noise components $C = 0$ and then individual current or voltage noise contributions add in quadrature, e.g.

$$V_{n,tot} = \sqrt{\sum_i V_{ni}^2}$$

4. Noise in Amplifiers

Consider a chain of two amplifiers (or amplifying devices), with gains A_1 and A_1, and input noise levels N_1 and N_2 .

A signal S is applied to the first amplifier, so the input signal-to-noise ratio is S / N_1 .



At the output of the first amplifier the signal is $A_1 S$ and the noise $A_1 N$.

Both are amplified by the second amplifier, but in addition the second amplifier contributes its noise, so the signal-to-noise ratio at the output of the second amplifier

$$\left(\frac{S}{N}\right)^2 = \frac{(SA_1A_2)^2}{(N_1A_1A_2)^2 + (N_2A_2)^2} = \frac{S^2}{N_1^2 + \left(\frac{N_2}{A_1}\right)^2}$$

$$\left(\frac{S}{N}\right)^2 = \left(\frac{S}{N_1}\right)^2 \frac{1}{1 + \left(\frac{N_2}{A_1 N_1}\right)^2}$$

The overall sign-to-noise ratio is reduced, but the noise contribution from the second-stage can be negligible, provided the gain of the first stage is sufficiently high.

Þ In a well-designed system the noise is dominated by the first gain stage.

Amplifier Noise Model

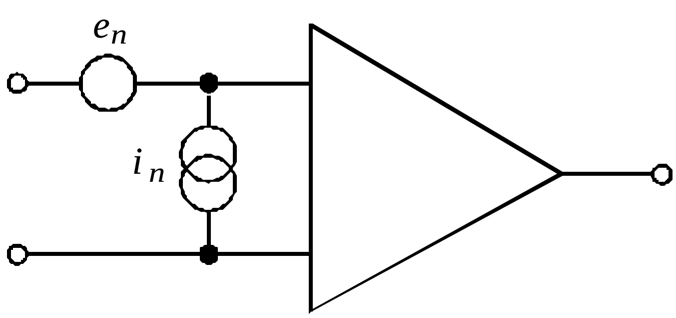
The noise properties of any amplifier can be described fully in terms of a

- voltage noise source

  and

- current noise source.

at the amplifier input. Typical magnitudes are $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and $\mathrm{pA}/\sqrt{\mathrm{Hz}}$.

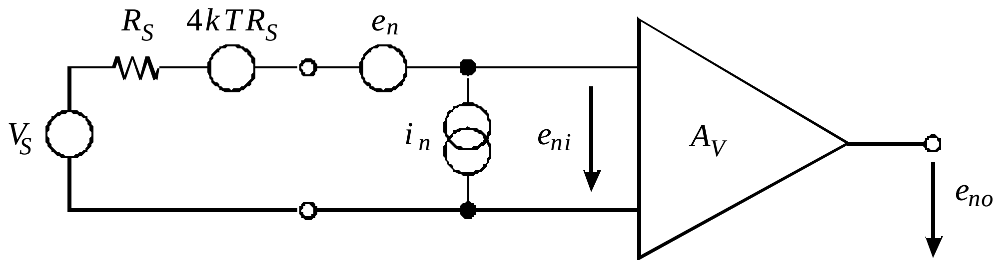


Here the magnitude of the noise sources is characterized by the spectral density

The noise sources do not have to physically present at the input. Noise also originates within the amplifier. Assume that at the output the combined contribution of all internal noise sources has the spectral density e_{no} . If the amplifier has a voltage gain A_V , this is equivalent to a voltage noise source at the input $e_n = e_{no} / A_V$.

It is convenient to express the input noise in terms of spectral density, so that the effect of amplifier bandwidth can be assessed separately.

Assume that a sensor with resistance R_S is connected to an amplifier with voltage gain A_V and an infinite input resistance, so no current flows into the amplifier.



The input noise current i_n flows through the source resistance R_S to yield a noise voltage $i_n R_S$, which adds to the thermal noise of the source resistance and the noise voltage of the amplifier.

All terms add in quadrature, since they are not correlated.

