II. Signal Formation and Detection Thresholds

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1. Detector Models

We consider detectors that provide electrical signal outputs.

To extract the amplitude or timing information the electrical signal is coupled to an amplifier, sent through gain and filtering stages, and finally digitized to allow data storage and analysis.

Optimal signal processing depends on the primary signal.

The signal can be

1. a continuously varying signal
2. a sequence of pulses, occurring
   - periodically
   - at known times
   - randomly

All of these affect the choice of signal processing techniques.

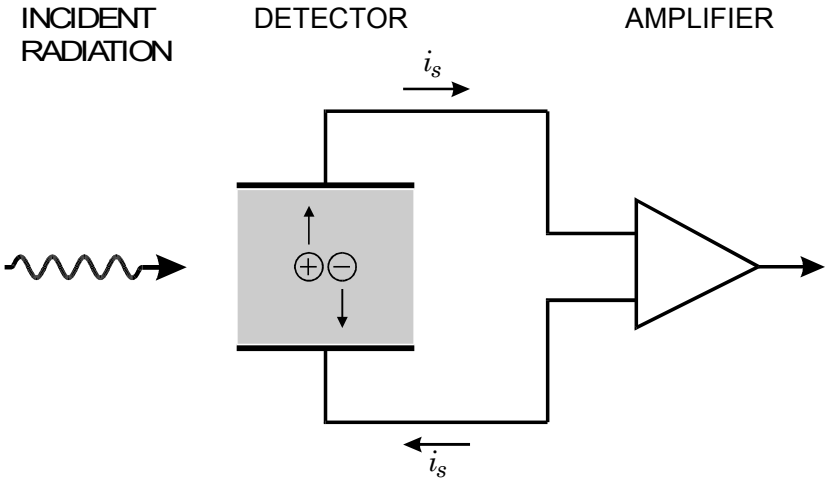
First steps in signal processing:
- Formation of the signal in the detector (sensor)
- Coupling the sensor to the amplifier

Detectors use either
- direct detection or
- indirect detection

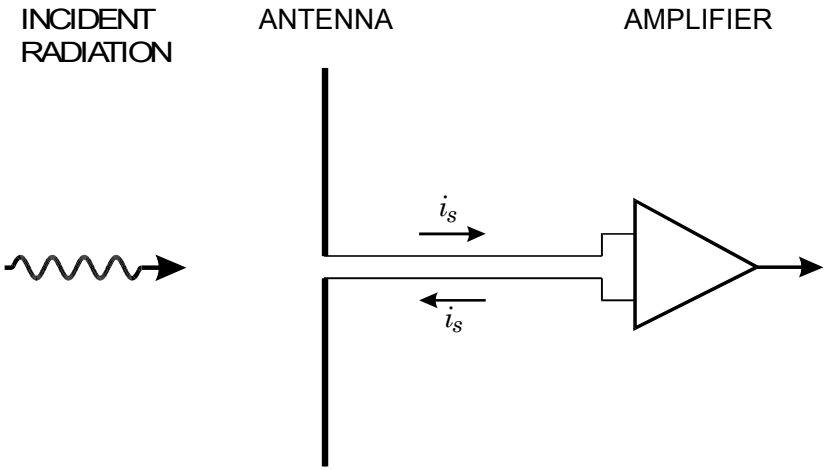
Examples:

1. Direct Detection

a) ionization chamber
(>eV photons, charged particles)

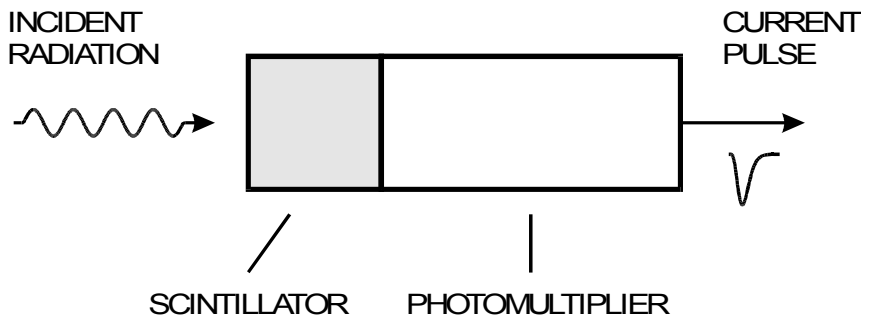


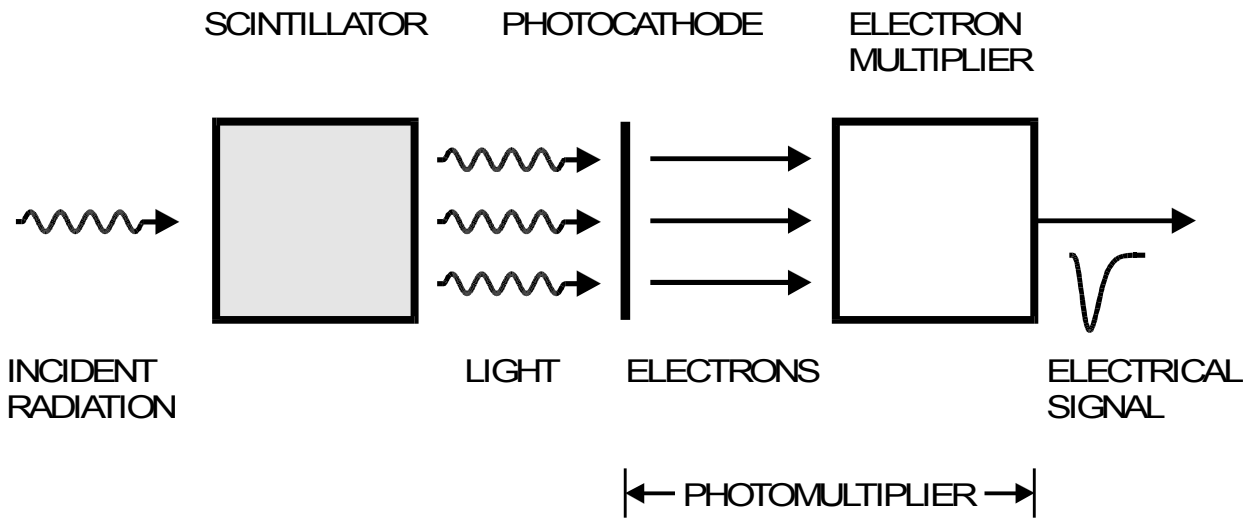
b) RF measurement (kHz ... THz), e.g. CMB



2. Indirect Detection

a) scintillation detector

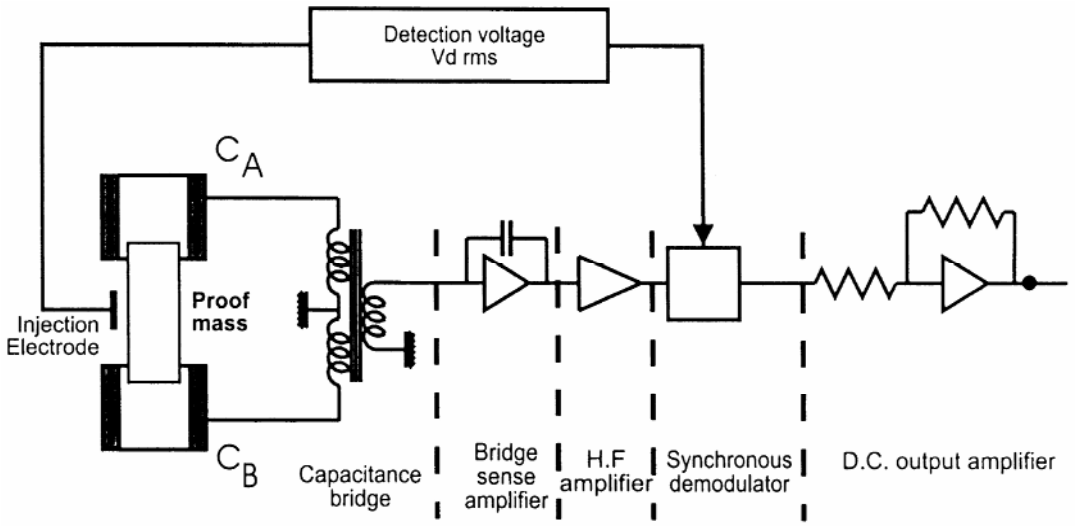




b) gravity wave detector

Motion of proof mass measured by capacitive sensor.

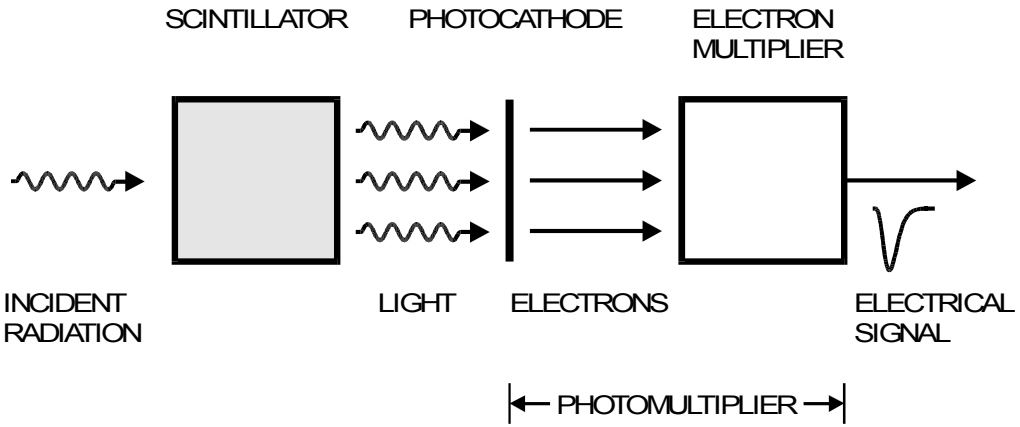
Schematic of LISA position sensor:



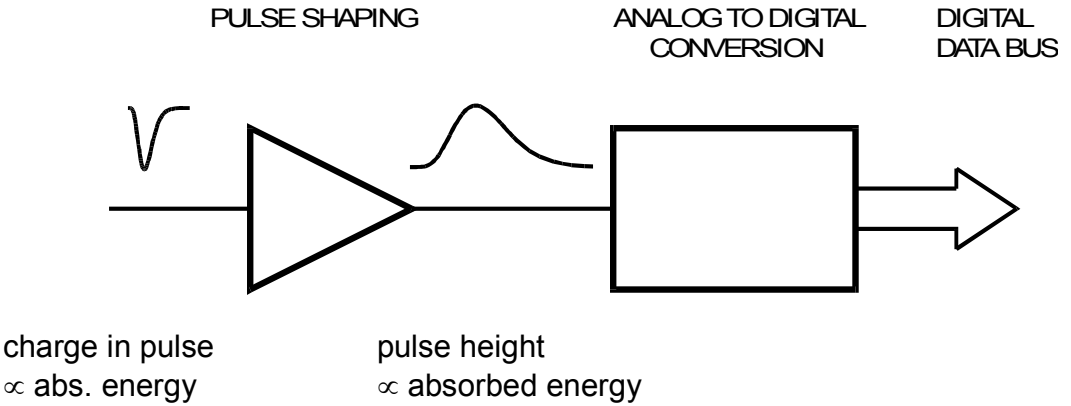
Capacitive sensors readily achieve sensitivities <100 pm.

Detector Functions

Processes in Scintillator – Photomultiplier



Signal Processing

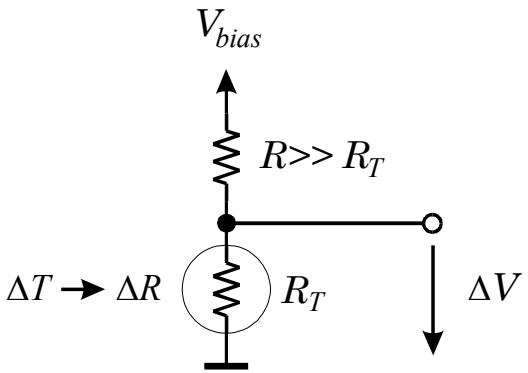


Example Detector Models

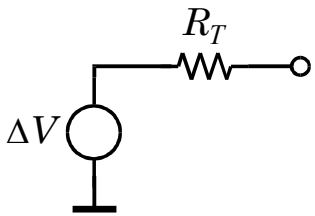
Although detectors take on many different forms, one can analyze the coupling to the amplifier with simple models.

1. Thermistor detecting IR radiation

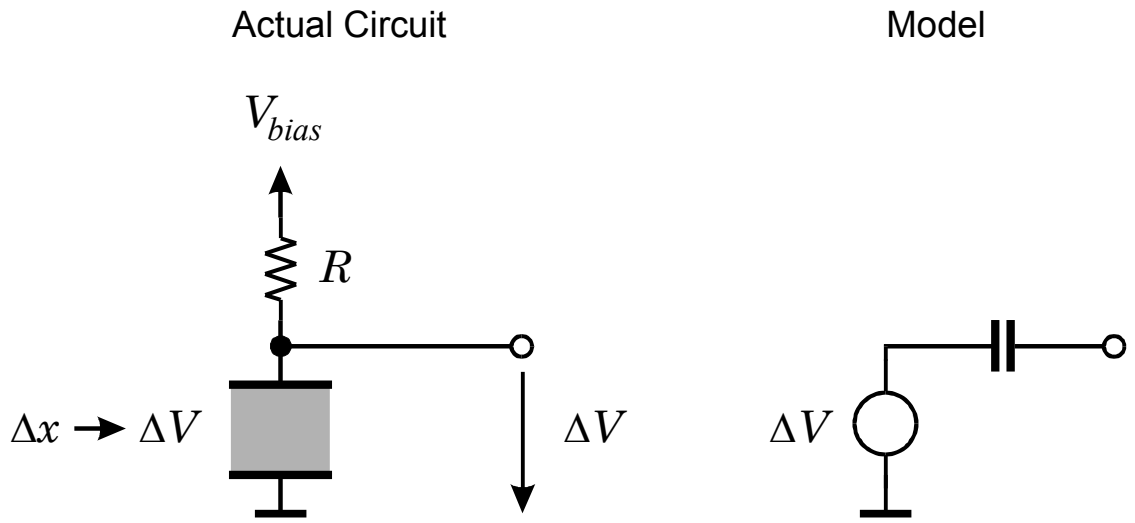
Actual Circuit



Model

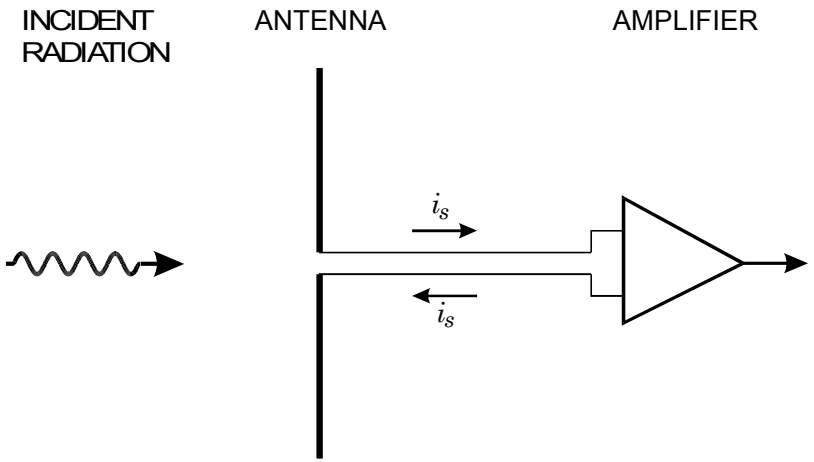


2. Piezoelectric Transducer



3. Radio Antenna

To derive the equivalent circuit apply the reciprocity principle, i.e. analyze the antenna as a radiator driven by an RF generator.



Radiated field:

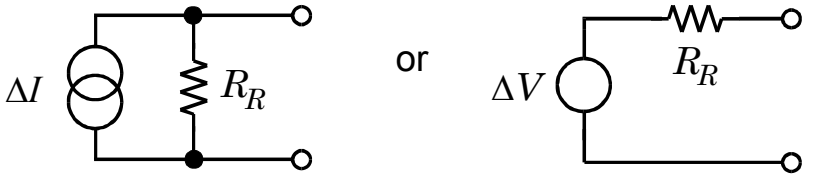
For simplicity assume the dipole length is a half wavelength λ/2.
The angular distribution of the radiated power (see J.D. Jackson, Classical Electrodynamics)

$$\frac{dP}{d\Omega} = \frac{I^2}{2\pi c} \frac{\cos^2\left(\frac{\pi}{2}\cos\Theta\right)}{\sin^2\Theta}$$

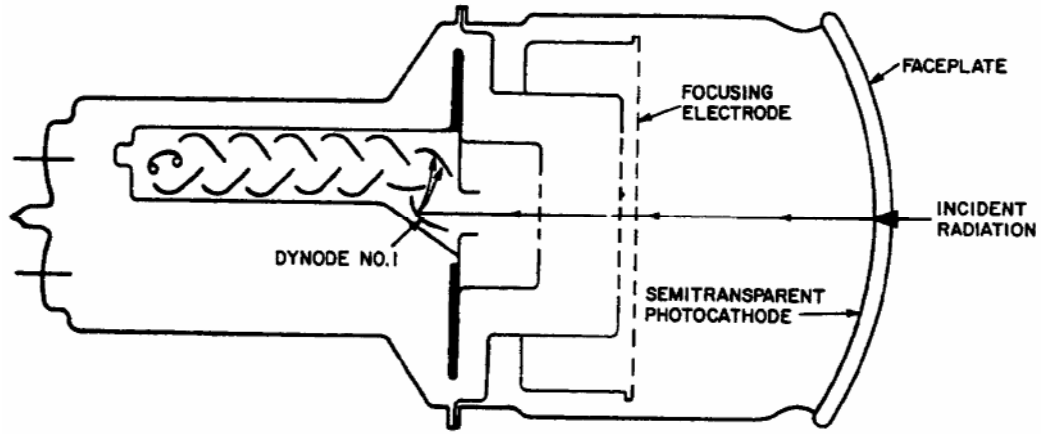
At the feed point the power fed to the dipole $P = I^2 R_R$

which is equal to the radiated power $P = \int_0^{2\pi} \frac{dP}{d\Omega} d\Omega$

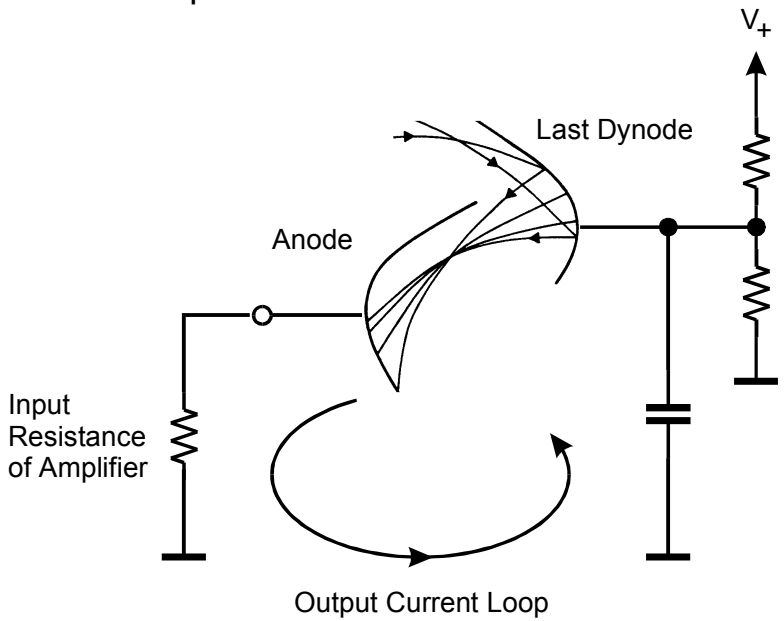
i.e. the dipole appears as a resistance, so the equivalent circuit



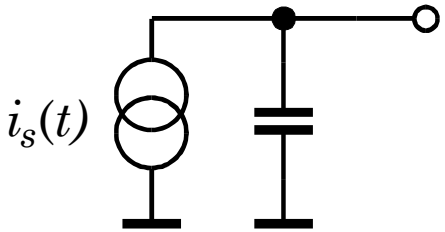
4. Photomultiplier Tube



Detail of output circuit



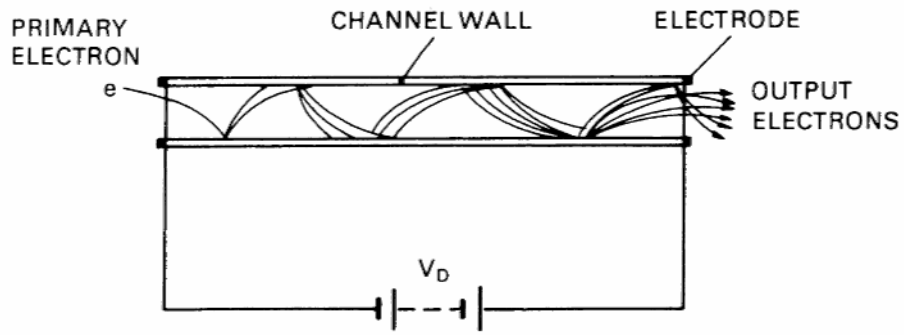
Model



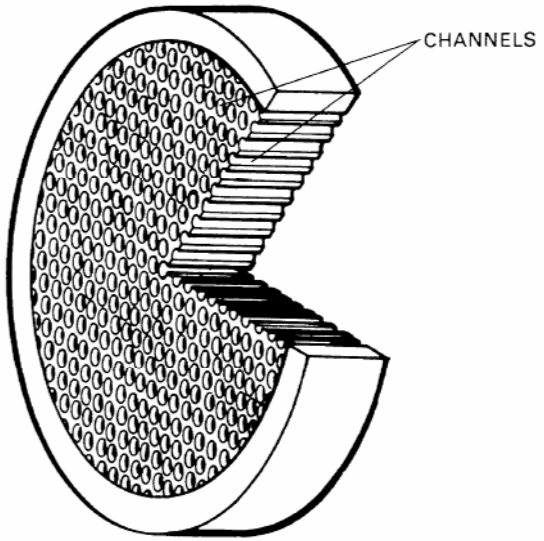
5. Channeltrons and Microchannel Plates

Channel electron multiplier

The inside of a glass capillary is coated with a secondary electron emitter that also forms a distributed resistance. Application of a voltage between the the two ends sets up a field, so that electrons in the structure are accelerated, strike the wall, and form secondaries.

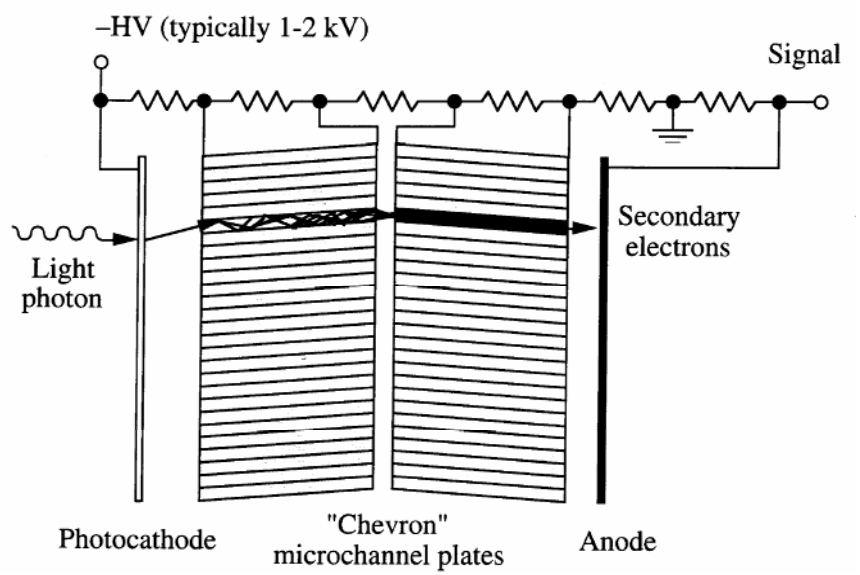


Channel electron multipliers are used individually ("channeltrons"), with tube diameters of ~1 mm, and in arrays called "micro-channel plates", which combine many small channels of order 10 μm diameter in the form of a plate. Microchannel plates are fabricated by stretching bundles of glass capillaries and then slicing the bundle to form 2 … 5 cm diameter plates of several hundred microns thickness.



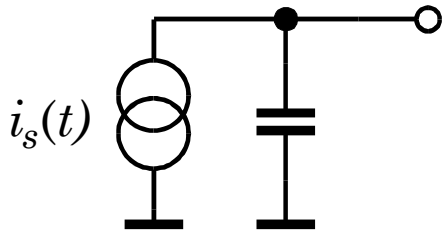
Microchannel plates are compact and fast. Transit time dispersion is < 1 ns due to the small dimensions of an individual channel. Pairs of microchannel plates can be combined to provide higher gain.

Connection scheme of a photon detector using microchannel plates



(from Derenzo)

Model

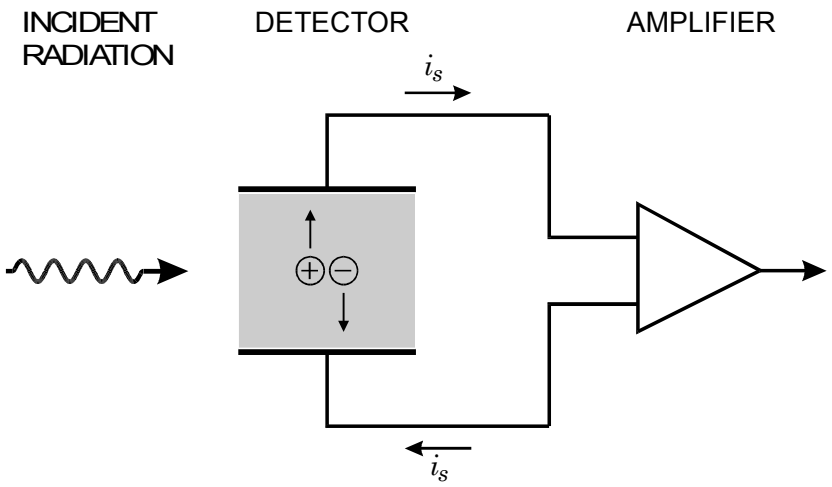


The shunt capacitor represents the capacitance between the exit face of the MCP and the anode.

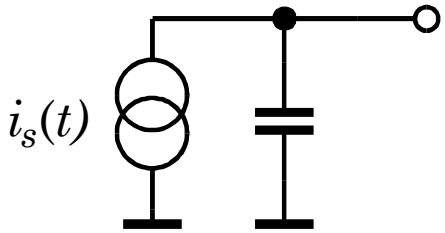
6. Ionization Chamber

semiconductor detectors (pad, strip, pixel electrodes)

gas-filled ionization or proportional chambers, ...



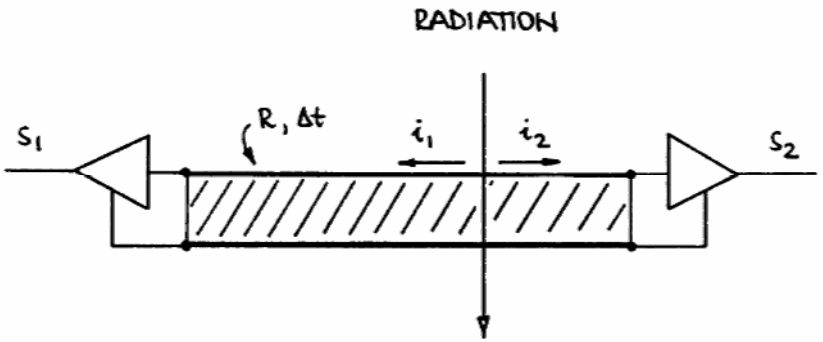
Model:



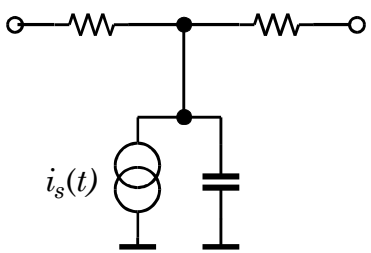
7. Position-Sensitive Detector with Resistive Charge Division

Electrode is made resistive with low-impedance amplifiers at each end. The signal current divides according to the ratio of resistances presented to current flow in the respective direction

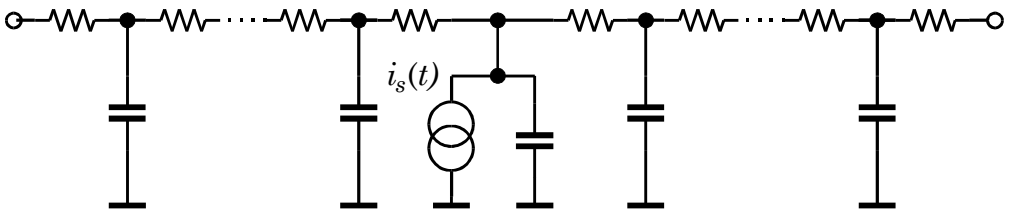
$$\frac{i_1(x)}{i_2(x)} = \frac{R_2(x)}{R_1(x)}$$



Simplest Model:



Depending on the speed of the amplifier, a more accurate model of the electrode includes the distributed capacitance:



2. The Signal

Any form of elementary excitation can be used to detect the radiation signal.

An electrical signal can be formed directly by ionization.

> Incident radiation quanta impart sufficient energy to individual atomic electrons to form electron-ion pairs (in gases) or electron-hole pairs (in semiconductors and metals).

Other detection mechanisms are

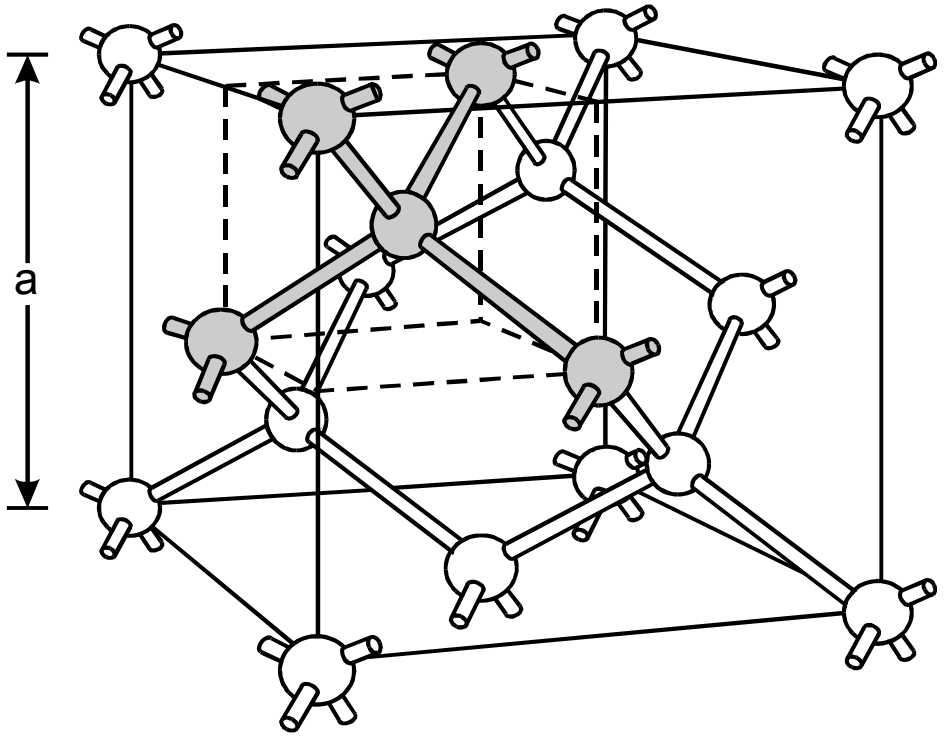
- Excitation of optical states (scintillators)
- Excitation of lattice vibrations (phonons)
- Breakup of Cooper pairs in superconductors
- Formation of superheated droplets in superfluid He

Typical excitation energies

Ionization in gases	~30 eV
Ionization in semiconductors	1 – 5 eV
Scintillation	~10 eV
Phonons	meV
Breakup of Cooper Pairs	meV

Band Structure in Crystals

Example: Lattice structure of diamond, Si, Ge (“diamond lattice”)

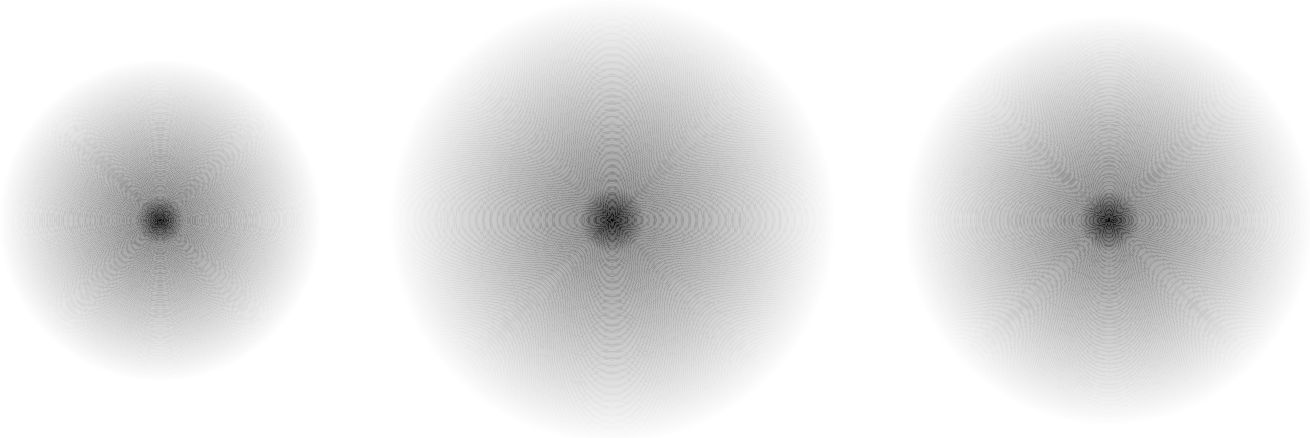


dimension a: lattice constant

Diamond:	3.56 Å
Ge:	5.65 Å
Si:	5.43 Å

Extent of wavefunctions of typical constituent atoms:

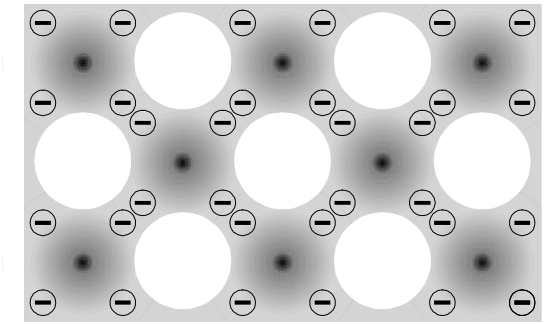
APPROXIMATE SCALE: ←1 Å→



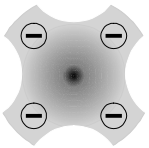
CARBON ($Z = 6$) SILICON ($Z = 14$) GERMANIUM ($Z = 32$)

(following Shockley)

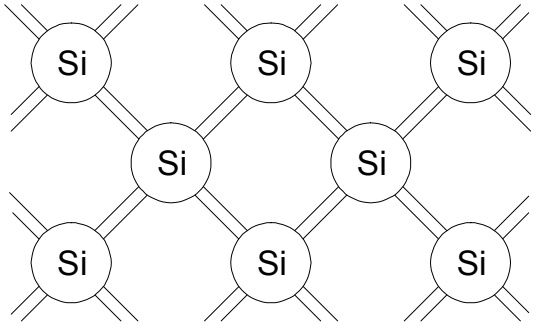
Crystal Bonds



SILICON “CORES” WITH ELECTRON “CLOUDS” SHOWING VALENCE PAIR BONDS

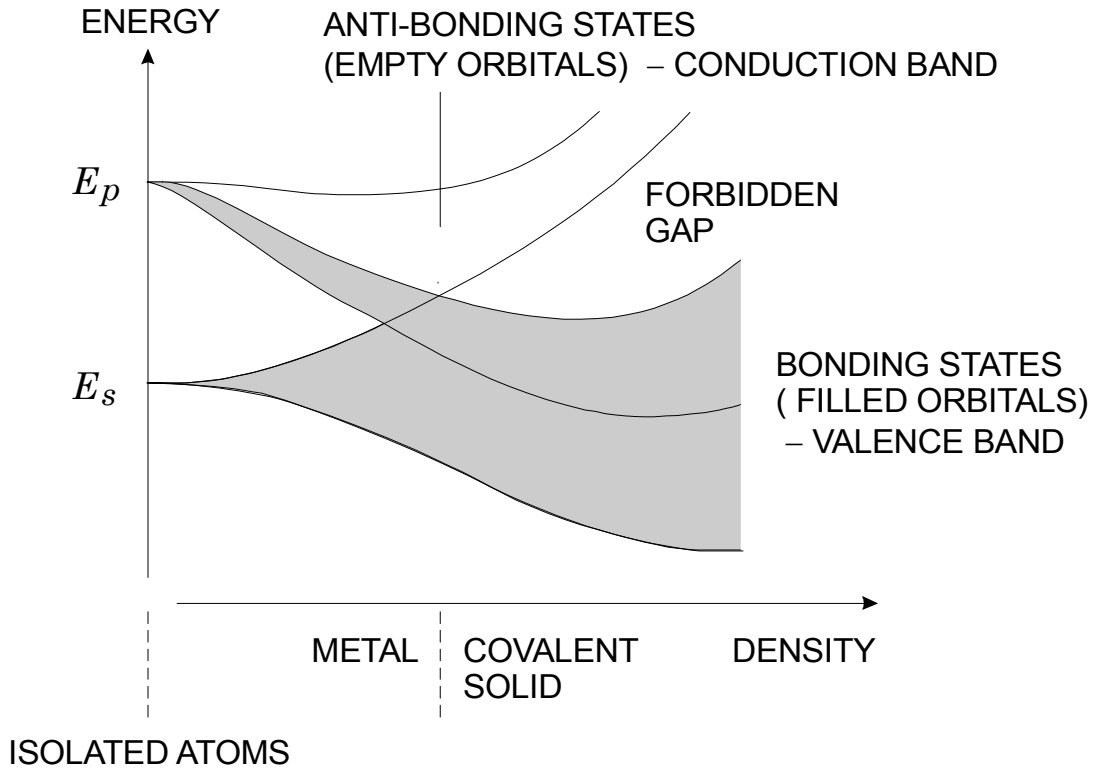


SILICON ATOM WITH FOUR VALENCE ELECTRONS



SYMBOLIC PLANE VIEW USING LINES TO REPRESENT BONDS

When isolated atoms are brought together to form a lattice, the discrete atomic states shift to form energy bands:



Filled band formed by bonding states: $\Psi \backeq \Psi_a + \Psi_a$

(Ψ_a = wavefunction of individual atom)

Empty band formed by anti-bonding states: $\Psi \backeq \Psi_a - \Psi_a$

(vanishing occupancy at mid-point between atoms)

Each atom in the lattice contributes its quantum states to each band:

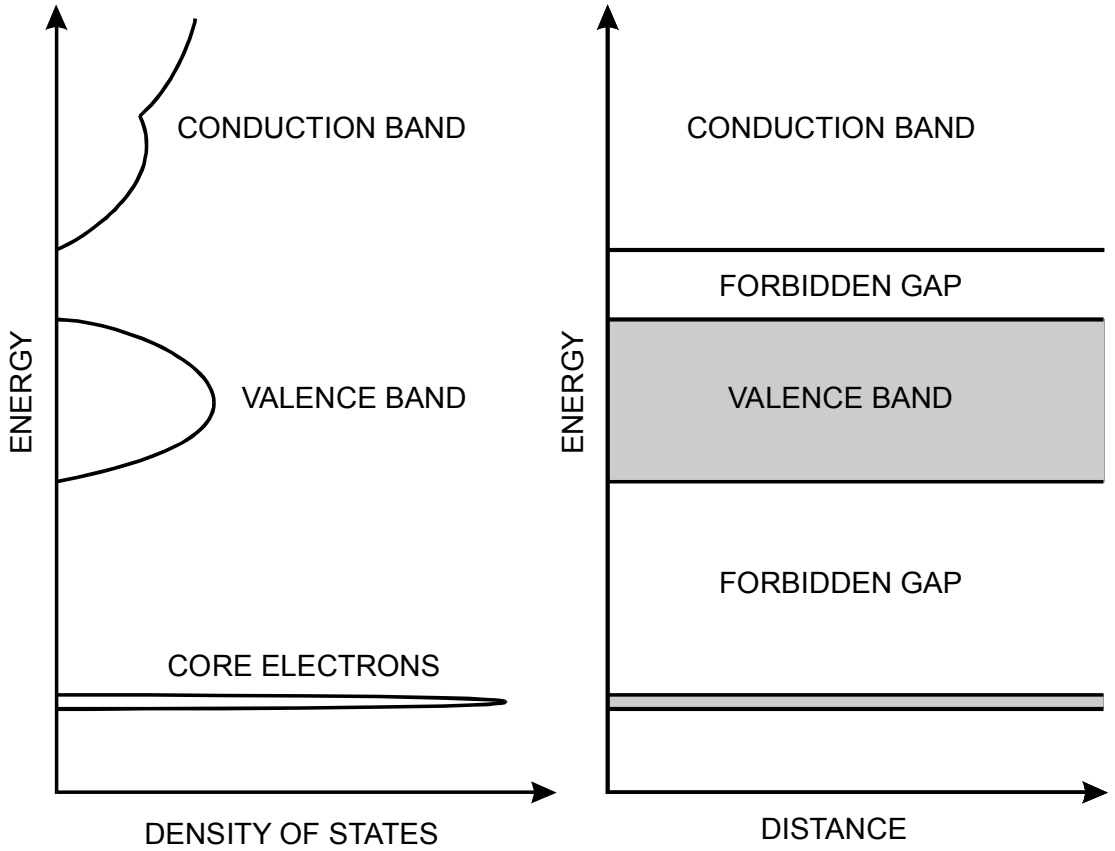
The number of quantum states in the band is equal to the number of states from which the band was formed.

The bands are extended states, i.e. the state contributed by an individual atom extends throughout the crystal.

Energy band structure

Typical band gaps
(valence – conduction band)

Ge	0.7 eV
GaAs	1.4 eV
Si	1.1 eV
Diamond	5.5 eV



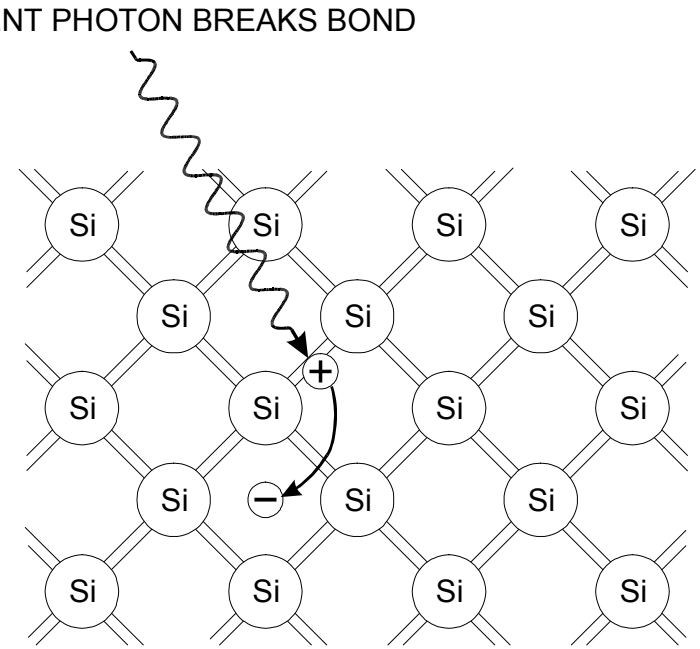
(following Shockley)

At 0K all electrons occupy bonding states, completely filling the valence band.

If an electric field is applied to the crystal, no current can flow, as this requires that the electrons acquire energy, which they can't, as no higher energy states are available in the valence band.

If energy is imparted to a bond by incident radiation, for example a photon, the bond can be broken,

- exciting an electron into the conduction band and
- leaving back a vacant state in the valence band, a "hole".



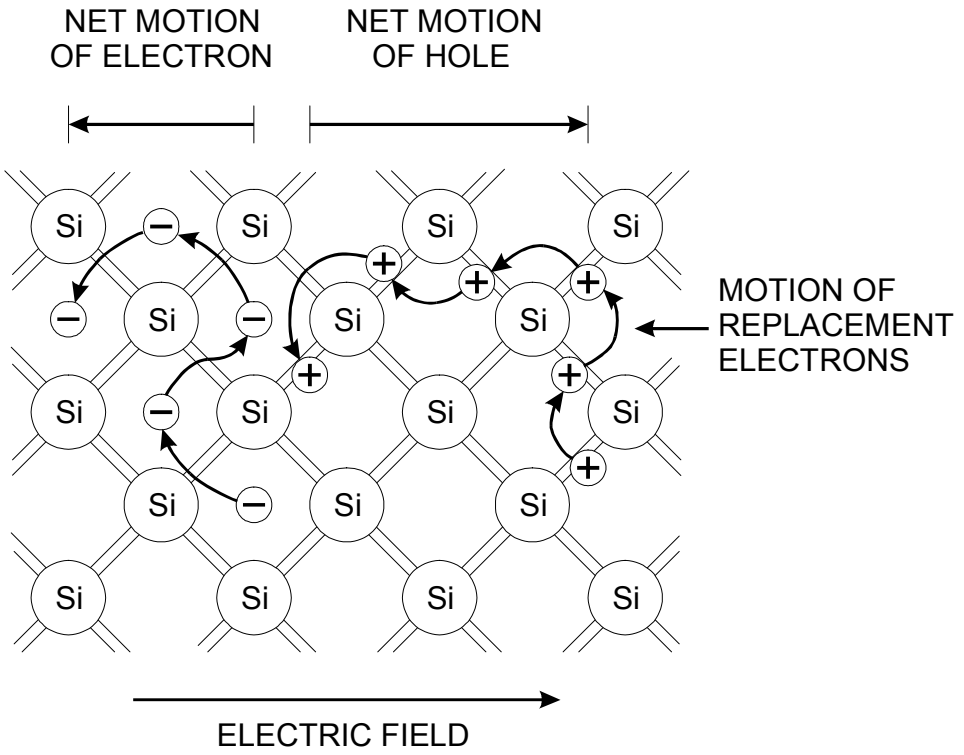
The electron can move freely in its extended state.

The hole can be filled by an electron from a nearby atom, thereby moving to another position.

The motion of the electron and hole can be directed by an electric field.

Holes can be treated as positive charge carriers just like the electrons

However, they tend to move more slowly as hole transport involves sequential transition probabilities (the wavefunction overlap of the hole and its replacement electron).



Ionization energy in solids is proportional to the band gap

small band gap ⇒ ~ conductor

electric field small

DC current >> signal current

large band gap ⇒ insulator

high electric field

small signal charge
+ small DC current

example: diamond

moderate band gap ⇒ semiconductor

high electric field

"large" signal charge
small DC current, but
"*pn*-junction" required.

examples: Si, Ge, GaAs

Although phonons have been represented as a penalty that increases the ionization energy, as mentioned above they are another form of elementary excitation that can be used to measure the signal.

More about this in the final lecture.