The total noise voltage at the input of the amplifier

$$e_{ni}^2 = 4kTR_S + e_n^2 + \left(i_n R_S\right)^2$$

and at the output of the amplifier

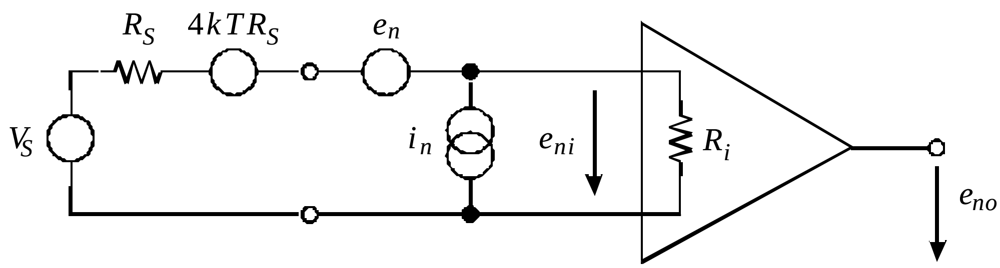
$$e_{no}^2 = (A_V e_{ni})^2 = A_V^2 \left[4kTR_S + e_n^2 + \left(i_n R_S\right)^2\right]$$

The signal-to-noise ratio at the amplifier output

$$\left(\frac{S}{N}\right)^2 = \frac{A_V^2 V_S^2}{A_V^2 \left[4kTR_S + e_n^2 + \left(i_n R_S\right)^2\right]}$$

is independent of the amplifier gain and equal to the input S/N, as both the input noise and the signal are amplified by the same amount.

In the preceding example the amplifier had an infinite input resistance, so no current flowed into the amplifier. Is the signal-to-noise ratio affected by a finite input resistance?



The signal at the input of the amplifier

$$V_{Si} = V_S \frac{R_i}{R_S + R_i}$$

The noise voltage at the input of the amplifier

$$e_{ni}^2 = \left(4kTR_S + e_n^2\right)\left(\frac{R_i}{R_i + R_S}\right)^2 + i_n^2\left(\frac{R_i R_S}{R_i + R_S}\right)^2$$

where the bracket in the i_n^2 represents the parallel combination of R_i and R_S. The signal-to-noise ratio at the output of the amplifier

$$\left(\frac{S}{N}\right)^2 = \frac{A_V^2 V_{Si}^2}{A_V^2 e_{ni}^2} = \frac{V_S^2\left(\frac{R_i}{R_S + R_i}\right)^2}{\left(4kTR_S + e_n^2\right)\left(\frac{R_i}{R_i + R_S}\right)^2 + i_n^2\left(\frac{R_i R_S}{R_i + R_S}\right)^2}$$

$$\left(\frac{S}{N}\right)^2 = \frac{V_S^2}{\left(4kTR_S + e_n^2\right) + i_n^2 R_S^2}\ ,$$

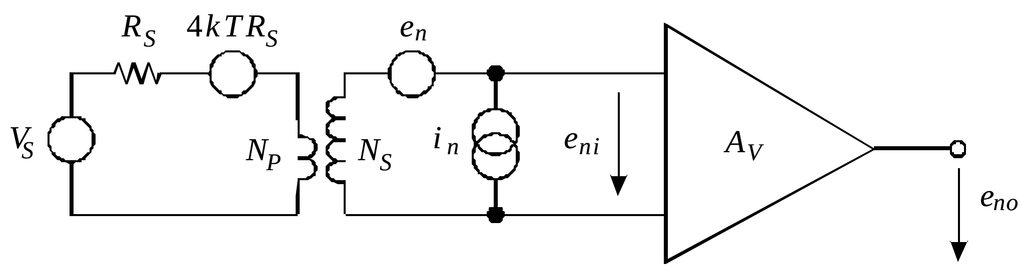
the same as for an infinite input resistance.

This result also hold for a complex input impedance, i.e. a combination of resistive and capacitive or inductive components.

Þ S/N independent of amplifier input impedance.

Noise matching with transformers

The sensor is coupled to the amplifier through a transformer with the turns ratio $N = N_S / N_P$.