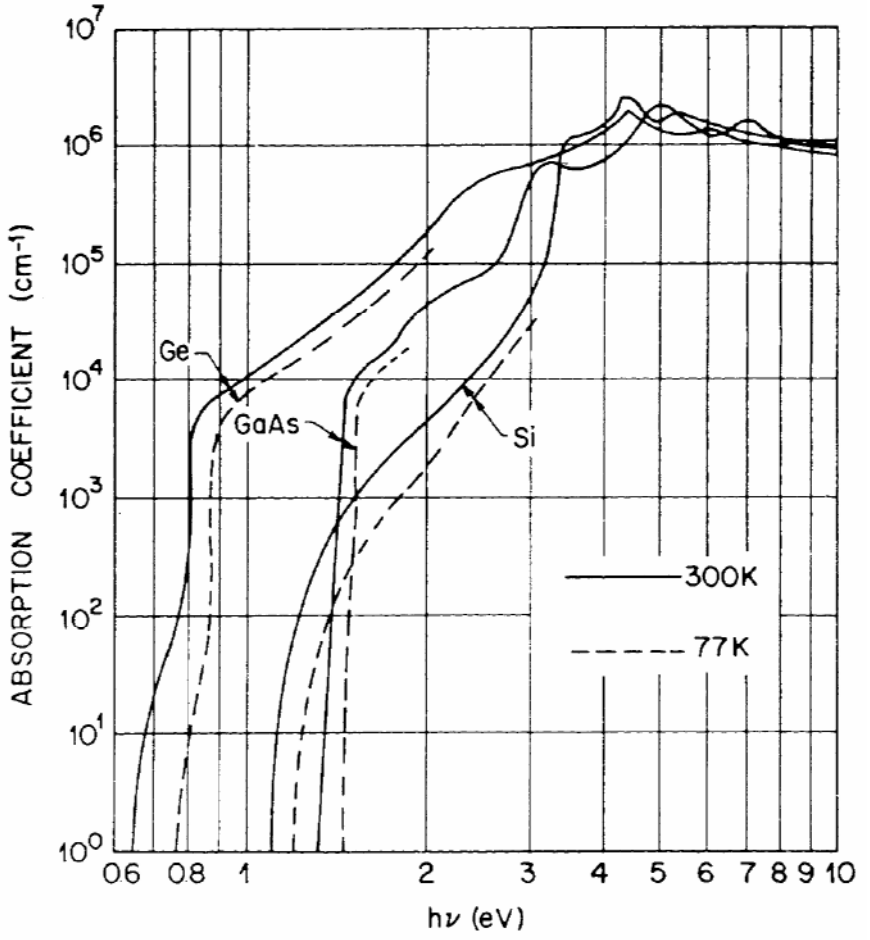
Detector Sensitivity

Example: Ionization signal in semiconductor detectors

a) visible light (energies near band gap)

Detection threshold = energy required to produce an electron-hole pair ≈ band gap

In indirect bandgap semiconductors (Si), additional momentum required:
provided by phonons



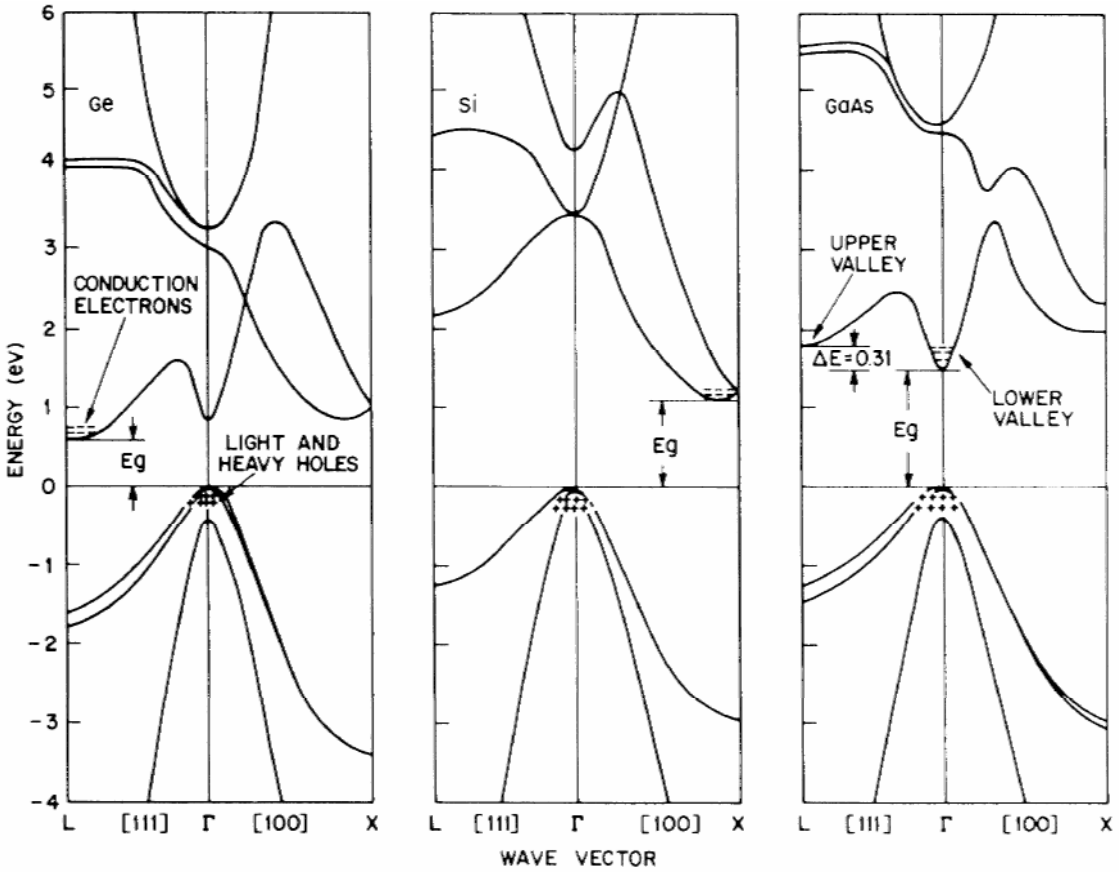
(from Sze)

Band Structure

Energy of the conduction and valence band edges vs. wave vector (momentum)
(from Sze)

Note that in Si and Ge the minimum of the conduction band is offset from the maximum of the valence band.

⇒ Promotion of an electron from the valence to the conduction band using an energy equal to the minimum gap spacing requires additional momentum transfer



b) high energy quanta ($E \gg E_g$)

It is experimentally observed that the energy required to form an electron-hole pair exceeds the bandgap.

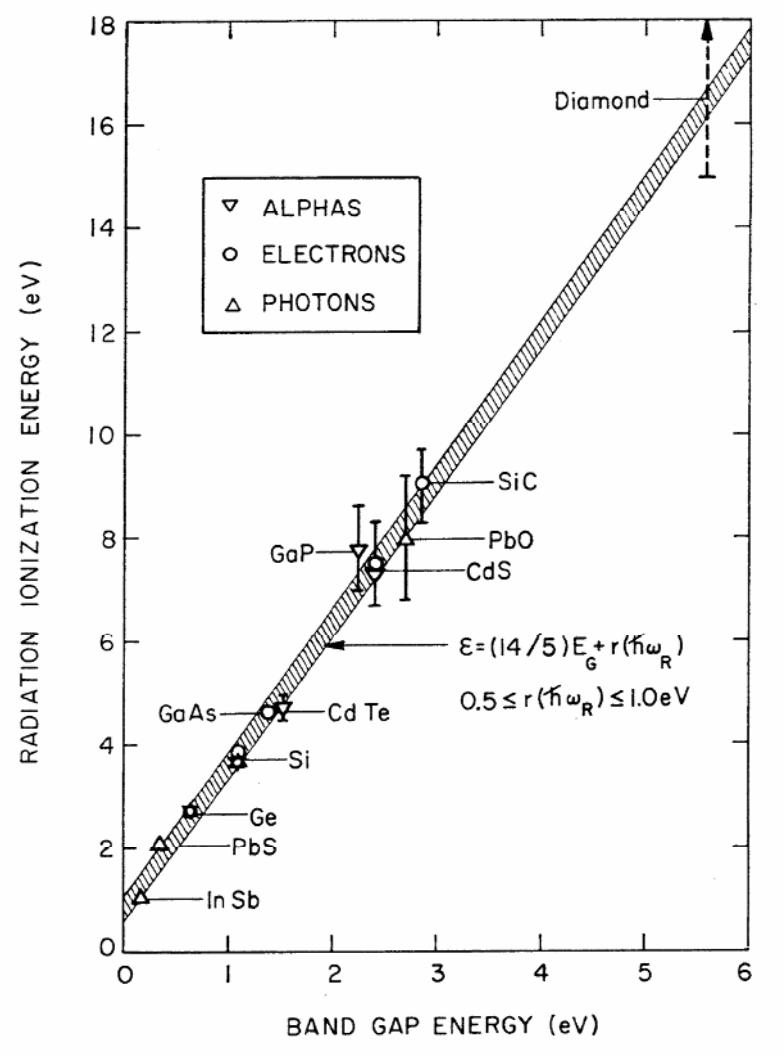
Why?

When particle deposits energy one must conserve both

energy and momentum

momentum conservation not fulfilled by transition across gap

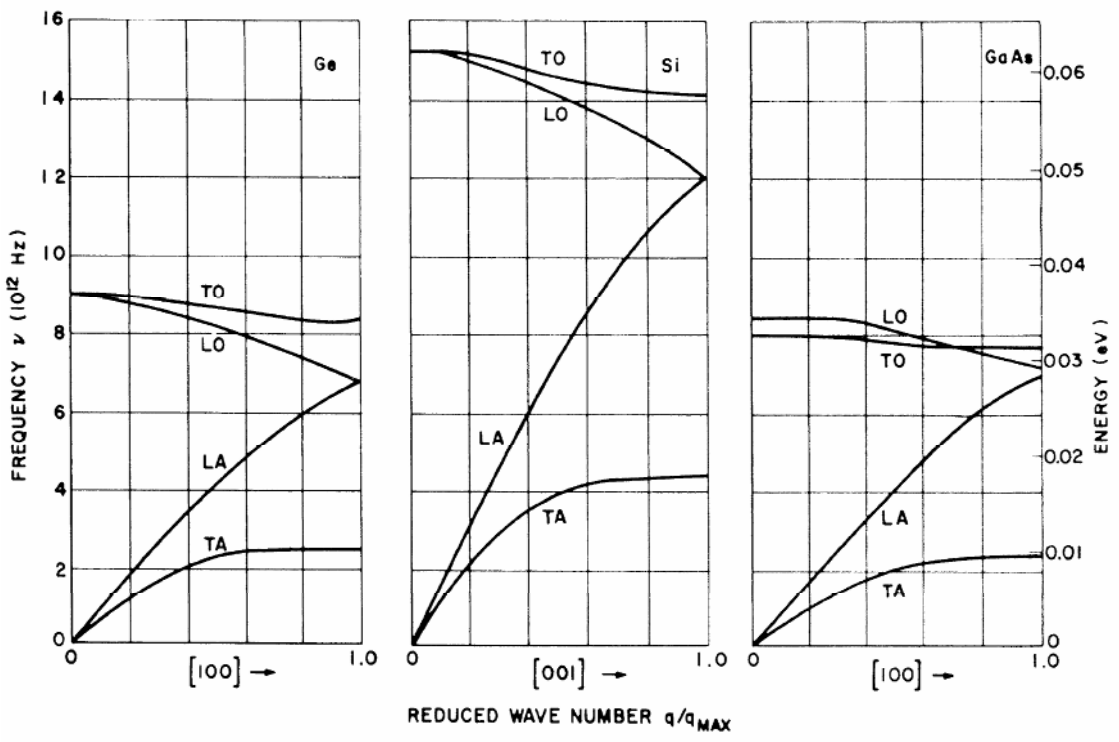
$\Rightarrow$ excite phonons



C.A. Klein, J. Applied Physics **39** (1968) 2029

Phonon energy vs. momentum (wavevector k)

In a semiconductor ionization detector ~60% of the deposited energy goes into phonon excitation.



Instead of detecting electron-hole pairs, detect heat or phonons

Energy scale: 10 meV $\Rightarrow$ lower energy threshold

Another possibility: Breakup of Cooper pairs in superconductors
The energy gap 2Δ (order 1 meV) is equivalent to the band gap in semiconductors.

Signal Fluctuations in a Scintillation Detector

Example: Scintillation Detector - a typical NaI(Tl) system (from Derenzo)

Resolution of energy measurement determined by statistical variance of produced signal quanta.

$$\frac{\Delta E}{E} = \frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

Resolution determined by smallest number of quanta in chain, i.e. number of photoelectrons arriving at first dynode.

In this example

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{3000}} = 2\% \text{ rms} = 5\% \text{ FWHM}$$

Typically 7 – 8% obtained, due to non-uniformity of light collection and gain.

511 keV gamma ray

⇓

25000 photons in scintillator

⇓

15000 photons at photocathode

⇓

3000 photoelectrons at first dynode

⇓

$3 \cdot 10^9$ electrons at anode

2 mA peak current

Fluctuations in the Signal Charge: the Fano Factor

The mean ionization energy exceeds the bandgap for two reasons

1. Conservation of momentum requires excitation of lattice vibrations
2. Many modes are available for the energy transfer with an excitation energy less than the bandgap.

Two types of collisions are possible:

a) Lattice excitation, i.e. phonon production (with no formation of mobile charge).

b) Ionization, i.e. formation of a mobile charge pair.

Assume that in the course of energy deposition

N_x excitations produce N_P phonons (or molecular vibrations, for example) and

N_{ion} ionization interactions form N_Q charge pairs.

On the average, the sum of the energies going into excitation and ionization is equal to the energy deposited by the incident radiation

$$E_0 = E_{ion} N_{ion} + E_x N_x$$

where E_{ion} and E_x are the energies required for a single excitation or ionization.

Assuming gaussian statistics, the variance in the number of excitations $\sigma_x = \sqrt{N_x}$

and the variance in the number of ionizations $\sigma_{ion} = \sqrt{N_{ion}}$

For a single event, the energy E_0 deposited in the detector is fixed (although this may vary from one event to the next).

If the energy required for excitation E_x is much smaller than required for ionization E_i, sufficient degrees of freedom will exist for some combination of ionization and excitation processes to dissipate precisely the total energy. Hence, for a given energy deposited in the sample a fluctuation in excitation must be balanced by an equivalent fluctuation in ionization.

If for a given event more energy goes into charge formation, less energy will be available for excitation.

$$E_x \Delta N_x + E_{ion} \Delta N_{ion} = 0$$

Averaging over many events this means that the variances in the energy allocated to the two types of processes must be equal

$$E_{ion} \sigma_{ion} = E_x \sigma_x$$

$$\sigma_{ion} = \frac{E_x}{E_{ion}} \sqrt{N_x}$$

From the total energy $E_{ion}N_{ion} + N_x E_x = E_0$: $$N_x = \frac{E_0 - E_{ion}N_{ion}}{E_x}$$

Inserted into the previous expression $\sigma_{ion} = \frac{E_x}{E_{ion}}\sqrt{N_x}$ this yields $$\sigma_{ion} = \frac{E_x}{E_{ion}}\sqrt{\frac{E_0}{E_x} - \frac{E_{ion}}{E_x}N_{ion}}$$

Since each ionization leads to a charge pair that contributes to the signal

$$N_{ion} = N_Q = \frac{E_0}{E_i}$$

where E_i is the average energy loss required to produce a charge pair,

$$\sigma_{ion} = \frac{E_x}{E_{ion}}\sqrt{\frac{E_0}{E_x} - \frac{E_{ion}}{E_x}\frac{E_0}{E_i}}$$

$$\sigma_{ion} = \sqrt{\frac{E_0}{E_i}} \cdot \sqrt{\frac{E_x}{E_{ion}}\left(\frac{E_i}{E_{ion}} - 1\right)}$$

The second factor on the right hand side is called the Fano factor F.

Since σ_i is the variance in signal charge Q and the number of charge pairs is $N_Q = E_0 / E_i$

$$\sigma_Q = \sqrt{FN_Q}$$

In Silicon E_x= 0.037 eV
$E_{ion} = E_g$= 1.1 eV
E_i = 3.6 eV

for which the above expression yields F= 0.08, in reasonable agreement with the measured value F =0.1.

⇒ The variance of the signal charge is smaller than naively expected: $\sigma_Q \approx 0.3\sqrt{N_Q}$

A similar treatment can be applied if the degrees of freedom are much more limited and Poisson statistics are necessary.

However, when applying Poisson statistics to the situation of a fixed energy deposition, which imposes an upper bound on the variance, one can not use the usual expression for the variance $\mathrm{var}\ N = \overline{N}$
Instead, the variance is $\overline{(N-\overline{N})^2} = F\overline{N}$ as shown by Fano [1] in the original paper.

An accurate calculation of the Fano factor requires a detailed accounting of the energy dependent cross sections and the density of states of the phonon modes. This is discussed by Alkhazov [2] and van Roosbroeck [3].

References: 1. U. Fano, Phys. Rev. **72** (1947) 26
2. G.D. Alkhazov et al., NIM **48** (1967) 1
3. W. van Roosbroeck, Phys. Rev. **139** (1963) A1702

Intrinsic Resolution of Semiconductor Detectors

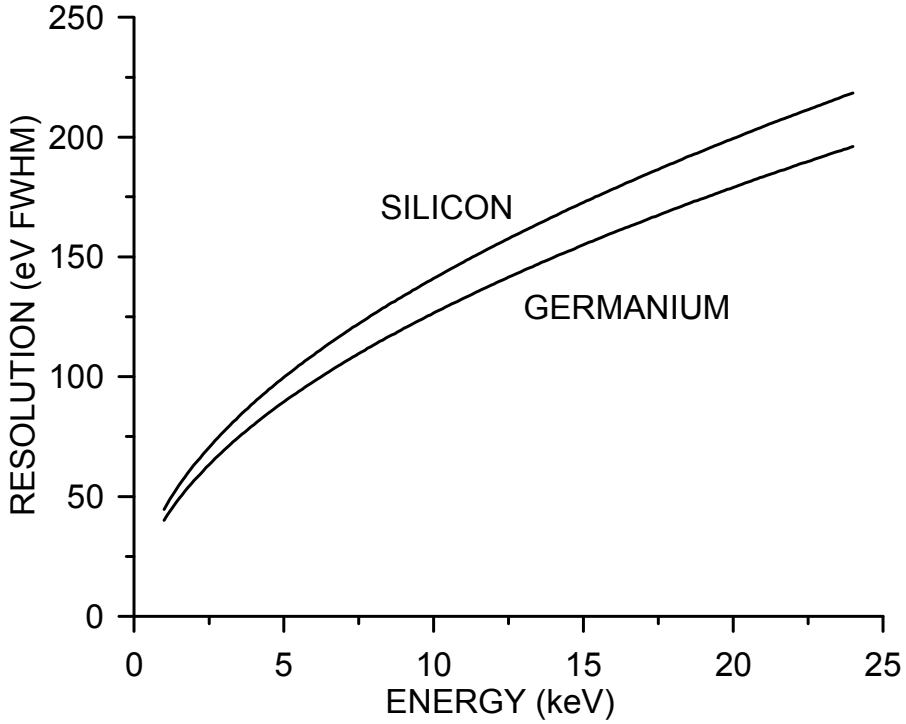
$$\Delta E_{FWHM} = 2.35 \cdot \varepsilon_i \sqrt{FN_Q} = 2.35 \cdot \varepsilon_i \sqrt{F \frac{E}{E_i}} = 2.35 \cdot \sqrt{FEE_i}$$

Si: $E_i = 3.6$ eV $\quad F = 0.1$

Ge: $E_i = 2.9$ eV $\quad F = 0.1$

Detectors with good efficiency for this energy range have sufficiently small capacitance to allow electronic noise of ~100 eV FWHM, so the variance of the detector signal is a significant contribution.

At energies >100 keV the detector sizes required tend to increase the electronic noise to dominant levels.



3. Signal Formation

First, we'll discuss direct signal conversion, i.e. the ionization signal and return to phonon detection towards the end of the lecture series.

Example: Semiconductor Diodes

> This allows us to introduce several key (and universal) concepts, but it also lays the groundwork for discussing microelectronics and noise mechanisms.

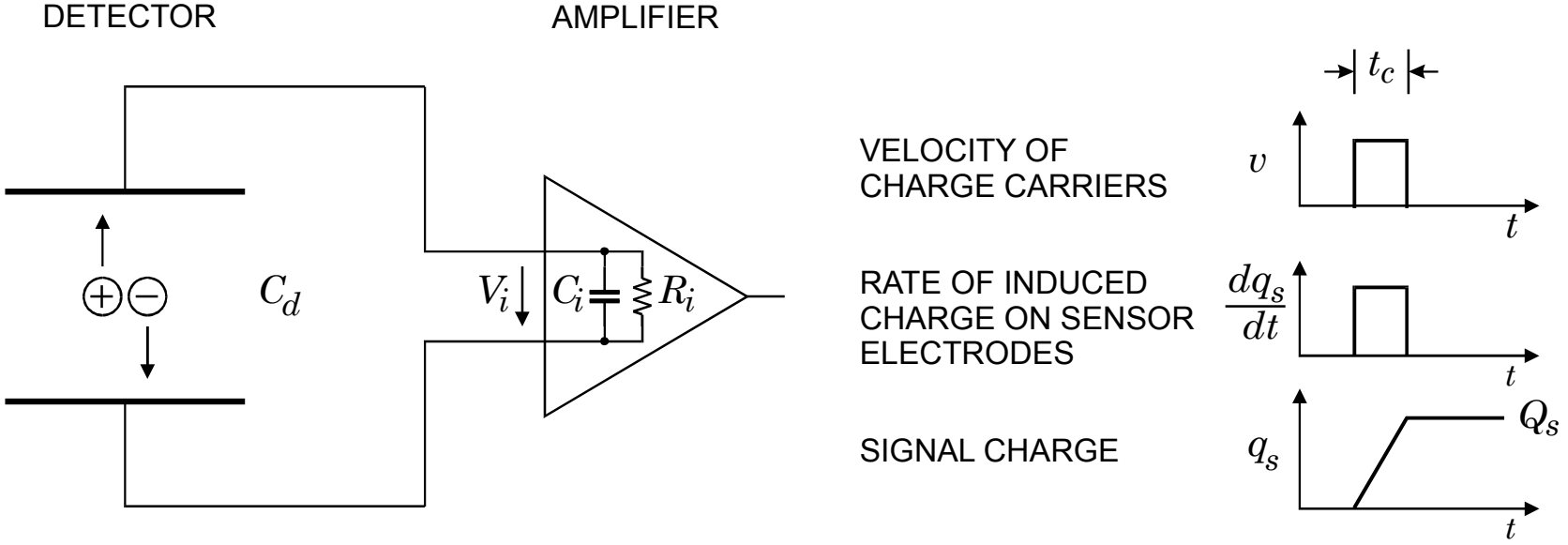
Semiconductor Detectors - Basic Operation

Semiconductor Detectors are Ionization Chambers:

Detection volume with electric field

Energy deposited $\rightarrow$ positive and negative charge pairs

Charges move in field $\rightarrow$ external electrical signal



If $R_i \cdot (C_d + C_i) \gg$ collection time t_c the peak voltage at the amplifier input $V_s = \dfrac{Q_s}{C_{\text{det}} + C_i}$

Ionization chambers can be made with any medium that allows charge collection to a pair of electrodes.

Medium can be gas
liquid
solid

Crude comparison of relevant properties

	gas	**liquid**	**solid**
density	low	moderate	high
atomic number Z	low	moderate	moderate
ionization energy ε_i	moderate	moderate	low
signal speed	moderate	moderate	fast

Desirable properties:

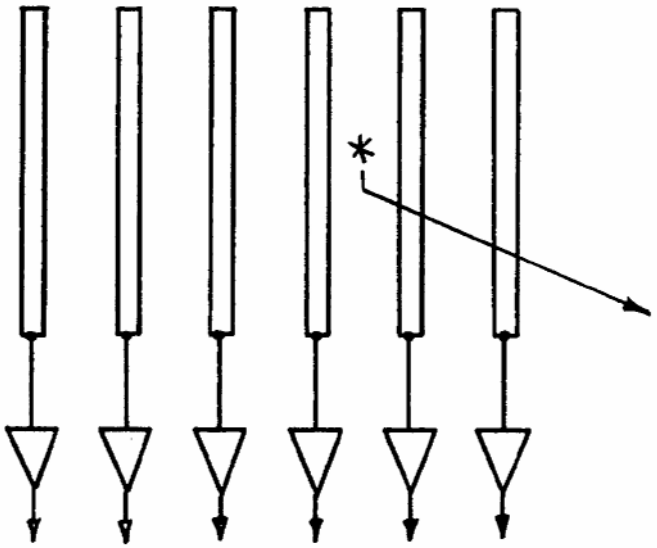
- low ionization energy $\Rightarrow$ 1. increased charge yield dq/dE
  2. superior resolution $\frac{\Delta E}{E} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E/E_i}} \propto \sqrt{E_i}$
- high field in detection volume $\Rightarrow$ 1. fast response
  2. improved charge collection efficiency (reduced trapping)

In addition to energy measurements, semiconductor detectors allow precision position sensing.

Resolution determined by precision of micron scale patterning of the detector electrodes (e.g. strips on 50 μm pitch).

Two options:

Binary Readout



to discriminators

Position resolution determined directly by pitch $\sigma_x = pitch/\sqrt{12}$

Analog Readout

Interpolation yields resolution < pitch

Relies on transverse diffusion

$$\sigma_x \propto \sqrt{t_{coll}}$$

e.g. in Si: $t_c \approx 10$ ns $\Rightarrow \sigma_x = 5$ μm

depends on S/N and p

$p = 25$ μm and $S/N = 50$

$\Rightarrow$ 3 – 4 μm resolution

In the first part of this section we discussed the magnitude of the signal charge and its fluctuations. Now we consider how the signal is extracted.

To form a current that can be measured in the external circuit, the signal charge carriers must be brought into motion. This is done by establishing a field in the detection volume. Increasing the field will sweep the charge more rapidly from the detection volume.

Formation of a High-Field Region

The conduction band is only empty at 0K.

As the temperature is increased, thermal excitation can promote electrons across the band gap into the conduction band.

Pure Si: carrier concentration ~ 10^{10} cm^{-3} at 300K (resistivity $\approx$ 400 kΩ·cm)

Since the Si lattice comprises $5 \cdot 10^{22}$ atoms/cm^3, many states are available in the conduction band to allow carrier motion.

In reality, crystal imperfections and minute impurity concentrations
limit Si carrier concentrations to ~10^{11} cm^{-3} at 300K.

This is too high for use in a simple crystal detector.