Assume unity coupling in the transformer. Then the sensor voltage appearing at the secondary

$$V_{SS} = NV_S$$

The thermal noise of the sensor at the secondary

$$e_{nSS}^2 = N^2 4kTR_S$$

Because the transformer also converts impedances, the source resistance appears at the secondary as

$$R_{SS} = N^2 R_S$$

Thus, the signal is increased, but so is the noise contribution due to the input noise current.

$$e_{ni}^2 = 4kTR_S N^2 + e_n^2 + R_S^2 N^4 i_n^2$$

and the signal-to-noise ratio

$$\left(\frac{S}{N}\right)^2 = \frac{V_S^2 N^2}{4kTR_S N^2 + e_n^2 + R_S^2 N^4 i_n^2} = \frac{V_S^2}{4kTR_S + \dfrac{e_n^2}{N^2} + N^2 R_S^2 i_n^2} ,$$

which attains a maximum for

$$R_S N^2 = \frac{e_n}{i_n}$$

Correlated Noise

The noise sources can be correlated, for example

$$e_n^2 = e_{n1}^2 + e_{n2}^2 + 2Ce_{n1}e_{n2}$$

where C is the correlation coefficient. C can range from –1 to +1 (anti-correlated to fully correlated).

If $C = 0$ the noise components are uncorrelated and they simply add in quadrature.

Thus, in the above example, if the input noise voltage and current are correlated, the input noise voltage

$$e_{ni}^2 = 4kTR_S + e_n^2 + i_n^2 + 2Ce_n i_n R_S$$

The total noise at the output is obtained by integrating over the spectral noise power $P_n(f) \propto e_{no}^2(f)$.

The frequency distribution of the noise is determined both by the spectral distribution of the input noise voltage and current and by the frequency response of the amplifier.

$$v_{no}^2 = \int_0^\infty e_{no}^2(f)\,df = \int_0^\infty e_{ni}^2(f)\left|A_V\right|^2 df$$

The amplifier gain factor is shown as magnitude squared, as in general the amplifier has a frequency-dependent gain and phase, so it is a complex number.

Noise Bandwidth vs. Signal Bandwidth

Consider an amplifier with the frequency response *A(f)*. This can be rewritten

$$A(f) \equiv A_0 G(f)$$

where A_0 is the maximum gain and $G(f)$ describes the frequency response.

For example, in the simple amplifier described above

$$A_V = g_m \left(\frac{1}{R_L} + iw\ C_o \right)^{-1} = g_m R_L \ \frac{1}{1 + iwR_L C_o}$$

and using the above convention

$$A_0 \equiv g_m R_L \quad \text{and} \quad G(f) \equiv \frac{1}{1 + i\ (2pf\ R_L C_o)}$$

If a “white” noise source with spectral density e_{ni} is present at the input, the total noise voltage at the output is

$$V_{no} = \sqrt{\int_0^\infty e_{ni}^2 \left| A_0 G(f) \right|^2 df} = e_{ni}\ A_0 \sqrt{\int_0^\infty G^2(f) df} \equiv e_{ni}\ A_0 \sqrt{\Delta f_n}$$

Df_n is the “noise bandwidth”.

Note that, in general, the noise bandwidth and the signal bandwidth are not the same. If the upper cutoff frequency is determined by a single *RC* time constant, as in the “simple amplifier”, the signal bandwidth

$$\Delta f_s = f_u = \frac{1}{2p\ RC}$$

and the noise bandwidth

$$\Delta f_n = \frac{1}{4RC} = \frac{p}{2} f_u$$

Equivalent Noise Charge

A detector readout systems that measure signal charge can be characterized in terms of Equivalent Noise Charge,

Equivalent Noise Charge (ENC) =
= the signal charge that yields a signal-to-noise ratio of one.

For a given detector material, the signal charge can be translated into absorber energy, so the noise can be express in terms of energy, i.e. eV or keV.

For an ionization energy e_i

$$\Delta E_n = e_i \cdot ENC$$

The relationship between ENC and basic amplifier noise parameters will be derived in the next chapter.

Continuous Signals vs. Individual Pulses

Consider a semiconductor detector detecting visible light.

At low intensities, where the mean time between successive photons is much longer than the collection time, the detector signal consists of individual pulses.

As the light intensity increases, so does the photon rate. At some point the signals from individual photons overlap and the detector output appears as a continuous current.

The average current of a sequence of pulses $i(t)$ of duration T occurring at a rate R

$$i_{av} = R\int i(t)\,dt$$

If each individual pulse has a DC component, the DC component of the pulse train will grow as the rate increases.

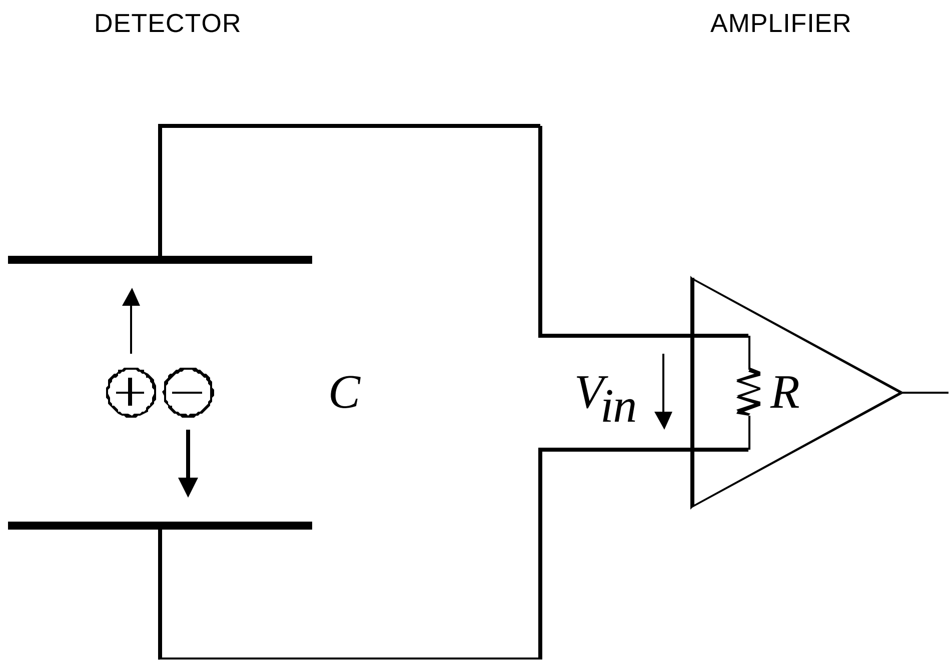
Each individual pulse has a characteristic Fourier spectrum. Since this is a linear superposition process, the sum of all pulses has the same frequency spectrum as an individual pulse.

⇒ signal-to-noise can be analyzed using either pulses or continuous signals.

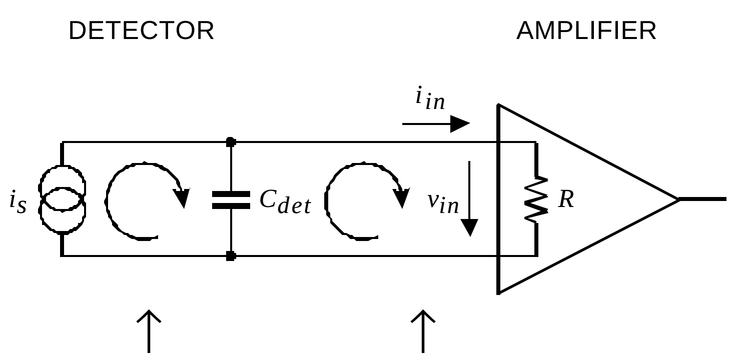
If a filter is chosen to optimize the signal-to-noise ratio for a single pulse, it will also optimize S/N at high rates.

However, the need to resolve individual pulses or measure their amplitude accurately adds an additional constraint that modifies the choice of filter at high rates.