A crystal detector is feasible with diamond,
but the charge yield is smaller due to the larger band gap.

High-field region with low DC current in semiconductors
is most easily achieved utilizing a *pn*-junction.

$\Rightarrow$ Introduction of impurities to control conductivity.

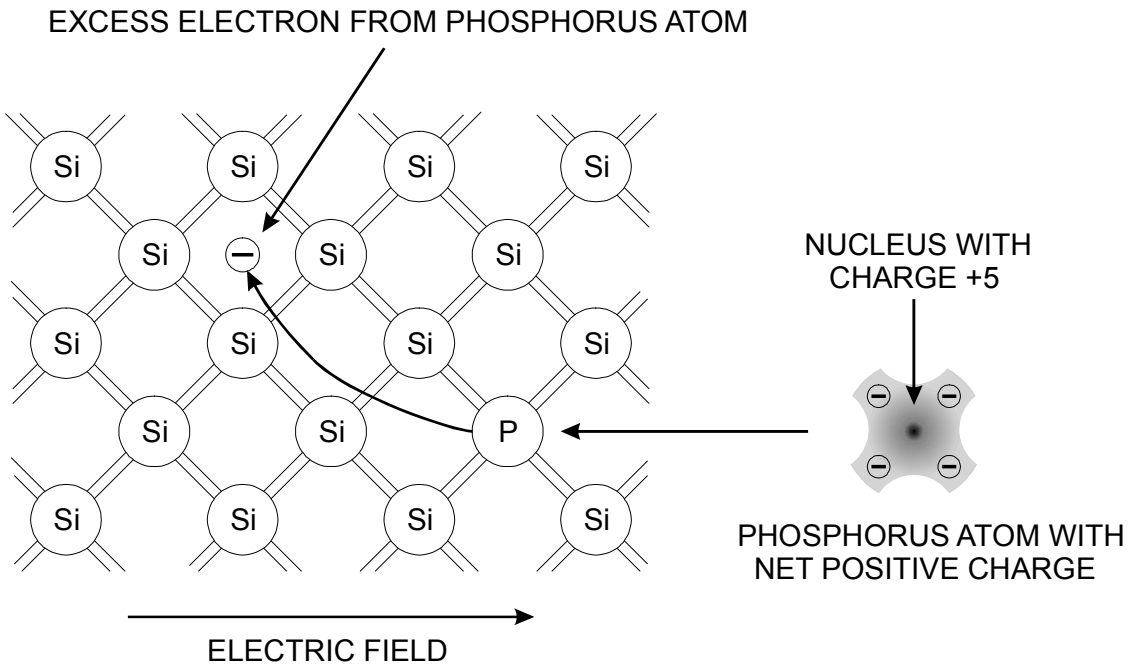
Doping

The conductivity of semiconductors can be controlled by introducing special impurities.

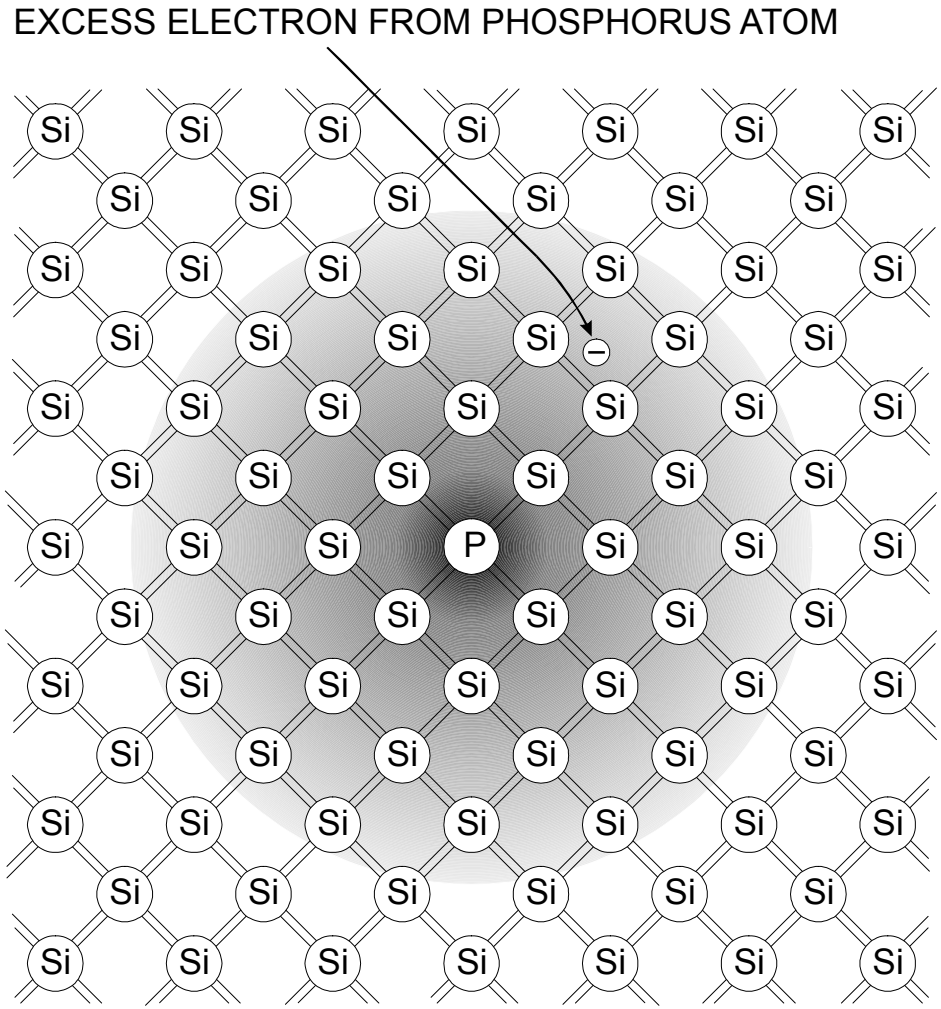
required concentrations: ~10^{12} – 10^{18} cm^{-3}

Replacing a silicon atom (group 4 in periodic table, i.e. 4 valence electrons) by an atom with 5 valence electrons, e.g. P, As, Sb, leaves one valence electron without a partner.

Since the impurity contributes an excess electron to the lattice, it is called a donor.



The wavefunction of the dopant atom extends over many neighbors.

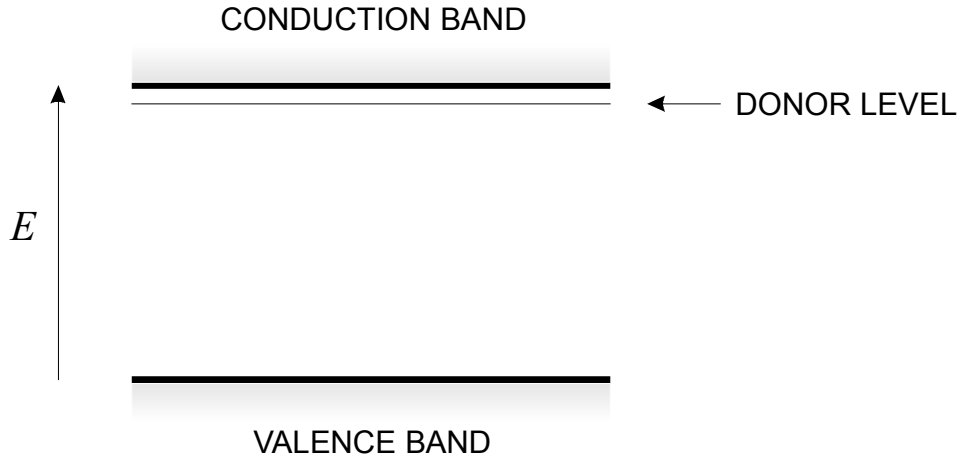


(following Shockley)

The excess electron is only loosely bound, as the coulomb force is reduced by the dielectric constant ε of the medium (ε=12 in Si).

$$E_i(lattice) \propto \frac{E_i(atom)}{\varepsilon^2}$$

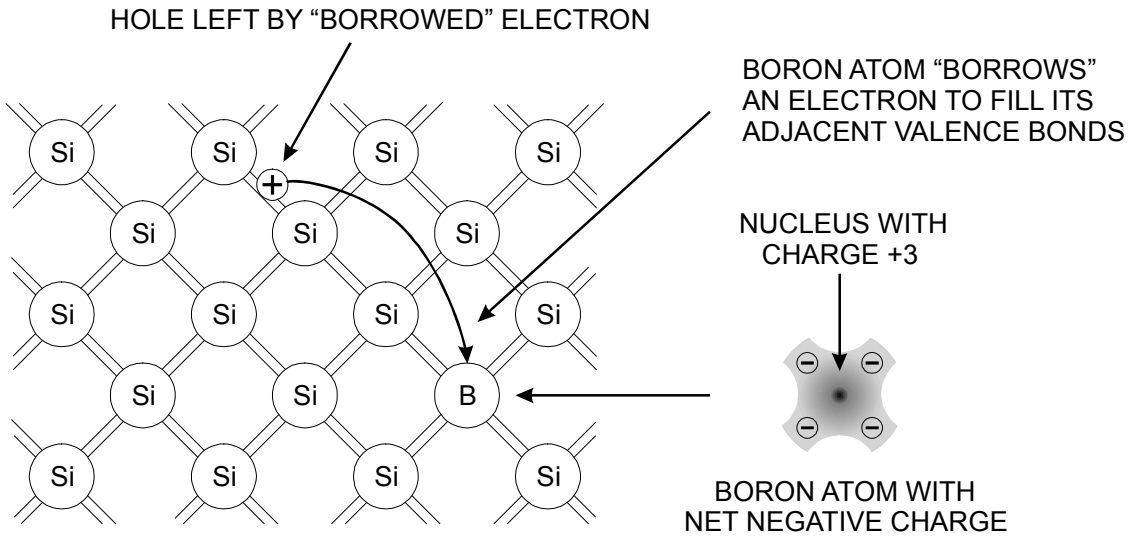
The bound level of this unpaired electron is of order 0.01 eV below the conduction band (e.g. for P: E_c - 0.045 eV).



$\Rightarrow$ substantial ionization probability at room temperature (E= 0.026 eV) – "donor"

$\Rightarrow$ electrons in conduction band

Conversely, introducing a group 3 atom (B, Al, Ga, In) leaves a Si valence electron without a partner.



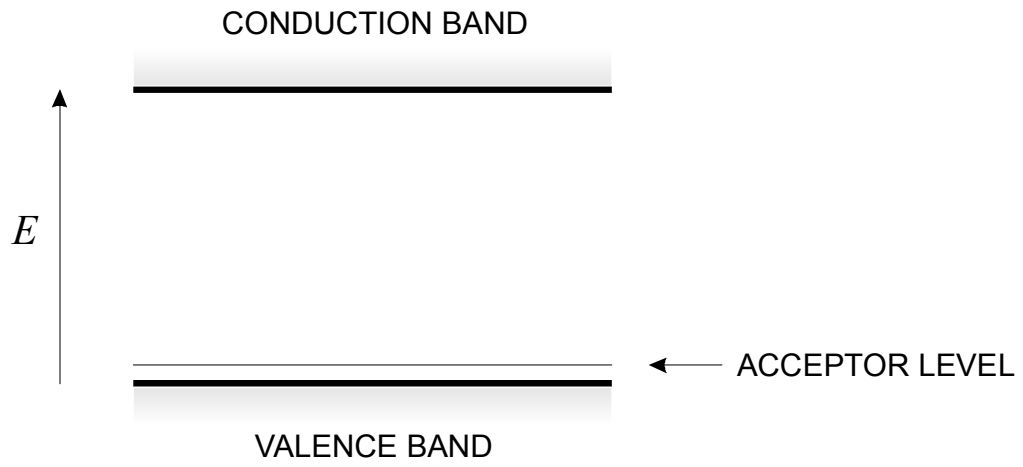
(following Shockley)

To close its shell the B atom “borrows” an electron from a lattice atom in the vicinity.

This type of dopant is called an “acceptor”.

The “borrowed” electron is bound, but somewhat less than other valence electrons since the B nucleus only has charge 3.

This introduces a bound state close to the valence band, also of order 0.01 eV from the band edge.



For example, a B atom in Si forms a state at E_v + 0.045 eV.

Again, as this energy is comparable to kT at room temperature, electrons from the valence band can be excited to fill a substantial fraction of these states.

The electrons missing from the valence band form mobile charge states called “holes”, which behave similarly to an electron in the conduction band, i.e. they can move freely throughout the crystal.

Since the charge carriers in the donor region are electrons, i.e. negative, it is called “n-type”.
Conversely, as the charge carriers in the acceptor region are holes, i.e. positive, it is called “p-type”.

pn-Junction

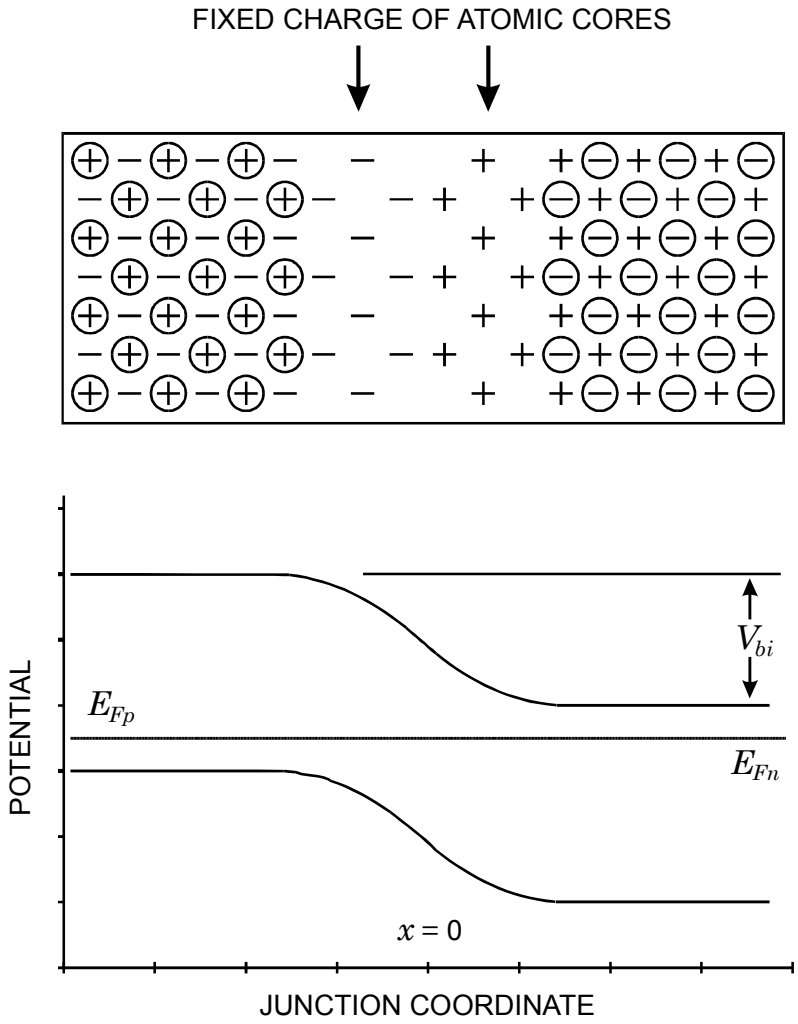
Consider a crystal suitably doped that a donor region and an acceptor adjoin each other, a "*pn*-junction".

Thermal diffusion will drive holes and electrons across the junction.

Although the *p* and *n* regions were originally electrically neutral, as electrons diffuse from the *n* to the *p* region, they uncover their respective donor atoms, leaving a net positive charge in the *n* region.

This positive space charge exerts a restraining force on the electrons that diffused into the *p* region, i.e. diffusion of electrons into the *p* region builds up a potential. The diffusion depth is limited when the space charge potential exceeds the available energy for thermal diffusion.

The corresponding process also limits the diffusion of holes into the *n*-region.



The diffusion of holes and electrons across the junction leads to a region free of mobile carriers – the "depletion region", bounded by conductive regions, which are n- and p-doped, respectively.

Strictly speaking, the depletion region is not completely devoid of mobile carriers, as the diffusion profile is a gradual transition.

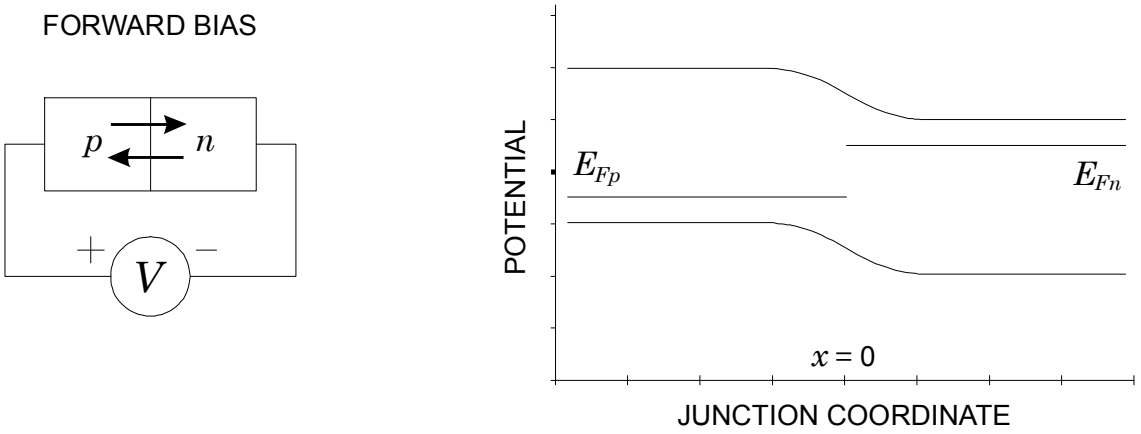
Nevertheless, since the carrier concentration is substantially reduced, it is convenient to treat the depletion zone as an abrupt transition between bulk and 0 carrier concentration.

Furthermore, the formation of the two adjacent space charge regions builds up a potential barrier between the n and p regions, which impedes the further flow of charge.

The magnitude of this potential barrier is typically 50 – 90% of the band-gap, depending on relative doping levels.

This represents the situation in thermal equilibrium. By application of an external potential, two distinctly different non-equilibrium modes can be established.

a) positive potential applied to the p region
negative potential applied to the n region

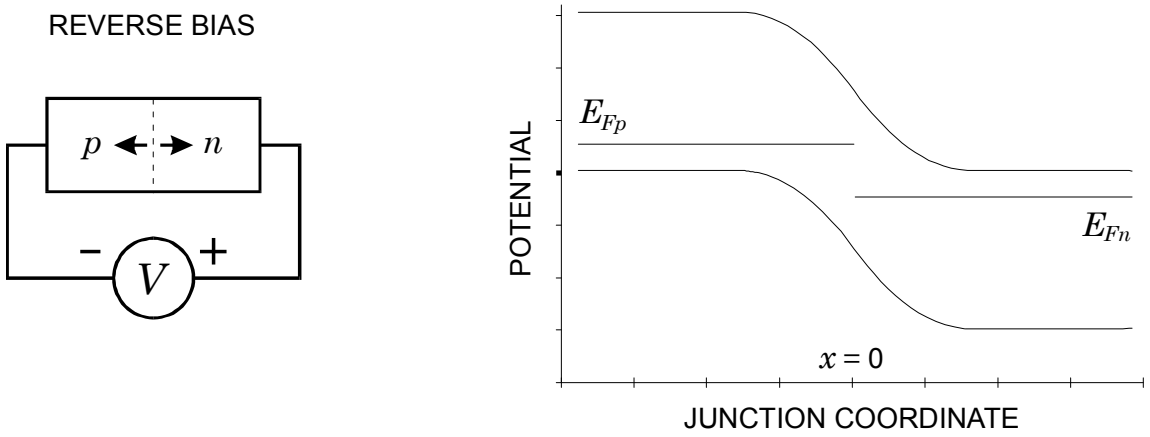


The externally applied voltage reduces the potential barrier, allowing increased charge transfer across the junction.

$\Rightarrow$ "forward bias"

Electrons flowing from the n-region across the junction are replenished from the external voltage supply and large current flow is possible.

b) negative potential applied to the p region
positive potential applied to the n region



This arrangement increases the potential barrier across the junction, impeding the flow of current.

⇒ "reverse bias"

Potential across junction is increased ⇒ wider depletion region

The p-n junction is asymmetric with respect to current flow (diode).

a) forward bias

positive supply connection $\rightarrow$ p contact
negative supply connection $\rightarrow$ n contact

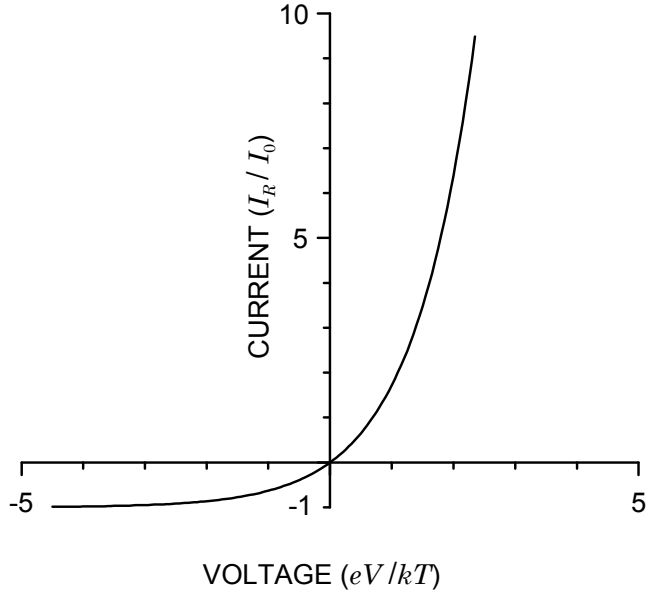
$\Rightarrow$ large current flow

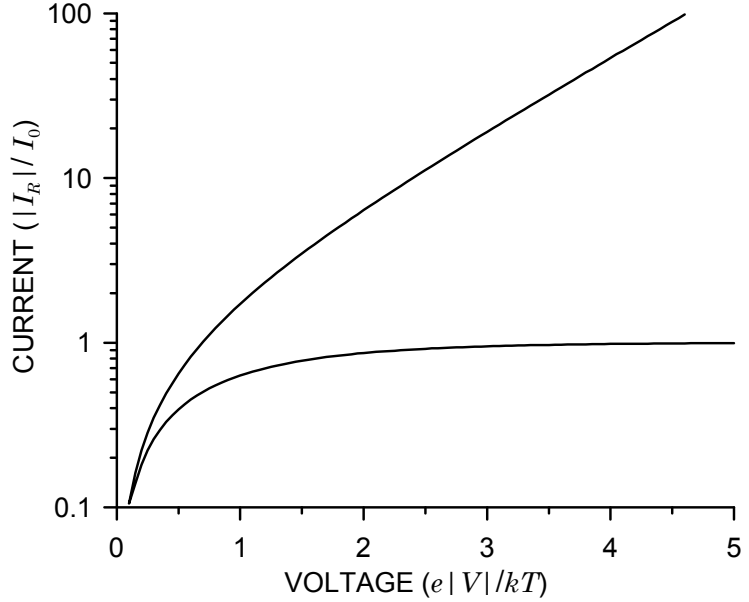
b) reverse bias

positive supply connection $\rightarrow$ n contact
negative supply connection $\rightarrow$ p contact

$\Rightarrow$ small current flow

Diode current vs. voltage $I = I_0(e^{q_e V/kT} - 1)$
(Shockley equation)





Since the depletion region is a volume with an electric field, it by itself could be used as a radiation detector.

- The width of the depletion region is increased by reverse bias.

Depletion width and electric field in p-n junction

Assume a reverse bias voltage V_b and that the potential changes only in the direction perpendicular to the n-p interface. Poisson's equation is then

$$\frac{d^2V}{dx^2} + \frac{Nq_e}{\varepsilon} = 0 \qquad (1)$$

where N is the dopant concentration and q_e the electron charge.

Consider an abrupt junction where charge densities on the n and p sides are $N_d\ q_e$ and $N_a\ q_e$, respectively.