S/N with Capacitive Signal Sources



Equivalent Circuit



charges moving in detector induce change of charge on detector electrodes

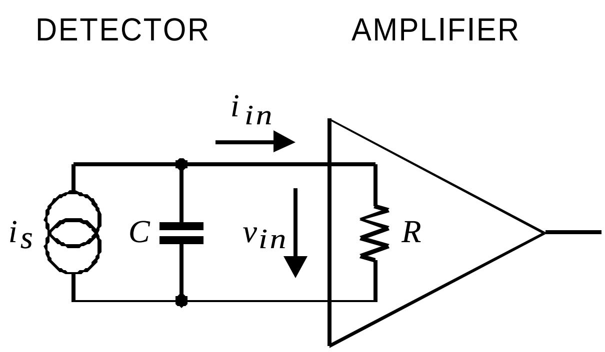
detector capacitance discharges into amplifier

Assume an amplifier with constant noise. Then signal-to-noise ratio (and the equivalent noise charge) depend on the signal magnitude.

Pulse shape registered by amplifier depends on the input time constant RC_{det}.

Assume a rectangular detector current pulse of duration T and magnitude I_s.

Equivalent circuit

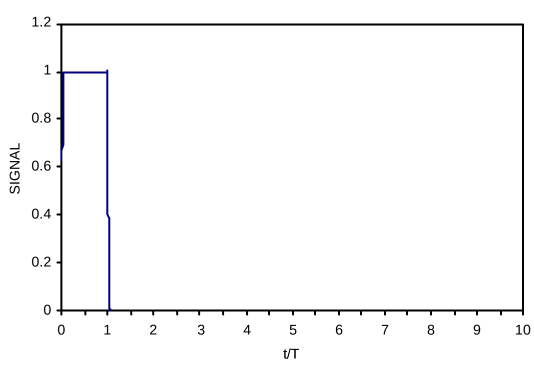


Input current to amplifier

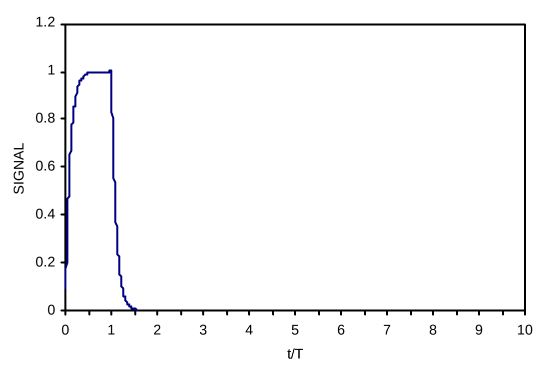
$$0 \le t < T: \qquad i_{in}(t) = I_S\left(1 - e^{-t/RC}\right)$$
$$T \le t \le \infty: \qquad i_{in}(t) = I_S\left(e^{T/RC} - 1\right) \cdot e^{-t/RC}$$

At short time constants $RC << T$ the amplifier pulse approximately follows the detector current pulse.

$RC = 0.01\ T$

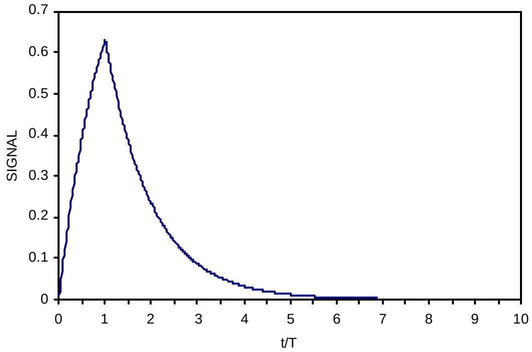


$RC = 0.1\ T$

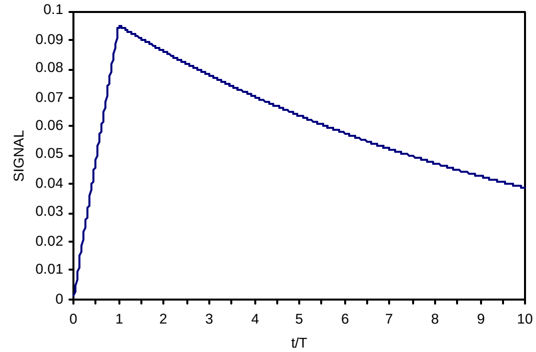


As the input time constant RC increases, the amplifier signal becomes longer and the peak amplitude decreases, although the integral, i.e. the signal charge, remains the same.

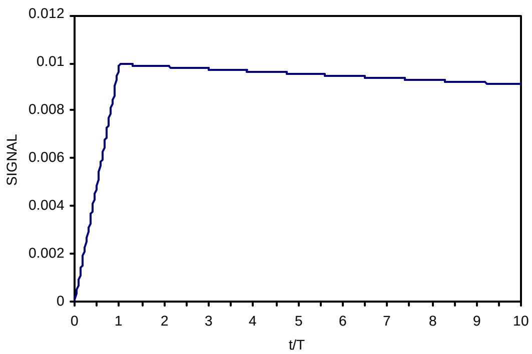
$RC = T$



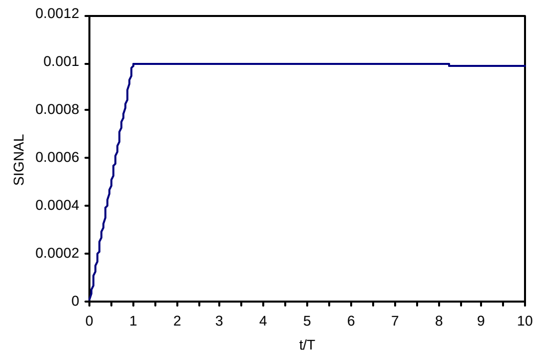
$RC = 10\ T$



$RC = 100\ T$



$RC = 10^3\ T$



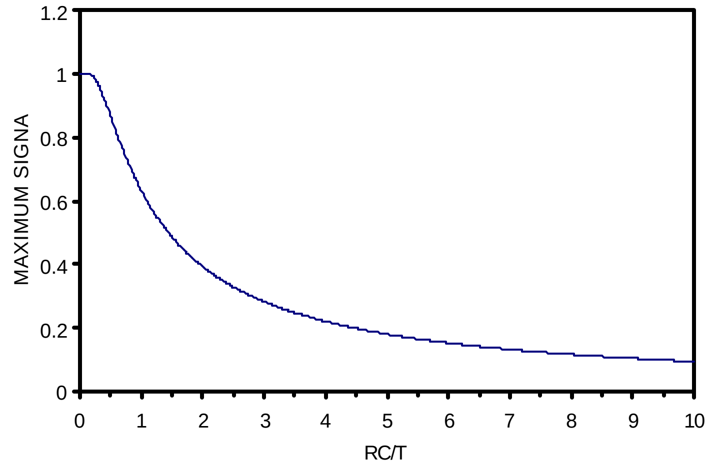
At long time constants the detector signal current is integrated on the detector capacitance and the resulting voltage sensed by the amplifier

$$V_{in} = \frac{Q_{\det}}{C} = \frac{\int i_s dt}{C}$$

Then the peak amplifier signal is inversely proportional to the **total capacitance at the input**, i.e. the sum of

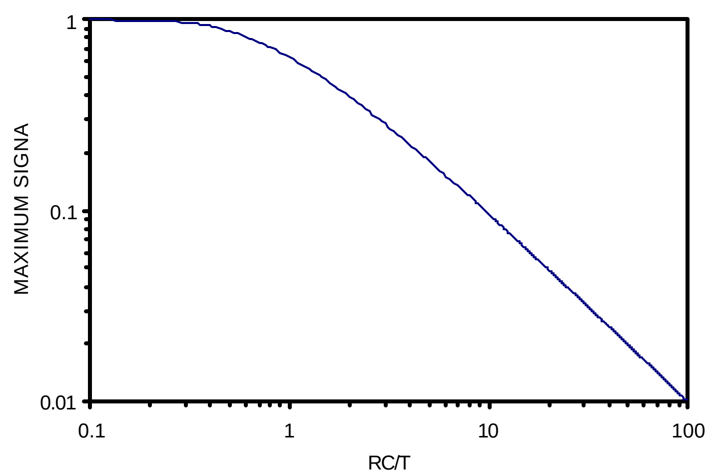
detector capacitance,
input capacitance of the amplifier, and
stray capacitances.