If the limits of the depletion region are x_n on the n-side and x_p on the p-side, after two successive integrations one obtains on the n-side

$$\frac{dV}{dx} = -\frac{q_e N_d}{\varepsilon}(x - x_n) \tag{2}$$

and

$$V = -\frac{q_e N_d}{\varepsilon}\frac{x^2}{2} + \frac{q_e N_d x x_n}{\varepsilon} + V_j \tag{3}$$

where V_j is the potential at the metallurgical junction. For $x = x_n$

$$V(x_n) = V_b = \frac{q_e N_d x_n^2}{2\varepsilon} + V_j \tag{4}$$

and the contribution of the n-region to the total reverse bias potential becomes

$$V_b - V_j = \frac{q_e N_d x_n^2}{2\varepsilon} \; . \tag{5a}$$

Correspondingly, in the p-region

$$V_j = \frac{q_e N_a x_p^2}{2\varepsilon} \tag{5b}$$

and the total potential becomes

$$V_b = \frac{q_e}{2\varepsilon}(N_d x_n^2 + N_a x_p^2). \quad (6)$$

Due to overall charge neutrality

$$N_d x_n = N_a x_p \quad (7)$$

and

$$V_b = \frac{q_e}{2\varepsilon}\left(1+\frac{N_a}{N_d}\right) N_a x_p^2 = \frac{q_e}{2\varepsilon}\left(1+\frac{N_d}{N_a}\right) N_d x_n^2 \ . \quad (8)$$

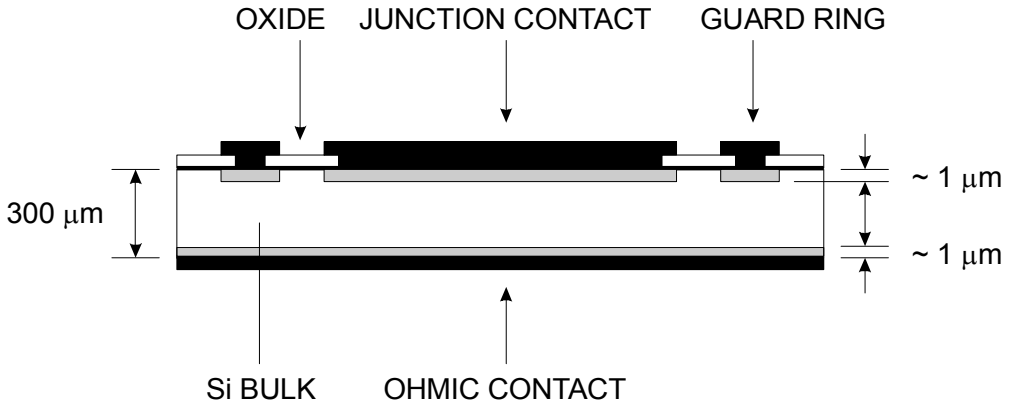
The depletion widths on the *n*- and *p*-side of the junction are

$$x_n = \sqrt{\frac{2\varepsilon V_b}{q_e N_d (1+N_d/N_a)}} \ ; \quad x_p = \sqrt{\frac{2\varepsilon V_b}{q_e N_a (1+N_a/N_d)}} \quad (9)$$

and the total depletion width becomes

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon V_b}{q_e}\,\frac{N_a+N_d}{N_a N_d}} \ . \quad (10)$$

Detector diodes are usually asymmetrically doped. The starting material (bulk) is lightly doped and the junction is formed by diffusing or ion-implanting a highly doped layer.



The external connection to the lightly doped bulk is made by an additional highly doped layer of the same type (non-rectifying, “ohmic” contact).

- The depletion region then extends predominantly into the lightly doped bulk.

Other details:

The guard ring isolates the wafer edge (saw cut) from the active region.

In the gap between the detector electrode and the guard ring it is critical to provide a neutral interface at the silicon surface to prevent formation of a conductive path.

This is best accomplished by oxide passivation (SiO_2).

If, for example, $N_a \gg N_d$, the depletion region extends predominantly into the n-side and the total depletion width is

$$W \approx x_n = \sqrt{\frac{2\varepsilon V_b}{q_e N_d}} \quad . \tag{11}$$

The doping concentration is commonly expressed in terms of resistivity

$$\rho = (\mu q_e N)^{-1},$$

because this is a readily measurable quantity. The parameter μ describes the relationship between the applied field and carrier velocity (to be discussed later).

Using resistivity the depletion width becomes

$$W = \sqrt{2\varepsilon \mu_n \rho_n V_b} \; . \tag{12}$$

Note that this introduces an artificial distinction between the n- and p-regions, because the mobilities μ for electrons and holes are different.

Since the mobility of holes is approximately 1/3 that of electrons, p-type material of a given doping concentration will have 3 times the resistivity of n-type material of the same concentration.

As discussed earlier, even in the absence of an external voltage electrons and holes to diffuse across the junction, establishing a "built-in" reverse bias voltage V_{bi}. If we take this inherent bias voltage into account and set for the bias voltage $V_b \rightarrow V_b + V_{bi}$, one obtains for the one-sided junction

$$W \approx x_1 = \sqrt{\frac{2\varepsilon(V_b + V_{bi})}{q_e N_d}} = \sqrt{2\varepsilon\mu_n\rho_n(V_b + V_{bi})}.$$

For example, in n-type silicon (V_b in volts and ρ in Ω·cm): $W = 0.5\ \mu m\ \mathrm{x}\ \sqrt{\rho(V_b + V_{bi})}$

and in p-type material: $W = 0.3\ \mu m\ \mathrm{x}\ \sqrt{\rho(V_b + V_{bi})}$

The depleted junction volume is free of mobile charge and thus forms a capacitor, bounded by the conducting p- and n-type semiconductor on each side.

The capacitance is $$C = \varepsilon \frac{A}{W} = A \sqrt{\frac{\varepsilon\ q_e N}{2(V_b + V_{bi})}}$$

For bias voltages $V_b \gg V_{bi}$ $$C \propto \frac{1}{\sqrt{V_b}}$$

In technical units $$\frac{C}{A} = \frac{\varepsilon}{W} \approx 1\ [\mathrm{pF/cm}]\ \frac{1}{W}$$

A diode with 100 μm thickness has about 1 pF/mm^2.

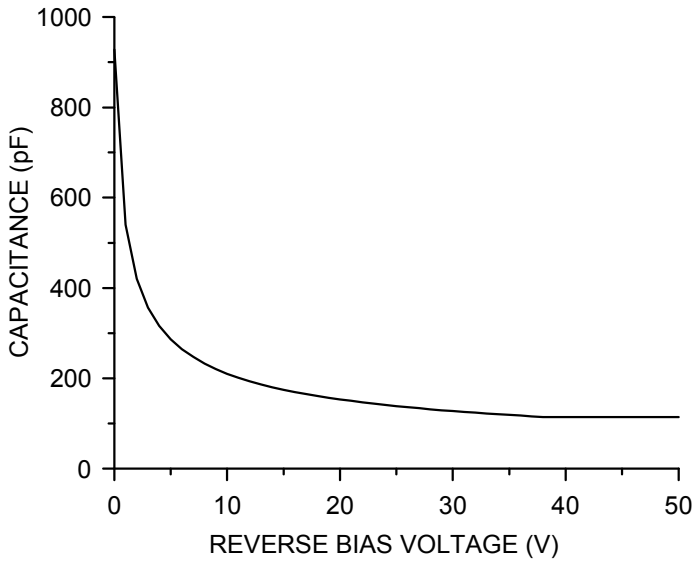
The capacitance vs. voltage characteristic of a diode can be used to determine the doping concentration of the detector material.

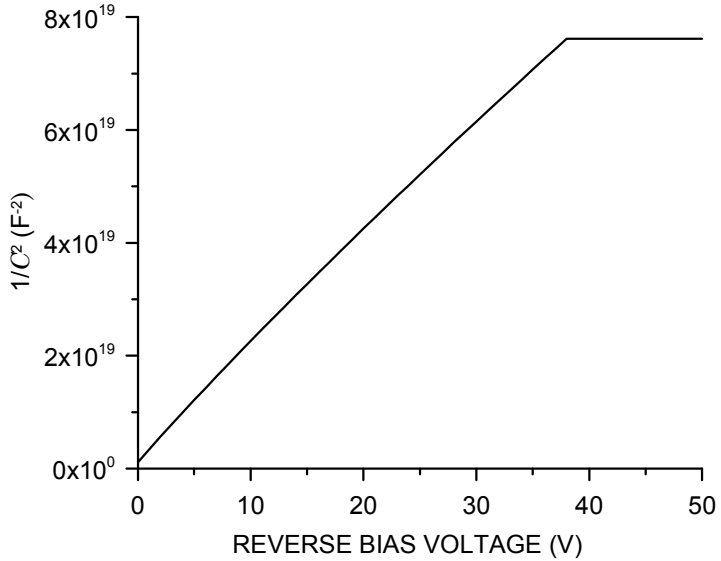
$$\frac{C}{A} = \sqrt{\frac{\varepsilon\, q_e N}{2(V_b + V_{bi})}}$$

In a plot of $(A/C)^2$ vs. the detector bias voltage V_b the slope of the voltage dependent portion yields the doping concentration N.

Example: Si pad detector, A= 1 cm^2, 100 μm thick

$$\frac{1}{N} = \frac{d(1/C^2)}{dV}\left(\frac{\varepsilon q_e}{2}\right) = \frac{1}{5\cdot 10^{12}}$$



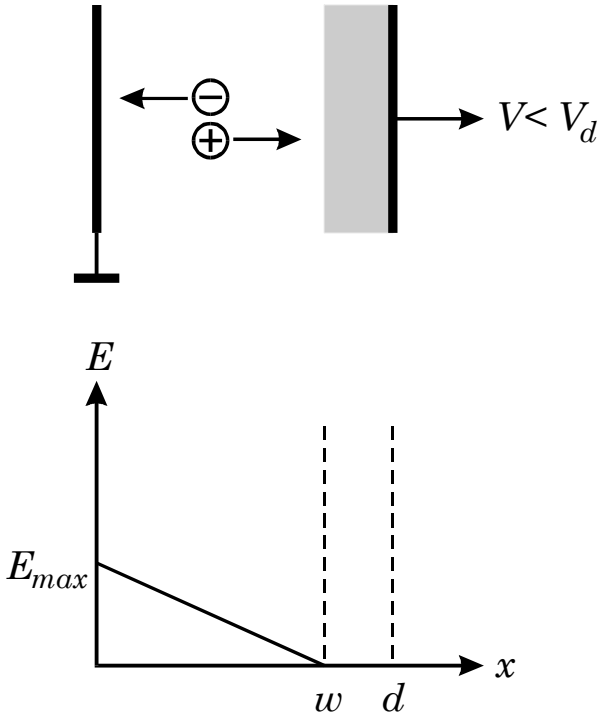


Charge Collection

Mobile electrons and holes formed by radiation move under the influence of the electric field in the junction.

Although electrons and holes move in opposite directions, their contribution to the signal current is of the same polarity.

The time required for a charge carrier to traverse the sensitive volume is called the collection time.



Using the depletion width eq. 13 one can rewrite eq. 2 for the electric field

$$E(x) = \frac{2(V_b + V_{bi})}{W}\left(\frac{x}{W} - 1\right) \qquad (14)$$

The detector bulk is completely depleted of mobile charge when $W = d$, the thickness of the substrate. This occurs at the externally applied depletion voltage

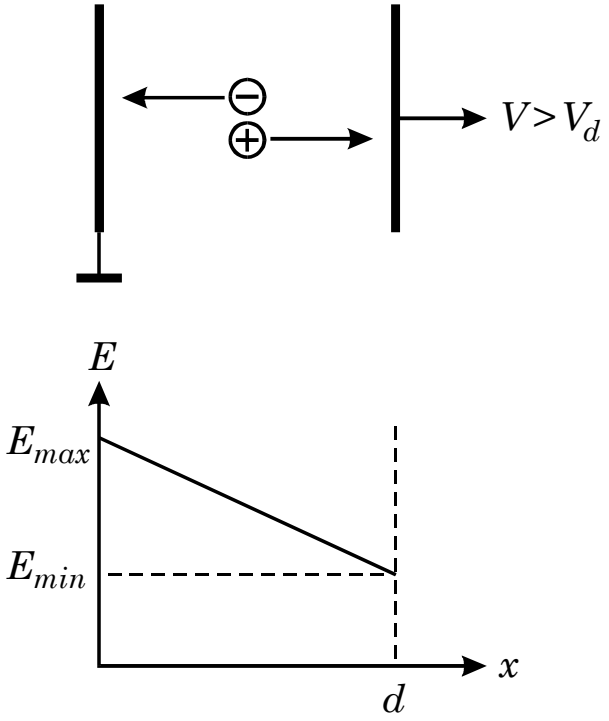
$$V_d = \frac{q_e N_d W^2}{2\varepsilon} - V_{bi} \,. \qquad (15)$$

The field drops linearly from its maximum value at the junction to zero at the opposite contact.

Increasing the bias voltage beyond this value adds a uniform field due to the voltage beyond depletion, yielding a distribution

$$E(x) = \frac{2V_{di}}{W}\left(1 - \frac{x}{W}\right) + \frac{V_b - V_{di}}{W} \qquad (16)$$

where $V_{di} \equiv V_d + V_{bi}$ has been defined as the internal depletion voltage.



First consider a detector operated at partial depletion $V_b > V_d$. The field

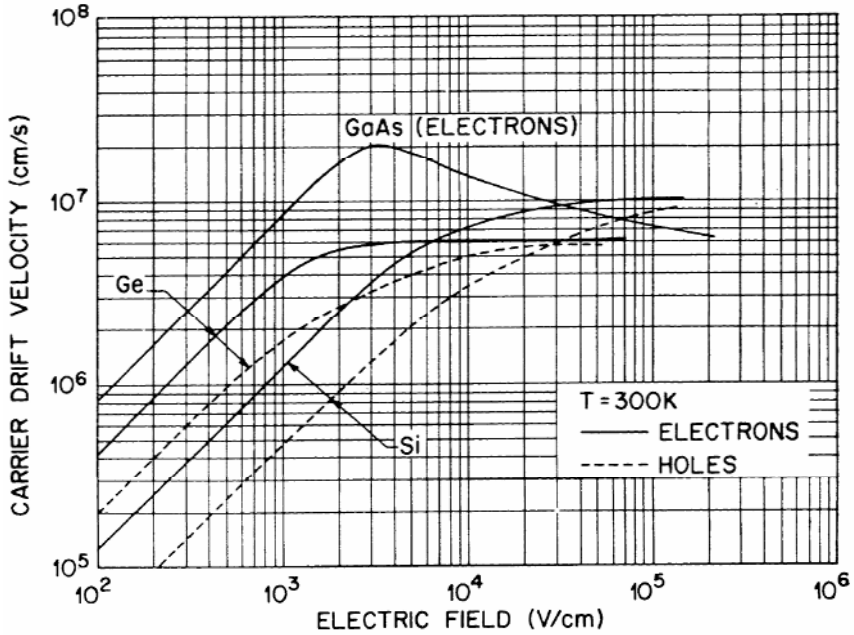
$$E(x) = -\frac{q_e N_d}{\varepsilon}(W - x) \equiv E_0(W - x) \qquad (17)$$

The local velocity of a charge carrier $\qquad v(x) = \mu E(x) = \mu E_0(W - x) \qquad (18)$

Note that the velocity does not depend on the time during which the charge carrier is accelerated, as in normal ballistic motion, since the charge carrier also interacts with the crystal lattice, exciting lattice vibrations (phonons). Since the characteristic times for phonon excitation are much smaller than the transport times, the carrier is always in equilibrium with the lattice, so the velocity is only a function of the electric field, at every position in the depletion region.

In Si at 300K the mobility at low fields is 1350 cm^2/ Vs for electrons and 480 cm^2/ Vs for holes.

The mobility is constant up to about 10^4 V/cm, but then increased phonon emission reduces the energy going into electron motion, so the mobility decreases. At high fields E > 10^5 V/cm the mobility $\mu \propto 1/E$ and carriers attain a constant drift velocity of 10^7 cm/s.



(from Sze)

The time required for a charge originating at x_0 to reach a point x is

$$t(x) = \int_{x_0}^{x} \frac{1}{v(x)} dx = \frac{1}{\mu E_0} \int_{x_0}^{x} \frac{1}{W - x} dx = -\frac{1}{\mu E_0} \left[\ln(W - x) \right]_{x_0}^{x}$$

$$t(x) = -\frac{1}{\mu E_0} \ln \frac{W - x}{W - x_0} = \frac{\varepsilon}{\mu q_e N_d} \ln \frac{W - x}{W - x_0} \qquad (19)$$

Consider a hole drifting toward the high-field region and collected at the p-electrode x= 0. Using the hole mobility μ_p eq. 19 yields

$$t(x_0) = -\frac{1}{\mu_p E_0} \ln \frac{W}{W - x_0} = \frac{\varepsilon}{\mu_p q_e N_d} \ln \frac{W}{W - x_0} \qquad (20)$$

If we define a characteristic collection time $\tau_p \equiv \frac{\varepsilon}{\mu_p q_e N_d}$,

then

$$t(x_0) = \tau_p \ln \frac{W}{W - x_0} \qquad (20a)$$

For example,

$$t(x_0 = 0.5W) = 0.7\tau_p$$

and

$$t(x_0 = 0.95W) = 3.0\tau_p .$$

For the electrons drifting toward the low-field electrode $x = W$, eq. 19 does not yield a solution. However, it can be rewritten to yield the position as a function of time

$$x(t) = W - (W - x_0)\, e^{-t/\tau_n} \qquad (21)$$

where τ_n has been defined analogously to τ_p.

For a charge originating at the metallurgical junction $x_0 = 0$ and drifting toward $x = W$

$$x(t) = W(1 - e^{-t/\tau_n}) \ . \qquad (22)$$

In this simple picture, a charge drifting toward the low field region is never collected (in reality this is accomplished by diffusion), although after a time $t = 3\tau_n$ the carrier will have traversed 95% of the detector. Note that in a partially depleted detector the collection time constants τ_n and τ_p are independent of the applied bias voltage (and depletion thickness), but determined only by the doping concentration of the bulk material and the carrier mobility.

τ_n is numerically equal to the dielectric relaxation time of the n-type bulk

$$\tau = \rho\varepsilon = \varepsilon_{Si}\ \varepsilon_0\ \rho = 1.05\left[\frac{ns}{k\Omega\cdot cm}\right]\ \rho\ . \qquad (23)$$

In n-type silicon of 10 kΩ·cm resistivity τ_n= 10.5 ns and τ_p= 31.5 ns, and typical collection times in partially depleted detectors are about 30 and 90 ns, respectively.

Voltage bias beyond depletion (overbias)

The collection time can be reduced by operating the detector at bias voltages exceeding the depletion voltage.

The field distribution was given in eq. 16, which can be rewritten as

$$E(x) = E_0 \left(1 - \frac{x}{W}\right) + E_1 \ . \qquad (24)$$

This yields a collection time

$$t(x) = \int_{x_0}^{x} \frac{1}{v(x)} dx = \frac{1}{\mu} \int_{x_0}^{x} \frac{1}{E_0 \left(1 - \frac{x}{W}\right) + E_1} dx$$

$$t(x) = -\frac{W}{\mu E_0} \left[\ln(E_0 + E_1 - E_0 \frac{x}{W} \right]_{x_0}^{x}$$

$$t(x) = \frac{W}{\mu E_0} \ \ln \frac{E_0 + E_1 - E_0 \frac{x}{W}}{E_0 + E_1 - E_0 \frac{x_0}{W}} \ . \qquad (25)$$

For holes originating at $x_0 = W$ and drifting to the p-electrode $x = 0$

$$t_{cp} = \frac{W}{\mu_p E_0} \ln\left(1 + \frac{E_0}{E_1}\right). \qquad (26a)$$

The corresponding result obtains for electrons originating at $x_0 = 0$ and drifting to the n-electrode $x = W$

$$t_{cn} = \frac{W}{\mu_n E_0} \ln\left(1 + \frac{E_0}{E_1}\right). \qquad (26b)$$

For large overbias $E_1 \gg E_0$: $\ln\left(1 + \frac{E_0}{E_1}\right) \approx \frac{E_0}{E_1}$

and $t_{cp} = \frac{W}{\mu_p E_1}$

as expected for a uniform field.

Rewritten in terms of voltages, eqs. 26a and 26b become $t_{cp} = \frac{W^2}{2\mu_p V_{di}} \ln\left(\frac{V_b + V_{di}}{V_b - V_{di}}\right)$

$$\text{and} \quad t_{cn} = \frac{W^2}{2\mu_n V_{di}} \ln\left(\frac{V_b + V_{di}}{V_b - V_{di}}\right) \qquad (27)$$

Example:

For n-type silicon of 10 kΩ·cm resistivity,

a detector thickness of 300 μm, and

a reverse bias voltage V_b= 60V= 2V_d (i.e. E_0=2·10^3 and E_1=10^3 V/cm)

Collection times for

Electrons: 12 ns

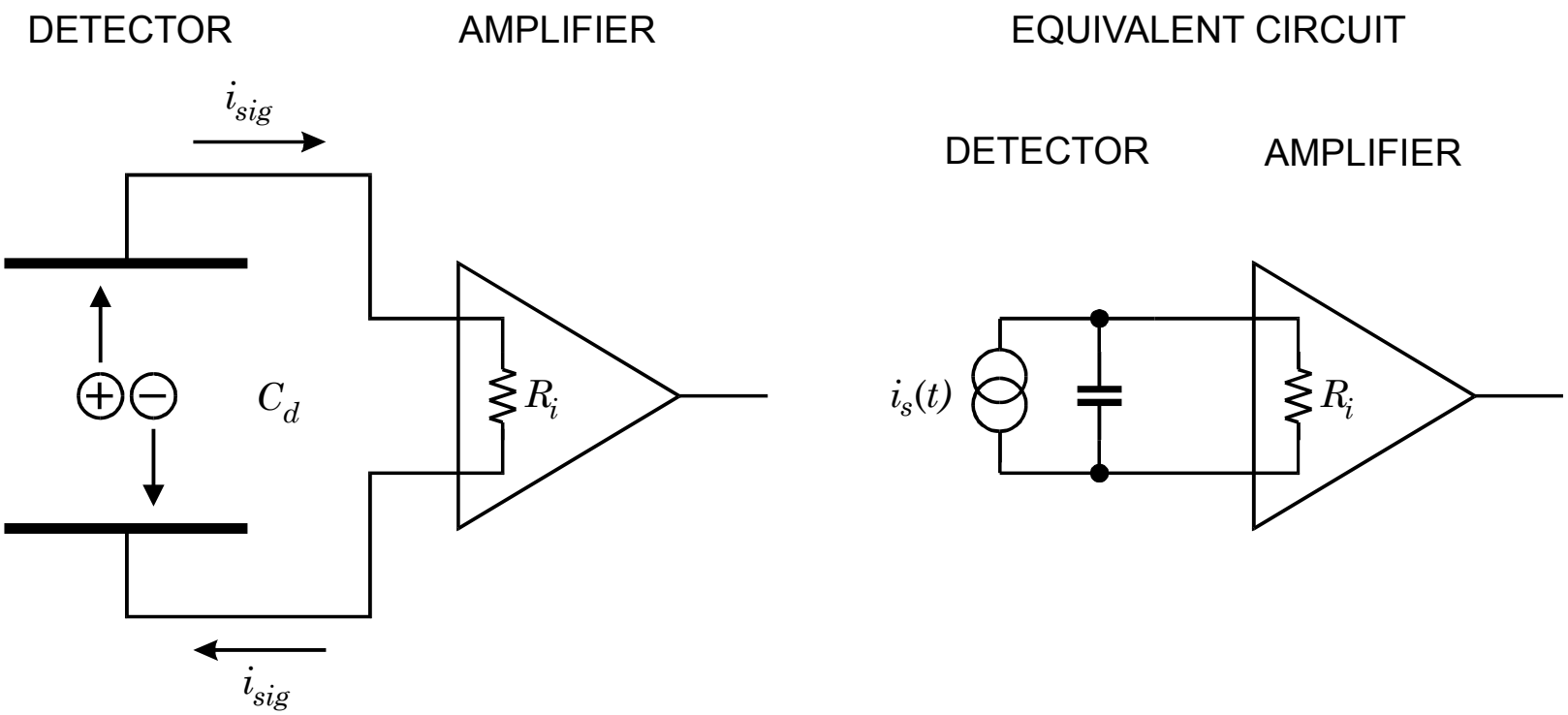
Holes: 36 ns.

This is substantially less than in the partially depleted device, where collection times for

Electrons: 30 ns

Holes: 90 ns.

Time Dependence of the Signal Current



When does the signal current begin?

a) when the charge reaches the electrode?

or

b) when the charge begins to move?

Although the first answer is quite popular (encouraged by the phrase “charge collection”), the second is correct.

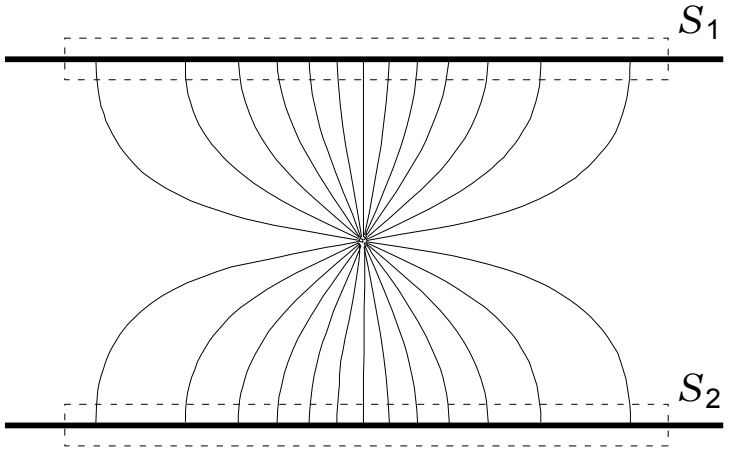
When a charge pair is created, both the positive and negative charges couple to the electrodes and induce mirror charges of equal magnitude.