Maximum signal vs. capacitance



At small time constants the amplifier signal approximates the detector current pulse and is independent of capacitance.

At large input time constants (RC/T > 5) the maximum signal falls linearly with capacitance.



Þ For input time constants large compared to the detector pulse duration the signal-to-noise ratio decreases with detector capacitance.

Caution when extrapolating to smaller capacitances:

If S/N = 1 at RC/T = 100, decreasing the capacitance to 1/10 of its original value (RC/T = 10), increases S/N to 10.
However, if initially RC/T = 1, the same 10-fold reduction in capacitance (to RC/T = 0.1) only yields S/N = 1.6.

Charge-Sensitive Preamplifier
Noise vs. Detector Capacitance

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,

  i.e. the *equivalent input noise voltage* $v_{ni} = v_{no} / A_v$.

- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if $C_i >> C_{det}$).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value $v_{no} = v_{ni} A_{v0}$.

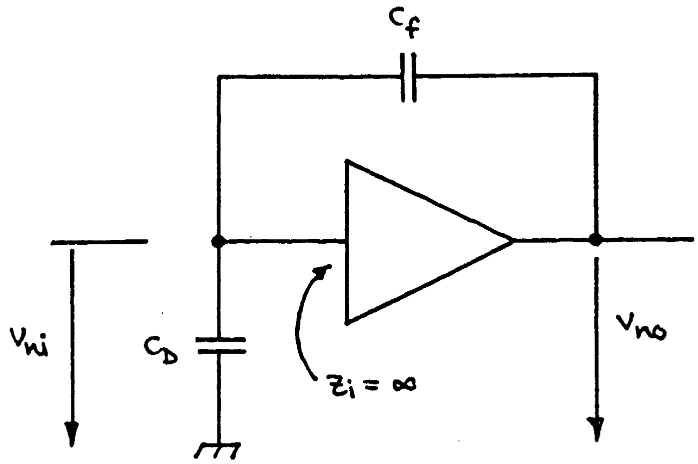
- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

Note, that although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier. Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input, to which the loop responds in the same manner as to a detector signal.

⇒ **S/N at the amplifier output depends on feedback.**

Noise in charge-sensitive preamplifiers

Start with an output noise voltage v_{no}, which is fed back to the input through the capacitive voltage divider $C_f - C_d$.



$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_D}}{X_{C_D}} = v_{ni} \frac{\dfrac{1}{w\ C_f} + \dfrac{1}{w\ C_D}}{\dfrac{1}{w\ C_D}}$$

$$v_{no} = v_{ni} \left(1 + \frac{C_D}{C_f} \right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

$$Q_{ni} = v_{ni} \left(C_D + C_f \right)$$

Signal-to-noise ratio

$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni}(C_D + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage-sensitive amplifier, but here

- *the signal is constant and*
- *the noise grows with increasing C.*

As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load C, because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with C just as for the charge-sensitive amplifier.

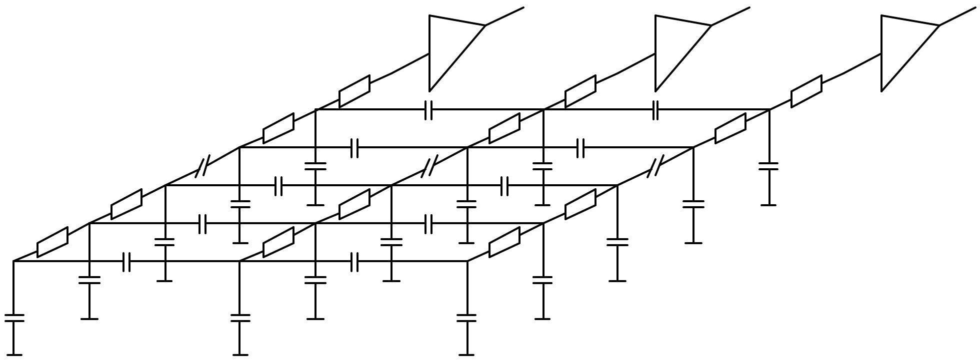
Conclusion

In general

- optimum S/N is independent of whether the voltage, current, or charge signal is sensed.
- S/N cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

Strip Detector Model for Noise Simulations



Noise coupled from neighbor channels.