The following discussion applies to ALL types of structures that register the effect of charges moving in an ensemble of electrodes, i.e. not just semiconductor or gas-filled ionization chambers, but also resistors, capacitors, photoconductors, vacuum tubes, etc.

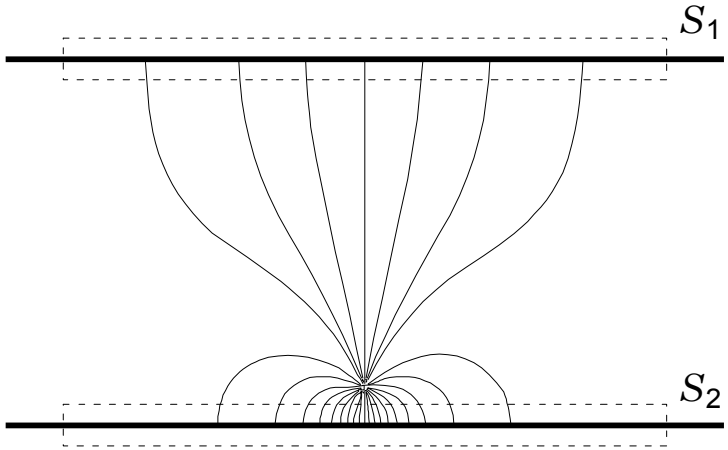
Induced Charge

Consider a charge q in a parallel plate capacitor:

When the charge is midway between the two plates, the charge induced on one plate is determined by applying Gauss' law. The same number of field lines intersect both S_1 and S_2, so equal charge is induced on each plate (= q / 2).



When the charge is close to one plate, most of the field lines terminate on that plate and the induced charge is much greater.



As a charge traverses the space between the two plates the induced charge changes continuously, so current flows in the external circuit as soon as the charges begin to move.
Mathematically this can be analyzed conveniently by applying Ramo's theorem.

Induced Charge – Ramo's Theorem

W. Shockley, J. Appl. Phys. **9** (1938) 635
S. Ramo, Proc. IRE **27** (1939) 584

Consider a mobile charge in the presence of any number of grounded electrodes.

Surround the charge q with a small equipotential sphere. Then, if V is the potential of the electrostatic field, in the region between conductors

$$\nabla^2 V = 0$$

Call V_q the potential of the small sphere and note that V= 0 on the conductors. Applying Gauss' law yields

$$\int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V}{\partial n} ds = 4\pi q$$

Next, consider the charge removed and one conductor A raised to unit potential.

Call the potential V_1, so that

$$\nabla^2 V_1 = 0$$

in the space between the conductors, including the site where the charge was situated. Call the new potential at this point V_{q1}.

Green's theorem states that

$$\int_{\substack{\text{volume between}\\ \text{boundaries}}} (V_1 \nabla^2 V - V\ \nabla^2 V_1)\ dv = \quad - \int_{\substack{\text{boundary}\\ \text{surfaces}}} \left[V_1 \frac{\partial V}{\partial n} - V \frac{\partial V_1}{\partial n} \right] ds$$

Choose the volume to be bounded by the conductors and the tiny sphere.

Then the left hand side is 0 and the right hand side may be divided into three integrals:

1. Over the surfaces of all conductors except A. This integral is 0 since on these surfaces $V = V_1 = 0$.

2. Over the surface of A. As $V_1 = 1$ and $V = 0$ this reduces to

$$- \int_{\text{surface A}} \frac{\partial V}{\partial n} ds$$

3. Over the surface of the sphere.

$$-V_{q1} \int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V}{\partial n} ds \quad + \quad V_q \int_{\substack{\text{sphere's}\\ \text{surface}}} \frac{\partial V_1}{\partial n} ds$$

The second integral is 0 by Gauss' law, since in this case the charge is removed.

Combining these three integrals yields

$$0 = -\int_{\text{surface A}} \frac{\partial V}{\partial n} ds \quad - \quad V_{q1} \int_{\substack{\text{sphere's} \\ \text{surface}}} \frac{\partial V}{\partial n} ds = 4\pi\ Q_A - 4\pi\ q V_{q1}$$

or $$Q_A = q V_{q1}$$

If the charge q moves in direction x, the current on electrode A is

$$i_A = \frac{dQ_A}{dt} = q \frac{dV_{q1}}{dt} = q \left(\frac{\partial V_{q1}}{\partial x} \ \frac{dx}{dt} \right)$$

Since the velocity of motion

$$\frac{dx}{dt} = v_x$$

the induced current on electrode A is $$i_A = q\ v_x \frac{\partial V_{q1}}{dx}$$

where V_{q1} is the “weighting potential” that describes the coupling of a charge at any position to electrode A.

The weighting potential is for a specific electrode is obtained by setting the potential of the electrode to 1 and setting all other electrodes to potential 0.

- If a charge q moves along any path s from position 1 to position 2, the net induced charge on electrode k is

$$\Delta Q_k = q(V_{q1}(2) - V_{q1}(1)) \equiv q\left(\Phi_k(2) - \Phi_k(1)\right)$$

- The instantaneous current can be expressed in terms of a weighting field

$$i_k = -q\ \vec{v} \cdot\ \overrightarrow{F_k}$$

  The weighting field is determined by applying unit potential to the measurement electrode and 0 to all others.

Note that the electric field and the weighting field are distinctly different.

- The electric field determines the charge trajectory and velocity

- The weighting field depends only on geometry and determines how charge motion couples to a specific electrode.

- Only in 2-electrode configurations are the electric field and the weighting field of the same form.

Example 1: Parallel plate geometry with uniform field

(semiconductor detector with very large overbias)

Assume a voltage V_b applied to the detector. The distance between the two parallel electrodes is d.

The electric field that determines the motion of charge in the detector is

$$E = \frac{V_b}{d}$$

Assume that the velocity of the charge carriers is collision limited, so the velocity of the charge

$$v = \mu E = \mu \frac{V_b}{d}$$

The weighting field is obtained by applying unit potential to the collection electrode and grounding the other,

$$E_Q = \frac{1}{d}$$

so the induced current

$$i = qvE_Q = q\mu \frac{V_b}{d}\frac{1}{d} = q\mu \frac{V_b}{d^2}$$

Since both the electric field and the weighting field are uniform throughout the detector, the current is constant until the charge reaches its terminal electrode.

Assume that the charge is created at the opposite electrode and traverses the detector thickness d.

The required collection time, i.e. the time required to traverse the detector thickness d

$$t_c = \frac{d}{v} = \frac{d}{\mu \frac{V_b}{d}} = \frac{d^2}{\mu V_b}$$

The induced charge

$$Q = it_c = q\mu \frac{V_b}{d^2} \frac{d^2}{\mu V_b} = q$$

Next, assume an electron-hole pair formed at coordinate x from the positive electrode.

The collection time for the electron

$$t_{ce} = \frac{x}{v_e} = \frac{xd}{\mu_e V_b}$$

and the collection time for the hole

$$t_{ch} = \frac{d-x}{v_h} = \frac{(d-x)d}{\mu_h V_b}$$

Since electrons and holes move in opposite directions, they induce current of the same sign at a given electrode, despite their opposite charge.

The induced charge due to the motion of the electron

$$Q_e = q_e \mu_e \frac{V_b}{d^2} \frac{xd}{\mu_e V_b} = q_e \frac{x}{d}$$

whereas the hole contributes

$$Q_h = q_e \mu_h \frac{V_b}{d^2} \frac{(d-x)d}{\mu_h V_b} = q_e \left(1 - \frac{x}{d}\right)$$

Assume that $x = d/2$. After the collection time for the electron

$$t_{ce} = \frac{d^2}{2\mu_e V_b}$$

it has induced a charge $q_e/2$.

At this time the hole, due to its lower mobility $\mu_h \approx \mu_e/3$, has induced $q_e/6$, yielding a cumulative induced charge of $2q_e/3$.

After the additional time for the hole collection, the remaining charge $q_e/3$ is induced, yielding the total charge q_e.

In this configuration

- Electrons and holes contribute equally to the currents on both electrodes
- The instantaneous current at any time is the same (although of opposite sign) on both electrodes

The continuity equation (Kirchhoff's law) must be satisfied:

$$\sum_k i_k = 0$$

Since k=2: $i_1 = -i_2$

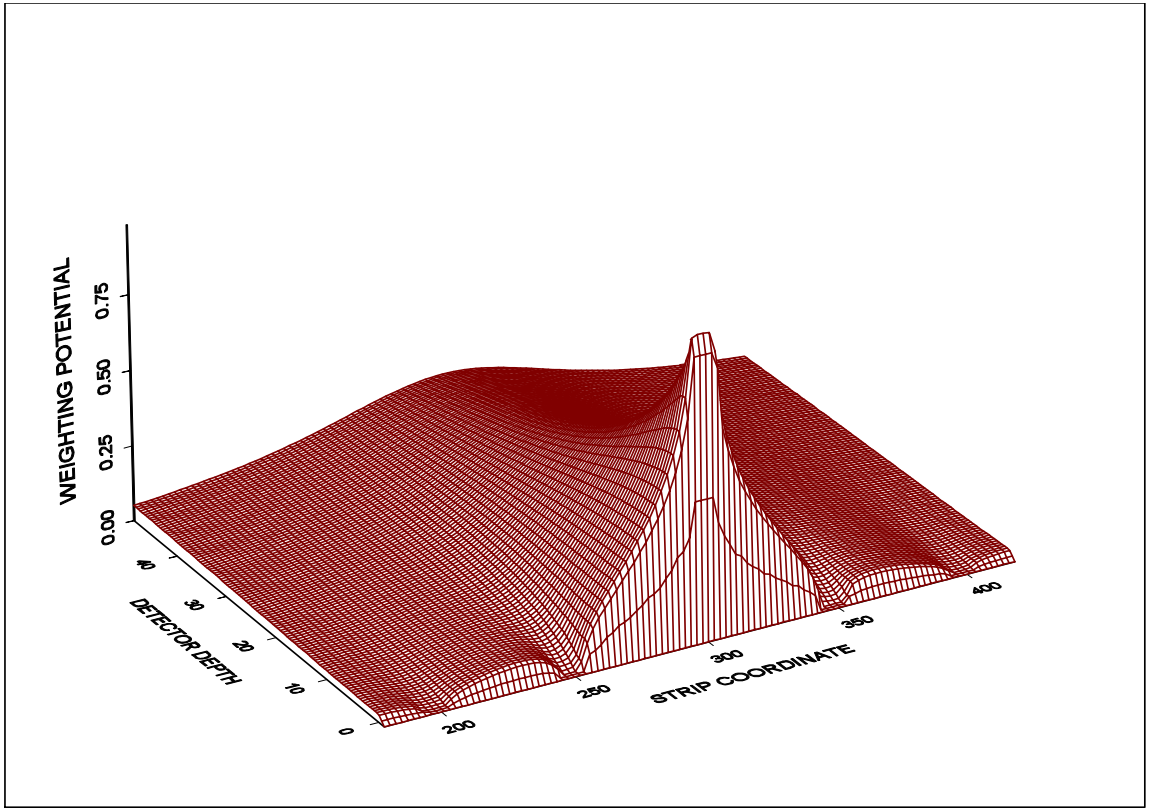
Example 2: Double-Sided Strip Detector

The strip pitch is assumed to be small compared to the thickness.

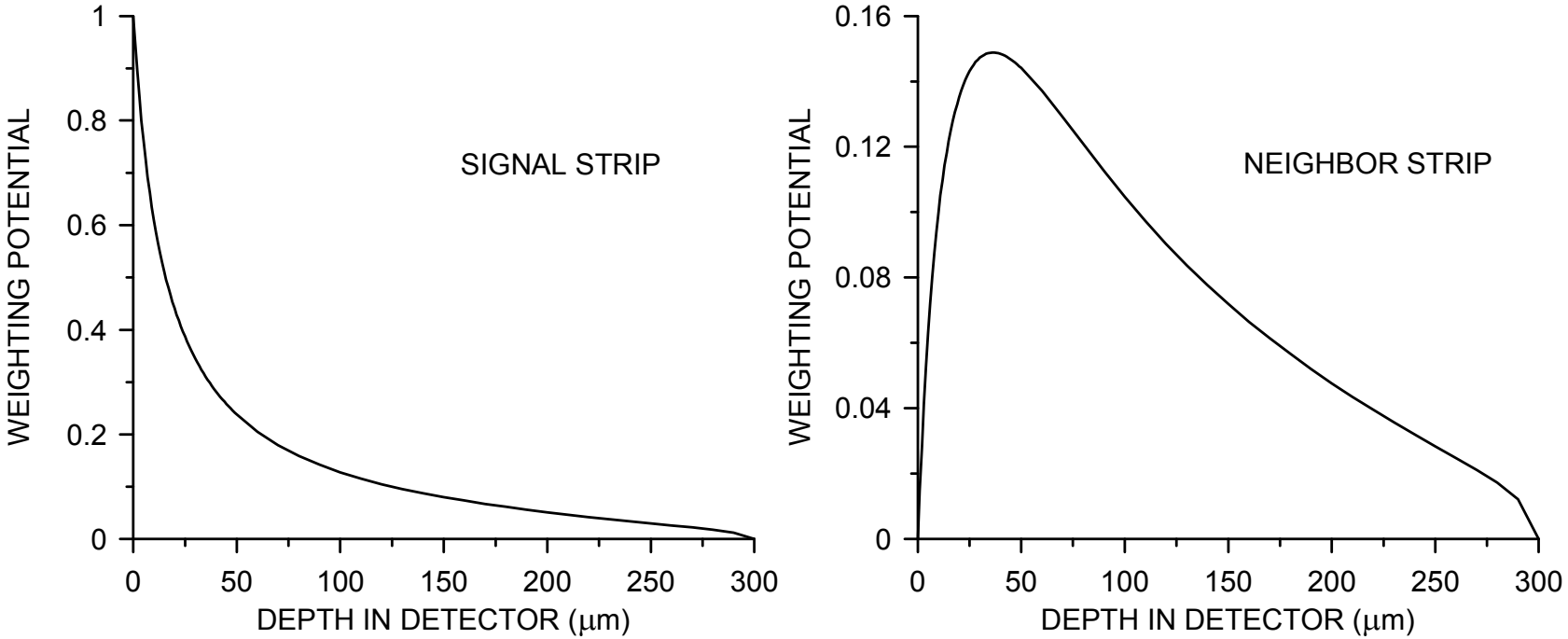
The electric field is similar to a parallel-plate geometry, except in the immediate vicinity of the strips.

The signal weighting potential, however is very different.

Weighting potential for a 300 μm thick strip detector with strips on a pitch of 50 μm. Only 50 μm of depth are shown.



Cuts through the weighting potential



Consider an electron-hole pair q_n, q_p originating on a point x_0 on the center-line of two opposite strips of a double-sided strip detector. The motion of the electron towards the n-electrode x_n is equivalent to the motion of a hole in the opposite direction to the p-electrode x_p. The total induced charge on electrode k after the charges have traversed the detector is

$$Q_k = q_p[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] + q_n[\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$

since the hole charge $q_p = q_e$ and $q_n = -q_e$

$$Q_k = q_e[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_0)] - q_e[\Phi_{Qk}(x_n) - \Phi_{Qk}(x_0)]$$
$$Q_k = q_e[\Phi_{Qk}(x_p) - \Phi_{Qk}(x_n)]$$

If the signal is measured on the p-electrode, collecting the holes,

$$\Phi_{qk}(x_p) = 1$$
$$\Phi_{qk}(x_n) = 0$$

and $Q_k = q_e$.

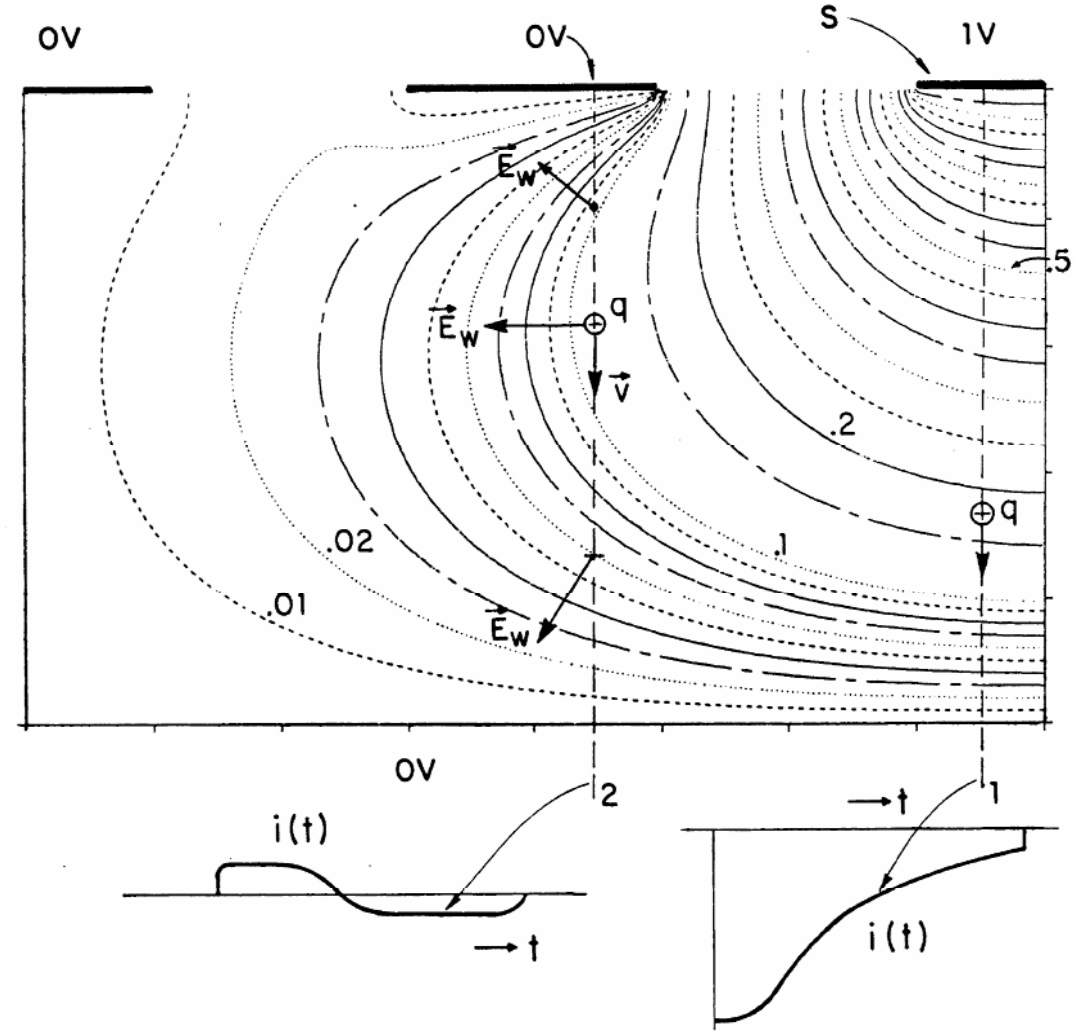
If, however, the charge is collected on the neighboring strip $k+1$, then

$$\Phi_{Q(k+1)}(x_n) = 0$$
$$\Phi_{Q(k+1)}(x_p) = 0$$

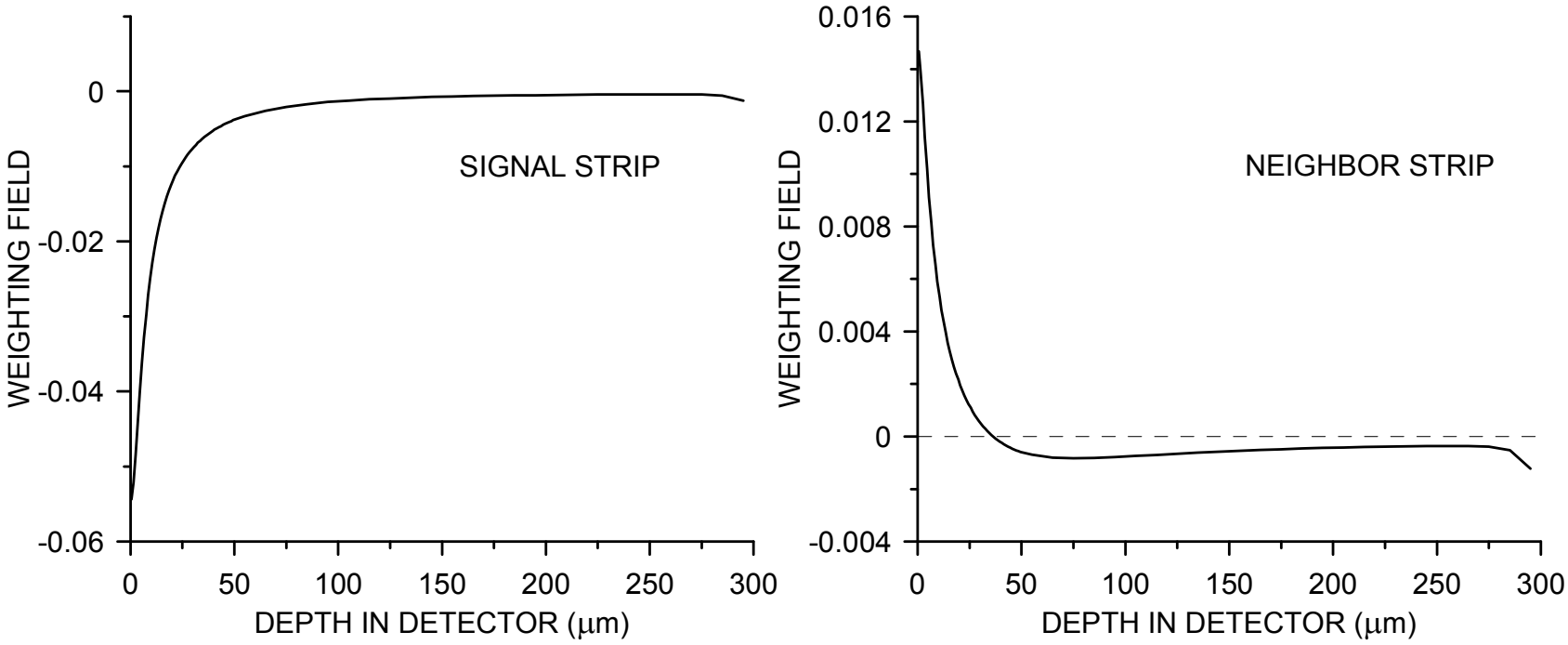
and $Q_{k+1} = 0$.

In general, if moving charge does not terminate on the measurement electrode, signal current will be induced, but the current changes sign and integrates to zero.

This is illustrated in the following schematic plot of the weighting field in a strip detector (from Radeka)



Cuts through the Weighting Field in a Strip Detector
(d= 300 μm, p= 50 μm)



Note, however, that this charge cancellation on “non-collecting” electrodes relies on the motion of both electrons and holes.

Assume, for example, that the holes are stationary, so they don't induce a signal. Then the first term of the first equation above vanishes, which leaves a residual charge

$$Q_k = q_e[\Phi_{Qk}(x_0) - \Phi_{Qk}(x_n)]$$

since for any coordinate not on an electrode

$$Q_k(x_0) \neq 0$$

although it may be very small.