Analyze signal and noise in center channel.

Includes: Noise contributions from neighbor channels
Signal transfer to neighbor channels
Noise from distributed strip resistance
Full SPICE model of preamplifier

Measured Noise of Module

Simulation Results:

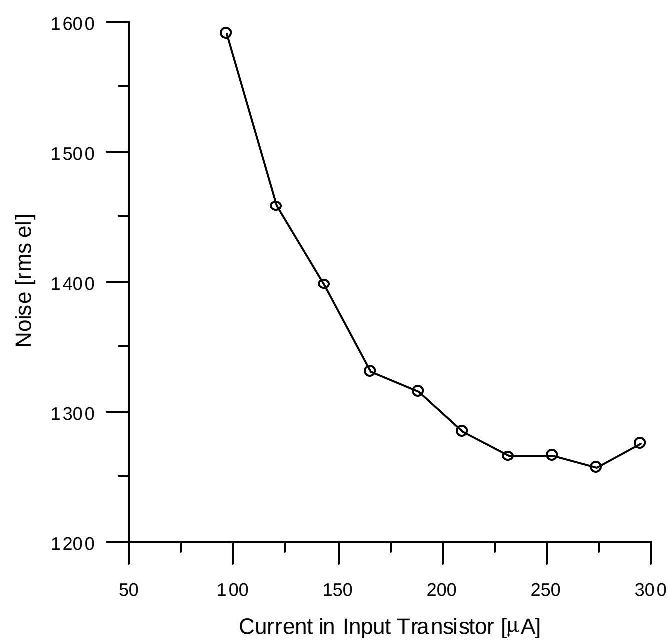
p-strips on n-bulk
BJT input transistor
(see Part V)

1460 el (150 μA)

1230 el (300 μA)

⇒

Noise can be predicted
with good accuracy.



Quantum Noise Limits in Amplifiers

What is the lower limit to electronic noise?

Can it be eliminated altogether, for example by using superconductors and eliminating devices that carry shot noise?

Starting point is the uncertainty relationship

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Consider a narrow frequency band at frequency w. The energy uncertainty can be given in terms of the uncertainty in the number of signal quanta

$$\Delta E = \hbar w \Delta n$$

and the time uncertainty in terms of phase

$$\Delta t = \frac{\Delta j}{w},$$

so that

$$\Delta j \Delta n \geq \frac{1}{2}$$

We assume that the distributions in number and phase are Gaussian, so that the equality holds.

Assume a noiseless amplifier with gain G, so that n_1 quanta at the input yield

$$n_2 = G n_1$$

quanta at the output.

Furthermore, the phase at the output j_2 is shifted by a constant relative to the input.

Then the output must also obey the relationship

$$\Delta j_2 \Delta n_2 = \frac{1}{2}$$

However, since $\Delta n_2 = G \Delta n_1$ and $\Delta j_2 = \Delta j_1$

$$\Delta j_1 \Delta n_1 = \frac{1}{2G} \ ,$$

which is smaller than allowed by the uncertainty principle.

This contradiction can only be avoided by assuming that the amplifier introduces noise per unit bandwidth of

$$\frac{dP_{no}}{dw} = (G-1)\hbar w \ ,$$

which, referred to the input, is

$$\frac{dP_{ni}}{dw} = \left(1 - \frac{1}{G}\right)\hbar w$$

If the noise from the following gain stages is to be small, the gain of the first stage must be large, and then the minimum noise of the amplifier

$$\frac{dP_{ni}}{dw} = \hbar w$$

At 2 mm wavelength the minimum noise corresponds to about 7K.

This minimum noise limit applies to phase-coherent systems. In systems where the phase information is lost, e.g. bolometers, this limit does not apply.

For a detailed discussion see
C.M. Caves, Phys. Rev. D **26** (1982) 1817-1839