An important consequence of this analysis is that one cannot simply derive pulse shapes by analogy with a detector with contiguous electrodes (i.e. a parallel plate detector of the same overall dimensions as a strip detector). Specifically,

1. the shape of the current pulses can be quite different,

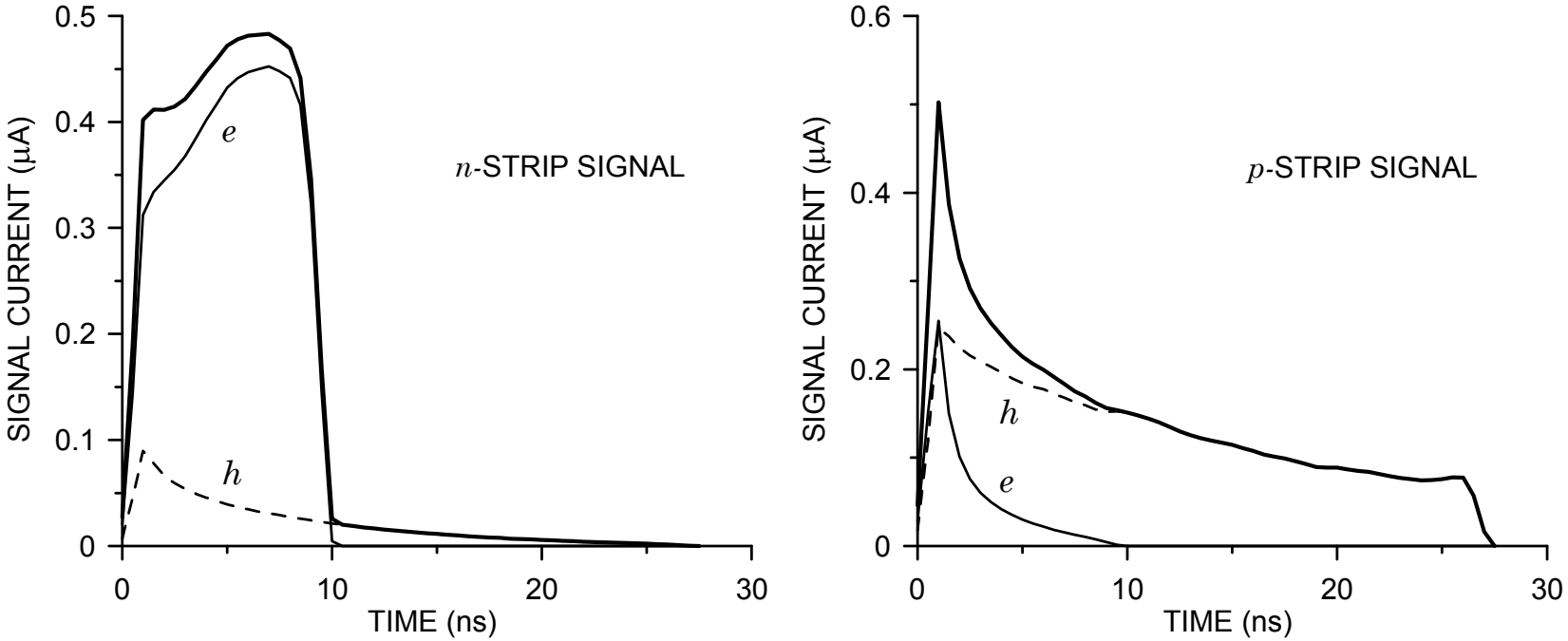
2. the signals seen on opposite strips of a double-sided detector are not the same (although opposite in sign), and

3. the net induced charge on the p- or n-side is not split evenly between electrons and holes.

   - Because the weighting potential is strongly peaked near the signal electrode, most of the charge is induced when the moving charge is near the signal electrode.

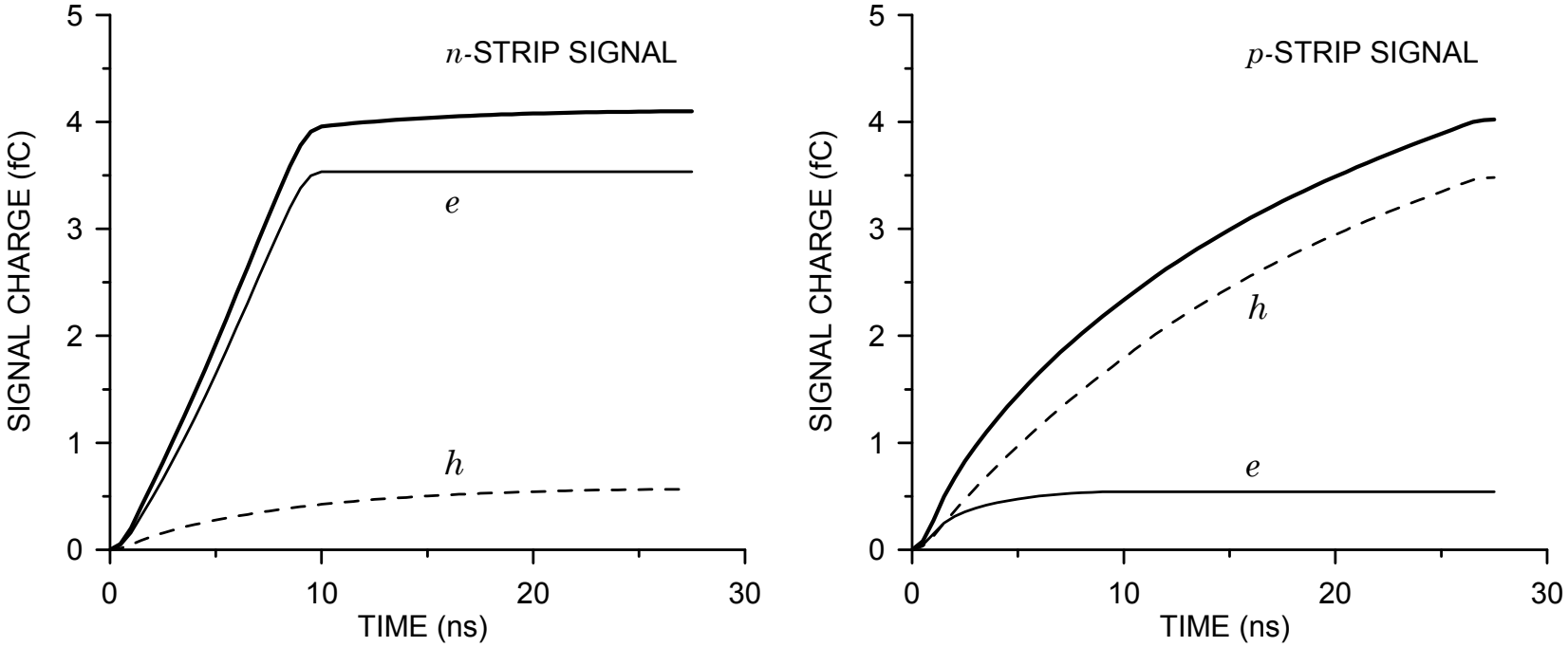
   - As a result, most of the signal charge is due to the charge terminating on the signal electrode.

Current pulses in strip detectors (track traversing the detector)



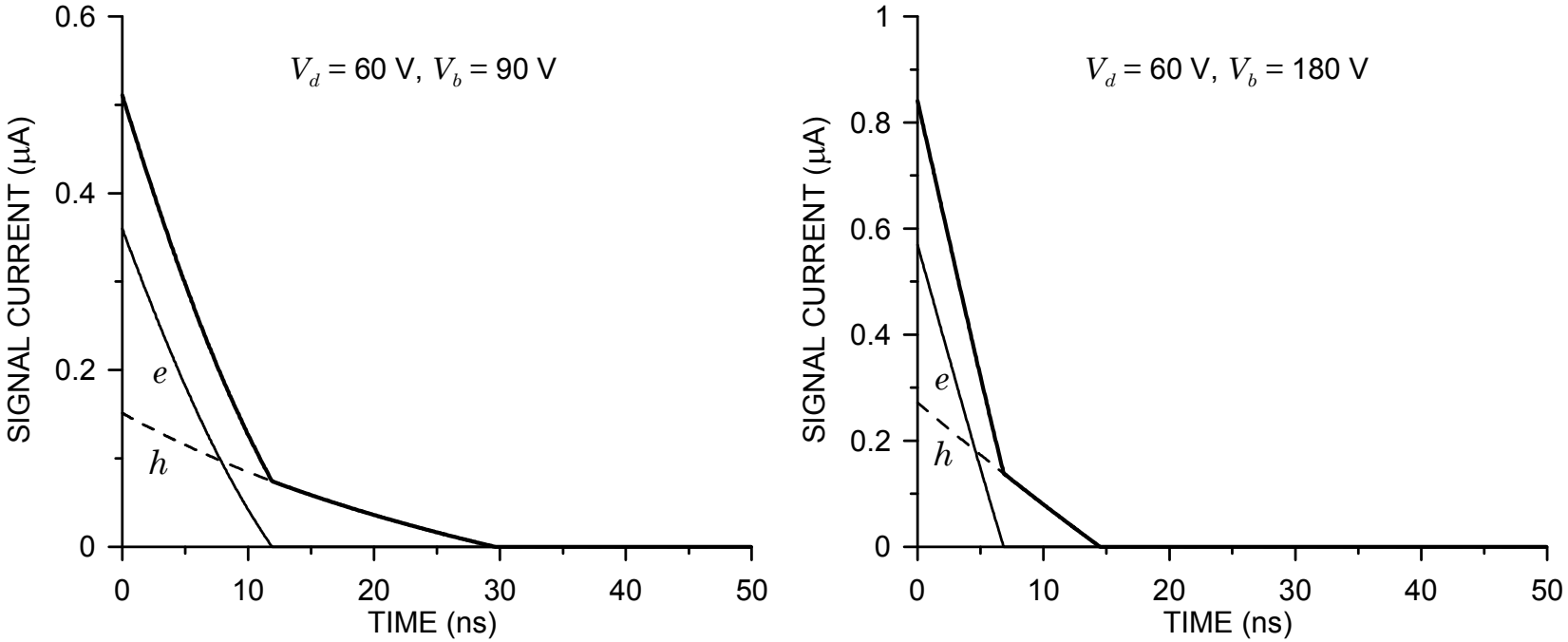
The duration of the electron and hole pulses is determined by the time required to traverse the detector as in a parallel-plate detector, but the shapes are very different.

Strip Detector Signal Charge Pulses



For comparison:

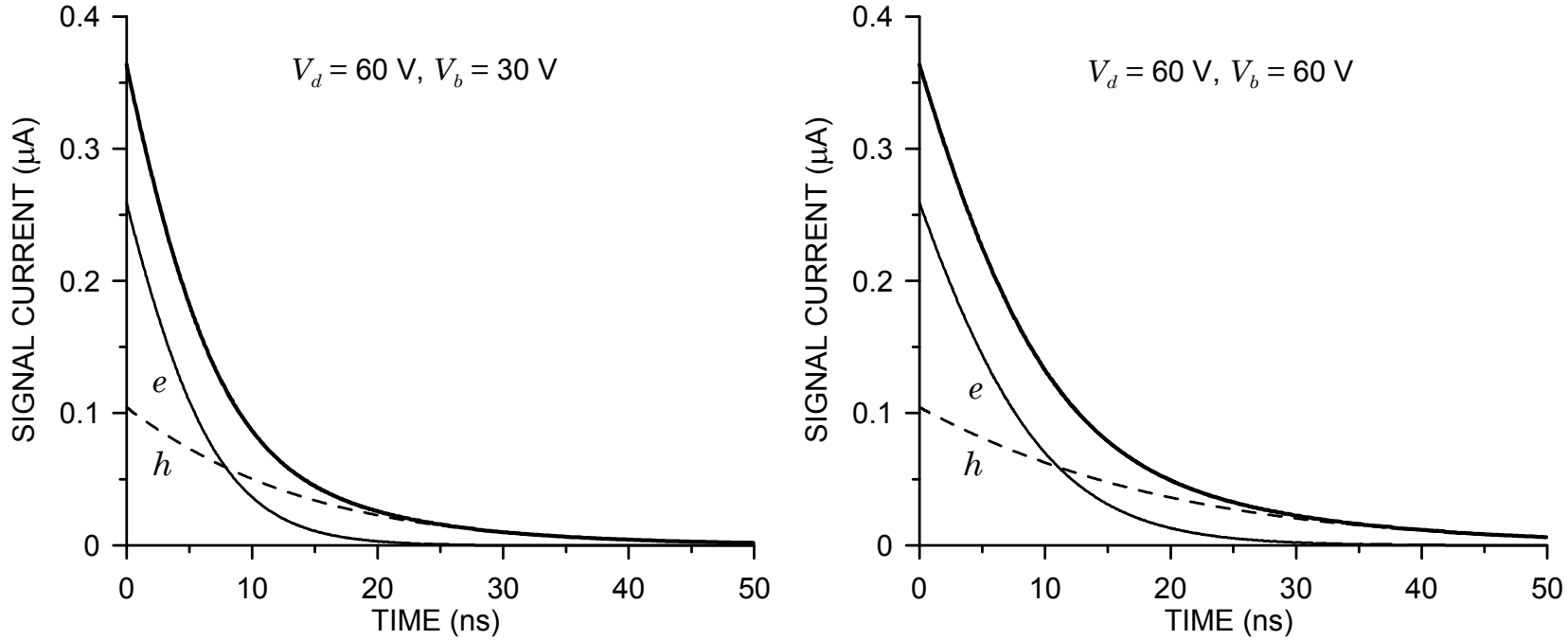
Current pulses in pad detectors (track traversing the detector)



For the same depletion and bias voltages the pulse durations are the same as in strip detectors, although the shapes are very different.

Overbias decreases the collection time.

Operation at or below full depletion leads to long “tails” from the low-field region.



Charge Collection in the Presence of Trapping

Practical semiconductor crystals suffer from imperfections introduced during crystal growth, during device fabrication, or by radiation damage.

Defects in the crystal

- impurity atoms
- vacancies
- structural irregularities (e.g. dislocations)
- radiation damage

introduce states into the crystal that can trap charge.

Charge trapping is characterized by a carrier lifetime τ, the time a charge carrier can "survive" in a crystal before trapping or recombination with a hole.

Trapping removes mobile charge available for signal formation.

Depending on the nature of the trap, thermal excitation or the externally applied field can release the carrier from the trap, leading to delayed charge collection.

Given a lifetime τ, a packet of charge Q_0 will decay with time: $Q(t) = Q_0 e^{-t/\tau}$

In an electric field the charge will drift. The time required to traverse a distance x is

$$t = \frac{x}{v} = \frac{x}{\mu E},$$

after which the remaining charge is

$$Q(x) = Q_0 e^{-x/\mu E\tau} \equiv Q_0 e^{-x/L}.$$

Since the drift length $L \equiv \mu\tau E$ is proportional to the mobility-lifetime product, $\mu\tau$ is often used as a figure of merit.

Assume a detector with a simple parallel-plate geometry. For a charge traversing the increment dx of the detector thickness d, the induced signal charge is

$$dQ_s = Q(x)\frac{dx}{d},$$

so the total induced charge

$$Q_s = \frac{1}{d}\int_0^d Q(x)dx = \frac{1}{d}\int_0^d Q_0 e^{-x/L}dx$$

$$Q_s = Q_0 \ \frac{L}{d} \ \left(1 - e^{-d/L}\right)$$

The magnitude of the recovered signal depends on the drift length relative to the width of the sensor's sensitive region.

$$d \gg L: \qquad \frac{Q_s}{Q_0} \approx \frac{L}{d}$$

$$d = 3L: \qquad \frac{Q_s}{Q_0} = 0.95$$

In high quality silicon detectors: $\tau \approx$ 10 ms

$\mu_e =$ 1350 V/cm·s^2

$E =$ 10^4 V/cm $\Rightarrow$ $L \approx$ 10^4 cm

In amorphous silicon $L \approx$ 10 μm (short lifetime, low mobility).

In diamond, however, $L \approx$ 100 – 200 μm (despite high mobility).

In CdZnTe at 1 kV/cm, $L \approx$ 3 cm for electrons, 0.1 cm for holes

Carrier lifetime also important for efficiency of solar cells!

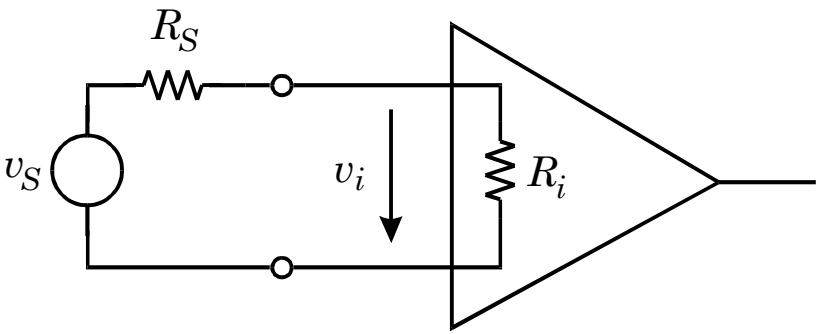
4. Signal Acquisition

Amplifier Types

a) Voltage-Sensitive Amplifier

The signal voltage at the amplifier input

$$v_i = \frac{R_i}{R_S + R_i} v_S$$



If the signal voltage at the amplifier input is to be approximately equal to the signal voltage

$$v_i \approx v_S \qquad \Rightarrow \qquad R_i \gg R_S$$

To operate in the voltage-sensitive mode, the amplifier's input resistance (or impedance) must be large compared to the source resistance (impedance).

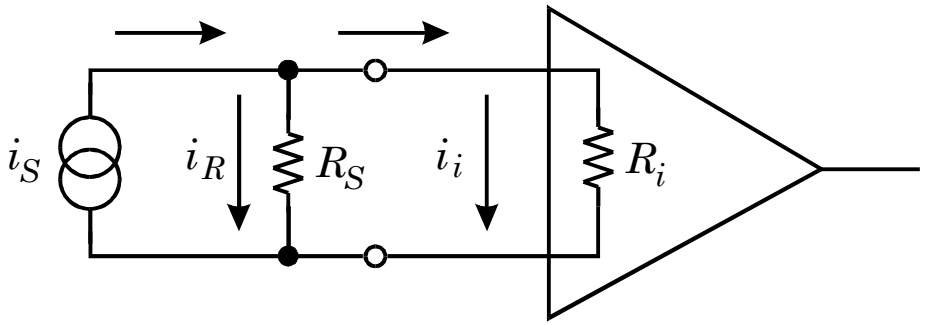
In ideal voltage amplifiers one sets $R_i = \infty$, although this is never true in reality, although it can be fulfilled to a good approximation.

To provide a voltage output, the amplifier should have a low output resistance, i.e. its output resistance should be small compared to the input resistance of the following stage.

b) Current-Sensitive Amplifier

The signal current divides into the source resistance and the amplifier's input resistance. The fraction of current flowing into the amplifier

$$i_i = \frac{R_s}{R_s + R_i} i_S$$



If the current flowing into the amplifier is to be approximately equal to the signal current

$$i_i \approx i_S \qquad \Rightarrow \qquad R_i \ll R_S$$

To operate in the current-sensitive mode, the amplifier's input resistance (or impedance) must be small compared to the source resistance (impedance).

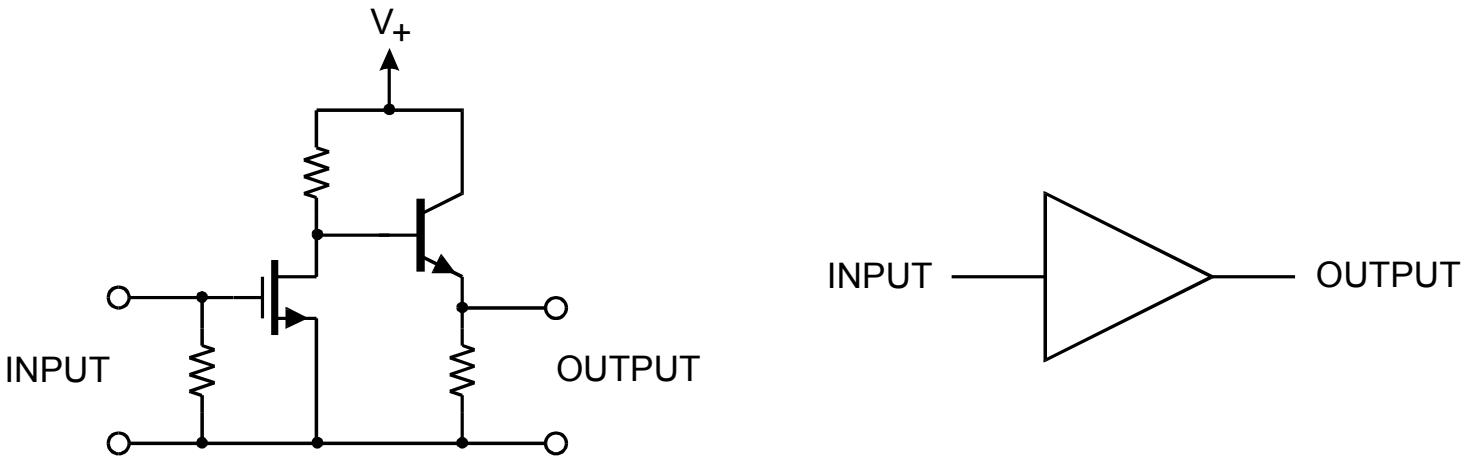
One can also model a current source as a voltage source with a series resistance. For the signal current to be unaffected by the amplifier input resistance, the input resistance must be small compared to the source resistance, as derived above.

At the output, to provide current drive the output resistance should be high, i.e. large compared to the input resistance of the next stage.

- Whether a specific amplifier operates in the current or voltage mode depends on the source resistance.

- Amplifiers can be configured as current mode input and voltage mode output or, conversely, as voltage mode input and current mode output. The gain is then expressed as V/A or A/V.

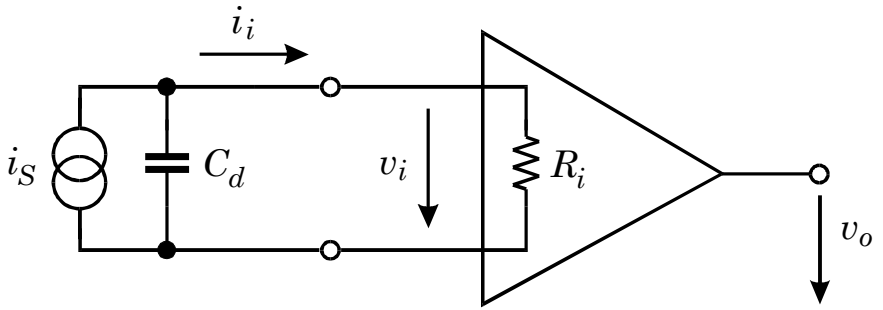
Although an amplifier has a pair of input and a second pair of output connections, since the two have a common connection a simplified representation is commonly used:



c) Voltage and Current Mode with Capacitive Sources

Output voltage:

v_o = (voltage gain A_v) $\times$ (input voltage v_i).



Operating mode depends on charge collection time t_c and the input time constant R_iC_d :

a) $R_iC_d \ll t_c$

detector capacitance discharges rapidly

$\Rightarrow \quad v_o \propto i_s(t)$

current sensitive amplifier

b) $R_iC_d \gg t_c$

detector capacitance discharges slowly

$\Rightarrow \quad v_o \propto \int i_s(t)dt$

voltage sensitive amplifier

Note that in both cases the amplifier is providing voltage gain, so output signal voltage is determined directly by the input voltage. The difference is that the shape of the input voltage pulse is determined either by the instantaneous current or by the integrated current and the decay time constant.

Goal is to measure signal charge, so it is desirable to use a system whose response is independent of detector capacitance.

Active Integrator ("charge-sensitive amplifier")

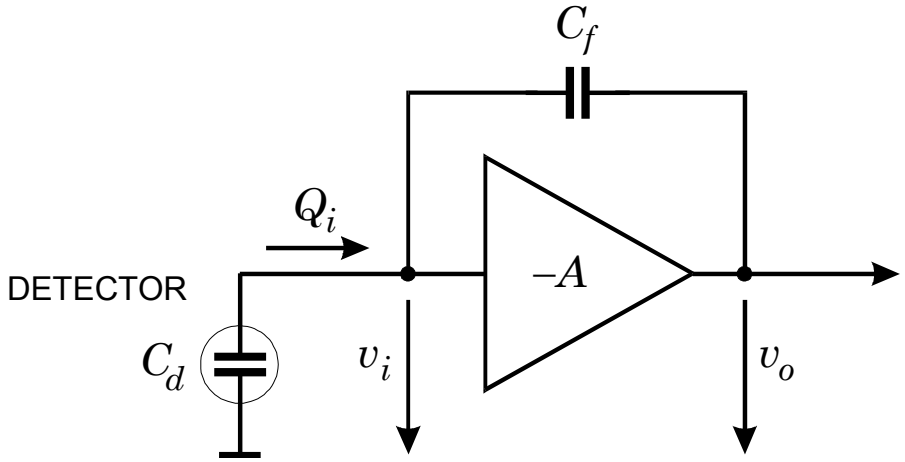
Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow$

$v_o = -Av_i$

Input impedance = ∞ (i.e. no signal current flows into amplifier input)

Connect feedback capacitor C_f between output and input.



Voltage difference across C_f: $\quad v_f = (A+1)v_i$

$\Rightarrow$ Charge deposited on C_f: $\quad Q_f = C_f v_f = C_f(A+1)v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

$\Rightarrow$ Effective input capacitance $\quad C_i = \dfrac{Q_i}{v_i} = C_f(A+1)$ ("dynamic" input capacitance)

Gain $\quad A_Q = \dfrac{dV_o}{dQ_i} = \dfrac{A \cdot v_i}{C_i \cdot v_i} = \dfrac{A}{C_i} = \dfrac{A}{A+1} \cdot \dfrac{1}{C_f} \approx \dfrac{1}{C_f} \quad (A >> 1)$

Dependent on a well-controlled quantity, the feedback capacitance.

Q_i is the charge flowing into the preamplifier .... but some charge remains on C_d.

What fraction of the signal charge is measured?

$$\frac{Q_i}{Q_s} = \frac{C_i v_i}{Q_d + Q_i} = \frac{C_i}{Q_s} \cdot \frac{Q_s}{C_i + C_d}$$

$$= \frac{1}{1 + \frac{C_d}{C_i}} \approx 1 \quad (\text{if } C_i >> C_d)$$

Example: A = 10^3

C_f = 1 pF $\Rightarrow$ C_i = 1 nF

C_{det} = 10 pF: Q_i/Q_s = 0.99

C_{det} = 500 pF: Q_i/Q_s = 0.67

↑

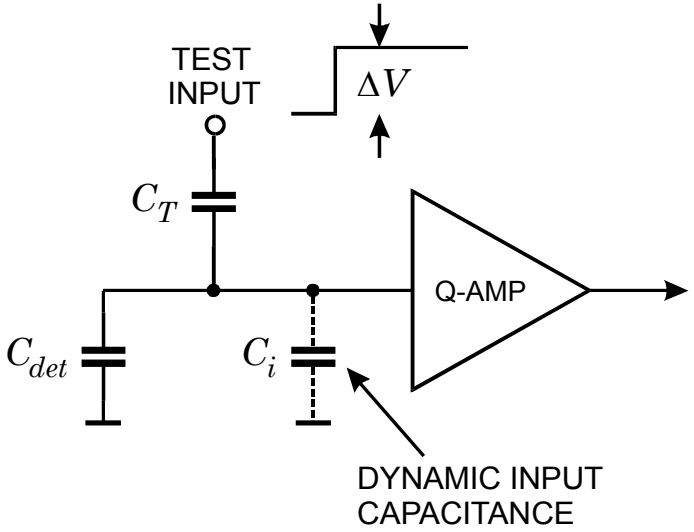
Si Det.: 50 μm thick, 250 mm^2 area

Note: Input coupling capacitor must be $\gg C_i$ >>C_i for high charge transfer efficiency.

Calibration

Inject specific quantity of charge - measure system response

Use voltage pulse (can be measured conveniently with oscilloscope)



$C_i >> C_T \Rightarrow$ Voltage step applied to test input develops over C_T.

$\Rightarrow \quad Q_T = \Delta V \cdot C_T$

Accurate expression: $$Q_T = \frac{C_T}{1+\frac{C_T}{C_i}} \cdot \Delta V \approx C_T \left(1-\frac{C_T}{C_i}\right) \Delta V$$

Typically: $C_T / C_i = 10^{-3} - 10^{-4}$

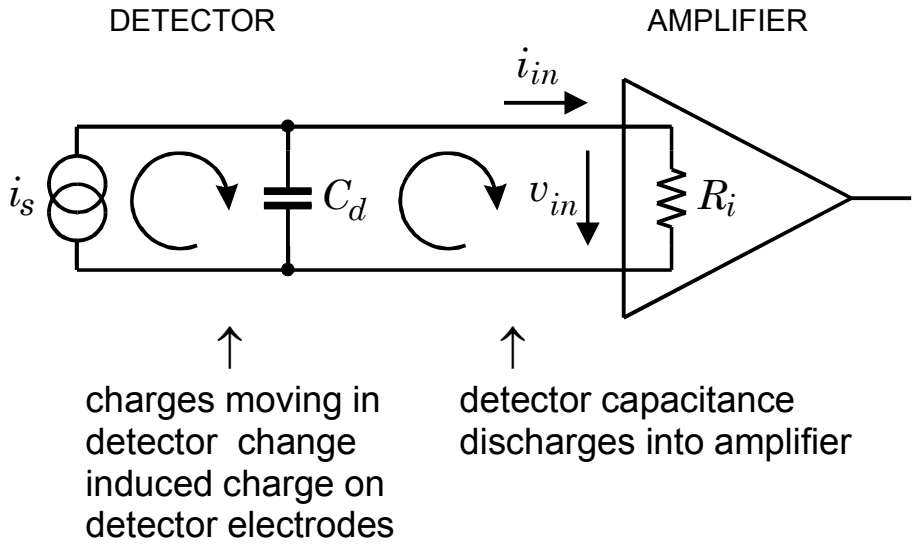
Realistic Charge-Sensitive Preamplifiers

The preceding discussion assumed idealized amplifiers with infinite speed.

In reality, amplifiers may be too slow to follow the instantaneous detector pulse.

Does this incur a loss of charge?

Equivalent Circuit:

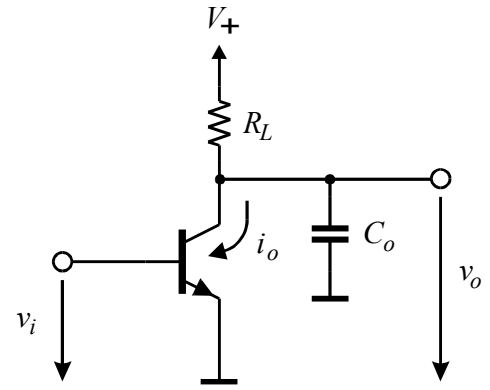


Signal is preserved even if the amplifier responds much more slowly than the detector signal.

However, the response of the amplifier affects the measured pulse shape.

- How do “real” amplifiers affect the measured pulse shape?
- How does the detector affect amplifier response?

A Simple Amplifier



Voltage gain: $$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

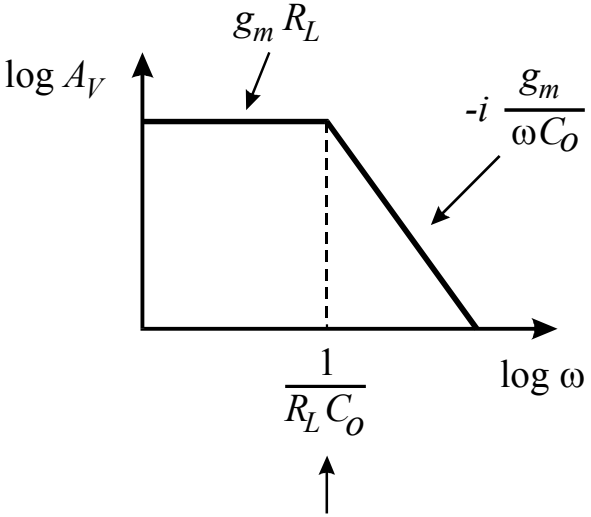
$g_m \equiv$ transconductance

$$Z_L = R_L \,//\, C_o$$

$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega\, C_o$$

$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\, C_o \right)^{-1}$$

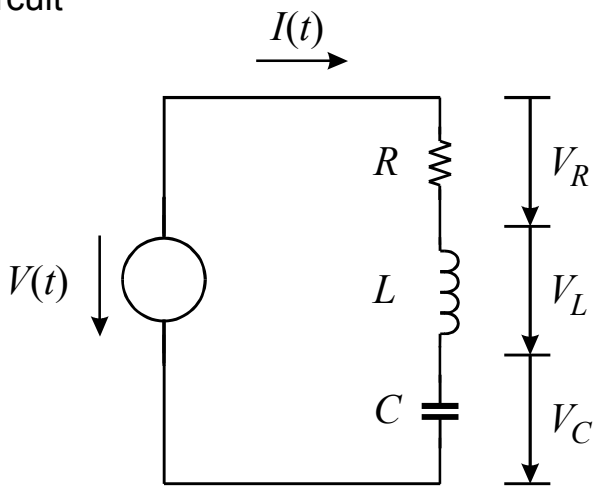
↑ low freq. ↑ high freq.



Appendix 1

Phasors and Complex Algebra in Electrical Circuits

Consider the RLC circuit



$$V = V_R + V_L + V_C$$

$$V = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{dI}{dt}R + L\frac{d^2I}{dt^2} + \frac{I}{C}$$

Assume that $V(t) = V_0 e^{\mathbf{i}\omega t}$ and $I(t) = I_0 e^{\mathbf{i}(\omega t+\varphi)}$

$$\mathbf{i}\omega V_0 e^{\mathbf{i}\omega t} = \mathbf{i}\omega R I_0 e^{\mathbf{i}(\omega t-\varphi)} - \omega^2 L I_0 e^{\mathbf{i}(\omega t-\varphi)} + \frac{1}{C} I_0 e^{\mathbf{i}(\omega t-\varphi)}$$

Then

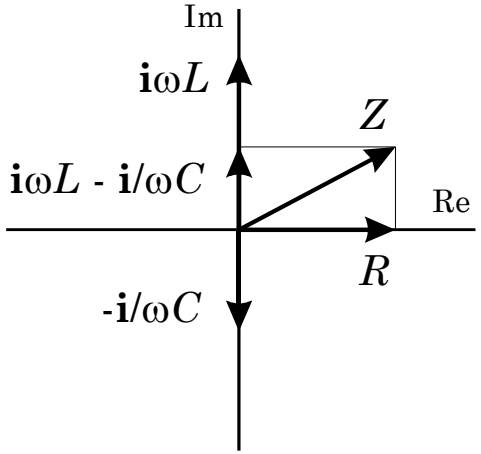
$$\frac{V_0}{I_0} e^{\mathbf{i}\varphi} = R + \mathbf{i}\omega L - \mathbf{i}\frac{1}{\omega C}$$

Thus, we can express the total impedance $Z \equiv (V_0 / I_0)\, e^{\mathbf{i}\varphi}$ of the circuit as a complex number with the magnitude $|Z| = V_0 / I_0$ and phase φ.

In this representation the equivalent resistances (reactances) of L and C are imaginary numbers

$$X_L = \mathbf{i}\omega L \quad \text{and} \quad X_C = -\frac{\mathbf{i}}{\omega C} \ .$$

Plotted in the complex plane:

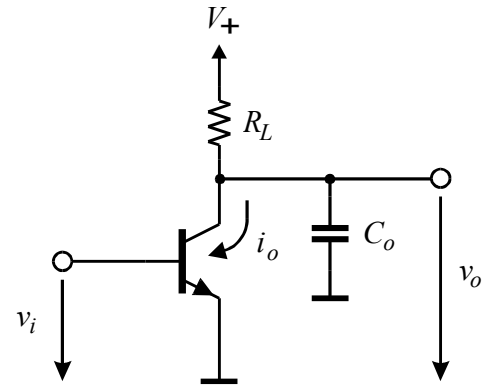


Relative to V_R, the voltage across the inductor V_L is shifted in phase by +90°.

The voltage across the capacitor V_C is shifted in phase by -90°.

Use to represent any element that introduces a phase shift, e.g. an amplifier. A phase shift of +90° appears as $+\mathbf{i}$, -90° as $-\mathbf{i}$.

A Simple Amplifier



Voltage gain: $$A_V = \frac{dv_o}{dv_i} = \frac{di_o}{dv_i} \cdot Z_L \equiv g_m Z_L$$

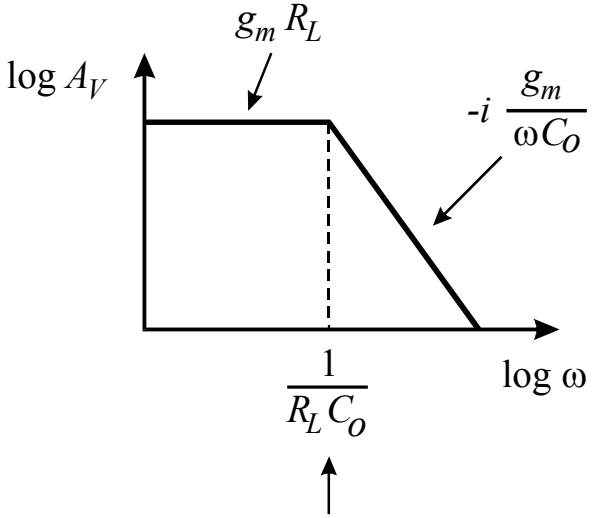
$g_m \equiv$ transconductance

$$Z_L = R_L \,//\, C_o$$

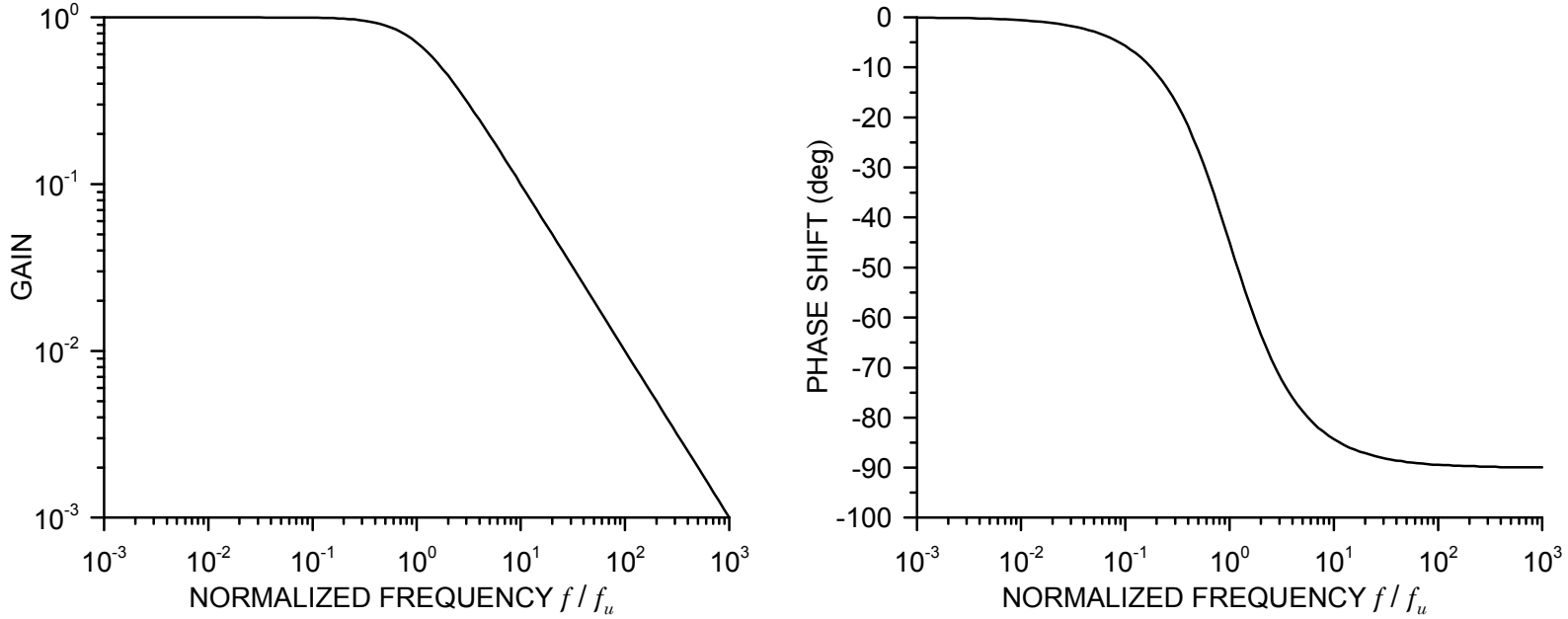
$$\frac{1}{Z_L} = \frac{1}{R_L} + \mathbf{i}\omega\, C_o$$

$$\Rightarrow \quad A_V = g_m \left(\frac{1}{R_L} + \mathbf{i}\omega\, C_o \right)^{-1}$$

low freq. high freq.



Frequency and phase response:



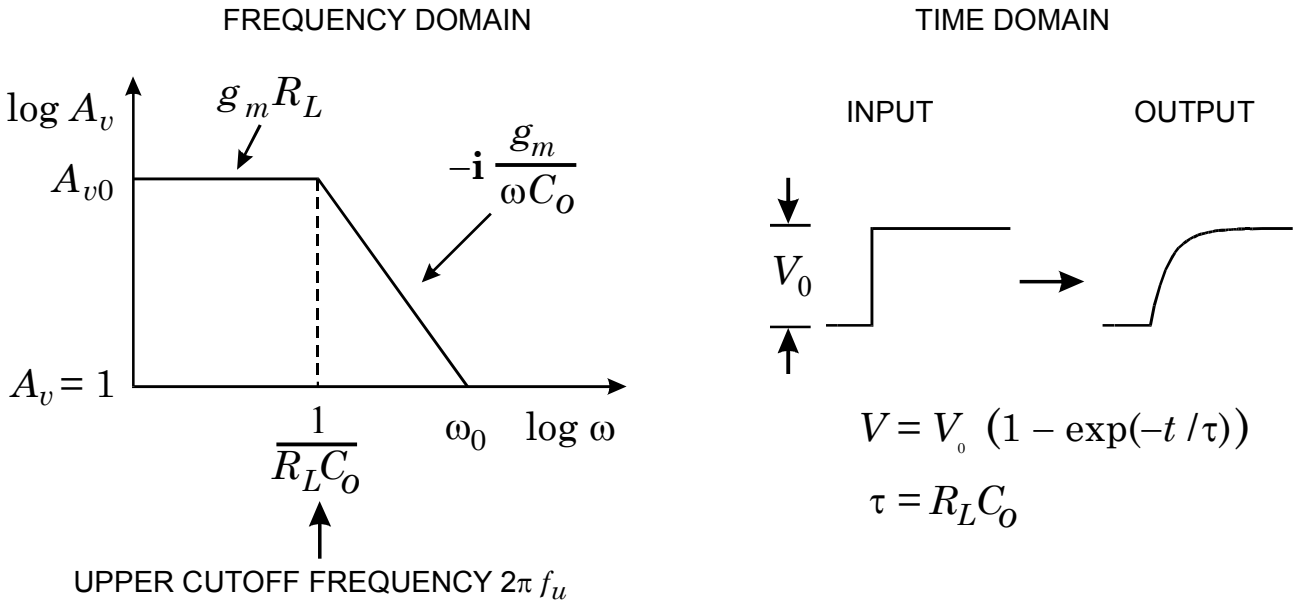
Phase shows change from low-frequency response. For an inverting amplifier add 180°.

Pulse Response of the Simple Amplifier

A voltage step $v_i(t)$ at the input causes a current step $i_o(t)$ at the output of the transistor.

For the output voltage to change, the output capacitance C_o must first charge up.

$\Rightarrow$ The output voltage changes with a time constant $\tau = R_L C_o$



The time constant τ corresponds to the upper cutoff frequency : $\tau = \dfrac{1}{2\pi\, f_u}$

Input Impedance of a Charge-Sensitive Amplifier

Input impedance $$Z_i = \frac{Z_f}{A+1} \approx \frac{Z_f}{A} \quad (A >> 1)$$

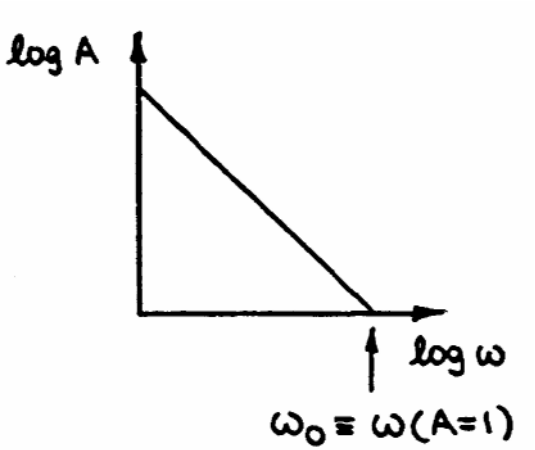
Amplifier gain vs. frequency beyond the upper cutoff frequency
Feedback Impedance

$$A = -\mathbf{i}\ \frac{\omega_0}{\omega}$$

Feedback impedance $$Z_f = -\mathbf{i}\ \frac{1}{\omega\ C_f}$$

$\Rightarrow$ Input Impedance $$Z_i = -\frac{\mathbf{i}}{\omega\ C_f} \cdot \frac{1}{-\mathbf{i}\frac{\omega_0}{\omega}}$$

$$Z_i = \frac{1}{\omega_0 C_f}$$



Gain-Bandwidth Product

Imaginary component vanishes $\Rightarrow$ *Resistance:* $Z_i \rightarrow R_i$

$\Rightarrow$ low frequencies ($f < f_u$): capacitive input
high frequencies ($f > f_u$): resistive input

Time Response of a Charge-Sensitive Amplifier

Input resistance and detector capacitance form RC time constant $\tau_i = R_i C_D$

$$\tau_i = \frac{1}{\omega_0 C_f} \cdot C_D$$

⇒ **Rise time increases with detector capacitance.**

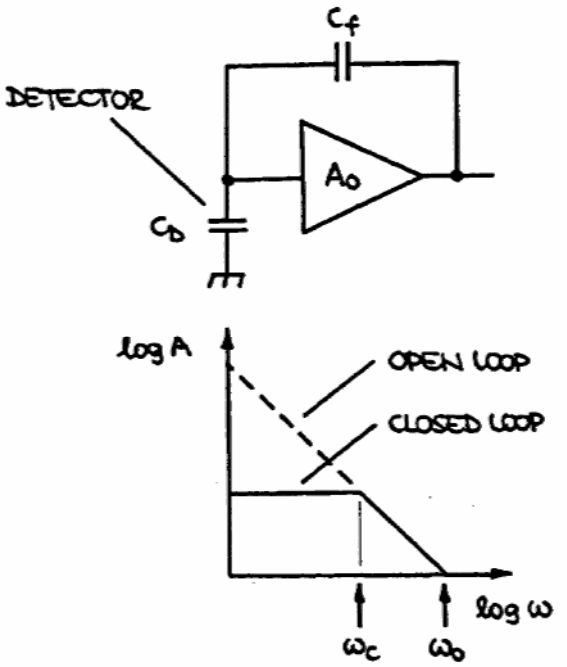
Or apply feedback theory:

Closed Loop Gain $A_f = \dfrac{C_D + C_f}{C_f} \quad (A_f << A_0)$

$$A_f \approx \frac{C_D}{C_f} \quad (C_D >> C_f)$$

Closed Loop Bandwidth $\omega_C A_f = \omega_0$

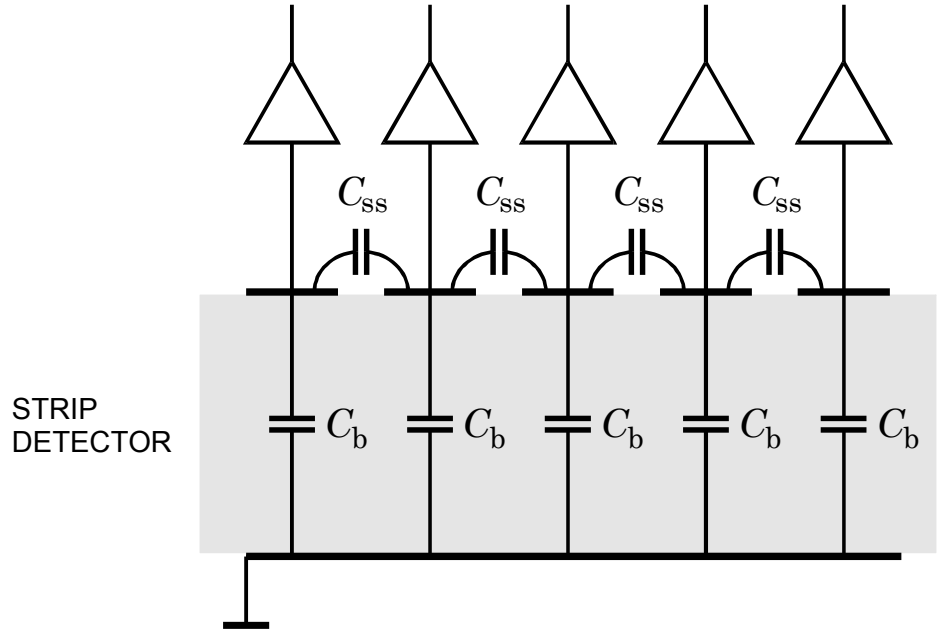
Response Time $\tau_{amp} = \dfrac{1}{\omega_C} = C_D \ \dfrac{1}{\omega_0 C_f}$



Same result as from input time constant.

Input impedance is critical in strip or pixel detectors:

Amplifiers must have a low input impedance to reduce transfer of charge through capacitance to neighboring strips



For strip pitches that are smaller than the bulk thickness, the capacitance is dominated by the fringing capacitance to the neighboring strips C_{SS}.

Typically: 1 – 2 pF/cm for strip pitches of 25 – 100 μm on Si.

The backplane capacitance C_b is typically 20% of the strip-to-strip capacitance.

Negligible cross-coupling at shaping times $T_P > (2 \ldots 3) \times R_i C_D$ and if $C_i \gg C_D$.